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Mathematics We Need

BOOK SIX

- MANUAL
- FACSIMILE OF TEXT
- ANSWERS

CURRICULUM

GINN MODERN MATHEMATICS SERIES

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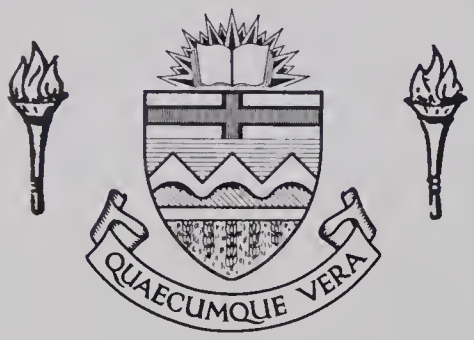

Teaching Mathematics We Need

GUIDE · FACSIMILE OF TEXT · ANSWERS

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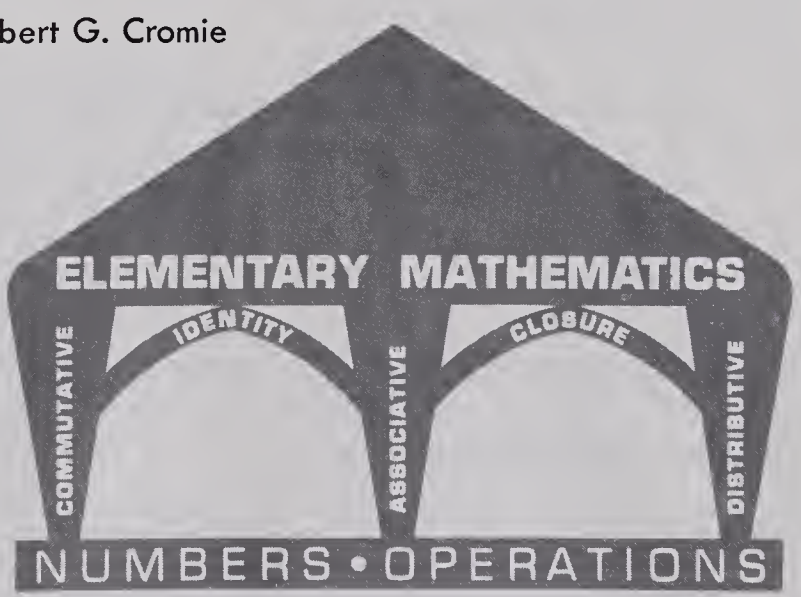

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Throughout the entire period when the School Mathematics Study Group was conducting its writing and experimentation for grades 1–8, several of the *Mathematics We Need* authors, as members of that group, participated in the program and made many contributions to the experimentation. In turn, the *Mathematics We Need* program has benefited from participation by its authors in the SMSG projects.

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Your Teachers' Edition and Modern Mathematics

Teaching Modern Mathematics Confidently

Completeness is the key word which describes a program that may be presented with confidence by a large majority of elementary-school teachers. Achievement of *completeness* is time-consuming in the writing stage; *completeness* is tremendously saving of time for the program in use.

Completeness, as represented in *Mathematics We Need*, implies

- careful blending of mathematics that is new in the curriculum for grades 1–8 and mathematics that is time-tested for those grades;
- adjustment that utilizes the excellent growth that teachers have made in the last three decades;
- well-planned use of the many important principles of teaching and learning of elementary mathematics that have become available in the last thirty years;
- a curriculum organization which provides an adequate variety of presentations at a sensible pace so that thorough understanding is achieved before highly-technical symbols and procedures are introduced;
- textbooks which, instead of merely being excellent mathematical outlines, guide the discovery of mathematical ideas superbly through sufficient and varied verbal questioning and maintain and extend what is learned through strong programs of enrichment, practice, application, and testing;
- teachers' editions so carefully prepared that day by day they provide for every need in building depth of background and in individualizing instruction by including a wide range of suggestions through pre-book, textbook, and post-book lessons;
- supplementary devices and printed materials skillfully integrated to make the total mathematics program a superb instrument for teaching and for learning;
- a program so carefully organized that it can be introduced into a school system with a minimum of confusion and without the sacrifice of time taken from other important subjects of the curriculum which, of course, are also demanding time.

Preparing to Teach a Modern Program

Before we examine more closely with you the details of the *Mathematics We Need* program, we would like to say more about the all-pervasive problem of preparing to teach a modern program.

It is possible to work out programs in elementary mathematics so advanced in character, so skeletonized in form, and geared to so rapid a pace of learning that *nearly all* teachers would encounter great difficulty in teaching them. This, though, is far from the situation you will encounter if you are teaching *Mathematics We Need*. It is the belief of the authors of this series that you have a program which is mathematically powerful, yet is also highly teachable, highly learnable by all children, and highly useful to them.

Some of those who are openly critical regarding the inadequate mathematical background of teachers propose that, if teachers will take a single short, hurried, highly technical in-service course in mathematics, they can be readied to teach modern mathematics. From this hasty survey, teachers are supposed to learn all the complexities and refinements of the subject so as to be able to identify the characteristics of the difficult concepts and to ask the right questions and plan the best activities for pupil discovery. More than that,—it is presumed that if teachers are to teach a program in which the printed material is hardly more than a mathematical outline, they will be sufficiently prepared so they can write the supplementary materials which are not available in the skeleton of prepared materials, and can themselves devise what is needed for practice, maintenance, problem-solving, caring for individual differences, and evaluation.

Those of us who were responsible for the preparation of *Mathematics We Need* recognize that brief courses have value in introducing concepts. Our concern is that such a once-over course, isolated from the classroom, cannot come anywhere near readying teachers for their new duties. This is so, because even those who have strong orientation toward mathematics absorb many misconceptions in the single exposure provided in the course. Even

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one or two regular mathematics courses leave serious gaps. The only solution that we could discern was to take extra years to prepare a program that will provide you each day with a maximum of help. Please note that we say "each day." Our program gives you day-by-day background suggestions to assist you in deepening and refining your mathematical understanding; day-by-day helps in guiding correct development of concepts and skills; and an abundance of day-by-day materials for maintenance, practice, problem-solving, individualizing instruction, and evaluation.

Facsimile of Pupil's Page	Teacher's Page
Oral developmental questions to assure asking of right questions for discovery and to clarify concepts for pupils and teachers Variety of written work, including practice to further clarify and to nudge learning in the direction of permanence References to reteaching, extra activities, extra practice, and extra problems in back of book to help make very complete the program of caring for individual differences	Objectives and Background to clarify and extend the ideas for teachers Pre-Book Lesson that provides the first stage for discovery Notes for Using the Text Page which provide guiding suggestions so that important aspects in the text are emphasized Individualizing Instruction which provides wide range of suggestions as a capstone of the overall program of individualization

Using the Day-by-Day Helps

The highlights of the general organization of a pupil's developmental page and corresponding teacher's page are outlined above. To make more evident to you the value of our suggestions and aids, we discuss with you below the stages that you may want to follow as you plan a lesson.

The items mentioned for the pupil's page are characteristic of developmental pages. Besides developmental pages, there are many pages devoted to resurveying, reteaching, exploring, enrichment, problems, practice, maintenance, and testing.

The facsimiles of the pupil's pages in this teachers' edition have the further help of answers and annotations printed thereon, over and above the content of the pupil's page as such.

• As you start to plan a lesson, you should have clearly in mind the underlying purposes of the authors in writing that text page (hereafter referred to as "pupil's page"). These purposes are identified on the pupil's page by titles and clue captions, and they are further clarified by carefully-stated **Objectives** on the facing page of teaching suggestions (hereafter referred to as "teacher's page"). In addition to orienting you to the underlying purposes, these several helps will enable you to determine whether the pupil's page is to be used to resurvey, reteach, explore, develop new concepts, extend, enrich, improve problem-solving ability, provide practice, maintain learning, or test understanding or skill.

• You will also be concerned about having a thorough understanding of each mathematical concept or idea which is to be taught. As a step in this direction, (a) read through the oral development and the associated reproduced answers on the pupil's page (oral work is designated by [O]; written work by [W]), and then (b) read the **Background** section of the teacher's page. Note that we have not chosen to discuss all mathematical ideas at once in a single place, as in a few pages in the end matter, because this form of treatment, separate from the context in which the ideas are taught, would probably be more confusing than helpful. Instead, in accordance with our day-by-day plan, we cover each idea thoroughly at the time when you teach it, thus enabling you to relate the idea directly to the content prepared for students. Moreover, this plan gives us the opportunity to approach the idea with you in a variety of ways so

that the idea is brought to the work on later pupil pages and, consequently, discussed again in Background discussions on many of those pages.*

• After reading the Background section, read again the oral development on the pupil's page to help you in planning your procedure for guiding discovery on the part of your pupils. Note the order of the statements made and the questions asked. Attempts to guide discovery are successful to the degree that learners are led logically step by step from what they know to the concluding idea or understanding which is sought. Stated differently, clumsy and non-sequential questioning gets nowhere and defeats the purpose of instruction.

We take pride in the developmental lessons in our program. In the writing of each lesson, questions have been worded carefully, only to be reworded and then reworded again in the interest of clarity; and questions have been tried out in one sequence after another in search of the best order attainable. As we have said, we take pride in the final product, but our pride is tempered with awareness that we have done no more than authors of texts should do. Many teachers starting on a new program will not find it possible, on their own, to devise a sequence of questions adequate to guide discovery effectively, and they are entitled to what help can be given them. Teachers who are more sophisticated in mathematics will see in what we offer a means of checking and possibly improving their own patterns of questioning.

The introductory question and the sequence of questioning in the oral development have these further values: (a) the introductory challenging question that is utilized to introduce the development often serves to motivate pupils; (b) the oral responses will help you keep track of your pupils' thinking; and (c) these developmental lessons enable you to help your pupils acquire skill in reading with understanding the closely-knit type of writing that is characteristic of mathematical material. Point (c) needs to be emphasized. Pupils who do not have mathematics texts that require this kind of reading will almost certainly encounter serious difficulties when they deal with the heavily verbal texts at later grade levels.

• Now with the purposes, a clear understanding of the content to be taught, and the general plan of guided discovery of the pupil's page in mind, read the **Pre-Book Lesson** on the teacher's page. Here, too, we felt that we should take whatever time was necessary to work out with exceeding care model plans of instruction to precede the study of the text page—model plans only in the sense of illustrative patterns to be adapted as desired.

Seek these things from the suggestions in the Pre-Book section: a further opportunity to enhance your own understanding; an excellent technique for inventorying pupil background prior to more systematic development; a first approach to the idea—an approach that offers optimum opportunity for guided discovery, and one that is less abstract and is closer to pupils' experiences than the text development; and an opportunity to strengthen earlier learnings that are needed for the new topic.

• As you turn to the section on the teacher's page entitled **Using the Text Page**, you will be closely relating the suggestions in this section to the [O] and [W] sections of the pupil's page. It is at this point that you crystallize the plan that you will use for the portion of the lesson that pertains to the basic purpose of the page.

• In completing your planning of the lesson, you will need to be concerned regarding the extensions, practice, maintenance, problem-solving, and testing requirements that are so fundamental in caring for individual differences. *Mathematics We Need*, because it is a complete program, provides all of these. First, look at the pupil's page itself. You frequently will find on that page one or more sets of the following: practice items presented in interesting variations; maintenance exercises for concepts developed earlier; a section of enrichment; a set of problems; a brief test. Also, on a pupil's page, you often will find references to one or more of the following: **Reteaching** (except for grades 1–3), **Extra Examples, Extra Problems, Extra Activities.** These are references to the valuable reservoir at the end of the pupil's book. Reteaching

* If you are interested in examining a number of the ideas at one time, first turn to **Terms with Illustrations,** page 372 of this Teachers' Edition. Then, for more detailed study, turn to pages xv-xviii to locate references in this Teachers' Edition to pages where further discussion of the ideas may be found.

is included in the back of the book since in many classes or for several pupils in a class it will not be needed.

• Although we have moved a long way toward completeness in building into the textbook itself a very comprehensive program for *caring for individual differences*, we have not been satisfied to stop there. You will find on many teacher's pages that there are helpful suggestions for *Slower Learners*, *All Pupils*, and *More Capable Children* under the general heading **Individualizing Instruction.** This wealth of materials for individualizing your teaching combined with all that is provided in the pupil's book means that you can help each pupil achieve at his highest level.

In summary, then, as exemplified by almost any pupil's page and its accompanying teacher's page, we have tried to give you a self-contained program that day by day relates the content of each day's work to the total mathematics program; that helps you, the teacher, to have the kind of clear and thorough understanding of the content that is needed for superior teaching; that has a model set of questions for each development to help each pupil, largely through his own efforts, discover concepts and procedures; that gives you guidance in setting up the highly important pre-book lessons; that provides for differences in learning rates; and that provides for you complete programs of practice, maintenance, problem-solving, enrichment, and testing, over and above a most carefully-worked-out sequence of mathematical content.

Notes Regarding the Mathematics We Need Program

Building on Mathematical Structure

In elementary-school mathematics, pupils use and study progressively more sophisticated sets of numbers—counting numbers, whole numbers, fraction numbers (non-negative rational numbers), integers, rational numbers, and real numbers. As new sets of numbers are investigated, operations on these sets are defined and properties of these operations are observed and generalized. In turn, these properties supply the justifications for various algorithms such as those that are used for finding sums or products. This explicit study of various number systems, of operations on their sets of elements, of properties of these operations, and of applications of these properties is frequently called *study of mathematical structure.**

In the *Mathematics We Need* series, stress is placed on a realistic developmental program leading to having pupils eventually see elementary mathematics as a structured subject. The program has a first cycle that introduces during the first four years the elements of the structure in a somewhat intuitive and inductive approach—this builds understandings and skills carefully, unencumbered by extensive symbolism. Then, in a second cycle, the program becomes more explicit and more symbolic so as to provide optimum progress toward a logical, formal, deductive approach to the study of mathematics.

In the early school years, the concept of a set is developed through well-planned study of natural objects, and then by extension to sets of numbers. The sets of counting, whole, and fraction numbers are carefully studied and the operations on these sets are progressively discovered. Similarly, the Commutative and Associative Properties of Addition and Multiplication, the Distributive Property of Multiplication over Addition, zero as the identity element for addition, one as the identity element for multiplication, and additive and multiplicative inverses are studied first through observation in many examples. Then after pupils have encountered these concepts repeatedly in a variety of situations and have generalized them at their own level of maturity and understanding, technical names are attached to various sets of numbers and to properties of the operations on these sets.

In the middle grades, the properties of operations are used explicitly to justify the procedures in multiplication algorithms and long-division algorithms. Properties of operations on non-negative rational numbers are explicitly stated and named and are used to justify procedures in adding, subtracting, multiplying, and dividing in this set of numbers.

In grades seven and eight the interrelation of the various sets of numbers studied previously is observed and diagrammed. The negatives of whole numbers are introduced and the set of integers is identified. The properties of the operations on these various sets are summarized and are symbolized in forms such as $a + b = b + a$. In these grades, too, other number systems are introduced, non-negative and negative rational numbers are recognized as making up the set of rational numbers, and numbers that correspond to points on the number line but that cannot be expressed in the form $\frac{a}{b}$, with a and b integers and $b \neq 0$, are named "irrational numbers." Finally, the union of the sets of rational and irrational numbers is found to be the set of real numbers. As these sets of numbers are introduced through applications, their properties are observed and generalized. Throughout this study, the properties of the operations on these sets are seen to justify more complex computations and are used to justify short cuts, mental procedures, and steps in solving open mathematical sentences.

It should be emphasized that the program as a whole involves a gradual, progressively more explicit development starting in the primary grades, extending through grade 8, and then continuing into the high-school books in the Ginn Modern Mathematics Series. The pupil is led naturally from intuitive reasoning to the threshold of formal

* The illustration on the title page of the textbook may be used to indicate to pupils the general goal of the book in the study of structure.

study of the number systems. Ideas are introduced when they can be made reasonable to the pupil. Once they are introduced, they are *used*. The pupil is not artificially or unrealistically subjected to formalism, technical terms, and symbols until the ideas are well established.

With such a program, the pupil is helped to understand what he is doing and sees the study of mathematics as a unified, rational whole. At the same time he has a sound basis for a logical deductive study of mathematics in the years that follow.

Building on Mathematical Strands

There are certain basic mathematical concepts, fundamental principles, and computational techniques which permeate and unify the entire mathematics program from grade 1 through grade 8. These we call "mathematical strands." Many strands are introduced in the early grades with provision for consistent reinforcement and extension at each succeeding grade level. At whatever point the first presentation is made, opportunities are given for building on these strands, a number of which are mentioned below:

1. Sets and Operations on Sets—studied as the basis for the development of whole-number concepts, for the understanding of operations on numbers, and for clarification of geometric ideas

2. Number—study of the set of whole numbers extended to the sets of non-negative rational numbers, integers, non-negative and negative rational numbers, and real numbers

3. Numeration Systems—thorough study of our base-10 place-value system; comparison with place-value systems having bases other than 10

4. Operations on Sets of Numbers—study of the meaning of operation, certain defined operations, and the properties of the operations and use of these ideas to make logical the computational techniques which are in general use

5. Estimation and Methods of Mental Computation

6. Mathematical Sentences—emphasis upon symbols for operations ($+, -, \times, \div, \sqrt{\ }$) and symbols which indicate equality, inequality, and order ($=, \neq, <, >, \leq, \geq$); true, false, and open mathematical sentences

7. Problem-Solving—extended from simple one-step situations to complex situations which require the statement and solution of involved mathematical sentences

8. Non-Metric Geometry—starting from the recognition of common geometric figures and progressing gradually to the study of properties of such figures as prisms, pyramids, cylinders, cones, and spheres

9. Measurement—a strand which utilizes many concepts of metric geometry

To illustrate more fully the development of a strand and the manner in which such a sequence contributes to a unified whole, let us examine in some detail a strand that is somewhat closer to applied mathematics—the measurement strand. Illustrations of certain other strands were provided in the section entitled *Building on Mathematical Structure*.

The study of measurement progresses from simple beginnings when pupils intuitively sense a difference in sizes of objects, to the need for comparison through the selection and use of appropriate units of measurement, to the development in the upper grades of ideas of precision. Experiences in the measurement of a variety of physical objects in the environment are provided, but the discussion that follows will be limited to the developmental sequence for the measurement of length, area, and volume.

Basic to the concept of measurement in the elementary grades is the idea of a unit of measurement which is of the same nature as the thing to be measured. The measure of a segment is in terms of a unit that is a line segment; the measure of an angle is in terms of a unit that is an angle; the measure of a region is in terms of a unit that is a region, and so on. However, the unit need not be a standard unit, and the initial phase for the study of each type of measurement is the use of a variety of arbitrary units. Thus, attention is focused upon the process rather than upon the unit.

Pupils see that they may express the length of their desks, of the table, or of the bookcase in such arbitrary units as "finger inch" or "pencil." They may find the length of the room in "paces." Emphasis is placed upon the idea that the measure of a segment is a number which indicates how

many times the unit is applied in the distance from one end point to the other end point of the segment measured. To name a length is to mention both the number that is the measure and the unit of measurement—for example, 5 feet.

As pupils experiment with arbitrary units they find that convenience in measuring requires the use of both large and small units. These and other experiences bring to pupils a realization of the necessity for standard units and the need to develop skill in the use of standard measuring devices scaled in inches, centimeters, feet, yards, and so on.

Since the measure of a segment may not always be a whole number, a logical approach to the study of rational numbers is provided. Throughout grades 4–8, measurement units and rational numbers are closely related through the study of concrete objects, regions, and the number line.

The part of the measurement strand which deals with the measure of the region builds on the basic concepts developed for the measurement work with segments. Pupils first use unit regions having various shapes—triangular, rectangular, circular, square. When a square region is recognized as the most advantageous unit, then the square inch, the square centimeter, the square foot, and so on are derived from the corresponding standard units for segments. In all phases of the measurement strand, there is emphasis upon the idea that the smaller the unit used, the more precise the measurement.

When measurement of volume is developed in grade 6 and then extended in grades 7 and 8, the basic measurement ideas described for segments and regions are utilized. Further extension of the work in measurement includes the development of such formulas as those for computing perimeter, area, and volume.

Values of Precision in Mathematical Language

In a modern program, more emphasis is placed on the ideas underlying mathematics and more attention is given to abstract patterns of thought. To achieve all this, new symbolism and terminology are needed in the early grades. However, this symbolism and terminology must not be arbitrarily rushed into the program—what is introduced must be precise but, at the same time, it must be introduced gradually. This introduction should occur after sufficient understanding has been achieved so that the use of the symbol or term will promote more complete understanding of the concept under study. Once introduced, such symbolism and terminology has to be maintained throughout the program with frequent reinforcement and extension.

The degree of precision used at a certain level should depend not only on the subject matter of that level but also on how well it will hold up under more advanced treatment. For example, the statement "When numbers are added, the sum is greater than any addend." seems very precise if the only numbers that are known to the pupil are the counting numbers 1, 2, 3, and so on. However, the lack of precision becomes evident when the pupil learns about the set of whole numbers, with its new member 0, and recognizes that for $3 + 0$, for instance, the sum is not greater than the addend 3.

Traditional mathematical areas—algebra, geometry, trigonometry—still form the core of the secondary-mathematics program, but this core is enveloped by modern notation, modern approaches and, to some extent, new concepts. Such a program is easily studied if good foundations have been laid in the elementary program. Thus, precision in an elementary program is necessary to clarify and simplify the material being studied at any particular level and to lay sound foundations for clear and concise definitions for extensive development of work in later grades.

Geometry provides a good illustration of the value of careful use of language. Precision in presenting geometric concepts is needed in order that the pupil understand, early in his training, the difference between metric and non-metric geometry and the difference between abstract sets of points and geometric shapes in the physical universe. Such precision is necessary not only for further study in geometry but also in order to understand better the subject matter that is being presented in many of the modern science programs.

Although the need for precise terminology in enabling pupils to communicate their thoughts with clarity is recognized, it is unfortunate that, even in the so-called modern approach to mathematics, there does not exist universally accepted

terminology. This leads to some confusion as one turns to different sources. However, we are closer to such an ideal situation than ever before. *Mathematics We Need* employs terminology that is up-to-date, generally accepted, and widely used. In cases where variations of terminology for a certain idea exist, *Mathematics We Need* introduces such variations when this does not lead to inconsistency.

Using Mathematics

The expression "using mathematics" means the employing of ideas from mathematics that help solve the many numerical and geometrical problems so common in this scientific age, as well as problems in more advanced phases of mathematics. In other words, mathematics taught in an elementary grade has two major applications. One is external as applied to realistic problems and the other is internal as developmental in instilling an attitude of creative attack on the mathematics of later grades and the mathematics of the future, some of which is now not even known.

We normally think of the external use when we refer to "applied" mathematics. Pupils, of course, do use mathematics in real-life situations outside the classroom, but the immediate practical situations do not constitute a complete curriculum. Pupils also need to study problem-solving situations of a more abstract and, in a sense, less realistic nature. Because of the pressure of many language patterns and combinations of situations, many as yet undreamed of, it is difficult to teach each pattern of words individually in an effort to help pupils interpret independently all the problems they might face.

Providing a powerful foundation for problem-solving is the concept of a mathematical model. A problem normally involves the application of mathematics to some physical situation. But to solve the problem, the pupil must translate the description in words into the language of mathematics. In technical terms, he needs to learn how to set up a mathematical model for the problem. At first, this model may be the basic idea: if the two addends are known, addition is used to find the sum; and if a sum and an addend are known, subtraction is used to find the unknown addend. Similarly, if the two factors are known, multiplication is used to find the product; and if a product and one factor are known, division is used to find the unknown factor. Later, the pupil moves to a more symbolic model—a mathematical sentence for the situation. In many cases, this sentence may be a statement of an addends-sum relationship (for example, $8 + 6 = n$) or a statement of a factors-product relationship (for example, $3 \times n = 24$); or perhaps a statement of a well-known formula such as $F = \frac{9}{5}C + 32$. This means that pupils must learn to express verbal statements as mathematical sentences, and be able to solve these sentences once they have been written. The final phase of problem-solving consists of the pupil's translating into its physical meaning in the problem situation the answer he obtained by working in the model.

Problems chosen to illustrate problem-solving techniques can be very interesting. Fortunately, pupils enjoy a wealth of applied problems which are significant to them. They like to know more about how mathematics is actually used in science, in technology, in sports, in business, in the social sciences, and so on. Even when not immediately practical for pupils, such uses of mathematics help them develop problem-solving skills and enable them to understand how other people use mathematics.

The second major aspect of using mathematics is often neglected when teaching a particular topic. Virtually all of the mathematics developed in the elementary grades is needed later in other mathematics courses and potentially in presently unrecognized uses. It is often difficult to explain in meaningful terms to the pupil that he is studying a topic one year so that he can use it another year or even several years in the future, in connection with other topics in mathematics presently unknown to him.

This study of mathematics for its own sake, which might be called internal applications, takes many forms: learning basic skills for future application, solving problems of a purely numerical nature, solving fun or recreational problems, and others. Indeed, most of the modern topics such as sets, properties, numeration systems, and non-metric geometry are justified as an important part of the curriculum, because of their usefulness in the

development of insight, in providing an orderly base for mathematical thinking, and in meeting the mathematical needs of the future rather than because of their immediately utilitarian value.

In a complete elementary-mathematics program, then, pupils encounter mathematics which they will use in several ways: actual applications to problems in their own life; applications to problems beyond their relatively limited experiences, techniques and understandings that will help them unlock mathematics in the future; and problems of a puzzle or recreational nature.

There is no sharp distinction between "pure" and "applied" mathematics, and that is as it should be. If mathematics is to be functional and worthwhile for a pupil, he should think of the subject as one which is used so often that it is practical, but also as one which is fascinating in its own right, without regard to application. This "dual but unified" nature of mathematics is excellently reflected in the *Mathematics We Need* series.

Providing for Individual Differences

The extraordinarily great range of differences within and among children at any given age level often is neither well defined nor understood. At the fifth-grade level, for instance, it is normal to find children ranging from early third-grade to late eighth-grade levels in interest, ability, and achievement. Psychologists did not discover individual differences, they merely described their range.

Most of us as teachers and parents are conditioned to judge progress or success in terms of some preconceived notion of a standard which is considered to be average or normal. Our levels of expectation in mathematics instruction and achievement are affected by some such "average." Making adjustments for children who deviate from this normal expectation is a major responsibility if the effectiveness of our instruction is to be at a maximum. The provisions and recommendations for attaining the highest level of interest and achievement by all pupils which are found in the *Mathematics We Need* series stem from the following philosophy of mathematical learning:

a. A sound, carefully prepared, sequential set of mathematical ideas and skills must be made available to all pupils.

b. Regardless of whether or not a school system uses acceleration as one way of providing for individual differences, there still must be a wide variety of activities provided in the basic printed program. This is so because even when pupils are accelerated, there will still be individuals who differ within the accelerated group and who must, therefore, have differentiated assignments.

c. Enrichment activities should be available to all pupils. Slower learners should not be denied the excitement and additional insight often found in enrichment activities. In general, enrichment work extends and supplements the basic mathematics program. It affords an opportunity to take "side trips" into the world of mathematics which may be equally enjoyed by all pupils. *Mathematics We Need* frequently provides enrichment suggestions which can be used with all pupils.

There is a wide range of activities and suggestions for individualizing instruction available in the *Mathematics We Need* series.

- In the pupil's textbook you will find a variety of extra materials. At intervals through the chapters of each book are sections which are clearly indicated as being intended for enrichment. Many other pages offer alternate ways of performing an operation or carrying out new activities for children to try.
- The reservoir at the end of each textbook for grades 3–8 provides three types of extra help. These are found under the following titles: (1) **Extra Problems;** (2) **Extra Examples;** and (3) **Extra Activities.** Further, in grades 4–8, some **Reteaching** helps are provided in the reservoir. Each of these sections can be an excellent and useful source of material for individuals or small groups of children as the occasion demands. References to these extra exercises are provided at the end of appropriate material in the basic pages of the textbook.
- A section entitled **Individualizing Instruction** is available in most of the lesson plans. Included in this section are suggestions for all pupils, slower learners, and more capable learners. Often, several different types of activities are suggested for a given group of children. Thus, a wide variety of experiences in learning mathematical concepts is provided.

• The carefully developed discussions in the Background section of lesson plans provide a source that can be often used for more enrichment.

• Pre-Book Lessons and Using the Text Page in the lesson plans suggest further resources in caring for individual differences.

• At the end of each chapter in the teachers' edition there are further suggestions for supplementing and extending the mathematics program. Some of these activities involve the alternate uses of pages of material in the pupil's text. This feature takes advantage of the fact that many of the activities on the pupil's pages may be modified or adapted slightly to create an exciting and valuable extended-learning or practice experience.

• The workbooks in the series afford many opportunities for individualizing instruction.

• Further extension is possible through using supplementary materials that are available from Ginn and Company. These include *Ginn Arithme-Sticks*, *Related-Facts Cards*, two kits of flannel items entitled *Introduction to Sets and Numbers* and *Operations on Sets and Numbers*, and a number-line model entitled *Modern Number Line*. All of these are designed to present modern mathematics ideas and all have complete guides. Further, for certain situations, we suggest that you write to the publishers, Ginn and Company, to inquire regarding supplementary publications that will provide further enrichment.

The Learning Process in Mathematics

Thirty years ago, when the idea of developing arithmetic through understanding rather than through memorization was being pioneered by Ginn and Company arithmetic authors and editors, a widely recognized expert in the teaching of secondary mathematics asked one of the authors to tell him something of the new movement. At the outset he admitted that he was unsympathetic with the little he had heard about it. As the author proceeded in his explanation of "meaningful arithmetic," this secondary specialist became steadily more impatient, finally breaking out with the statement, "All nonsense! Just see to it that children in the elementary grades learn to add, subtract, multiply, and divide. Then when they come to high school, we'll teach them how to think!" He did not say how and why children who had been learning arithmetic blindly and mechanically for seven or eight years would suddenly be ready, disposed, and able to blossom into the ability to reason mathematically upon becoming high-school freshmen.

The anecdote is worth the telling for two reasons. In the first place, this critic's brief statement epitomized the conception of arithmetic and of the processes of teaching and learning the subject that was dominant thirty years ago: arithmetic was thought to consist primarily of computation skills, to be learned and maintained by incessant practice. In the second place, the conversation reveals the conditions under which "meaningful arithmetic," the precursor of "modern elementary mathematics," made its great contributions in developing the climate for and making excellent progress in the learning of mathematics via understanding—contributions for which it is not too generally given credit.

The key word in any account of the learning process in mathematics is *understanding*. This statement holds true even in the case of attitudes, for how can one be confident in using intellectual skills one does not understand or respect a body of knowledge that possesses little meaning?

Now, understanding is not an all-or-none affair, either present or absent; rather, it is a matter of degree. A child is told that there are five horses in a field. Does he understand "five"? Yes, to some extent, if he realizes that "five" is a correct word to use for the particular set of horses he sees; but No, in a fuller sense of the word, if to him "five" refers to these special animals and is not understood as a characteristic of all sets that are in one-to-one correspondence with that set of horses.

The degrees of understanding just illustrated are rarely adequate in mathematical learning, save as starting points. For the idea of "five," we are dealing with abstract ideas which have no physical existence whatsoever; but, if we understand the idea thoroughly, we can impose this idea on a great variety of external representations. However, note the word "impose." The ideas are not *in* the representations. Instead, we *put* them there.

For every idea there is usually, but not always, a word or other symbol ("five," "5," "3 + 2,"

"8 − 3") which stands for it; and more complex abstractions, such as generalizations which involve a relationship among several ideas are also usually couched in language. But neither an idea nor a generalization can have much meaning if one knows only the corresponding symbol or symbolic expression. And here lies one of the most serious pitfalls in teaching mathematics, or anything else, for we may infer that a learner who can correctly employ the appropriate language in response to a limited number of specific unvarying questions has actually acquired all the understanding we want him to have. It is for this reason that too early introduction of symbols, terms, and definitions defeats sound learning—too early in the sense of an inadequate base of understanding so that memorization is the only recourse.

Do you want an example? If so, read the following: Gom flas mev hukl. What does the sentence mean? Read the series of syllables over and over until you can repeat it without referring to the syllables themselves. Now what does the whole thing mean? Is the sentence correct or incorrect? Does it apply to a game? to economics? to clothing? to a map? Have you not memorized a succession of verbal symbols without acquiring much by way of meaning? And will continued repetition of the series increase its meaning? Finally, do you perceive the applicability of your experiences here to much that passes for learning in the classroom?

As you have seen, repetitive practice does not enhance meaning; but it does do something else: it increases your skill or makes you more efficient in saying the series of syllables, and it may enable you to recall the series for a longer period of time than would a single repetition. And repetitive practice performs these same functions in the learning of mathematics. Needless to say, however, it should be used, not with nonsense material like that above, but with content (generalizations and skills, for example) that are already well understood. Hence, the statement that there is no place for drill in modern mathematics is erroneous. On the other hand, it should be deferred in each instance to a time when the desired degree of meaning has been engendered.

So, we are left with the question, How are we to assist children to develop or to broaden, deepen, sharpen, and enrich understanding? We do so by providing through guided discovery a variety of relevant experiences—not essentially the same experience over and over again as in drill. All these experiences contain the element of meaning to be taught, but in differing contexts.

Thus, in learning "five," children are guided to enumerate in order to find the number of members in sets of objects at home and at school—sets of pencils, marbles, dolls, chairs, and so on; they manipulate movable objects and imaginatively manipulate pictured objects, noting sets which have the number characteristic of "fiveness" and sets that do not have this "fiveness" characteristic; they produce sets which have "fiveness" by making marks, drawing oranges, tapping a bell; they come to recognize at a glance conventionalized groupings like Fig. 5–7, as having the number property of "fiveness." Each time, attention is directed to the

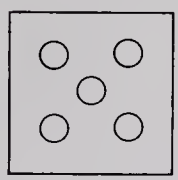

Fig. 5

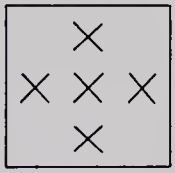

Fig. 6

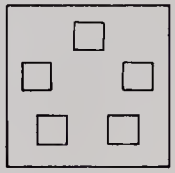

Fig. 7

one idea to be taught, in contrast to other inappropriate ideas. Eventually, children arrive at an abstraction which is symbolized by numerals such as 5, five, V, 7 − 2, and 4 + 1. It should be noted that over and above the necessity of a variety of approaches for refining an idea, such variety is also imperative if those who fail to understand through one approach are to have opportunities to attain understanding through other approaches.

Understanding grows in a manner which, if pictured, would resemble stairs of uneven steps, unlike in height and depth. Children do not need always to traverse all the possible steps, starting invariably with something equivalent to the enumeration of concrete objects. Instead, depending upon their degree of understanding, they may begin with step 3 or step 6 in a hypothetical set of stairs with ten steps. (It would be foolish indeed to teach children the meaning of 307,486 beginning with enumerating objects.) Understanding at any stage rests and builds upon understandings previously acquired and in turn contributes to the next stage. It is, in

fact, this very characteristic which makes it possible to guide pupils in discovery of "new" ideas—"new" because the ideas are actually only extensions of earlier understandings. Much has been said about this earlier in the sections pertaining to building on structure and on mathematical strands.

To conclude, first negatively, then positively: children do not increase understanding of mathematical content by memorizing the language in which it is expressed or by engaging repeatedly in what is practically the same experience; they advance in understanding by undergoing a variety of guided discovery and other experiences, all of them embodying the same content but otherwise different, and all leading by easy stages to progressively more abstract and generalized forms of thought.

In sum, a complete program should be written so that understanding is assured through a careful development that includes an intelligently planned variety of approaches. In essence, such a plan is accomplished if there is true balance in the program. This implies careful selection from the content used in experimental programs; judicious retention of time-tested content; constant use of ideas taught, both in developing other ideas and in application to practical situations; utilization of variety even when repetition is needed for building a skill; and having balance in methodology as represented by guided discovery which blends the efficiency of telling, without the deadliness of telling, and the effectiveness of discovery, without the confusion and time loss of random discovery. The *Mathematics We Need* program is noteworthy for its remarkable balance that assures optimum learning through understanding.

Notes Regarding the Grade-6 Program

Introduction

While the underlying purpose of this teachers' manual is to be of maximum assistance to teachers day by day, it also provides a good general background in modern mathematics. In this section, we will present brief summaries of the important mathematical ideas found in the grade-6 program, and then refer you to Teacher's Pages throughout this Teachers' Edition where additional information is available.

General Plan of the Grade-6 Program

• The books for grades 5 through 8 constitute the second major cycle of the Mathematics We Need program. Many widely accepted features of the first cycle, grades 1 through 4, are continued with learning experiences appropriate for pupils in grades 5 through 8. Some of these are as follows:

—ideas are first explored intuitively

—the development of each topic is gradual from the intuitive to the symbolic

—more than one approach is introduced when this technique increases understanding

—modern language and symbolism is used to clarify ideas and to express ideas more precisely

—maximum provision is made for guided discovery

—special experiences for developing competence in problem-solving and for fixing skills are included

—numerous opportunities are provided for individualizing instruction, for maintenance of skills, and for evaluation

—the organization permits the teacher to vary the pace to fit the ability of her class

• The grade-6 text is a transition between the first book of the last cycle and the grade-7 text which is a first step toward mathematics of the high school. While it resurveys and re-establishes basic mathematical concepts and algorithms extending from these concepts, it presents new ideas with emphasis on logical reasoning leading to more mature mathematical thinking. Precise vocabulary, mathematical symbolism, properties of number systems, geometric ideas, and sets of numbers expressed in many ways are presented with an eye toward establishing greater confidence in mathematics.

• As in the other texts of this series, the grade-6 text has a comprehensive resurveying program, especially in Chapters 1 and 2. Resurvey of set terminology, bases other than ten, place value, operations of addition and subtraction on whole numbers, and number properties is developmental in Chapter 1 and tests are provided at intervals to determine if difficulties or weaknesses exist. More thorough reteaching is available from the material in the back of the book as are sets of extra examples for those who need more work to develop skills in the basic operations. In Chapter 2, set terminology applied to geometry; multiplication and division of whole numbers; and multiplication and division algorithms are resurveyed. Also resurveyed are number properties, prime and composite numbers, products of primes, and finding the arithmetic mean. Statistics is extended by the introduction of the concepts of mode and median.

• Chapter 3 stresses addition and subtraction of rational numbers using fractions. Measurement includes a comparison between standard and non-standard units, applying linear measurement to models of geometric figures, and the measurement of angles using a protractor. Regular polygons and congruent figures are also discussed.

• Chapter 4 emphasizes measurement; using standard units of length, weight, and time; and becoming familiar with metric units of measurement. Addition and subtraction of measures, finding perimeters of polygons, unit of area, area of regions, finding the unit of precision, and greatest possible error are also explored. Addition and subtraction of rational numbers named with decimals, and rounding rational numbers are also presented.

• Work with rational numbers named with fractions is extended to include multiplication and division using the common-denominator method in Chapter 5. Finding the reciprocal of a number leads to using the reciprocal method in division with fractions. The meaning of ratio is also presented through the use of scale drawings.

• In Chapter 6, work with rational numbers named with decimals is extended to include multiplication and division with decimals and using the vertical algorithm for multiplication and division. Ratio is extended to per cent forms, including naming rational numbers with per cent forms, addition and subtraction with per cent forms, using the factors-product relationship, and finding the ratio and base factors. Most of the geometry in this chapter is related to circles and finding diameters, radii, and circumferences.

• Understanding and using graphs is developed in Chapter 7 through reading and constructing pictographs, bar graphs, double-bar graphs, line graphs, divided-bar graphs, and circle graphs. The number system is extended to include positive and negative numbers, and relationships among sets of numbers is shown through the use of Venn diagrams. Through use of diagrams and illustrative devices, geometric ideas are developed as they relate to area of regions, space figures, polyhedrons, prisms and right prisms, area of polyhedrons, solids, volume of a solid, and construction of congruent segments, congruent angles, congruent triangles, and bisector of an angle.

• The entire book 6 provides for a balance among the development of basic skills; the ability to apply mathematics to practical situations; and the ability to apply basic concepts, principles, and understandings to more mature mathematical thinking. Special sections of the teaching pages such as the Pre-Book Lesson and Individualizing Instruction make ample provision for these objectives.

• A brief overview of the first chapter will be found on the page having the heading "Overview—Chapter 1" preceding Teacher's Page 1. Overviews for the remaining chapters will be found on Teacher's Pages 52, 94, 140, 184, 228, and 274.

Overview of Mathematical Ideas

On pages xvi–xviii you will find selected headings arranged in alphabetical order. Related subtopics are classified under these headings. Accompanying each heading and subtopic are references to lesson plans where discussions can be read in direct relationship to the pupil's pages.

If you want to read about the basic ideas in a logical sequence before you start teaching the grade-6 program, read the references for each heading and its related subtopics in this order: Sets; Numeration System; Numbers; Mathematical Sentences; Properties; Measurement; Geometric Ideas; Statistics and Graphs; and Per Cent and Ratio.

Page references are indicated by numerals. You should first read the mathematical development in the **Background.** Further discussions of how the topics or concepts may be used in teaching the text page are sometimes found in the **Pre-Book Lesson** and **Using the Text Page.**

• Geometric Ideas, *point*, *line*, *ray*, (see 36–39, 43, 44); *plane figures*, (see 41, 42, 45–48, 88–90, 134, 135, 173, 252–256, 292–295, 303); *space figures*, (see 296–302, 304, 305); *solids*, (see 306–309); *construction*, (see 314–317). The study of sets of points in space is used to progress from simple concepts to more difficult concepts of space figures and solids. While the approach is at first intuitive, precise language and logical mathematical thinking is used.

• Mathematical Sentences, (see 27, 29–31, 52, 58–59, 106, 118, 119, 167, 184, 185, 192, 193, 204, 205, 228, 229, 238, 240, 262, 263). Many ideas in the grade-6 program are approached through the language of mathematical sentences. Problem-solving, properties, maintenance of skills, and generalizations are developed through their use. Their use in simple addends-sum and factors-product relationships, in per cent, and with rational numbers helps to unify presentation throughout the text.

• Measurement, *basic ideas*, (see 124–126, 140, 141, 145, 146); *approximate nature of*, (see 154–157); *metric system*, (see 142–144); *perimeter*, (see 150–153, 254); *angles*, (see 128–133, 136); *area*, (see 174–177, 180, 292–295, 301, 302, 304, 305); *volume*, (see 310, 311). A strong emphasis on the concept of measurement, including metric units, leads to a genuine understanding of practical applications of measurement. Familiar units of measurement are presented along with new concepts of metric units, precision in measurement, and measurement of perimeter, area, and volume.

• Numbers, *sets of*, (see 6–8); *operations with whole numbers*, (see 27, 29, 30, 31, 54, 55, 60–62, 71–75); *prime and composite*, (see 66–69, 108);

positive and negative, (see 312, 313); *rational numbers named with fractions*, (see 94–101, 103–107, 110–115, 184–189, 192–198, 204–206, 208–212, 214–215); *rational numbers named with decimals*, (see 120–123, 160, 162–164, 228–234, 236–238, 240–244, 246, 248, 249). In grade-6, subsets of the set of whole numbers are investigated. The set of numbers within a pupil's understanding is extended to include rational numbers and negative numbers. Pupils learn to differentiate between prime and composite numbers and apply these ideas to factoring, finding multiples, and to work with fractions.

• Numeration System, *place value*, (see 16–18, 20, 120–121); *other systems*, (see 13–15, 26); *bases other than ten*, (see 21–25). An understanding of our numeration system and place value is critical to using and understanding the algorithms which are developed and used throughout the text. Work in other bases and with other systems of place value and numeration reinforces understanding of, and leads to an appreciation of the base-ten system.

• Per Cent and Ratio, *scale drawings*, (see 220, 221, 224); *meaning of*, (see 222, 223, 257–263); *ratio factor and base factor*, (see 264–267); *problems with*, (see 266–269). This topic is introduced and developed through the idea of ratio and rational numbers. The factors-product relationship provides a means for understanding and finding ratio and base factors, performing operations with per cent forms, and solving problems with per cent. While the topic of per cent itself is traditional, the approaches to it are modern and its application in many fields is readily evident.

• Properties, (see 28, 54–56, 107, 187, 189, 197, 198, 232). An important feature of a modern mathematics program is to make the properties of sets of numbers under an operation more explicit. In the grade-6 program, the main emphasis is upon properties of rational numbers named with fractions and decimals. This emphasis allows the pupils to better grasp the structure of the number system and to approach ordinary computation more logically.

• Sets, *defining sets*, (see 1–4); *subsets*, (see 83); *finite and infinite sets*, (see 7); *union and intersection of*, (see 8, 9); *sets in geometry*, (see 84–86); *Venn diagrams*, (see 290, 291). The language of sets is a unifying idea in mathematics since it is used in explaining many ideas which seem to be unrelated.

• Statistics and Graphs, *statistics*, (see 79–81); *pictograph*, (see 274–277); *bar graph*, (see 278, 279); *double-bar graph*, (see 280); *line graph*, (see 282–285); *divided-bar graph and circle graph*, (see 286, 287). Graphing is extended to include reading and constructing statistical graphs so that various data can be plotted, read, and interpreted. Pupils learn to integrate other mathematical concepts into their work with graphs, and use graphs in problem-solving situations.

Summary of Other Features

In addition to the mathematical ideas discussed above, other characteristics make this a text from which pupils will learn effectively. These features are summarized below; selected page numbers refer to a Teacher's Page and the accompanying Pupil's Page where the characteristics are illustrated.

• Concrete Aids, (see xix).

• Enrichment, (see 12, 26, 31, 63, 82, 108, 158, 172, 179, 180, 219, 305, 307–309, 311, 313–317). Besides these pages of enrichment, you will find frequent suggestions in the text and on Teacher's Pages supplementing provisions for more capable pupils. Numerous individual problems require the more capable pupils to explore the more subtle concepts. Many of the Extra Activities in the back of the text provide excellent enrichment.

• Evaluation. Since understanding by the pupil is one of the most important outcomes of this program, a planned systematic evaluation is essential. Each chapter has a thorough testing program that includes Diagnostic, Computation, Problem, and Information and Meaning Tests.

• Individualizing Instruction. No list of references is provided because this title is one of the major sections for each page of the Teacher's Book. On various pages will be found suggestions for more concrete experiences for slower learners and extensions for the more capable pupils. Special instructions on ways to handle the oral and written work for each page are included to provide most effectively for each pupil.

In addition, at the end of each chapter, there are extensive lists of Supplementary Activities. These

are found on Teacher's Pages 49–51, 91–93, 137–139, 181–183, 225–227, 271–273, 318–321. There are many uses for these pages: They may be used with all pupils to increase interest or more capable pupils may work on them when they have finished their regular assignment or upon completion of the end-of-chapter tests.

• Oral Devclopment. No page references are given because most of the pupil's text pages illustrate this plan. For the development of abstract concepts like those that are being brought into elementary mathematics programs today, it is particularly important to have careful oral development provided in the textbooks. Many advantages could be listed for this plan, but if teaching can be defined as "the guiding of learning," then a prime requisite is that teachers know what pupils are thinking. Oral questioning and discussion that has been carefully planned is often the only means of gaining this vital information. Further, the oral aspect of the program provides an opportunity for the pupil to organize and verbalize his thinking as he proceeds toward discovering a new idea.

• Practice. No page references are given because nearly all pages contain some amount of work that is practice. There are enough exercises so that each pupil can develop essential skills. The variety of exercises permits the teacher to individualize the practice program to meet the needs of individual pupils.

• Problem-Solving, (see a selection of these pages: 32–35, 52, 53, 57, 64, 65, 77–80, 109, 116, 118–119, 165, 169, 170, 178, 179, 190, 199–201, 207, 217, 218, 239, 245, 250, 251, 268, 269). This list is an indication of the wide variety of approaches in problem-solving in the grade-6 program. The authors have recognized that much must be done to help pupils associate mathematics with the problems that arise in their quantitative world.

Pupils must also learn a method of analyzing mathematical problems that will be helpful in future mathematics courses. The abstract nature of mathematical sentences is recognized and specific experiences are provided to help pupils analyze problems, delete unnecessary information, and learn to express mathematical relationships in the situation with a mathematical sentence.

Pupils learn to draw pictures to represent relationships in a problem and to collect and organize data in a table. Pupils are encouraged to arrive at plausible conclusions from the data and in some cases to prove their answers.

The chapters on geometry contain problem-solving activities which not only help the pupil interpret his environment, but also encourage him to use precise language in expressing his ideas.

Elementary Mathematics Aids

Throughout the Teachers' Edition, included in the lesson plans, there are many references to certain of the items listed below. You may want to study this list early in the year in order to arrange well in advance for obtaining the items you will want to use. It should be noted that this represents a complete listing of the aids referred to throughout the *Mathematics We Need* series so some of these items will not be suitable for use at this grade level.

The following may be purchased from Ginn and Company (Home Office, Boston, Massachusetts 02117):

1. Flannel Board Fractional Parts
2. Flan-O-Graph
3. Ginn Arithme-Sticks (with guide)
4. Introduction to Sets and Numbers (flannel items and guide)
5. Modern Mathematics Kindergarten Kit (flannel items and guide)
6. Modern Number Line (with guide)
7. Number Concept Cards
   (includes
   Picture Number Cards
   Pattern Number Cards
   Pattern Cards for Number Facts)
8. Related-Facts Cards (with guide)
   8A Addition and Subtraction (one box)
   8B Multiplication and Division (one box)
9. Operations on Sets and Numbers (flannel items and guide)

9J. Modern Mental Computation (flannel items and guide)

9K. Modern Geometric Ideas (flannel items and guide)

9R. Plastic Pegboard and Pegs

9S. Pegboard Discovery Guides, Numbers and Numerals, 0–20 (to be used with 9R above)

The following are available from Milton Bradley Company. Comparable items are often available from other sources.

10. Calcuframe
11. Clock Dial
12. Plastic Counting Discs
13. Cubical Counting Blocks
14. Day by Day Calendar
15. Educational Thermometer
16. Educational Toy Money
17. Flannel Board Cut-Outs
18. Flannel Board Numbers
19. Geometric Figures and Solids
20. Hundred Chart
21. Place Value Indicator
22. Primary Number Line
23. Self-Teaching Flash Cards: addition
24. Self-Teaching Flash Cards: subtraction
25. Self-Teaching Flash Cards: multiplication
26. Self-Teaching Flash Cards: division
27. Tick-Tock Primary Clock
28. New Math Symbols

Contents of Pupil's Text

CHAPTER		PAGE
1	MAINTAINING AND EXTENDING UNDERSTANDINGS AND SKILLS: SETS – finite and infinite; intersection and union • NUMBER • NUMERATION – Roman numerals; Egyptian numerals; place value; numerals for numbers through trillion; exponent form; base-ten system; other bases: base five, base four, base seven, base eight • MATHEMATICAL SENTENCES – true; false; equalities and inequalities • PROPERTIES • OPERATIONS – A. and S. with whole numbers • NON-METRIC GEOMETRY – point; segment; line; ray; angle; plane; spatial relationships; curves; polygons; circles ENRICHMENT • PROBLEM-SOLVING • TESTING	1
2	MAINTAINING AND EXTENDING UNDERSTANDINGS AND SKILLS: SETS – subsets; sets in geometry • NUMBER – factors of a number; prime; composite; prime factors; products of primes • OPERATIONS – M. and D. of whole numbers; multiplication and division algorithms • STATISTICS – arithmetic mean; mode; median • MATHEMATICAL SENTENCES • PROPERTIES • NON-METRIC GEOMETRY – simple closed curves; interior and exterior; regions ENRICHMENT • PROBLEM-SOLVING • TESTING	52
3	ADDITION AND SUBTRACTION OF RATIONAL NUMBERS USING FRACTIONS: RATIONAL NUMBERS – named with fractions: A. and S. with fractions, vertical form for A. and S.; named with decimals: rounding • MEASUREMENT – standard and non-standard units; measuring lengths; measuring angles • GEOMETRY – regular polygons, congruent figures MAINTENANCE THROUGH USE: Sets; Mathematical sentences; Properties; Estimating; Non-metric geometry ENRICHMENT • PROBLEM-SOLVING • TESTING	94
4	MEASUREMENT; ADDITION AND SUBTRACTION USING DECIMALS: RATIONAL NUMBERS – A. and S. with decimals; rounding • MEASUREMENT – standard units of length; weight, time, liquid measure; metric units; A. and S. of measures; perimeters of polygons; unit of precision; greatest possible error; area of regions; unit of area; area of rectangular regions MAINTENANCE THROUGH USE: A., S., M., and D. of whole numbers; A. and S. with fractions; Sets; Mathematical sentences; Properties; Estimating; Non-metric geometry ENRICHMENT • PROBLEM-SOLVING • TESTING	140

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— 5 — MULTIPLICATION AND DIVISION USING FRACTIONS: 184
RATIONAL NUMBERS (named with fractions) – M. with fractions; D. with fractions; common-denominator method; reciprocal of a number; reciprocal method • RATIO – scale drawings; meaning; expressing ratios

MAINTENANCE THROUGH USE: A., S., M., and D. of whole numbers; A. and S. with fractions and decimals; Sets; Mathematical sentences; Properties; Estimating; Geometry; Measurement

ENRICHMENT • PROBLEM-SOLVING • TESTING

— 6 — PER CENT; MULTIPLICATION AND DIVISION USING DECIMALS: 228
RATIONAL NUMBERS (named with decimals) – M. with decimals, vertical algorithm; D. with decimals, vertical algorithm • GEOMETRY – diameter; radius; circumference • PER CENT – meaning; naming rational numbers; A. and S. with per cent forms; using factors-product relationship; ratio factor; base factor

MAINTENANCE THROUGH USE: A., S., M., and D. of whole numbers; A., S., M., and D. with fractions; A. and S. with decimals; Sets; Mathematical sentences; Properties; Estimating; Non-metric geometry; Measurement; Ratio

ENRICHMENT • PROBLEM-SOLVING • TESTING

— 7 — GRAPHING AND GEOMETRY 274
GRAPHING – pictograph; bar graph; double-bar graph; line graph; divided-bar and circle graphs • SETS – sets of numbers; Venn diagrams • GEOMETRY – regions; area of regions; space figures; polyhedron; prism; right prisms; lateral area of right prism; area of polyhedrons; solids; volume of a rectangular solid • POSITIVE AND NEGATIVE NUMBERS • GEOMETRY – construction of congruent segments, congruent angles, congruent triangles, bisectors

MAINTENANCE THROUGH USE: A., S., M., and D. of whole numbers; A., S., M., and D. with fractions and decimals; Sets; Mathematical sentences; Properties; Statistics; Measurement

ENRICHMENT • PROBLEM-SOLVING • TESTING

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1

Overview - Chapter 1

• Most of Chapter 1 is devoted to resurvey and extensions of such basic concepts and strands of modern topics as sets, numeration systems, problem solving involving the addends-sum relationship, and geometric figures.

• **Resurvey.** Some topics resurveyed include sets, set operations, terminology and descriptions, and develop to include ideas of number, numeral, numeration systems, numerals in other bases, expanded form, and place value. Also resurveyed are Roman numerals; rounding numbers; greater numbers; the identity element and properties of addition; true and false mathematical sentences; and geometric ideas. The geometry includes naming figures, properties of figures, points of intersection, and types of angles, curves, and polygons.

• **Extension.** Ideas resurveyed are extended to include equivalent sets; one-to-one correspondence; meaning of number and numeral; number of a set; finite and infinite sets; symbols for union and intersection of sets; exponent and expanded form to name place values; Egyptian numeration symbols; other place-value systems; and types and properties of triangles and quadrilaterals.

• **Problem-Solving.** In this chapter, the problems involve an addends-sum relationship which is to be resolved by first expressing the relationship in the form of a mathematical sentence, and then solving the sentence. Problems cover such topics as science, geology, sets, school and social situations, and 2-step problems. On pages 33 and 35 reference is made to the problem sets at the back of the book.

• **Maintenance.** Some of the practice pages include such topics as meaning of and operations on sets and using set terminology properly; rounding numbers; Roman numerals; other numeration systems and numerals in other bases; place value; exponent and expanded form; the addends-sum relationship; mathematical sentences; and the properties of addition. Throughout the chapter, reference is made to Reteaching and Extra Examples in the back of the book.

• **Enrichment.** Suggestions for enrichment are found in each lesson plan in this teacher's manual, and also under the Supplementary Activities at the end of each chapter. Topics include the fact that the number of the union set of two sets is related to the sum of the numbers of each of the sets minus the number of their intersection set; the use of a bar in writing Roman numerals to indicate multiplication by 1,000; and the use of the symbols $\geq$ and $\leq$ to indicate both inequality and equality in mathematical sentences.

• **Tests.** The four end-of-chapter tests on pages 49–51 are designed to evaluate the extent to which the skills and understandings presented in Chapter 1 have been mastered. Topics covered include place value; numerals in other bases; expanded and exponent form for numbers; rounding numbers; Roman numerals; the addends-sum relationship and the number-line picture; properties of addition, and computation in and differentiating between addition and subtraction; geometric ideas; and problems involving social situations. The summary exercises *How Well Do You Remember* on pages 12 and 26, and the practice exercises *To Keep In Practice* on pages 35, 40, and 43 may also be used as tests if desired.

Teaching Page 1

Pupil's Objective

To become aware of the expanding uses of mathematics in scientific inquiry.

Background

In this scientific age, the expression "mathematics in their future" will certainly be of a scientific nature for many pupils. For some, it may even be a form of mathematics with which we are not familiar. The exercises on this page give the teacher an opportunity to point out that many of the topics that will be presented to the pupils are an important part of the curriculum not because of their immediate value in solving real-life situations outside the classroom, but in developing a base for mathematical thinking to meet the pupils' future needs.

Pre-Book Lesson

Ask, "Why do we need to study mathematics?" Encourage the pupils to speak freely and honestly and do not attempt to discredit any pupil's opinion honestly advanced in this matter. In discussing the answers given, you may add or suggest that we study mathematics now to (a) handle real-life situations outside the classroom; (b) to prepare for mathematics courses in higher grades, which in turn are necessary for courses in other fields such as science and business; and (c) to develop orderly mathematical thinking necessary for solving problems in one's chosen profession or employment. Supplemcnt these suggestions with actual examples of the rolc mathematics may play in each area.

Using the Text Page

• Ex. 1 and 2. In addition to the bathyscaph you may wish to introduce the term bathysphere. The bathyscaph is the submarine-shaped underwater vessel picturcd on the opposite page. The bathysphere is a sphere-shaped underwater vessel much smaller than the bathyscaph and able to dcscend to depths only one tenth as great as that of the bathyscaph. These two exercises can be used to stimulate the pupils' appetite for scientific knowledge, including knowledge of a mathematical nature. For example, at a depth of 35,800 feet, the water pressure is about 15,000 pounds per square inch. A window with a surface area of 100 square inches in the bathyscaph is subjected to a force of $1\frac{1}{2}$ million pounds at that depth.

Pupils may use reference books to investigate such scientific topics as undersea trenches, islands and mountain tops, drilling for oil under the sea, mysteries of the ocean floor, minerals in sea water, salt from sea water, and fresh water from sea water.

• Ex. 3 and 4. It is interesting to note that the top of Mauna Kea is about 31,000 feet above its base at the bottom of the ocean. This is about 2,000 feet greater than the elevation of Mt. Everest above sea level.

• Ex. 5. Ask what use scientists would make of mathematics in conducting these investigations.

• Ex. 6 and 7. Take advantage of the increasing activity and interest in connection with space exploration to have children suggest science topics in this area and mathematical problems related to each topic.

Individualizing Instruction

• Ask for volunteers among the *more capable pupils* to look up one or more of the following topics in *National Geographic* or a similar publication, and report to the class on interesting scientific facts, particularly those of a mathematical nature: the bathyscaph *Trieste* and its use in searching for the submarine *Thresher*; the Marianas Trench; Mt. Everest; ocean floors (Atlantic and Pacific); and our atmosphere.

• Have *all pupils* bring to class interesting scientific facts of a mathematical nature found in newspapers and magazines.

At the beginning of the year, you may wish to suggest or require that *all pupils* maintain a "mathematics notebook." Such a notebook would contain completed sample exercises, diagrams, charts, and tables, brief explanations of difficult concepts, rules, and generalizations, recorded at the discretion of each pupil. In addition, you may require that specific material be recorded from time to time.

Keeping Aware of Uses of Mathematics

[o]

Devices have been developed not only for the exploration of space but also for the exploration of the ocean floor. Mathematics is used in designing and constructing such devices, in giving information regarding their performance, and in the recording and the analysis of the scientific findings.

1. A bathyscaph like the one shown in the picture on the opposite page can descend to the deepest parts of the ocean floor. In January 1960, the bathyscaph *Trieste* descended to a depth of 35,800 ft. to the bottom of the Marianas Trench. Is 35,800 ft. greater than 6 miles? Yes How much greater? (Hint: 5,280 ft. = 1 mi.) 4,120 ft.

2. In a horizontal direction, a bathyscaph can travel only about 6,080 feet per hour. Then in $\frac{1}{2}$ hour, a bathyscaph can travel about how many feet? 3,040 ft.

3. The height of Mt. Everest, the highest mountain on Earth, is 29,028 ft. The depth of the Marianas Trench (Ex. 1) is how much greater than the height of Mt. Everest? 6,722 ft.

4. The Hawaiian volcano Mauna Kea is a great lava dome whose height above the ocean floor is about 31,000 ft. The top of Mauna Kea is about 13,800 ft. above the surface of the ocean. What is the height of the part of Mauna Kea that is below the surface of the ocean? 17,200 ft.

5. Using bathyscaphs, scientists can study the changes in temperature and pressure at different depths. Name some other things that scientists would be interested in knowing about the ocean depths. Sample answers: What kinds of plants live at various depths? What kinds of fish live at various depths?

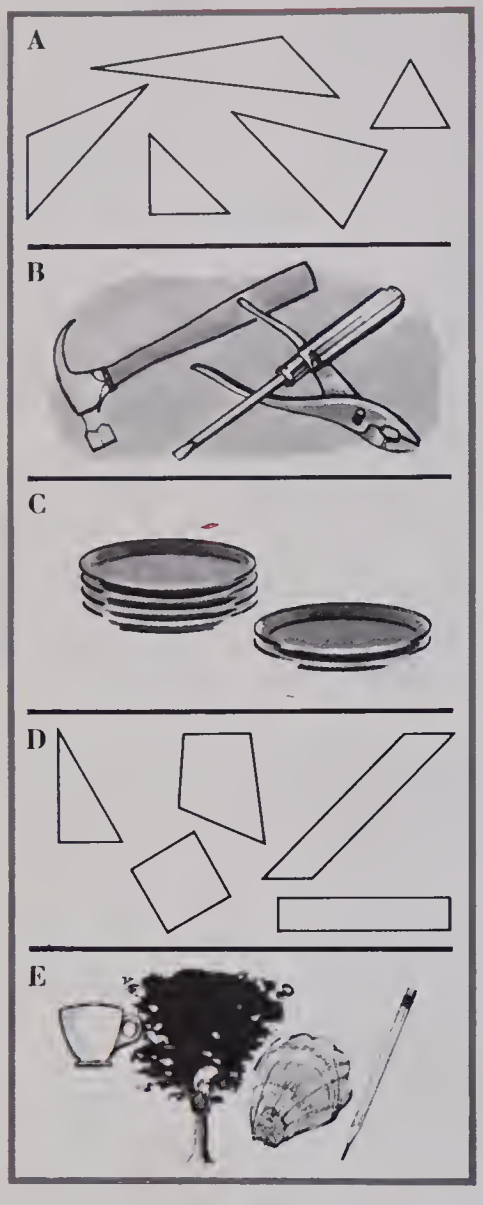
6. Name some of the devices that scientists use to study the atmosphere. Sample answers: balloons; rockets

7. Name some science topics you would be interested in studying and give some related mathematical problems. Answers will vary.

* A collection is a set only if it is well-defined. A collection is well-defined if it is always possible to determine if any given element is or is not a member of the collection. These two requirements can be met by describing the set precisely in words or by listing its members.

Sets

Resurvey [O]



* 1. A *set* is a collection of things. The things are called the *members* of the set. The members of the set pictured in box A are geometric figures called triangles. For each of the sets pictured in boxes B–D, tell what the members have in common. B, tools; C, dishes; D, polygons

2. Sometimes the members of a set do not seem to have anything in common other than the fact that they belong to the same set. Such a set is shown in box E. Name the members of the set. cup, shell, tree, pencil

3. How many members of the set of polygons shown in box D are also members of the set of all triangles? 1 quadrilaterals? 4 pentagons? 0

4. How many members of the set pictured in box C belong to the set of all blue dishes? 4 Is this another way of asking how many blue dishes are shown in box C? Yes

5. How many members are there in the set of letters of the English alphabet? 26 How many members are there in the set of states of the United States? 50

6. Are you a member of the set of all children? Yes Are you also a member of the set of all people? Yes Can something be a member of several sets at the same time? Yes

[W]

7. Write five names that are members of the set of all girls' names that begin with the letter *M*. Answers will vary.

8. List three objects in your classroom that are members of the set of all metal objects. Answers will vary.

9. List three sets of which you are a member. Answers will vary.

Teaching Page 2

Pupil's Objectives

To review (a) the concept of set; (b) the idea that members of a set need not have anything in common; and (c) that a member of a set may also be a member of several other sets at the same time.

Background

Any well-defined collection of things may be called a set. The things that make up a set, called its members, may be concrete objects such as the checkers in a set of checkers, or they may be abstract ideas such as a set of points in a line, or a set of hours in a day.

There is an important difference between a member of a set and the set itself. A set may have characteristics and properties that are different from the characteristics and properties of the members of the set. For example, the set of *all even counting numbers less than 3* contains only one member, the number *2*. The *set* has the property of containing an odd number of members, yet no *member* of the set has the property of being an odd number.

Usually, members of a set have something in common, but such need not be the case. A collection of such dissimilar things as *pupil*, *triangle*, and *7* may constitute a set. To constitute a set requires only that the *members* of a collection be clearly defined despite their dissimilarity.

Pre-Book Lesson

• For pupils who have not had previous contact with set terminology, discuss the ideas of: member and number of members of a set; two or more simple sets with one or more members in common; listing members of a set given the description; and describing a set from its listing.*

• You may wish to review the meaning of *polygon*, *triangle*, *quadrilateral*, and *pentagon* or you may prefer to wait until these come up in connection with Ex. 3 on the text page.

* See item 4 on page xix. Use of this aid may be beneficial to *all pupils* in establishing basic concepts.

Using the Text Page

• Ex. 1. Be sure pupils know the meaning of the words *in common* as used in this exercise.

• Ex. 2. Have pupils give other examples of a set whose members seem to have nothing *in common* except that they are members of the same set. Ask them if the members of the set pictured in box A have anything in common with the members of the set pictured in box B, or with the members of the set pictured in any one of the other boxes. Lead them to express these common characteristics precisely.

• Ex. 3. For every member shown in box D, have pupils name other sets of which the polygon is a member.

• Ex. 4–5. Give pupils a number such as two, or five, and have them describe a set whose number of members is the same as the given number.

• Ex. 6–8. One thing may be a member of several sets at the same time. Name an object in the room and see how many different sets can be described that contain that object as a member.

Individualizing Instruction

• *More capable pupils* can be given a set whose members at first appear to be very dissimilar, and be asked to find some common property of the members. For example, the members of the set consisting of a baseball bat, a breadboard, a tree, and a rowboat have the common property of all being made of wood. The numbers 2, 9, 16, and 23 all have remainder 2 when divided by 7.

They can be asked to describe in words as many different sets of numbers as they can think of.

• *All pupils* can be asked for examples of sets consisting of very few members, very many members, or no members at all.

• *Slower learners* can be asked to give examples of sets composed of things found in the classroom. Ask them to describe as many sets as possible of which they themselves are members. For example: the set of all pupils in the school; the set of pupils who are boys; the set of pupils who have black hair; the set of pupils 11 years old.

Teaching Page 3

Pupil's Objectives

(a) To review describing a set; and (b) to practice describing sets.

Background

A set may be described by listing each member of the set and enclosing this listing within a pair of braces. The set of all counting numbers less than 4 may be described by writing {1, 2, 3}. A second method involves describing in words the requirements that must be met for an object to be a member of the set. The description may or may not be enclosed within braces. Examples of this are "the set of all girls in the room with black hair" and {all states of the United States west of the Mississippi River}.

When a set is described in words, the description must be explicit enough to leave no doubt as to whether or not any given object is a member of the set. For example, the description "the set of all beautiful girls in the class" is not explicit enough as ideas of beauty vary with the beholder. When a collection may be classified as being well-defined, it is a set. If a collection is not well-defined, it is not a set.

In the listing of the members of a set, the same member is not listed more than once. The set of all letters in the word *see* is {*e*, *s*} or {*s*, *e*}. The order is immaterial.

There may be several ways to describe any given set. For example, the set whose members are *a*, *b*, and *c* can be described by any one of the following:

{*a*, *b*, *c*}
{all the letters in the word *cab*}
{all the letters that appear in both *back* and *absence*}
{first three letters of the English alphabet}

It is possible for a set to have no members. There are no members in the set of all girls who play on the New York Mets baseball team or in the set of all living four-headed dinosaurs. This set is called the *empty set* or the *null set*. The symbol for this set is { } or ∅, but { } will be used throughout this text.

Since there is only one empty set, it is always called *the* empty set or *the* null set.

Pre-Book Lesson

- Have pupils suggest different ways to describe a set.
- Have pupils consider a set such as "the set of all people who are in this school." Think about the set at different times of day. At midnight, there is probably no one in the school. Therefore, there are times when the set will have no members in it.

Using the Text Page

- Ex. 1–3. Discuss the advantages of listing letters in alphabetical order and numbers in proper sequence when they are members of a set. List members of a set and have pupils describe it.
- Ex. 4–6. Have pupils think of their own examples of the empty set. Discuss the fact that since every empty set is identical, we use the term *the* empty set, not *an* empty set.
- Ex. 7–10. After the pupils have described a set by listing the members within braces, have them use words to describe the same set.

Individualizing Instruction

- *More capable pupils* can give descriptions of collections that leave some doubt as to whether or not certain things are or are not members. Then, have them modify their descriptions so that the collection becomes well defined.

Have them give examples of sets in which it is impossible to list all the members.

- *All pupils* can be given sets whose members are listed within braces and be asked to write word descriptions for these sets.
- *Slower learners* can describe sets with which they are familiar such as days of the week, months of the year, and letters of a given word. Given a word, make sure they understand that each different letter in the word appears only once in the listing of the set.

* Point out that members should not be repeated. Also, emphasize that the order of listing the members is irrelevant.

Describing a Set

Resurvey [O]

1. Sometimes a set is described by listing its members within braces. For example, $\{a, b, c\}$ (read, *the set a, b, c*) describes the set whose members are *a*, *b*, and *c*. Does $\{b, a, c\}$ also describe this set? Yes ∧ On the board, show four other ways to describe this set using braces. {c, a, b}; {b, c, a}; {a, c, b}; {c, b, a}

2. The set of letters of the English alphabet used in writing the word *class* can be described by $\{c, l, a, s\}$. Why is the letter *s* listed only once? ∨ Does $\{s, a, l, c\}$ also describe this set? Yes

We are describing a set of different letters.

* **3.** On the board, list the set of letters used in writing the word *mathematics*. Remember to use braces. {m, a, t, h, e, i, c, s}

4. If a set has no members, it is called the *empty set*. We write { } to denote, or name, the empty set. Does { } denote the set of children in your classroom who are only six months old? Yes

5. Does { } denote the set of all horses that can fly? Yes

6. On the board, use braces to describe the set of letters whose members are in both $\{a, d\}$ and $\{c, g\}$. { }

[W]

Ex. 7–13. Describe the set by listing its members within braces.

7. The set of letters in the word *element* {e, l, m, n, t}

8. The set of letters that appear both in the word *set* and in the word *member* {e}

9. The set of states of the United States whose names begin with the letter *A* {Alabama, Arkansas, Alaska, Arizona}

10. The set of states of the United States whose names begin with the letter *X* { }

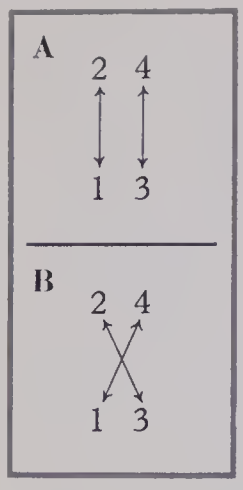
11. The set of subjects you are taking in school this year Answers will vary.

** **12.** The set of whole numbers from 100 through 120
{100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120}

13. The set of whole numbers between 18 and 19 { }

** You may want to review "whole numbers" before assigning Ex. 12-13. The whole numbers are 0, 1, 2, 3, and so on. See page 322.

* Study of a one-to-one correspondence between members of a set also serves as an introduction to counting and finding the number of a set.

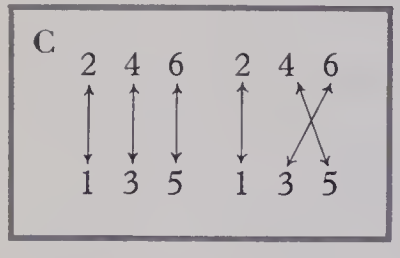


Equivalent Sets

Extension; one-to-one correspondence [O]

* 1. Box A. The members of {2, 4} are shown matched, or paired, with the members of {1, 3}. Is each member of {2, 4} matched with one and only one member of {1, 3}? Yes ^ Is each member of {1, 3} matched with one and only one member of {2, 4}? Yes ^ Such a matching shows that a *one-to-one correspondence* exists between the two sets.

2. Box B. Does this matching also show that a one-to-one correspondence exists between {2, 4} and {1, 3}? Yes



3. Box C. The members of {2, 4, 6} are shown matched with the members of {1, 3, 5} in two different ways. Does each matching show that a one-to-one correspondence exists between the two sets? Yes ^ On the board, show four other ways to match the members to show a one-to-one correspondence between the two sets.

** 4. When a one-to-one correspondence exists between two sets, the sets are said to be *equivalent*. Is {2, 4, 6} equivalent to {1, 3, 5}? Yes ^ Is {2, 4} equivalent to {1, 3}? Yes

5. Can you show a one-to-one correspondence between {2, 3} and {4, 6, 7, 8}? No ^ Are these two sets equivalent? No

[W]

Ex. 6–9. Write whether the two sets are *equivalent* or *not equivalent*.

6. The set of whole numbers less than 3
   The set of letters in the word *set* Equivalent

7. {5, 7, 9}, {*a*, *b*, *c*, *d*} Not equivalent

8. The set of students in your class
   The set of desks in your classroom Answers will vary.

9. The set of pairs of shoes in your classroom
   The set of people in your classroom Probable answer: equivalent

** Notice that an equivalence between sets does not constitute an equality between the sets. The sets are equal only when they contain the same members, not just the same number of members.

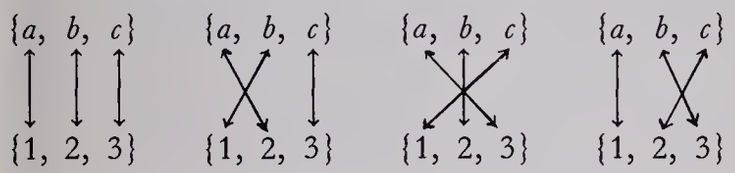
Teaching Page 4

Pupil's Objectives

(a) To learn the meaning of one-to-one correspondence; and (b) to learn the meaning of and be able to identify equivalent sets.

Background

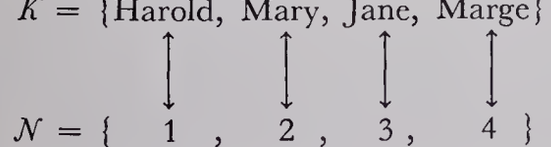
If the members of two sets can be matched in such a way that for each member in the first set there corresponds one member in the second set and for each member in the second set there corresponds one member in the first set, then a one-to-one correspondence exists between the two sets. This matching may be shown in several different ways using doubleheaded arrows. When a one-to-one correspondence exists between two sets, the sets are said to be *equivalent.*



A set which has a whole number of members is called a *finite* set. The number of members of a finite set is called the cardinal number of the set. Since equivalent, finite sets have the same number of members, they will have the same cardinal number.

If two equivalent sets contain exactly the same members, they are called *equal* sets. {Phil, George, Carl} and {Carl, George, Phil} are equal sets. Equal sets are also equivalent sets, but equivalent sets are not necessarily equal sets.

We often use the idea of one-to-one correspondence without realizing it. For example, to determine the cardinal number of the set K below, we place it in a one-to-one correspondence with a set of counting numbers beginning with 1.



The last counting number in the correspondence names the cardinal number of the set.

Pre-Book Lesson

Discuss the concept of determining the cardinal number of a set by matching the members of the given set with a set of counting numbers. Have pupils use doubleheaded arrows to show matching of the members of one set with those of an equivalent set in all possible ways. Start with sets having two members, then sets with three members, and so on. Illustrating one-to-one correspondence using flannel items on a flannelboard will be helpful.*

Using the Text Page

- Ex. 1–2. Lead pupils to discover that there is no one particular way of showing correspondence. If two sets match in one way, they will match in all possible ways.
- Ex. 3. Before asking a pupil to find all possible ways in which two sets can be shown to correspond, bear in mind that two sets of five members each will correspond in 120 different ways, two sets of four members each will correspond in 24 different ways, and two sets of three members each will correspond in six different ways.
- Ex. 4–9. Pupils may confuse equivalent sets and equal sets. Emphasize that, to be equivalent, sets must contain the *same number* of members, regardless of the nature of the membership.

Individualizing Instruction

- *More capable pupils* can be asked for examples of one-to-one correspondence such as the correspondence between the set of dinner plates on a table and the set of people eating at the table; the set of pupils in the classroom and the set of pupil's desks; the set of odd numbers less than 10 and the set of even numbers less than 10.
- *Slower learners* can be given physical things and asked to arrange them into two sets so that there is a one-to-one correspondence between the two sets. They may diagram the correspondence.

*Colorful classroom illustrations of set concepts can help in stimulating class interest. See items 2, 4, and 9 on page xix.

Pupil's Objectives

(a) To resurvey the ideas of one-to-one correspondence and equivalent sets; and (b) to use these ideas in understanding *number* as being the property of a set.

Background

One concept of *number* is its being the property of a set which names how many elements are in the set. The number property of the set $\{a, b, c\}$ is *three* and all sets equivalent to this set have the number property *three*.

This concept of a number applies to whole numbers. There are many numbers other than whole numbers which do not name the property of a set. Among the classifications of numbers that do not name the property of a set are negative numbers, rational numbers that are not whole numbers, irrational numbers, and complex numbers.

Because the question "What is a number?" is very profound, discussion at this grade level is necessarily restricted to whole numbers. It is also important that pupils understand that number is an idea. The idea of a specific number and of numbers in general is understood by people all over the world who may speak and write about numbers in many different ways. You may ask if pupils know of any other kind of number, but avoid a detailed discussion at this time.

The whole numbers have uses other than naming the number property of a set. They also indicate order or placement within a set. In "Grade 6," the "6" is not being used to name the number property of a set, it names the grade after Grade 5 and before Grade 7. Other examples of the ordinal use of numbers are "Page 321" and "72 Maple St."

Pre-Book Lesson

• Pick out two sets in the room that have the same number property. Have the pupils list as many properties as they can for each set and select any that are common to both sets. This should lead to the idea of the common number property of the two sets.

• Let pupils pick out other pairs of sets in the room that share a common number property.

• You may wish to indicate a set and have the pupils identify the number property of that set. You may wish to have pupils compare the number property of two sets that are not equivalent.*

Using the Text Page

• Ex. 1–6. Help pupils understand that (a) every set has a number property; (b) equivalent sets have the same number property; and (c) a number is an idea and is not something we can "see." It exists only in our minds.

• Some pupils may feel that a *3* or *seven* or *XII* written on the board is a number. Help these pupils to see that they are only *names* for numbers. Point out the difference between a number and the symbols we use to name the number (numerals). We can see the symbols but we cannot see the number or the idea the symbol represents.

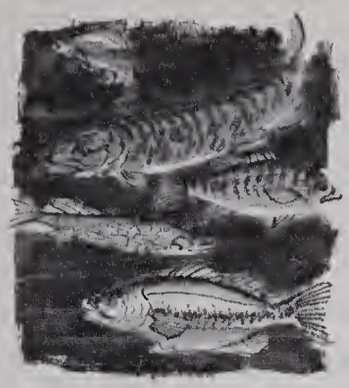
Individualizing Instruction

• Ask the *more capable pupils* for examples of whole numbers that are not used to name the property of a set but rather to name a position in a sequence (Grade 6, Channel 2, Page 36).

• Have *all pupils* describe sets in the room and then name their number property. Restrict this activity to sets for which a one-to-one correspondence may be easily established with the set of whole numbers. Ask, "What is the number property of the set of all live elephants in the room?" (zero). "What is the number property for the empty set?"

• For the *slower learners*, name a number and ask them to describe a set whose number property is the number named. Restrict this activity to sets readily visible in the room, and sets about whose number property there could be no confusion or dispute: the set of doors; the set of windows; the set of panes in one window; the set of pencil sharpeners; and so on.

* See item 7 on page xix. These cards can be used to relate set concepts to numbers.

A 

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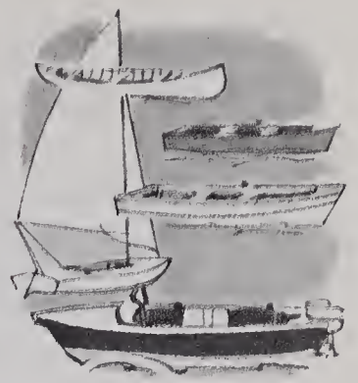
C 

What Is a Number?

Meaning [O]

1. Does a one-to-one correspondence exist between the set of fish shown in box A and the set of trees shown in box B? Yes Then are these two sets equivalent? Yes

2. One property that the set of fish and the set of trees have in common is that their members are living things. Does the set shown in box C have this property? No

3. Is the set of boats equivalent to the set of fish? Yes ∧ to the set of trees? ∨ Why? Yes A one-to-one correspondence exists between them.

4. Is *five* a common property of the three sets? Yes ∧ Name some other sets which have this property. Answers will vary.

5. Use sets to explain the meaning of *ten*. Sample answer: The number property of sets equivalent to {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

6. Is a number something we can see? No ∨ hear? No ∨ smell? No ∨ touch? No ∨ taste? No ∧ Do you understand that a number is an *idea*? Yes

[W]

Ex. 7–10. Explain the meaning of the number by naming two sets which have the number as a common property. Answers will vary.

7. Sample answer: The set of sides of a triangle; the set of whole numbers less than 3.

7. Three 8. Twelve 9. Fifty 10. One hundred

11. Is *four* a common property of the set of birds shown at the right and the set of states in the United States whose names begin with the letter *A*? Yes

* When describing a set using three dots, list enough members (usually in natural order) so that a pattern can be recognized. Later pupils will learn that the three dots may be used between members of a set to indicate missing members as in {0, 1, 2, · · ·, 6}.

Sets of Numbers

Resurvey and extension [O]

* 1. The set of numbers 1, 2, 3, and so on is called the set of counting numbers. We write $\{1, 2, 3, \cdots\}$ to describe this set. The three dots $(\cdots)$ mean "and so on." The pattern of the listed members indicates the other members. Does the set of counting numbers contain 0? No 5? Yes 3? Yes $\frac{1}{2}$? No

2. The set of numbers 0, 1, 2, 3, and so on is called the set of whole numbers. On the board, describe the set of whole numbers using braces and three dots. Is zero a whole number? Yes Is each counting number also a whole number? {0, 1, 2, 3, · · ·} Yes

3. A capital letter may be used to name a set. Tell which set named in the box at the left may be described as

$P = \{4, 6, 8\}$
$Q = \{0, 2, 4, 6, \cdots\}$
$R = \{1, 3, 5, \cdots\}$
$S = \{2, 4, 6, \cdots\}$
$T = \{5, 6, 7, 8, 9\}$

a. the set of even counting numbers. S
b. the set of even whole numbers. Q
c. the set of odd counting numbers. R
d. the set of odd whole numbers. R
e. the set of even whole numbers from 4 through 8. P
f. the set of whole numbers between 4 and 10. T
g. the set of multiples of 2. Q

[W]

Ex. 4–10. Describe the set by using braces.

4. The set of even whole numbers between 5 and 11 {6, 8, 10}
5. The set of odd whole numbers greater than 10 {11, 13, 15, · · ·}
6. The set of whole numbers that are not counting numbers {0}
7. The set of whole numbers less than 7 {0, 1, 2, 3, 4, 5, 6}
8. The set of whole numbers greater than 9 and less than 19 {10, 11, 12, 13, 14, 15, 16, 17, 18}
9. The set of even whole numbers greater than 19 {20, 22, 24, · · ·}
10. The set of multiples of 9 {0, 9, 18, · · ·}

▲ **Reteaching.** Be sure you understand the meaning of whole numbers. Study Set 1 on page 322.

** By "multiple," we mean "whole number multiple." Notice that 0 is a member of this set.

Teaching Page 6

Pupil's Objectives

(a) To review counting numbers and whole numbers; and (b) to use a capital letter to name a set.

Background

There are many sets that cannot be described by listing every member of the set because the listing continues indefinitely. The set of counting numbers and the set of whole numbers are examples of such sets. Each of these sets can be described by listing the first few members of the set and then writing three raised dots, "· · ·," to indicate that the pattern of the numbers continues indefinitely. The set of counting numbers may be described as {1, 2, 3, · · ·} and the set of whole numbers as {0, 1, 2, 3, · · ·}. The three raised dots may be read "and so on." This notation is also used in listing sets which do not continue indefinitely. For example, the set of even numbers 2, 4, 6, and so on through 24 may be listed as {2, 4, 6, · · · , 24}.

The only difference between the set of counting numbers, or natural numbers, and the set of whole numbers is that zero is a member of the set of whole numbers only. Thus, every counting number is a whole number, but not every whole number is a counting number.

It is important to realize that zero is a number and can be a member of a set. The symbol for a set that has no members is the empty set, { }, *not* {0}. The solution set for the mathematical sentence $\square + 1 = \square$ would be the empty set.

When working with sets, it may be impractical or too time consuming at times to describe each set completely either in words or by listing all its members. Instead, a set may be named by use of a capital letter. In referring to such a set, you may write the letter that represents that set. For example, if $P = \{3, 7, 9, 12\}$ and $Q = \{\text{all counting numbers less than } 25\}$, these sets can be referred to simply as P and Q.

Unless indicated otherwise, the set of multiples of a number means the set of *whole-number multiples* of that number. The set of multiples of 2 is {0, 2, 4, 6, 8, · · ·}. Notice that 0 is included in the set. The set of *counting-number multiples* of 2 is {2, 4, 6, · · ·}.

Pre-Book Lesson

Review the meaning of the terms used in the exercises such as "whole numbers," "counting numbers," "even numbers," "odd numbers," "multiples of 2," "greater than 16," "less than 25," "from 9 through 21," "between 4 and 10."

Using the Text Page

• Ex. 1–2. Discuss the problem of listing the members of a set when the number of members is very great, and the solution of this problem by the use of the three raised dots.

Discuss some meanings of zero. It is a whole number; it names the number of members of the empty set; it is a member of every set of whole-number multiples, and so on.

• Ex. 3. Point out that zero is an even number since the product of 2 and any whole number is an even number. In connection with "betweenness," make certain that pupils do not list the *given* numerals.

Individualizing Instruction

• *More capable pupils* can be shown how the three raised dots can be used in listing members of a set that has a great many members, but *not* infinitely many. For example, the set of all whole numbers less than 25 may be written {0, 1, 2, 3, · · · , 24}. Have them describe empty sets such as "the set of all whole numbers that are greater than 16 and also less than 9."

Ask pupils to list some different sets of numbers other than the set of counting numbers and the set of whole numbers.

• *All pupils* can practice recognizing sets of even numbers, odd numbers, and multiples. Devise sets of numbers such as {1, 5, 9, 13, · · ·}, {1, 2, 4, 7, 11, 16, · · ·}, and {1, 2, 4, 8, 16, 32, · · ·} in which the sequence of numbers follows a definite pattern. Ask pupils to try to discover the pattern and indicate the next few numbers in the sequence. The first Reteaching exercise may be worked here.

Pupil's Objectives

(a) To learn the meaning of finite and infinite sets; and (b) to learn to name the number of members in a set.

Background

The members of a set may have individual properties or properties in common with other members of the set. For example, each member of the set {△, □, ▭} has the property of being a polygon, but only △ has the property of having three sides. The set itself also has properties such as the number of members. This property is called the *number property* of the set. Every set, including the empty set, has a number property and this property is called the cardinal number of the set. If the cardinal number of a set is a whole number, the set is said to be a finite set. The cardinal number of {△} is the whole number 1, so it is a finite set. The cardinal number of the empty set is the whole number zero, so the empty set is also a finite set.

If the listing of the members of a set continues on and on without end so that there is no whole number that represents the cardinal number of the set, the set is called an infinite set. Examples of infinite sets are: "the set of all counting numbers," "the set of all points in a given line," and "the set of all whole numbers greater than 12,345."

Pre-Book Lesson

• Have the pupils volunteer to describe and discuss different kinds of sets and classify them according to three categories: (a) those that are the empty set; (b) those whose listing of members has an end; and (c) those whose listing of members continues on and on without end.

• Discuss the idea of number of a set for sets with a finite number of members and for sets with an infinite number of members.

• Correct pronunciation of the word finite is fī′nīt (Barnhart).

Using the Text Page

• Ex. 1. Emphasize that the empty set is a finite set. The number of members in the set is the whole number zero.

• Ex. 3. There is often confusion between a finite set that has a very great number of members and an infinite set. The cardinal number of a finite set may be astronomically great, such as the set of grains of sand on Daytona Beach, or blades of grass in a field, or the set of all leaves on the trees in Central Park. However, there exists a whole number that is the number of members in this set.

• Ex. 4–6. Pupils may become confused by the fact that the empty set, { } and {0} are two different sets. The empty set has no members while {0} has one member, 0.

Individualizing Instruction

• *More capable pupils* could be led in a discussion concerning whether certain sets in our universe are finite or infinite. Use previous references to extremely large sets to start off the discussion.

• *All pupils* may be asked for examples of sets that are finite but whose number of members is very great.

Describe for *all pupils*, or have them describe, pairs of sets one of which is the empty set and the other a set whose only member is 0.

• *Slower learners* may be asked for the number of members for sets of things in the classroom, such as the set of books on a desk, or the set of pictures on the wall, or the set of panes of glass in a window, or the set of papers on a bulletin board, and so on.

The first set of Extra Examples in the back of the book is referenced here. You may wish to assign them at this time or wait until later review reveals specific weakness in the area.

* For many sets it would be physically impossible to find the whole number that is the number of members in the set; however, if it can be established that such a whole number exists, then the set is finite.

Finite and Infinite Sets

Extension [O]

1. Study the sets named in the box. How many members are there in each set? 0; 2; 4

$M = \{\ \}$
$S = \{A, B\}$
$T = \{3, 4, 5, 9\}$

* If the number of members in a set is a whole number, then the set is a *finite set.*

2. Tell why each set named in the box is a finite set. Each set has a whole number of members.

3. If we started to list the members of the set of all even counting numbers, would we be able to finish the listing? No Why not? The set of even counting numbers, $\{2, 4, 6, \cdots\}$, is an example of an *infinite set.* The set goes on and on. There is no whole number great enough to indicate the number of members in an infinite set. Tell which of the following sets are finite and which are infinite.

a. $\{5, 10, 15, \cdots\}$ Infinite

b. The set of whole numbers between 5 and 12 Finite

c. The set of words on this page Finite

** **d.** The empty set Finite

e. The set of whole numbers greater than 25 Infinite

f. The set of trees in Sequoia National Park Finite

[W]

Ex. 4–7. Name the number of members in the set.

4. $\{\ \}$ 0 **5.** $\{0\}$ 1 **6.** $\{0, 10, 100\}$ 3 **7.** $\{2, 3, 5, 7, 11, 13\}$ 6

8. The set of the days in a week has how many members? 7

9. The set of even whole numbers greater than 3 and less than 15 has how many members? 6

Ex. 10–13. Write whether the set is *finite* or *infinite.*

10. The set of counting numbers infinite

11. The set of whole numbers less than 10 million finite

12. The set of multiples of 5 infinite

13. The set of words in this book finite

◆ **Extra Examples.** Set 14. These sets start on page 335.

** The empty set is a finite set because the number of members in the set is 0 and 0 is a whole number.

* Intersection and union are operations on sets in much the same way addition and multiplication are operations on numbers. Since intersection or union can be applied to only two sets at a time, they are called binary set operations.

* Intersection and Union of Sets

Resurvey [O]

A
$R = \{2, 3, 4\}$
$S = \{3, 5\}$

1. Study the sets named in box A.

a. R is the set of whole numbers greater than _? 1 and less than _? 5.

b. The members of S are the numbers _? 3 and _? 5

c. Is 3 a member of both sets? Yes

d. Is 3 the only number that is in both sets? Yes

The intersection of two sets is the set of all things that are members of both sets.

2. The intersection of sets R and S is the set which contains the number _? 3 We write $\{3\}$ to describe the intersection of sets R and S.

3. Are there any numbers that are members of both $\{2, 5\}$ and $\{3, 5, 8\}$? Yes The intersection of these two sets is _? {5}

4. On the board, list within braces the members of the intersection of $\{2, 3, 4, 7, 9\}$ and $\{4, 5, 6, 7\}$. {4, 7}

5. Study the sets named in box B.

B
$V = \{4, 5, 6\}$
$W = \{5, 6, 7, 8\}$

a. Set V is the set of whole numbers between _? 3 and _? 7. Describe set W. The set of whole numbers between 4 and 9**

b. Does $\{4, 5, 6, 7, 8\}$ describe the set of numbers that are members of V or of W or of both V and W? Yes

The union of two sets is the set of all things that are members of either set or of both sets.

c. Is $\{4, 5, 6, 7, 8\}$ the union of sets V and W? Yes

d. The intersection of sets V and W is _? {5, 6}

[W]

Ex. 6–8. Describe the set by using braces.

6. The intersection of $\{4, 7, 9\}$ and $\{1, 9\}$ {9}

7. The intersection of $\{0, 4, 7, 9\}$ and $\{1, 2, 3, \cdots\}$ {4, 7, 9, }

8. The union of $\{2, 5\}$ and $\{5, 9, 10\}$ {2, 5, 9, 10}

♦ **Extra Examples.** Set 15.

** Other acceptable descriptions might be "between 4 and 9" or "from 5 through 8" or "greater than 4 and less than 9."

Teaching Page 8

Pupil's Objectives

To review (a) intersection of sets; and (b) union of sets.

Background

The intersection of two sets is the set of all things that are members of both sets. If $A = \{1, 3, 5, 7, 9\}$ and $B = \{3, 5, 10, 15\}$, the intersection of set A and set B is $\{3, 5\}$, since the only members contained in *both* sets are 3 and 5.

If two sets have no member in common, the intersection of the two sets will be the empty set. Such sets are said to be disjoint with respect to each other. If $C = \{\text{Mary, Jane, Joan}\}$ and $D = \{\text{Bill, Eric, Mark}\}$, the intersection of set C and set D is the empty set.

The union of two sets is the set of all things that are members of either set or of both sets. Any member that is in the first set, or the second set, or both sets, is in the union set of the two sets. For example, if $F = \{a, b, c, d\}$ and $G = \{c, d, e, f\}$, then the union of sets F and G is the set $\{a, b, c, d, e, f\}$. Notice that even though a member may be in both sets, it is listed only *once* in the union set.

Union and intersection are operations on sets just as addition and subtraction are operations on numbers. Two sets may be operated on by union or intersection to form a third set, just as two numbers may be operated on by addition to result in a unique third number.

Union and intersection of sets are important in that they can be used to clarify many concepts in mathematics. Both have important applications in geometry. All geometric figures may be thought of as unions or intersections of sets of points in space. For this reason it is important to master the concept of these operations before undertaking the study of geometry.

Pre-Book Lesson

• Discuss what is meant by operating on two numbers to result in a unique third number. This will serve as a foundation for the understanding of operations on sets.*

• Have pupils compare pairs of sets to discover if they have any or no members in common. Let them describe both types of sets.

Using the Text Page

• Ex. 1–3. Ask for examples of sets whose members are physical things, and ask that the intersection of two such sets be determined.

Have pupils give examples of sets whose intersection is the empty set.

• Ex. 4–8. Point out that the intersection of two sets is a third set, and that the union of two sets is also a third set.

Individualizing Instruction

• The similarities between union of disjoint sets and addition can be explained by the *more capable pupils*. They should explain that union is an operation on sets while addition is an operation on numbers. Have them give examples of each.

• *All pupils* may be given problems involving union of two sets and intersection of two sets whose members relate to other courses, such as geography or English. Examples of such exercises would be: Find the union of the set of all states east of the Mississippi and the set of all states west of the Mississippi; Find the intersection of all vowels of the English alphabet and all consonants of the English alphabet.

• *Slower learners* can be asked to find the union and the intersection of sets of things with which they are familiar in the classroom or at home, such as: the intersection of the set of all boys in the room and those 7 feet tall; the intersection of the set of all pupils in the room and the set of pupils who have brothers or sisters in the school.

* See item 9 page xix. This aid may help establish concepts of set operations and relate them to operations on numbers.

Teaching Page 9

Pupil's Objectives

To learn the symbol for (a) the intersection of two sets; and (b) the union of two sets.

Background

The symbol $\cap$ is used to indicate the intersection of two sets. For example, if $A = \{0, 2, 4\}$ and $B = \{1, 2, 3\}$ then $A \cap B = \{2\}$. The notation $A \cap B$ is read "the intersection of sets A and B."

If two sets have no members in common, the intersection of the two sets will be the empty set. For example, $\{a, b, c\} \cap \{d, e, f\} = \{\ \}$. Similarly, if $A = \{\triangle, \square\}$ and $B = \{\square\}$, then $A \cap B = \{\ \}$. The intersection of any set and the empty set is the empty set.

The symbol $\cup$ is used to indicate the union of two sets. For example, if $A = \{f, g\}$ and $B = \{g, h\}$, then $A \cup B = \{f, g, h\}$.

Notice that the members common to both sets are listed only once in the union set. The union of any given set and the empty set will be the given set.

If $A = \{3, 9\}$ and $B = \{\ \}$, then $A \cup B = \{3, 9\}$. A general statement of this idea is $A \cup \{\ \} = A$.

The union of equal sets will contain only the members in either of the two given sets. For example, {Mary, Sloane, Joanne} $\cup$ {Sloane, Joanne, Mary} = {Mary, Sloane, Joanne}.
The same will be true of the intersection of two equal sets.
{Mary, Sloane, Joanne} $\cap$ {Sloane, Joanne, Mary} = {Mary, Sloane, Joanne}

In only these two instances will the intersection of two sets be the same as the union of the two sets.

Pre-Book Lesson

• Review briefly set terminology for union and intersection of sets. Show some sets and their members on the board and have pupils describe their union and intersection.

• Ask pupils if they can think of shorter ways to describe intersection and union of sets. Ask about the possibility of using symbols for these operations on sets and have pupils describe some imaginary symbols.

Using the Text Page

• Ex. 1. Have the pupils practice finding the intersection and the union of two sets when the two sets are: equal, equivalent, disjoint, and also when one of the sets is the empty set.

• Ex. 2–4. Have pupils give examples of two sets whose members are physical objects and whose intersection is the empty set.

Individualizing Instruction

• *More capable pupils* could be asked to find the union or intersection of three sets. Let them work in two steps to do this. Let them find first the intersection or union of any two of the sets. Then find the intersection or union of the resulting set and the third set.

Let them describe or list pairs of sets whose union would be the same as their intersection.

Let them find also the union of the empty set and any other set, and describe the resulting set.

• As an activity for *all pupils*, one pupil may be asked to describe a set of objects from sets previously prepared by the teacher, a second pupil may be asked to describe another similarly prepared set of objects, a third pupil may be asked to determine their intersection, and a fourth pupil, to determine their union.*

• *Slower learners* may be permitted to manipulate the objects in the sets indicated above. By doing this, they could discover what objects the two sets contained in common (intersection set). Similarly by manipulation, they could regroup the objects in the two sets to form a third set (union set) to contain one of each object in the two original sets. The manipulative process would precede the listing or recording operation.

* A flannel board and various flannel objects would be useful here. See items 2, 9, and 28 on page xix.

* In later work, the concepts of intersection and union will be used in developing ideas in geometry and also to clarify the concepts of greatest common factor and least common multiple.

* Using Symbols for Intersection and Union

Extension [O]

The symbol $\cap$ is used to indicate the intersection of sets M and N by writing $\{6, 8\} \cap \{4, 8, 12\}$, or $M \cap N$. This notation is read, *the intersection of sets M and N.*

The symbol $\cup$ is used to indicate the union of sets M and N by writing $\{6, 8\} \cup \{4, 8, 12\}$, or $M \cup N$. This notation is read, *the union of sets M and N.*

$M = \{6, 8\}$
$N = \{4, 8, 12\}$
$R = \{5, 10, 15\}$
$S = \{10, 15, 20\}$

1. Study the sets named in the box. Is it true that $M \cap N = \{8\}$? Yes Name the members of $M \cup N$. 4, 6, 8, 12 On the board, list within braces the members of each of the following:

a. $M \cap R$ { } **b.** $M \cup R$ {5, 6, 8, 10, 15} **c.** $R \cap S$ {10, 15}

d. $R \cup S$ {5, 10, 15, 20} **e.** $N \cap S$ { } **f.** $N \cup S$ {4, 8, 10, 12, 15, 20}

** **2.** If the intersection of two sets is the empty set, then do the two sets have any members in common? No

3. Which pairs of sets named in the box do not have any members in common? M and R; M and S; N and R; N and S

4. Tell whether or not each of the following pairs of sets have any members in common.

a. The set of letters in the word *glad*
The set of letters in the word *late* Yes

b. The set of odd whole numbers
The set of even whole numbers No

c. The set of counting numbers
The set of whole numbers less than 1 No

[W]

Ex. 5–10. Copy and complete, using the sets named at the right.

$D = \{10, 12, 14, 16, 18\}$
$E = \{5, 10, 15, 20\}$
$F = \{4, 8, 12, 16, 20\}$
$G = \{3, 4, 5, 6, 7, 8\}$

5. $D \cup E =$ _?_ {5, 10, 12, 14, 15, 16, 18, 20}

6. $E \cap G =$ _?_ {5}

7. $D \cup G =$ _?_ {3, 4, 5, 6, 7, 8, 10, 12, 14, 16, 18}

8. $D \cap F =$ _?_ {12, 16}

9. $D \cup F =$ _?_ {4, 8, 10, 12, 14, 16, 18, 20}

10. $D \cap G =$ _?_ { }

◆ **Extra Examples.** Set 16.

** If the intersection of two sets is the empty set, the sets are said to be disjoint. Ex. 2-4 are designed to give pupils intuitive experience with the concept of disjoint sets. In earlier grades, disjoint sets were used to illustrate addition.

* Sometimes numbers which are exactly halfway are rounded so that the digit in the place to which the number is being rounded names an even number. For example, 35 would still be rounded to 40, but 65 would be rounded to 60.

Rounding Numbers

Resurvey and extension [O]

A
{0, 10, 20, 30, 40, 50, · · ·}

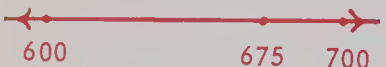
1. The set described in box A is the set of multiples of _?10_. The *greatest* multiple of 10 that is less than 32 is _?30_. The *least* multiple of 10 that is greater than 32 is _?40_.

2. The three numbers 30, 32, and 40 are shown in the number-line picture at the left. Is 32 nearer 30 than 40?Yes We say that 32 *rounded to the nearest ten* is 30.

3. On the board, draw a number-line picture like the one for Ex. 2 and then locate 37. Is 37 nearer 30 than 40?No Is it nearer 40 than 30?Yes Then 37 rounded to the nearest ten is _?40_.

* 4. Now locate 35 on your number-line picture. Do you see that 35 is located halfway between the location of 30 and the location of 40?Yes In such a case, we round to the greater multiple of ten. Explain why 35 rounded to the nearest ten is 40. 35 is located halfway between 30 and 40 so we round to 40, the greater multiple of ten.

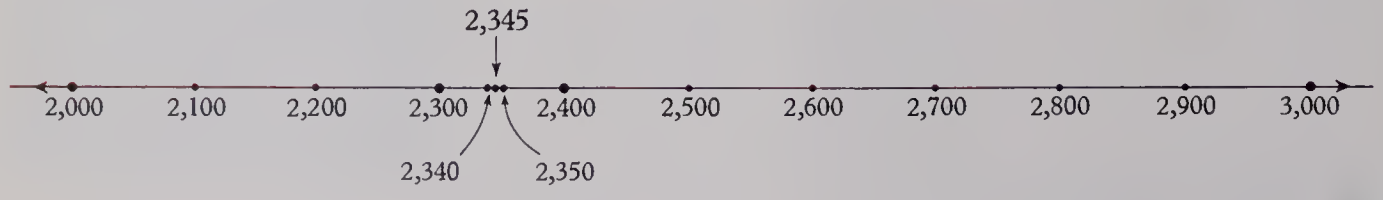
5. On the board, describe a set of multiples of 100 as in box A. {0, 100, 200, 300, 400, 500, · · ·} The greatest multiple of 100 that is less than 675 is _?500_ and the least multiple of 100 that is greater than 675 is _?600_.



6. On the board, draw a number-line picture and locate 600, 675, and 700. 675 rounded to the nearest hundred is _?700_.

** 7. Study the number-line picture below. 2,345 rounded to the nearest ten is _?2,350_; to the nearest hundred is _?2,300_; to the nearest thousand is _?2,000_.

8. Using the number-line picture, round Ex. a–c to the nearest thousand. **a.** 2,650 3,000 **b.** 2,490 2,000 **c.** 2,500 3,000



** This is an excellent example of why we do not round again a number that has been rounded. If 2,345 is rounded to 2,350 and then 2,350 is rounded to 2,400, the result is the same as 2,345 to 2,300.

Teaching Pages 10 and 11

Pupil's Objectives

(a) To review the rounding of numbers; and (b) to review some practical reasons for rounding numbers.

Background

It is often more convenient to present number facts as approximations rather than as exact figures. This is particularly true when the exact number is difficult or impossible to determine, constantly changing, or when other numbers used in comparison or computation are either rounded or vary greatly in their precision. For example, the census may indicate the population of a city is 589,242. Obviously, this number was correct only for a given instant of time and it would be just as meaningful to refer to the population of this city as 589,000.

In teaching the concept of rounding numbers, the number-line picture is very useful. When rounding to tens, a picture showing multiples of ten may help pupils visualize the position of the number to be rounded with respect to the least multiple greater than and greatest multiple less than. A similar approach using number-line pictures for multiples of 100 and 1,000 may be used where rounding to hundreds or to thousands.

Care must be taken in rounding a number that has been previously rounded. For example, a pupil rounded 153,499 to the nearest hundred and obtained 153,500. He then realized that he should have rounded to the nearest thousand, so he rounded 153,500 to the nearest thousand, or 154,000. However, 153,499 to the nearest thousand is 153,000, not 154,000.

Rounding numbers has many uses. The distance from the sun to the earth is usually expressed as 93,000,000 miles because the exact distance is difficult to determine, and is constantly changing. Even if a nearly exact distance could be ascertained, it would have little more meaning or use for most persons than the approximate distance and would be almost impossible to remember.

If the Federal budget is $99,076,432,119.77, it is far more convenient for all practical purposes to refer to it as a *$99 billion budget*, rounding the number to the nearest billion dollars.

Another important use of rounding numbers is in estimating the answers to calculations. To estimate the product 502 × 29, the number 502 may be rounded to 500 and 29 rounded to 30. We then can easily estimate that the answer will be about 500 × 30, or about 15,000.

Pre-Book Lesson

- Have the pupils give examples of numbers which are impossible to determine exactly, which are constantly changing, or are very cumbersome to use. Suggest that they give examples of such numbers from their work in history, geography, and science. Discuss the need for approximate numbers.
- List several numerals on the board naming numbers in the hundreds. Have pupils round the numbers using multiples of 100.
- Use a yardstick and, choosing a measurement, have a pupil name it to the nearest foot.

Using the Text Pages

- Ex. 1–5. The concept of rounding numbers is presented by use of a number-line picture. Make sure the pictures are good representations of multiples of 10, 100, and 1,000.* Discuss what is meant by "greatest multiple less than . . ." and "least multiple greater than . . ."
- Ex. 7–8. Rounding is performed by first examining the digit to the right of the place to which the number is being rounded. If this digit is 5, 6, 7, 8, or 9, the number named in the place to which the number is being rounded is increased by one and the digits to the right are replaced by zeros. If the digit names a number less than 5, the number named in the place to which the number is being rounded is not increased, but the digits to the right are replaced by zeros.
- Ex. 9. Have the pupils give other examples of numbers which would be just as meaningful if rounded.

* A classroom size model of a number line would be especially useful here. See item 6, page xix.

• Ex. 10. Have the pupils perfom other calculations, estimating their answers by use of numbers that have been rounded.

• Ex. 11. . Be sure that the pupils do not round a number that has already been rounded. If they are to round a number to tens, and then to hundreds, make sure they round the original number each time.

Individualizing Instruction

• The *more capable* pupils can be asked to round numbers such as the following:

(a) round 49,499 to the nearest thousand
(b) round 29,999 to the nearest ten
(c) round 5,959 to the nearest hundred

• *All pupils* can be asked to find examples in magazines and newspapers of numbers that have been rounded. Have them use these examples to summarize the reasons why rounding numbers is so useful.

• *Slower learners* may need assistance from the teacher when rounding in several of the examples. They may benefit from additional practice with number-line pictures showing multiples of ten, hundred, or thousand.

NOTES

9. Instead of using a number-line picture, we can round a whole number by examining the digits in the standard numeral for the number. For example, to round to the nearest ten, we examine the digit in one's place. If that digit is 0, 1, 2, 3, or 4, we round to the lesser multiple of 10. If the digit is 5, 6, 7, 8, or 9, we round to the greater multiple of 10. Give similar statements for rounding to the nearest hundred; the nearest thousand; the nearest ten thousand. ∨ ...examine the digit in ten's place.... ∨ ...examine the digit in hundred's place.... ...examine the digit in thousand's place....

10. In 38,648 the digit in hundred's place is _?_ 6, so 38,648 rounded to the nearest thousand is _?_ 39,000. 38,648 rounded to the nearest ten thousand is _?_ 40,000.

11. The number of people residing in a city is usually rounded. For example, the population of a certain city may be given as 470,000 people even though the exact population on a certain day was 471,208 people. Why do you think population figures are usually rounded? Sample answer: They change so often.

12. Rounding often helps us check our answers for addition and subtraction problems. For example, for addends 642, 54, and 392, we can round each addend to the nearest 10 and add the rounded numbers for an estimate of the correct answer. At the board, estimate the answer. 1,080 ∧ Then find the correct answer 1,088 ∧ and compare it with your estimate.

[W]

Ex. 13–20. Round to the nearest ten; to the nearest hundred.

13. 954 950; 1,000 14. 2,546 2,550; 2,500 15. 503 500; 500 16. 6,781,570 6,781,600 68,220,345 68,220,350; 68,220,300

17. 1,694 1,690; 1,700 18. 32,605 32,610; 32,600 19. 6,781,568 ∧ 20. 891,197 891,200 891,200

Ex. 21–23. Round to the nearest hundred thousand.

21. 400,576,884 ∨ 400,600,000 22. 2,770,652,202 ∨ 2,770,700,000 23. 825,000 ∨ 800,000

Ex. 24–27. Round to the nearest dollar; to the nearest ten dollars.

24. $43.20 ∨ $43.00; $40.00 25. $435.64 ∨ $436.00; $440.00 26. $56.66 ∨ $57.00; $60.00 27. $800.35 ∨ $800.00; $800.00

▲ **Reteaching.** Set 2. ◆ **Extra Examples.** Set 17.

How Well Do You Remember?

[W]

Ex. 1–14. List within braces the members of the set.

1. The set of counting numbers {1, 2, 3, . . .}
2. The set of whole numbers {0, 1, 2, 3, . . .}
3. The set of whole numbers between 7 and 20 {8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19}
4. The set of letters in the word *remember* {r, e, m, b}
5. The set of even whole numbers greater than 10 {12, 14, 16, . . .}
6. The set of counting numbers less than 1 { }
7. The union of $\{2, 5, 7, 9, 11\}$ and $\{6, 7, 8, 10, 15, 16\}$ ∨ {2, 5, 6, 7, 8, 9, 10, 11, 15, 16}
8. The intersection of $\{5, 7, 8, 9\}$ and $\{6, 7, 8\}$ {7, 8}

9. $\{2, 5\} \cup \{5, 7, 8\}$ {2, 5, 7, 8}
10. $\{3, 9, 10\} \cup \{1, 6, 7, 8\}$ ∨ {1, 3, 6, 7, 8, 9, 10}
11. $\{\ \} \cup \{8\}$ {8}
12. $\{6, 7\} \cap \{1, 3, 20\}$ { }
13. $\{1, 5, 7\} \cap \{1, 2, 3, 7\}$ {1, 7}
14. $\{\ \} \cap \{0, 1\}$ { }

Ex. 15–20. Round to the nearest ten.

15. 68 70
16. 42 40
17. 35 40
18. 189 190
19. 445 450
20. 6,622 6,620

Ex. 21–26. Round to the nearest thousand.

21. 2,007 ∨ 2,000
22. 32,702 ∨ 33,000
23. 67,500 ∨ 68,000
24. 345,220 ∨ 345,000
25. 983 ∨ 1,000
26. 30,500 ∨ 31,000

Try These Exercises

Enrichment [W]

$S = \{3, 5, 6, 9\}$
$T = \{4, 5, 7, 8, 9\}$
$S \cap T = \{5, 9\}$

If A and B represent finite sets, then the number of members in $A \cup B$ is equal to the sum of the number of members in A and the number of members in B less the number of members in $A \cap B$. For example, S has 4 members, T has 5 members, and $S \cap T$ has 2 members. $(4 + 5) - 2 = 7$ $S \cup T$ has 7 members.

Ex. 1–3. Compute to find the number of members in the union of the sets. Check by listing the members in the union.

1. $A = \{3, 6, 8, 9\}$, $B = \{1, 2, 3, 4\}$
   (4 + 4) − 1 = 7
   12 A ∪ B = {1, 2, 3, 4, 6, 8, 9}
2. $M = \{5, 9, 10, 15\}$, $N = \{3, 7, 13\}$
   (4 + 3) − 0 = 7
   M ∪ N = {3, 5, 7, 9, 10, 13, 15}
3. $R = \{2, 3, 4, 5, 6\}$, $S = \{2, 3, 4, 5\}$
   (5 + 4) − 4 = 5
   R ∪ S = {2, 3, 4, 5, 6}

Teaching Page 12

Pupil's Objectives

(a) To satisfactorily complete the test; and (b) to review areas of weakness that may be revealed by the test.

Background

A short review may be held prior to the test so that any concept not understood by some pupils may be presented again. The main concepts being tested are:

(a) Use of braces in set notation
(b) Union of two sets
(c) Intersection of two sets
(d) Rounding numbers

For errors in	*Review pages*
Ex. 1–6	5, 6
8–14	8, 9
15–26	10, 11

This enrichment exercise shows the relationship between the number properties of two sets and the number property of their union set. The number of elements in the union set of two sets is equal to the sum of the number of elements in each of the sets less the number of elements in the intersection set of the two sets.

In earlier grades when addition of numbers and union of sets was associated, only disjoint sets were considered. That is, the sets had no members in common. On this page, the intersection of the sets is not empty. This is why it is necessary to subtract the number of members in the intersection set. If the symbol $n(A)$ is used to denote the number of elements in set A, then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. This demonstrates the difference between addition which is an operation on numbers, and union, which is an operation on sets. The number of elements in the union set may vary according to the number of members in the intersection set. If set A has 5 elements and set B has 7 elements, then their union set will have 12 members only if the intersection set is the null set indicating that the two sets have no members in common.

Pre-Book Lesson

You may wish to have a brief review of topics presented on the test prior to the test or the test may be assigned immediately without review. Indicate how you wish the test completed and what pupils are to do when they complete their work. Some pupils may be assigned Extra Examples, others may work the enrichment section or may work Extra Activities.

Using the Text Page

• Ex. 1–26. The test should be corrected as soon as possible and an analysis made of the type and number of errors so that concepts in which pupils show weakness can be retaught before going on to new work. For the convenience of the teacher, Reteaching pages covering certain topics have been presented at the back of the book. These may be used in conjunction with Extra Examples.

• Ex. 1–3. After these examples have been completed, you may work together with pupils who have done the work in presenting the ideas to the remainder of the class. They may work other examples similar to the ones in this enrichment exercise.

Individualizing Instruction

• Individual help may be provided to *all pupils* by grouping those with the same weakness and working Extra Examples or by working special selected examples.

• *More capable pupils* who may finish the test early should be permitted to try the exercises at the bottom of the page.

Enrichment exercises are designed for these pupils and can be used at various times with them. However, when other pupils have completed their required work you may assign them the exercises or you may wish to discuss them orally with the entire class.

Teaching Page 13

Pupil's Objectives

(a) To review reading and writing Roman numerals; and (b) to review the ideas of repetition, addition, and subtraction as used in the Roman system of numeration.

Background

Roman numerals are shown by the capital letters, *I*, *V*, *X*, *L*, *C*, *D*, and *M*. These correspond to our numerals 1, 5, 10, 50, 100, 500, and 1,000 respectively. The Roman system of numeration is repetitive, additive, subtractive, and multiplicative; it has no symbol for zero, and the symbols are written from left to right, with the greatest number named first.

The system is repetitive in that each of the symbols, *I*, *X*, *C*, and *M* may be repeated, but no letter except *M* is written more than three times in succession. Instead of writing a letter four times in succession, each of the letters *I*, *X*, and *C* may be written before the letters naming the next *two* greater numbers, indicating the lesser value is to be subtracted from the greater value. The numeral *IV* means *I* is to be subtracted from *V*. In this way, the system is said to be subtractive. There are only six such possible subtractions shown by *IV*, *IX*, *XL*, *XC*, *CD*, and *CM*. Each of these examples of subtraction is to be considered as a single numeral. *V*, *L*, and *D* are never subtracted.

The Roman system is additive in that the value of a numeral is determined by adding the value of the individual addends. *CCCXXIV* means $C + C + C + X + X + IV$, or $100 + 100 + 100 + 10 + 10 + 4$, or 324.

In present day descriptions, the Roman system is considered to be multiplicative. A bar over a symbol or group of symbols within a numeral may mean multiplication by 1,000. $\overline{XII}$ means $12 \times 1{,}000$ or 12,000, $\overline{M}$ means $1{,}000 \times 1{,}000$ or 1,000,000, and $\overline{XX}CCXIX$ means 20,219.

Roman Numerals are still used today for dates, volume numbers, and clocks, but have fallen into disuse because they do not permit rapid calculation.

Pre-Book Lesson

Discuss some places where Roman numerals are still in use today. Point out that Roman Numerals were used mainly to record numbers and not for calculation. You may want to try a simple addition, subtraction, multiplication or division to prove this point.

Using the Text Page

- Ex. 1–3. Use these exercises to point out and explain that the Roman system may be said to be repetitive, additive, and subtractive. Only the letters *I*, *X*, *C*, and *M* may be repeated, and only *M* may be written more than three times in succession. The symbols are meant to show addends, and each of the six examples of subtraction is to be treated as a single symbol.
- Ex. 4. Let pupils point out examples of the additive and subtractive idea in this exercise.
- Ex. 5–9. If the pupils experience difficulty in writing standard numerals for the numbers named by Roman numerals, it may be caused by the examples of subtraction. It may help pupils having such difficulty to first examine the numeral and identify examples of subtraction within the numeral.

Individualizing Instruction

- *More capable pupils* may be asked to summarize the principles of the Roman system of numeration, including the multiplicative principle, and provide examples of each principle. Ask for comparisons with our system of numeration with respect to use of zero, repetitive, additive and subtractive ideas, number of symbols, place value, facility of use for calculation, and so on.
- *All pupils* can be given examples of Roman numerals containing errors. Ask them to identify the rule or principle that is being violated. Such examples may include:

XXXX = forty	*VVV* = fifteen
IL = forty-nine	*MMMM* = five thousand
IC = ninety-nine	$\overline{III}$ = three thousand
IM = nine hundred fifty	*VC* = ninety-five

* The main reasons for studying the Roman-numeral system is to emphasize the distinction between numerals and numbers and to illustrate a numeration system without place value.

Roman Numerals

Resurvey and extension [O]

* Numerals in our numeral system (the Hindu-Arabic system) are composed of the basic numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Numerals in the *Roman-numeral system* are composed of the basic numerals I, V, X, L, C, D, and M.

A			B	
	I	1	IV	4
	V	5	IX	9
	X	10	XL	40
	L	50	XC	90
	C	100	CD	400
	D	500	CM	900
	M	1,000		

1. Box A. Give the numbers named by the Roman numerals. 1; 5; 10; 50; 100; 500; 1,000

2. Box B. Each of these Roman numerals indicates a subtraction. For example, IV = 5 − 1, or 4. Give similar sentences for the other five Roman numerals in box B.
IX = 10 − 1, or 9; XL = 50 − 10, or 40; XC = 100 − 10, or 90; CD = 500 − 100, or 400; CM = 1,000 − 100, or 900

In reading and writing Roman numerals, we use the six numerals IV, IX, XL, XC, CD, and CM as single numerals and add their values to the values of the other symbols used.

3. To find the number named by a Roman numeral, we add the numbers named by the symbols in the numeral. For example, XVI = 10 + 5 + 1, or 16. Also, since IV names the number 4, XIV = 10 + 4, or 14.

a. LXXIII = _?_^{50} + _?_^{10} + 10 + 1 + 1 + 1, or _?_^{73}

b. DLV = _?_^{500} + _?_^{50} + _?_^{5}, or _?_^{555}

c. MCDIX = 1,000 + 400 + _?_^{9}, or _?_^{1,409}

4. Give the numbers named by the Roman numerals in box C.

C		
a.	XXV	25
b.	LXXX	80
c.	DVII	507
d.	XIX	19
e.	DXLIII	543
f.	MCM	1,900
g.	MCMXL ∨	
h.	MM ∨	1,940 2,000

[W]

Ex. 5–9. Find the number named by the Roman numeral.

5. XXXIV ∨ 34 6. LVII ∨ 57 7. MDCVI ∨ 1,606 8. MCMLIV ∨ 1,954 9. CCXLV ∨ 245

Ex. 10–19. Write a Roman numeral for the number.

10. 37 ∨ XXXVII 11. 19 XIX 12. 67 LXVII 13. 274 ∨ CCLXXIV 14. 1,943 MCMXLIII

15. 745 ∨ DCCXLV 16. 287 ∨ CCLXXXVII 17. 1,696 ∨ MDCXCVI 18. 927 ∨ CMXXVII 19. 3,000 MMM

◆ **Extra Examples.** Set 18.

* Although order is irrelevant, encourage pupils to write symbols naming greater numbers to the left of symbols naming lesser numbers.

An Ancient Numeration System

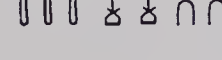
Extension [O]

	1
	10
	100
	1,000
	10,000
	100,000
	1,000,000

Thousands of years ago, the Egyptians wrote numerals using , , , , , , and which were pictures of a staff, a heelbone, a scroll, a lotus flower, a bent finger, a fish, and a man showing astonishment.

1. Give the numbers named by the Egyptian numerals in the box. 1; 10; 100; 1,000; 10,000; 100,000; 1,000,000;

2. The Roman-numeral system is both an additive system and a subtractive system; additive because XI = 10 + 1 and subtractive because IX = 10 − 1. The ancient Egyptian system was only additive. For example, = 10 + 1, or 11, and = 1 + 10, or 11.

3. In  , each names 10,000, each names 1,000, each names 10, and each names 1, so = 32,022. At the board, find the numbers named by the following:

a. **b.** **c.** **d.** **e.**

11,010 10,014 3,010 1,001,101 2,000,020

* 4. For the number 12, the Egyptians could write , or , or , or . Write an Egyptian numeral for each of Ex. a–e.

Arrangements will vary.

a. 4 **b.** 8 **c.** 13 **d.** 34 **e.** 1,243

Ex. 5–9. Find the number named by the Egyptian numeral.

5. **6.** **7.** **8.** **9.**

1,000,026 10,014 40,020 203 211,300

Ex. 10–14. Write an Egyptian numeral for the number.

10. 23 **11.** 66 **12.** 112 **13.** 144,213 **14.** 1,003,042

15. Write Egyptian numerals for the numbers from 3 through 213 counting by 30's.

Teaching Page 14

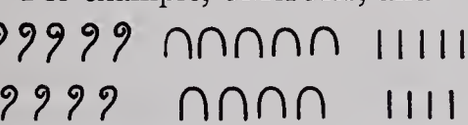
Pupil's Objectives

(a) To learn the principles of the ancient Egyptian system of numeration; and (b) to compare the Roman and Egyptian systems of numeration.

Background

The purpose of studying the Egyptian system of numeration is not to develop proficiency in working with Egyptian numerals, but to study one more step in the evolution of numeration systems and to better understand and appreciate our own system of numeration.

The ancient Egyptians used picture numerals which are called hieroglyphic numerals. Like the Roman system, the Egyptian system has seven symbols, is repetitive and additive, has no symbol for zero, and does not employ the principle of place value. However, it differs from the Roman system in that it is not subtractive or multiplicative. Also, the relationship among the symbols differs from that of the Roman system in that each symbol names a power of ten, and can be written up to nine times in succession. In the Roman system, with its symbols for five (*V*), fifty (*L*), and five hundred (*D*), together with the subtractive principle, repetition of a symbol more than twice is not necessary. This means that often a Roman numeral is shorter than the corresponding Egyptian numeral. For example, *CMXCIX*, and



name the same number, yet the Roman numeral contains far fewer symbols than the Egyptian numeral.

The Egyptian system is not subtractive and does not have place value. The order of the symbols in a numeral does not matter. The numeral , the numeral , and the numeral all name the same number. Like the Roman numerals, the Egyptian numerals were used for recording numbers and not for calculations.

Pre-Book Lesson

- Discuss the need the Egyptians had for numerals, such as in building the pyramids, astronomy, commerce, and solving problems pertaining to the determination of land boundaries each year after the flooding Nile receded.
- Discuss difficulties in using their "picture" system for calculations.

Using the Text Page

- Ex. 1. Point out that both the Roman and Egyptian systems of numeration have seven symbols, but the relationship among the symbols is different in each of the systems.
- Ex. 2. Point out the similarities and differences of the Roman and Egyptian systems with respect to the repetitive, subtractive and additive principles, and the relationship among the symbols.

Individualizing Instruction

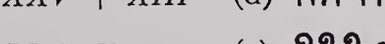
- *More capable pupils* could compare the Roman, Egyptian, and Hindu-Arabic systems of numeration with respect to: number of symbols; relationship among the symbols; repetitive, additive, subtractive, and multiplicative properties; symbol for zero; place value; calculation; advantages; and disadvantages.
- *All pupils* could be asked to attempt to perform some *simple* multiplications using Roman and Egyptian numerals to point out how difficult such calculations would be. In connection with these calculations, point out that children may be using, to some extent, the place-value property that these systems *did not possess.*
- *Slower learners* can work some additions using Egyptian numerals or Roman numerals. Some examples are:

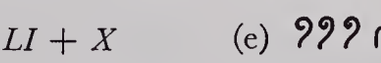
(a) $XXV + XIII$

(b) $LI + X$

(c) $IX + LXX$

(d)

(e) 

(f) 

Teaching Page 15

Pupil's Objectives

(a) To review numeral and digit; and (b) to learn the meaning of standard numeral.

Background

Primitive man's records of numbers may have been simple marks on a wall, notches in a stick, or knots in a rope to represent the number of people in his family or the number of sheep he owned. These were the first numerals.

Numerals are symbols for numbers, and a number may be represented by several different numerals. The number twenty-four may be represented by the Egyptian numeral by the Roman numeral XXIV, by 24 in our system, or by as in tallying. It can also be represented by $20 + 4$, 6×4, or $48 \div 2$. These are only seven of the many numerals that may name the number twenty-four yet there is only one number twenty-four.

If a numeral does not indicate an operation such as addition or subtraction, it is said to be the *standard numeral*. The standard numeral for the number twenty-four is 24 but it may also be named by the numerals $30 - 6$, 6×4, $48 \div 2$, $18 + 6$, and others.

Our system of numeration is called the Hindu-Arabic system. It was developed by the Hindu people in India, and spread to Europe by the Arabs. All numerals in this system use one digit or combinations of the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

Pre-Book Lesson

• Discuss how primitive man used simple ways to represent numbers. Discuss how he may have represented these numbers by marks on a wall, notches in a stick, knots in a rope, or pebbles in a pile.

• Emphasize the distinction between number and numeral, idea and representation.

Using the Text Page

• Ex. 1. To emphasize the difference between a numeral and a number, you may ask questions such as: "If you draw a picture of a clock on your paper, is the clock on your paper? What is on your paper? If you write 3 on your paper and then erase it, do you destroy the number three? What do you destroy?"

• Ex. 2–3. When a numeral does not indicate any operations, and is not represented using parentheses or other such symbols, it is said to be a standard numeral. Make sure pupils understand that while $6 + 4$, 5×2, and $20 - 10$ are certainly ways to represent the number ten, only 10 is the standard numeral.

• Ex. 4. The ten symbols we use to form our numerals are called digits. Ask the pupils why they think ten symbols were selected for our system of numeration.

• Ex. 5–6. In order to emphasize the distinction between numeral and digit; list some three-place numerals on the board and have pupils indicate the digits in each numeral.

Individualizing Instruction

• *More capable pupils* could be given numerals from an imaginary system of numeration and asked to figure out the value shown by each of the digits. For example, if they were given $\triangle + \square = 3$; $\triangle + \bigcirc = 5$; $\bigcirc + \square = 6$; $\triangle + \bigcirc + \square = 7$; and $\bigcirc + \bigcirc + \triangle = 9$, they might discover that $\triangle = 1$; $\square = 2$; and $\bigcirc = 4$.

• *All pupils* can explain the improper use of *number* and *numeral* as, "Half of 8 is 3 ($8 = 3$)," and "3 is larger than 8." Half the numeral 8 is 3, but half the number 8 is not 3. The numeral 3 is larger than the numeral 8, but the number three is not greater than the number eight.

• *Slower learners* can be asked to consider what life would be like without numerals. They might keep track of how many times in a day they actually use numerals of some kind. These pupils should be encouraged to keep a record of definitions, together with examples of number, numeral, and standard numeral. Their notebooks will be an excellent review and relearning device if kept from year to year. Also, they will be able to refer to that material which is difficult for them specifically.

* If pupils have studied other bases, it should be mentioned that the statements on this page refer to base ten. An interesting activity might be to have pupils give answers to the exercises in base-five.

* Numerals

Resurvey [O]

1. We know that a number is an idea. A *numeral* is a symbol, or expression, that names a number. For example, both 7 and $3 + 4$ are numerals which name the number seven. Which numerals in the box name the number eight? sixteen? b, e, f a, c, d, g, h

a. $2 + 6$
b. $20 - 4$
c. 2×4
d. $16 \div 2$
e. 16
f. 4×4
g. 8
h. $56 \div 7$

2. Numerals such as 0, 5, 24, 632, and so on are called *standard numerals* for the numbers zero, five, twenty-four, six hundred thirty-two, and so on.

a. Is 88 a standard numeral? Yes Is $2 + 7$ a standard numeral? No

b. Does the numeral 88 name the number that is named by $90 - 2$? Yes by $80 + 8$? Yes by 8×8? No by $8 + 8$? No by 22×4? Yes

3. On the board, write the standard numeral and four other numerals for the number ten. 10 Sample answers: 2×5 $12 - 2$ $8 + 2$ $20 \div 2$

4. All standard numerals are composed of some or all of ten basic numerals called *digits*. On the board, write the ten digits. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

5. Although the numeral 24 is not a digit, it is composed of the two digits _?2 and _?4.

6. How many different digits appear in the numeral 356? 3 5,657? 3 20,009? 3 Is each of these numerals a standard numeral? Yes

[W]

Ex. 7–10. Copy and complete the expression so that it will be a numeral for the number twelve.

7. $7 + \_?5$ 8. $\_?12 + 0$ 9. $24 \div \_?2$ 10. $\_?15 - 3$

Ex. 11–14. Write the standard numeral and four other numerals for the number. Answers will vary.

11. Fifteen 15 12. Twenty-two 22 13. Seventy 70 14. One hundred 100

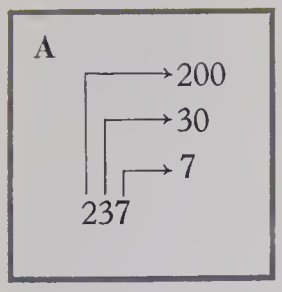
15. Is 0×1 a numeral for the number zero? Yes

16. Is $0 + 1$ a numeral for the number zero? No

17. List within braces the multiples of 8 that are less than 100. {0, 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96}

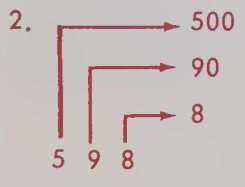
Place Value

Resurvey [O]



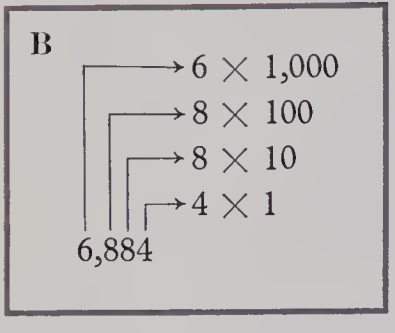
1. Box A. As the diagram shows, each digit in its place in the numeral 237 indicates a certain number. Give the number indicated by each digit in its place. The digit 2 indicates 200. The digit 3 indicates 30. The digit 7 indicates 7.

2. Draw a diagram as in box A for the numeral 598. Give the number indicated by each digit in its place. 5 - 500, 9 - 90, 8 - 8

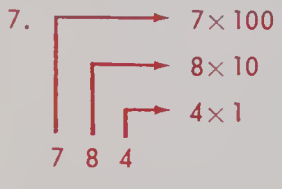


3. In the numeral 237, the digit 7 is in one's place, the digit 3 is in ten's place, and the digit 2 is in _?_ hundred's place.

4. In the numeral 35,678, is the digit 5 in thousand's place or in ten-thousand's place? Give the names of the places occupied by the other digits. 3 ten-thousand's, 6 hundred's, 7 ten's, 8 one's

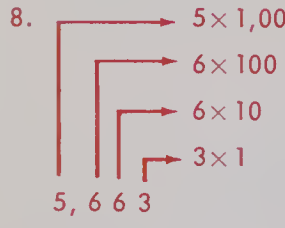


5. The value of one's place is 1, of ten's place is 10, of hundred's place is 100, and so on. The number indicated by a digit in its place is the product of the number named by the digit and the value of the place. For example, in the numeral 6,884, the digit 6 is in thousand's place so $6 \times 1{,}000$, or 6,000, is the number indicated by the 6 in its place. Explain the diagram in box B. 8 is in hundred's place, so 8 × 100 or 800 is the number indicated. 8 is in ten's place, so 8 × 10 or 80 is the number indicated. 4 is in one's place, so 4 × 1 or 4 is the number indicated.



6. Because the number indicated by a digit in its place in a numeral is partly determined by the value of the place, our numeral system is called a *place-value system*.

a. Is the Roman numeral system a place-value system? No Explain. Answers will vary. A symbol indicates the same number regardless of its place in the numeral.

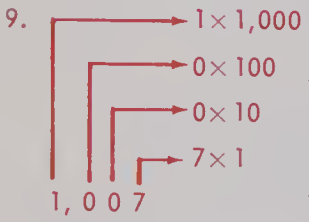


b. Was the ancient Egyptian system a place-value system? No

[W]

Ex. 7–10. Diagram the numeral as in box B.

7. 784 8. 5,663 9. 1,007 10. 238,773

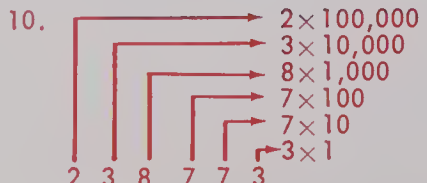


11. For the numeral 475,893, write the digit that is in

a. ten's place. 9 b. thousand's place. 5

c. hundred-thousand's place. 4 d. ten-thousand's place. 7

▲ **Reteaching.** Set 3.



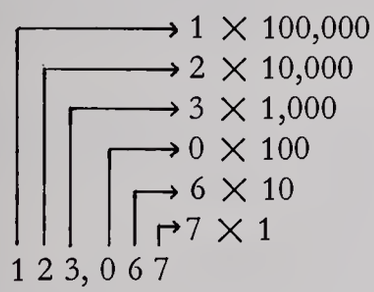
Teaching Page 16

Pupil's Objective

To review the principle of place value in our system of numeration.

Background

Unlike the Roman and Egyptian systems of numeration, the Hindu-Arabic system is not repetitive. Instead of repeating digits, the principle of place value is used. The number indicated by a digit in a numeral depends on the digit and its place within the numeral. In any numeral naming a whole number, the value of the first place (on the right) is *one*, and the value of each succeeding place to the left is ten times as great as that of the place to its right. Therefore,



In a place-value system, among its many other important uses, the symbol for zero, serves the important function of *place* holder, as every place in the numeral must contain a digit.

The principle of place value lends several advantages to our system of numeration. The pattern of place values continues without end, so there is no limit to the numbers that can be represented by numerals in this system. Furthermore, the place-value principle facilitates computations not possible in other numeration systems.

Pre-Book Lesson

• Have the pupils express 3,333 as a numeral in the Roman system and also as a numeral in the Egyptian system. The Roman numeral MMMCCCXXXIII, and the Egyptian numeral would each contain 12 symbols. This will illustrate that the decimal system requires fewer digits in recording numbers.

• Use these three numerals to discuss the lack of place value in the Roman and Egyptian systems, and the consequent drawbacks.

Using the Text Page

• Ex. 1–4. Point out that although all three systems of numeration (Roman, Egyptian, and Hindu-Arabic) are additive, the Hindu-Arabic system is distinctively different. means + | + | . XXIII means X + X + I + I + I but 237 does not mean 2 + 3 + 7; it means 200 + 30 + 7.

• Ex. 5–6. The pupil should be able to describe in his own words the meaning of a place-value system and supplement his definition with examples.

• Ex. 9. Use this exercise to illustrate the place-holder role played by zero.*

Individualizing Instruction

• *More capable pupils* can look up different sysitems of numeration in an encycopedia or other reference books, and report on these ancient systems. You may want to suggest such systems as the Babylonian, the Greek, and the Mayan.

• *All pupils* can be given a set of digits in any one of the three systems of numeration studied and be encouraged to determine the greatest and the least possible whole number that can be represented using each of these digits once. For example, the greatest whole number that can be represented using the digits 1, 5, 3, 2 is 5,321 and the least is 1,235. Given the symbols *I*, *V*, *X*, and *C*, the greatest number that can be represented is *CXVI* and the least number that can be represented is *XCIV*.

* See items 6 and 21 on page xix. Let pupils use these aids to illustrate concepts concerning place value.

Teaching Page 17

Pupil's Objectives

(a) To review reading numerals in our place-value system; and (b) to understand the use of periods in reading numerals.

Background

In our system of numeration, each place in a numeral has a name. If a numeral represents a whole number, the place at the extreme right is called the one's place, the place to its left is the ten's place, the next place to the left is the hundred's place, and so on.

To make numerals easier to read, the digits forming the numeral are separated by commas into groups of three digits each beginning at the right. These groups of digits show periods. The names of the first eight periods for numerals representing whole numbers are, from right to left: one, thousand, million, billion, trillion, quadrillion, quintillion, and sextillion.

When reading a numeral representing a whole number, the word *and* is not used. The reason is that *and* is said for the decimal point in decimal numerals in mixed form. For example, the importance of the word *and* is clear in the following: four hundred three thousandths (0.403); four hundred *and* three thousandths (400.003). It is advisable to have the pupils form the habit of not using the word *and* in reading numerals showing whole numbers.

The ancient Egyptians did not use commas because the symbols formed groups which separated the numeral for easy reading. The Egyptian numeral

is a good example to show this natural grouping.

Because of the subtractive and multiplicative properties of the Roman system, the symbols in the Roman numerals often do not form natural groups for easy reading. The following numeral illustrates this fact: *CDXLIX*

Pre-Book Lesson

Have the pupils try to read numerals such as 1425367 which contain no commas. Discuss the importance of grouping to show periods, the need for commas, and need for period names.

Using the Text Page

- Ex. 1. Explain why the word *and* is not used in reading a numeral that names a whole number.
- Ex. 2. Point out that the pattern continues indefinitely. There is no limit to the value of a number that can be expressed in our place-value system.

To show the importance of period names, have the pupils try to read a numeral such as 187,455,682 without using period names.

Individualizing Instruction

- *More capable pupils* can be given the names of periods beyond trillion and be asked to read and write numerals showing numbers greater than trillions. You may want to point out that there is very little use made of the names beyond billion. In just a short while, the pupils will learn how to express very great numbers in a simpler way.
- *All pupils* can be asked to find examples in newspapers and magazines of expressions for great numbers that are a combination of numeral and period name such as 16 million or 132 billion.
- *Slower learners* can be asked to read in class examples of great numbers they have found named in newspapers and magazines. Have them try to explain in different ways how many a million of anything is.

* The period name is used after reading the individual numerals in each period.

Reading Numerals for Greater Numbers

Resurvey [O]

1. The numeral 647 may be read, *six hundred forty-seven.* Note that we do not say "six hundred *and* forty-seven." Read the following numerals: two hundred seventy-eight

a. 68 ∨ sixty-eight **b.** 306 ∨ three hundred six **c.** 278 ^ **d.** 888 ∨ eight hundred eighty-eight **e.** 101 ∨ one hundred one

* **2.** Standard numerals, such as those in the box below, are easier to read if the digits are grouped by threes, beginning at the right. These groups show *periods* and are separated by commas. Name the periods shown. The first numeral is read, *five billion eight hundred thirty-seven million six hundred forty-three thousand eight hundred forty-two.* Note that the word *and* is not used. Read the other numerals. Twenty billion six hundred forty-seven million one hundred sixty-four thousand nine hundred fourteen; Thirty-four trillion two hundred fifty-nine billion one hundred five million one hundred seventy-eight thousand seven hundred eighty-five

Place Value	Ten trillion	Trillion	Hundred billion	Ten billion	Billion	Hundred million	Ten million	Million	Hundred thousand	Ten thousand	Thousand	Hundred	Ten	One
					5,	8	3	7,	6	4	3,	8	4	2
				2	0,	6	4	7,	1	6	4,	9	1	4
	3	4,	2	5	9,	1	0	5,	1	7	8,	7	8	5
Period Name	Trillion		Billion			Million			Thousand			One		

3. Read the numerals in the box at the right.

[W]

Ex. 4–6. Write the standard numeral for the number.

4. Thirty-two thousand three hundred five 32,305

5. Twenty-two million two thousand fifty-five 22,002,055

6. Six trillion two hundred forty-six billion nine hundred eighty-eight million three hundred fifty-two thousand nine hundred seventy-eight 6,246,988,352,978

110,000
1,210,005
3,689,164
10,502,463
893,691,689
10,004,450,000
4,692,064,082

3. One hundred ten thousand
One million two hundred ten thousand five
Three million six hundred eighty-nine thousand one hundred sixty-four
Ten million five hundred two thousand four hundred sixty-three
Eight hundred ninety-three million six hundred ninety-one thousand six hundred eighty-nine
Ten billion four million four hundred fifty thousand
Four billion six hundred ninety-two million sixty-four thousand eighty-two

* Expressing place values as products of 10's is studied on this page in preparation for the introduction of expressing place values with exponent forms on the next page.

* Renaming Place Values

Resurvey [O]

1. The value of ten's place is 10, of hundred's place is _?_, 100 of thousand's place is _?_, 1,000 and so on.

Yes

2. Is 10×10 another name for the number 100? ^ Is $10 \times 10 \times 10$ another name for the number 1,000? Yes

A

The value of hundred's place is 100, or 10×10.
The value of thousand's place is 1,000, or $10 \times 10 \times 10$.

Explain as in box A the value of

a. ten-thousand's place. 10,000 or $10 \times 10 \times 10 \times 10$ **b.** hundred-thousand's place. 100,000 or $10 \times 10 \times 10 \times 10 \times 10$

c. million's place. 1,000,000 or $10 \times 10 \times 10 \times 10 \times 10 \times 10$ **d.** ten-million's place. 10,000,000 or $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

3. Box B shows three ways to name the number 3,468.

B

$3{,}000 + 400 + 60 + 8$
$(3 \times 1{,}000) + (4 \times 100) + (6 \times 10) + (8 \times 1)$
$(3 \times 10 \times 10 \times 10) + (4 \times 10 \times 10) + (6 \times 10) + (8 \times 1)$

a. 30
3×10

b. 3,000
$3 \times 1{,}000$
$3 \times 10 \times 10 \times 10$

c. 30,000
$3 \times 10{,}000$
$3 \times 10 \times 10 \times 10 \times 10$

d. 300,000
$3 \times 100{,}000$
$3 \times 10 \times 10 \times 10 \times 10 \times 10$

As in box B, show three ways to name the number 63,644. v See below.

4. On the board, write the standard numeral for the number.

a. $(9 \times 100{,}000) + (4 \times 10{,}000) + (5 \times 1{,}000) + (3 \times 100) + (6 \times 1)$ 945,306

b. $(7 \times 10 \times 10 \times 10) + (7 \times 10 \times 10) + (3 \times 10) + (5 \times 1)$ 7,735

C

7,379
→ 300
→ 3×100
→ $3 \times 10 \times 10$

[W]

5. As in box C, diagram the number indicated by the digit 3 in its place for each of Ex. a–d. See left above.

a. 737 **b.** 3,987 **c.** 39,627 **d.** 1,306,880

Ex. 6–8. Write the standard numeral.

6. $(8 \times 10 \times 10) + (3 \times 10) + (4 \times 1)$ 834

7. $(9 \times 10 \times 10 \times 10 \times 10) + (5 \times 10 \times 10)$ 90,500

8. $150{,}000 + 2{,}000 + 600 + 40 + 7$ 152,647

3. $60{,}000+3{,}000+600+40+4$
$(6\times 10{,}000)+(3\times 1{,}000)+(6\times 100)+(4\times 10)+(4\times 1)$
$(6\times 10\times 10\times 10\times 10)+(3\times 10\times 10\times 10)+(6\times 10\times 10)+(4\times 10)+(4\times 1)$

Teaching Page 18

Pupil's Objective

To review renaming place values as a product of tens.

Background

In our place-value system of numeration, the number indicated by each digit in its place in a numeral is the product of the number named by the digit and the value of the place occupied by the digit. Therefore, 4,357 = 4 thousands + 3 hundreds + 5 tens + 7 ones or, 4,357 = $(4 \times 1{,}000) + (3 \times 100) + (5 \times 10) + (7 \times 1)$.

The value of the ten's place, hundred's place, thousand's place, and so on may be expressed as a product whose only factors are tens (except for 1×10). For example,

$$10 = 1 \times 10$$
$$100 = 10 \times 10$$
$$1{,}000 = 10 \times 10 \times 10$$
$$10{,}000 = 10 \times 10 \times 10 \times 10$$

Therefore, 4,357 may be expressed in the form $(4 \times 10 \times 10 \times 10) + (3 \times 10 \times 10) + (5 \times 10) + (7 \times 1)$.

Two reasons for learning to express place values as a product of a number of tens are:

a. to understand later work with the exponent form for showing place value and;

b. to apply the principle of place value to numeration systems with bases other than ten.

Using the Text Page

Ex. 1–2. Help pupils to see that the number of zeros in the numerals naming the place values is the same as the number of tens used as factors when the numeral is expressed as a product of tens. For example, 1,000 (three zeros) is the product of three tens, $10 \times 10 \times 10$.

Individualizing Instruction

• *More capable pupils* can practice expressing values of the different places in the million's and billion's periods as a product of tens.

• Applying the principle that 1,000,000,000 may be expressed as $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$, *all pupils* may be asked to offer suggestions for a shorter or more convenient method of expressing numbers of this type.

NOTES

Teaching Page 19

Pupil's Objective

To learn to read and write numerals using the exponent form.

Background

The purpose of having the pupils learn to express a number such as 100,000 using expanded notation ($10 \times 10 \times 10 \times 10 \times 10$) is that it serves as a preparation for the introduction of the concept of *exponent form*. The exponent form for the number 100,000 is 10^5. The small raised numeral names the *exponent*. When the exponent is a counting number, it tells how many times the number is being used as a factor.

$$10^5 = 10 \times 10 \times 10 \times 10 \times 10$$

The number being used as a factor is sometimes called the *base*. In the example above, 10 is the base, and 10^5 expresses the *fifth power of the base ten*, or *ten to the fifth power*. When the exponent is 2, the base may be referred to as being *squared*. 6^2 may be read *six to the second power*, or *six squared*. When the exponent is 3, the base may be referred to as being *cubed*, so 7^3 may be read *seven to the third power* or *seven cubed*. Whenever the exponent is 1, we shall use the simpler form of showing the base without showing the exponent. When no exponent is indicated after a numeral, the exponent is understood to be 1. In that case, the numeral would name just the base. That is, $10^1 = 10$; $5^1 = 5$; and $176^1 = 176$.

Pre-Book Lesson

• Review the meaning of factor. Show how cumbersome such numbers as 100 trillion are when shown in standard form, and even more so in expanded notation.

• Modern telescopes have detected objects in our universe 48,000,000,000,000,000,000,000,000,000 miles from the earth. Show a need for more compact forms for numerals to express such numbers, if only for the astronomer's use.

Using the Text Page

• Ex. 1. Explain that the exponent form may be used with any whole number as base, including zero and one. Explain also that the exponent form is a short form helpful in naming greater numbers.

• Ex. 3. Call attention to the special terms *squared* and *cubed* used in connection with the second power and the third power.

• Ex. 5. The exponent may be any whole number although the exponent 1 is usually not shown.

Individualizing Instruction

• *More capable pupils* may be taught to determine and study sequences by examining a pattern of powers such as the following:

$2^3 = 8$	$3^3 = 27$	$5^3 = 125$
$2^2 = 4$	$3^2 = 9$	$5^2 = 25$
$2^1 = 2$	$3^1 = 3$	$5^1 = 5$
$2^0 = ?$	$3^0 = ?$	$5^0 = ?$

Since the exponent 0 is not mentioned in the text, you may wish to take up this topic only with *more capable pupils*. From a study of the above sequences, ask these pupils if they can tell the value of any number to the zero power. $n^0 = 1 (n \neq 0)$

• For the *slower learners*, diagrams may help in explaining why 3^2 is called three squared and why 2^3 is called two cubed.

Fig. 1 shows $3^2 = 3 \times 3$, which can be considered as three rows of three objects each, forming a square.

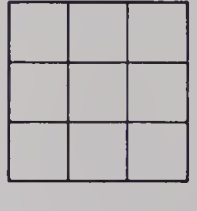


Fig. 1

$3^2 = 3 \times 3 = 9$

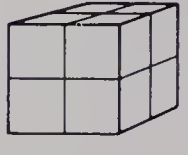


Fig. 2

$2^3 = 2 \times 2 \times 2 = 8$

Fig. 2 shows $2^3 = 2 \times 2 \times 2$, which can be considered as two layers of two rows of two objects each, forming a cube.

* In order to add, subtract, multiply, and divide exponents, it is mathematically sound to consider them as numbers; however, some books do refer to exponents as small raised numerals.

Exponent Forms

Extension [O]

* **1.** Numerals such as those in the box are called *exponent forms*. In an exponent form, the small raised numeral names the *exponent*. Name the exponent for each exponent form in the box. 3; 2; 4

a. 2^3 **b.** 5^2 **c.** 6^4

** **2.** If the exponent is a counting number, it indicates how many times the number named by the larger numeral is used as a factor. For example, $2^3 = 2 \times 2 \times 2$, so 2^3 names the number 8. $5^2 = \_? \times \_?$, so 5^2 names the number $\_?$. 5; 5; 25

3. The numeral 2^3 is read *two to the third power* or *two cubed*. The numeral 5^2 is read *five to the second power* or *five squared*. The numeral 6^4 is read *six to the fourth power*. Read each of the following: Two to the fifth power; Five to the second power or Five squared; Five to the fourth power; Ten to the seventh power

a. 2^{10} ∨ **b.** 2^5 ∧ **c.** 5^2 ∧ **d.** 8^6 ∨ **e.** 5^4 ∧ **f.** 10^7 ∧ **g.** 12^4 ∨

Two to the tenth power; Eight to the sixth power; Twelve to the fourth power

4. Write exponent forms and standard numerals for Ex. a–f.

a. Six to the second power 6^2; 36
b. Ten to the fifth power ∨
c. Seven cubed ∨ 10^5; 100,000 7^3; 343
d. Ten squared 10^2; 100
e. $6 \times 6 \times 6 \times 6$ 6^4; 1,296
f. Twelve to the fourth power 12^4; 20,736

5. The numbers 2, 2^2, 2^3, 2^4, ··· are called *powers* of 2. Note that we do not need to name the exponent when writing *two to the first power* in exponent form. The numbers 10, 10^2, 10^3, 10^4, ··· are powers of $\_?$. 10

[W]

Ex. 6–12. Write the standard numeral for the number.

6. 3^2 9 **7.** 3^4 81 **8.** 2^5 32 **9.** 5^3 125 **10.** 7^2 49 **11.** 10^4 ∨ 10,000 **12.** 30^2 900

Ex. 13–18. Write an exponent form for the number.

13. 25 5^2 **14.** 8 2^3 **15.** 81 9^2 or 3^4 **16.** 49 7^2 **17.** 27 3^3 **18.** 100 10^2

Ex. 19–21. Copy and complete.

19. $144 = \_?^2$ 12 **20.** $625 = 5^{\_?}$ 4 **21.** $8^2 = 2^{\_?}$ 6

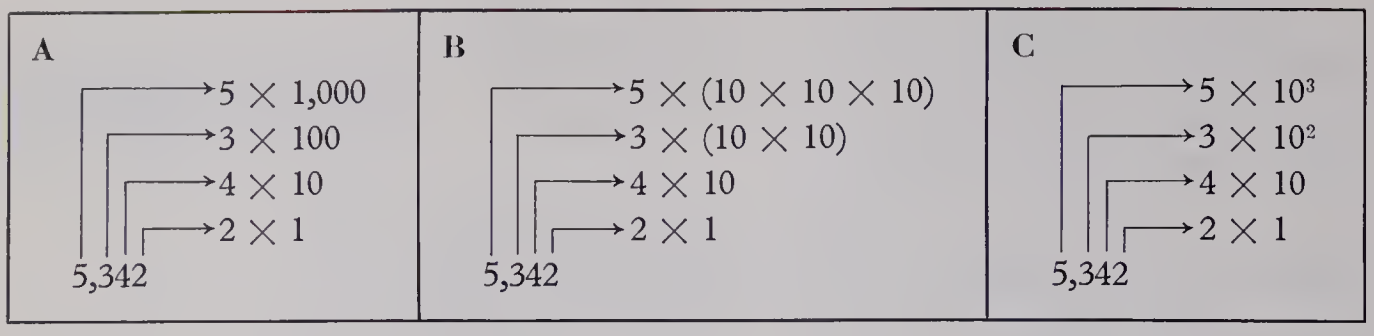
♦ **Extra Examples.** Set 19.

** In later mathematics, pupils will study roots of numbers such as square root and cube root. The exponents for roots are not counting numbers; they are named with fractions.

* If we consider the one's place as the 1st place and number to the left, 2d, 3d, · · ·, then the power of the base is one less than the ordinal number of the place, e.g. the 3d place is 10^2, the 4th place is 10^3, and so on.

Using Exponent Forms to Name Place Values

Extension [O]



1. Study the diagrams in boxes A and B. Explain why we may name the number 5,342 in the following two ways.

$(5 \times 1{,}000) + (3 \times 100) + (4 \times 10) + (2 \times 1)$

$(5 \times 10 \times 10 \times 10) + (3 \times 10 \times 10) + (4 \times 10) + (2 \times 1)$

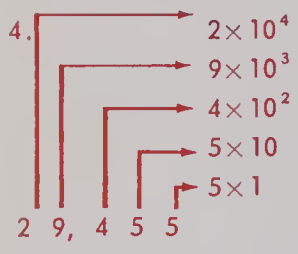
Five is in thousand's place so it indicates $5 \times 1{,}000$ or $5 \times 10 \times 10 \times 10$; . . .

2. On the board, write an exponent form for 10×10; for $10 \times 10 \times 10$; for 100,000. Explain the diagram in box C.

10^2 10^3 10^5

Key idea: 100 has been named 10^2; 1,000 has been named 10^3.

3. $5{,}342 = (\_ ? 5 \times 10^3) + (\_ ? 3 \times 10^2) + (\_ ? 4 \times 10) + (2 \times 1)$



4. On the board, diagram 29,455 as in box C and then write an expanded form for this number as in Ex. 3.

Since the place values in our numeration system are powers of 10, our system is called a *base-ten system* and our numerals are called *base-ten numerals*.

$29{,}455 = (2 \times 10^4) + (9 \times 10^3) + (4 \times 10^2) + (5 \times 10) + (5 \times 1)$

* 5. For 10^2, the exponent is the same as the number of times the digit 0 appears in the standard numeral 100. Give a similar statement for 10^3; 10^4; 10^5; 10^6.

Key idea: The exponent is the same as the number of times the digit 0 appears in the standard numeral.

6. In the numeral 1,000,000,000, the digit 0 appears how many times? 9 On the board, write an exponent form for the number. 10^9

[W]

Ex. 7–10. Write an expanded form as in Ex. 3. See below.

7. 38,775 8. 229,653 9. 38,356,554 10. 5,555

Ex. 11–13. Write an exponent form for the number.

11. 10,000 10^4 12. 1,000,000 10^6 13. 1,000,000,000,000 10^{12}

♦ Extra Examples. Set 20.

Ex. 7. $(3 \times 10^4) + (8 \times 10^3) + (7 \times 10^2) + (7 \times 10) + (5 \times 1)$

8. $(2 \times 10^5) + (2 \times 10^4) + (9 \times 10^3) + (6 \times 10^2) + (5 \times 10) + (3 \times 1)$

9. $(3 \times 10^7) + (8 \times 10^6) + (3 \times 10^5) + (5 \times 10^4) + (6 \times 10^3) + (5 \times 10^2) + (5 \times 10) + (4 \times 1)$

10. $(5 \times 10^3) + (5 \times 10^2) + (5 \times 10) + (5 \times 1)$

Teaching Page 20

Pupil's Objectives

(a) To learn to express place values using exponent form; (b) to learn to express numbers in expanded form using exponents; and (c) to learn to express very great numbers in exponent form.

Background

Place values in our system of numeration can be expressed as powers of ten. The values of the first six places of a numeral that names a whole number are as follows:

$$100{,}000 = 10 \times 10 \times 10 \times 10 \times 10 = 10^5$$
$$10{,}000 = 10 \times 10 \times 10 \times 10 = 10^4$$
$$1{,}000 = 10 \times 10 \times 10 = 10^3$$
$$100 = 10 \times 10 = 10^2$$
$$10 = 1 \times 10 = 10^1$$
$$1 = 1 = 10^0$$

The pattern continues indefinitely.

Although the value of the one's place is 10^0 and that of the ten's place is 10^1, these are almost always written simply as 1 and 10.

A numeral may be written in expanded form in several different ways. For example, 4,376 may be written in any one of the following ways:

(a) 4 thousands + 3 hundreds + 7 tens + 6 ones

(b) $(4 \times 1{,}000) + (3 \times 100) + (7 \times 10) + (6 \times 1)$

(c) $(4 \times 10 \times 10 \times 10) + (3 \times 10 \times 10) + (7 \times 10) + (6 \times 1)$

(d) $(4 \times 10^3) + (3 \times 10^2) + (7 \times 10) + (6 \times 1)$

In example (d), the place values are expressed in exponent form.

When we write 2^3, 2 is called the *base* and 3 is called the *exponent*. 10 is the base of the numeral 10^2 in exponent form. A numeration system whose place values are successive powers of 10 is called a *base-ten system*. A numeration system whose place values are successive powers of 2 is called a *base-two system*.

Pre-Book Lesson

• Help pupils to understand and master the concept of expanded form by having them write on the board in expanded form such numerals as 52,173; 40,365; 901; and 1,000,101.

• Let them use Ex. 7–10 to show place values in exponent form. For example, the standard form for 1,000 has 3 zeros and the exponent for 1,000 when expressed as a power of 10 is 3.

Using the Text Page

• Ex. 1. Have children compare box B and box C to discover the relationship between the number of *tens* and the exponent shown.

• Ex. 2. Point out that the number of zeros in a numeral in standard form that is a power of ten is the same as the exponent that shows the power. For example, 1,000 has 3 zeros and is also 10^3. Use additional examples.

Place values in our system are successive powers of ten. This is the reason it is called a *base-ten* system.

Help the pupils try to realize that the pattern of place values when expressed as powers of the base is:

$(\text{base})^3$	$(\text{base})^2$	$(\text{base})^1$	$(\text{base})^0$
10^3	10^2	10^1	10^0
1,000	100	10	1

This principle applies to place-value systems in other bases, which are to be studied next.

Individualizing Instruction

• *More capable pupils* may be asked why our numeration system has place values which are powers of ten. If we make the assumption that it is because we have ten fingers, discuss what our system of numeration would be like if humans had only five fingers in all. Challenge these pupils to write a table of place values for such a system. Have them use exponent forms to name the place values.

• *All pupils* should be given additional practice writing numerals in expanded form using exponents. The mastery of this is essential to understanding the concept of place value in numeration systems using other bases.

Teaching Page 21

Pupil's Objective

To resurvey applying the principles of place value to a base-five system of numeration.

Background

In our base-ten system of numeration, the place values are powers of ten. The place value of the first place at the right is 1, and as we move to the left the value of each succeeding place is ten times the value of the place to its right. Place values in other numeration systems are also powers of the base. In a base-five system, the place values are powers of five. The value of the first place is 1, and the place value of each succeeding place is five times that of the place to its right. The numeral 43_{five} means 4 fives + 3 ones.

Care must be taken to read the numerals correctly. The numeral 34_{five} is to be read "three-four, base five". This numeral is another name for the number *nineteen*.

Pre-Book Lesson

Tell the following story:

The Athletic Director at Parkwood School devised a system for recording how many teams could be formed for various games from different size groups of pupils. He used three blanks to record the information. In the first blank, he wrote the number of teams that could be formed from the group of pupils, in the second blank, he wrote the number of pupils left over that could be used as substitutes, and in the third blank, he wrote the number of pupils required to form one team. When he wrote $16 = 31_{five}$ it meant he could form from a group of 16 pupils 3 teams and have 1 substitute for a team game requiring five members on a team.

Fill in the blanks, indicating what numeral he would put in each blank to indicate the number of teams and substitutes that could be formed from a group of 24 pupils for (a) baseball, (b) basketball, (c) football.

(a) 24 = ___ ___ nine
(b) 24 = ___ ___ five
(c) 24 = ___ ___ eleven

In this system, what is the total number of pupils represented by

(a) 23_{five}?
(b) 31_{five}?
(c) 42_{five}?
(d) 86_{nine}?
(e) 22_{eleven}?

Using the Text Page

• To help explain why in Ex. 4 the numeral is written "43_{five}" rather than "43_5", ask

(a) How would the number five be named with a base-five numeral? (10_{five})

(b) What is the greatest number that can be named by just one digit in base five? (4)

(c) Why is the numeral "43_5" not acceptable? (There is no symbol "5" in base five.)

• Give more practice as needed for the pupils to understand base-five numerals before assigning the written work. You may wish to have pupils interpret sums of money up to 24¢ in terms of number of nickels and number of pennies.

Individualizing Instruction

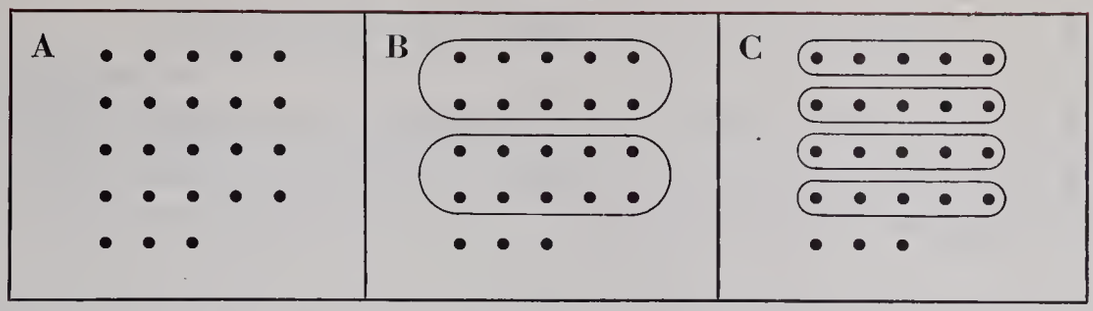
• Have the *more capable pupils* draw a number-line picture and label it with numerals in base five and base ten up to 100_{five}.

• Have *all pupils* express as a base-five numeral (a) their age, (b) the number of fingers on one hand, (c) the numbers of fingers on two hands, (d) the number of people in their family.

• Let *slower learners* draw two columns on other paper. In one column, let them write the decimal numerals 1 through 24. Beside each of these numerals, let them write the base-five numeral that names the same number. If necessary, they may use toothpicks, assembling them into groups of five, or draw dots and draw rings around groups of five. Representing the numerals by showing beads for the digit in each place can be very helpful in understanding the need for recording 1 in the next place when the number of ones is equal to the base.*

* An excellent device for illustrating place value is the Ginn Arithme-Stick. See item 3 on page xix.

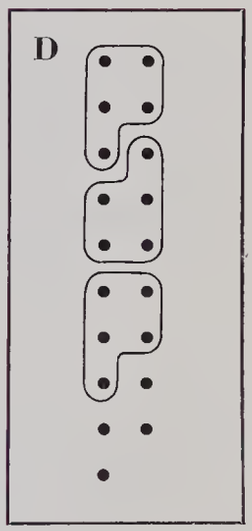
* For 43_{five}, the subscript "five" is not really part of the numeral; it simply indicates that 43 is a base-five numeral. Encourage pupils to use subscripts.



A Different Place-Value System

Resurvey; base five [O]

1. Count the dots in boxes A–C. How many are there? 23



2. In box B, the dots are shown in two groups of ten with _?3 extra. When we write *23*, we mean _?2 tens and _?3 ones.

3. The numeral 23 is a base-ten numeral. In a base-ten system, the place values are powers of _?10 and the value of a place is _?10 times the value of the place to the right. For base ten, give the value of the first place; 1 2d place; 10 3d place. 100

4. In box C, the dots are shown in _?4 groups of five with _?3 extra. There are _?4 fives and _?3 ones. We may name the number of dots in box C with the base-five numeral 43.

* To show that 43 is a base-five numeral, we usually write, 43_{five}.

** This numeral is read, *four-three, base five.*

5. In a *base-five system*, the place values are powers of 5 and the value of a place is five times the value of the place to the right. The value of the first place is 1 as in base ten. Give the value of the 2d place; 5 the 3d place. 25

6. Box D. There are _?3 groups of five with _?4 extra. On the board, write a base-five numeral to indicate the number of dots in box D. √Explain the meaning of your numeral. 3 fives and 4 ones

34_{five}

[W]

Ex. 7–10. On your paper, draw the number of dots indicated and then show them in groups of five as in boxes C and D. Then write a base-five numeral to indicate the number of dots.

7. 6 dots

11_{five}

8. 10 dots

20_{five}

9. 17 dots

32_{five}

10. 24 dots

44_{five}

** "Forty-three" should not be used—it is base-ten language.

* Since single digits such as 0, 1, 2, 3, and 4 in base five name the same numbers in any other base in which they are used, the base is usually not designated for 1-place numerals.

Renaming with Base-Ten Numerals

Resurvey [O]

1. Box A. Three base-five numerals are shown. Name each place shown and give its value. Read aloud the three numerals. Two-three, base five; Three-four, base five; Four-three-two, base five. one's place; 1; five's place; 5; twenty-five's place; 25

* 2. Box B. The number named by 23_{five} can be renamed 2 fives + 3 ones, or 13. When we name the number with 13, we are naming it with a base-ten numeral. This could be shown by writing 13_{ten}. However, we usually do not show the base when the numeral is a base-ten numeral. Tell what to think to rename 34_{five} and 432_{five} with base-ten numerals. 3 fives + 4 ones, or 19; 4 twenty-fives + 3 fives + 2 ones, or 117

3. On the board, diagram 34_{five} as in box B and then write a base-ten numeral for the number. 3 fives, 4 ones; 34_{five}; 19

4. Without using paper and pencil, give a base-ten numeral for each of Ex. a–d.

a. 12_{five} 7 **b.** 22_{five} 12 **c.** 10_{five} 5 **d.** 41_{five} 21

5. Copy and complete the list of base-five numerals in box C for the whole numbers from 1 through 10. See box C.

6. In 432_{five}, the digit 4 is in _?_ (twenty-five's) place, the digit 3 is in _?_ (five's) place, and the digit 2 is in _?_ (one's) place. Explain the expanded form in box D and the work shown for finding a base-ten numeral for the number. 432_{five} can be renamed 4 twenty-fives + 3 fives + 2 ones or $(4\times25) + (3\times5) + (2\times1)$ [W]

Ex. 7–12. Find a base-ten numeral for the number as in box D.

7. 213_{five} 58 **10.** 441_{five} 121

8. 222_{five} 62 **11.** 42_{five} 22

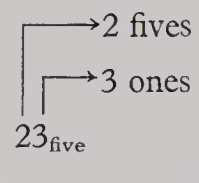
9. 404_{five} 104 **12.** 104_{five} 29

A

Base Five

	Twenty-five's Place	Five's Place	One's Place
a.		2	3
b.		3	4
c.	4	3	2

B



C

Base Ten	Base Five
1	1
2	2
3	_?_ 3
4	_?_ 4
5	10
6	_?_ 11
7	_?_ 12
8	13
9	_?_ 14
10	_?_ 20

D

$$432_{five} = (4 \times 25) + (3 \times 5) + (2 \times 1)$$
$$= 100 + 15 + 2$$
$$= 117$$

◆ **Extra Examples.** Set 21.

Teaching Page 22

Pupil's Objectives

(a) To resurvey writing the expanded form for base-five numerals; and (b) to rename numbers named by base-five numerals with base-ten numerals.

Background

In a system of numeration that uses the place-value principle, the number named by a numeral is equal to a sum for which each addend is the product of the number named by a digit and the value of the place in which the digit appears.

To change from base five to base ten, write the base-five numeral in expanded form, and then perform the indicated operations. The base-five place values shown below are named with base-ten numerals.

$$\begin{aligned} 234_{\text{five}} &= (2 \times 25) + (3 \times 5) + (4 \times 1) \\ &= 50 + 15 + 4 \\ &= 69 \end{aligned}$$

Therefore, 234_{five} and 69 name the same number.

Since we are working in a base-five system, to be absolutely correct, the expanded form above should be $(2 \times 100_{\text{five}}) + (3 \times 10_{\text{five}}) + (4 \times 1)$. However, to simplify the thinking, we name the place values with the base-ten numerals, 1, 5, 25, and so on.

Pre-Book Lesson

Ask a pupil to write on the board the expanded form for the numeral 234. Then ask someone to write the expanded form for the base-five numeral 234_{five}. If any difficulty is encountered, give the pupils a hint by writing

$$234_{\text{five}} = (2 \times \_) + (3 \times \_) + (4 \times \_)$$

and ask what number must be named in each blank. Let them work several other examples such as

(a) $342_{\text{five}} = (3 \times \_) + (\_ \times \_) + (\_ \times 1)$
(b) $432_{\text{five}} = (\_ \times \_) + (\_ \times \_) + (\_ \times \_)$

Using the Text Page

Give the pupils as much practice as is necessary in renaming with a base-ten numeral numbers named by base-five numerals before assigning the written work. Write on the board some three-place base-five numerals and ask *more capable pupils* to try to rename the numbers with base-ten numerals without using pencil and paper.

Individualizing Instruction

• Have *more capable pupils* rename numbers named by four-place and five-place base-five numerals with base-ten numerals after preparing a base-five place value chart to five places.

(a) 2334_{five}
(b) 3232_{five}
(c) 1234_{five}
(d) 14321_{five}
(e) 11223_{five}

• If the *slower learners* have dfficulty with written exercises 7–12, let them work sets of exercises of graduated difficulty such as the following:

(a) $132_{\text{five}} = (\_ \times 25) + (\_ \times 5) + (\_ \times 1)$
(b) $24_{\text{five}} = (2 \times \_) + (4 \times \_)$
(c) $234_{\text{five}} = (\_ \times \_) + (\_ \times \_) + (\_ \times \_)$
(d) $144_{\text{five}} = (\_ \times \_) + (\_ \times \_) + (\_ \times \_)$

Teaching Page 23

Pupil's Objective

To learn to rename numbers expressed in base ten with base-five numerals.

Background

To rename a number expressed in base ten with a base-five numeral, determine what digits should be written in each blank:

$$32 = (\_ \times 25) + (\_ \times 5) + (\_ \times 1)$$

When the above exercise is completed, the expanded form for the base-five numeral will be shown. To determine what numeral should be written in the first blank at the left (the third place), find the number of 25's that can be subtracted from 32.

$$\begin{array}{r} 1 \\ 25\overline{)32} \\ \underline{25} \\ 7 \end{array}$$

There is one 25 in 32 and a remainder of 7. Therefore, $32 = (1 \times 25) + 7$.

To find the numeral that should be written in the second place, find the number of 5's that can be subtracted from 7. Note the remainder.

$$\begin{array}{r} 1 \\ 5\overline{)7} \\ \underline{5} \\ 2 \end{array}$$

Therefore, $32 = (1 \times 25) + (1 \times 5) + (2 \times 1)$. From this we can see that $32 = 112_{\text{five}}$

Box C on the pupil's page shows how this work can be written more concisely.

To name a whole number greater than 124 but less than 625 in base five, proceed as follows:

$$579 = (\_ \times 125) + (\_ \times 25) + (\_ \times 5) + (\_ \times 1)$$

$$\begin{array}{r} 4 \\ 125\overline{)579} \\ \underline{500} \\ 79 \end{array} \qquad \begin{array}{r} 3 \\ 25\overline{)79} \\ \underline{75} \\ 4 \end{array} \qquad \begin{array}{r} 0 \\ 5\overline{)4} \\ \underline{0} \\ 4 \end{array} \qquad \begin{array}{r} 4 \\ 1\overline{)4} \\ \underline{4} \\ 0 \end{array}$$

$579 = 4304_{\text{five}}$

It must be borne in mind that the first division is performed using the greatest possible place value as a divisor. For all numbers between 124 and 625, the first divisor would be 125.

Pre-Book Lesson

Remind pupils that if no base is designated, then the base is ten. Ask pupils to determine the missing digit in the expanded form, and write the appropriate base-five numeral.

(a) $17 = (\_ \times 5) + (\_ \times 1)$
(b) $23 = (\_ \times 5) + (\_ \times 1)$
(c) $25 = (\_ \times 25) + (\_ \times 5) + (\_ \times 1)$
(d) $124 = (\_ \times 25) + (\_ \times 5) + (\_ \times 1)$
(e) $588 = (\_ \times 125) + (\_ \times 25) + (\_ \times 5) + (\_ \times 1)$

Using the Text Page

• Ex. 1a–e. To help *slower learners* with these examples, express the following partial sets of multiples on the board:

	twenty-fives	fives	ones
①	25	5	1
②	50	10	2
③	75	15	3
④	100	20	4

Then, have the pupils use this expanded form:
56 ___ twenty-fives___ fives___ ones ___five

• Ex. 4. Explain the work in box C carefully. Pupils may find the table of multiples shown for Ex. 1 helpful in understanding the work in box C and in working Ex. 4–6.

Individualizing Instruction

• Let the *more capable pupils* rename the following with base-five numerals:

(a) 125 (b) 281 (c) 498 (d) 619 (e) 944

• Ask *all pupils* to name the next consecutive number after each of the following:

44_{five}; 24_{five}; 404_{five}; 344_{five}; 4444_{five}; 244_{five}.

They may name the number that precedes each of the following:

100_{five}; 40_{five}; 10_{five}; 310_{five}; 3000_{five}; 204_{five}.

Show each of the following in the base indicated:

$11 =$ ___five; $27 =$ ___five; $14_{\text{five}} =$ ___ten; $131_{\text{five}} =$ ___ten.

* "1000_{five}" should not be written "$1,000_{five}$" —no comma should be used.

Renaming with Base-Five Numerals

[o]

1. To find a base-five numeral for the number named by the base-ten numeral 32, we may think, *32 is equal to 1 twenty-five + 1 five + 2 ones, so a base-five numeral for the number 32 is* _?_. 112
Find base-five numerals for the following in this way.

a. 18 33_{five} **b.** 28 103_{five} **c.** 56 211_{five} **d.** 82 312_{five} **e.** 124 444_{five}

2. On the board, copy and complete the list of base-five numerals in box A for the whole numbers from 11 through 26. See box A.
Name the five digits used in writing base-five numerals. 0, 1, 2, 3, 4

3. Explain how the work in box B shows that both 32 and 112_{five} name the same number. 32 is renamed as 1 twenty-five + 1 five + 2 ones.

4. Box C. To rename the number named by the base-ten numeral 32, we divide by the place values of base five. Explain the work for finding the base-five numeral. Each remainder is divided by the next lower place value of base five.

A

Base Ten	Base Five
11	21
12	_?_ 22
13	_?_ 23
14	_?_ 24
15	_?_ 30
16	_?_ 31
17	32
18	_?_ 33
19	_?_ 34
20	_?_ 40
21	_?_ 41
22	_?_ 42
23	_?_ 43
24	_?_ 44
25	100
26	_?_ 101

B

$$32 = (1 \times 25) + (1 \times 5) + (2 \times 1)$$
$$= 112_{five}$$

C

1	1	2
$25\overline{)32}$	$5\overline{)7}$	$1\overline{)2}$
25	5	2
R 7	R 2	R 0

* **5.** On the board, rename Ex. a–e as in box B. If necessary, show your work as in box C.

a. 62 222_{five} **b.** 89 324_{five} **c.** 100 400_{five} **d.** 112 422_{five} **e.** 125 1000_{five}

[w]

** **6.** Write base-five numerals for the whole numbers from 27 through 50. Base-five numerals:
102, 103, 104, 110, 111, 112, 113, 114, 120, 121, 122, 123, 124, 130, 131, 132, 133, 134, 140, 141, 142, 143, 144, 200

Ex. 7–12. Write a base-five numeral for the number.

7. 17 32_{five} **8.** 24 44_{five} **9.** 55 210_{five} **10.** 88 323_{five} **11.** 90 330_{five} **12.** 100 400_{five}

13. Express the value of the 4th place in a base-five system with **a.** a base-ten numeral. 125 **b.** an exponent form. 5^3

♦ **Extra Examples.** Set 22.

** In this text, when no base is indicated for a numeral with two or more digits, it is understood to be base-ten numeral.

* Period names are not used in reading numbers shown in other bases. Be careful that pupils do not read such numerals as 2130_{four} as two thousand one hundred thirty. Such numbers do not exist in the base-four system.

Some Other Place-Value Systems

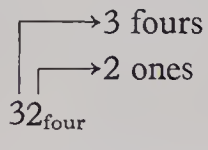
Extension [O]

A

Base Four

	Sixty-four's Place	Sixteen's Place	Four's Place	One's Place
a.			3	2
b.		2	1	2
c.	2	1	3	0

B



C

Base Ten	Base Four
1	1
2	2
3	3
4	10
5	_?_ 11
6	_?_ 12
7	_?_ 13
8	20
9	_?_ 21
10	_?_ 22
11	_?_ 23
12	_?_ 30
13	_?_ 31
14	32
15	_?_ 33
16	_?_ 100
17	101
18	_?_ 102

1. Box A. Three *base-four* numerals are shown. Name each place shown and give its value. one's place, 1; four's place, 4; sixteen's place, 16; sixty-four's place, 64 ^ In a *base-four system*, the place values are powers of _?_ 4 and the value of a place is _?_ 4 times the value of the place to the right.

* 2. Box A. The numeral for Ex. a is read, *three-two, base four*. Read the other numerals in this way. two-one-two, base four; two-one-three-zero, base four. 32_{four} = 3 fours + 2 ones

3. Box B. Explain the diagram. ^ Is it true that $32_{four} = (3 \times 4) + (2 \times 1)$, or 14? Yes ^ Find base-ten numerals for the numbers named by the other numerals in box A. $212_{four} = (2 \times 16) + (1 \times 4) + (2 \times 1)$, or 38 $2130_{four} = (2 \times 64) + (1 \times 16) + (3 \times 4) + (0 \times 1)$, or 156

4. On the board, copy and complete the list of base-four numerals in box C for the whole numbers from 1 through 18. See box C.

5. In a *base-seven system*, the place values are powers of _?_ 7 and the value of a place is _?_ 7 times the value of the place to the right.

6. In 102_{seven}, the digit 1 is in forty-nine's place, the digit 0 is in seven's _?_ place, and the digit 2 is in one's _?_ place. $102_{seven} = (1 \times$ _?_ 49 $) + (0 \times 7) + (2 \times$ _?_ 1 $)$ On the board, write a base-ten numeral for the number named by 102_{seven}. 51

7. On the board, express 534_{seven} in expanded form using powers of 7 and base-ten numerals. Then rename the number with a base-ten numeral. $(5 \times 7^2) + (3 \times 7) + (4 \times 1)$, or 270

8. In a *base-eight system*, the place values are powers of _?_ 8 and the value of a place is _?_ 8 times the value of the place to the right. Give the value of the first place; 1 the 2d place; 8 the 3d place. 64

9. On the board, express the whole numbers from 1 through 18, first in base seven and then in base eight.

Base seven: 1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15, 16, 20, 21, 22, 23, 24

Base eight: 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22

Teaching Pages 24 and 25

Pupil's Objectives

(a) To learn to apply the principles of place value to place-value systems other than those using base ten and base five; (b) to learn to read and write numerals in other bases; and (c) to write base-ten numerals for numbers expressed in other bases.

Background

Pupils should understand why they are studying different place-value systems. The purpose is not to become proficient in performing calculations in a given base, but rather to better understand our own base-ten system of numeration and to better understand other systems of numeration.

The teacher may wish to refer to the Background for Teaching Page 21. The principle that place values are powers of the base can be applied to systems of numeration in any base. Below are a few examples of place values up to the 4th place, expressed with base-ten numerals:

any base	$(base)^3$	$(base)^2$	*base*	*1*
base two	2^3 or 8	2^2 or 4	2	1
base three	3^3 or 27	3^2 or 9	3	1
base four	4^3 or 64	4^2 or 16	4	1
base six	6^3 or 216	6^2 or 36	6	1
base twelve	12^3 or 1,728	12^2 or 144	12	1

The base-five numeral 10_{five} represents the number five. The base-nine numeral 10_{nine} represents the number nine. In fact any numeral in the form 10_{base} represents the number that names the base. For example: 10_{five} = five; 10_{seven} = seven; 10_{two} = two, and so on. In any system, the greatest number that can be named by a single digit will be one less than the base: 1, in base *two*; 4, in base *five*; 9, in base *ten*; and so on.

The number of digits in a particular system is equal to the number named by the base. One of the digits will always be 0. Base ten has ten digits, base five has five digits, and base twelve has twelve digits. In base twelve, the letters *T* and *E* (the initial letters of *ten* and *eleven*) are often used as the two necessary additional digits. The digits in base twelve may be written as 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, T, E.

Any type of symbol can be used for a digit. The following is an example of counting in a base five system whose digits are represented by ○ (0), △ (1), □ (2), ▭ (3), and ▱ (4): △, □, ▭, ▱, △○, △△, △□, △▭, △▱, □○, □△, . . .

Pre-Book Lesson

Review the pattern of place values expressed as powers of the base of the system: . . . $(base)^3$, $(base)^2$, base, 1, . . . Ask pupils to illustrate the meaning by using different specific bases.

Using the Text Pages

• Ex. 1 and 2. Point out that the place values are powers of the base in the same pattern as in our base-ten system.

Show that each place in a numeral has a name, just as in our base-ten system. Make certain that pupils understand how to read a numeral in different bases. 111_{two} is read "*one-one-one, base two.*"

• Ex. 3. The type of notation used on pages 16 and 20 may be used in converting to numerals in the base-ten system, keeping in mind the place values in the specific numeration system involved. For example,

$$\begin{aligned} &\rightarrow 1 \times 8 = 8 \\ &\rightarrow 1 \times 4 = 4 \\ &\rightarrow 1 \times 2 = 2 \\ &\rightarrow 1 \times 1 = 1 \end{aligned}$$

$$1111_{two}$$

This notation should be used only after the standard horizontal notation has been mastered:

$$\begin{aligned} 1111_{two} &= (1 \times 2^3) + (1 \times 2^2) + (1 \times 2) + (1 \times 1) \\ &= (1 \times 8) + (1 \times 4) + (1 \times 2) + (1 \times 1) \\ &= 8 + 4 + 2 + 1 \\ &= 15 \end{aligned}$$

• Ex. 12–14, and Ex. 16–19. It may be of help if the pupil first tries to express a number in an expanded form in several different bases. For example:

$$126 = \_\ ?\ \__{\text{four}}$$

$$\begin{aligned} 126 &= (? \times 64) + (? \times 16) + (? \times 4) + (? \times 1) \\ &= (1 \times 64) + (3 \times 16) + (3 \times 4) + (2 \times 1) \\ &= 1332_{\text{four}} \end{aligned}$$

or

$$126 = \_\ ?\ \__{\text{eight}}$$

$$\begin{aligned} 126 &= (? \times 64) + (? \times 8) + (? \times 1) \\ &= (1 \times 64) + (7 \times 8) + (6 \times 1) \\ &= 176_{\text{eight}} \end{aligned}$$

• Ex. 20–27. Have the pupils use the following form of expanded notation in working these:

$$\begin{aligned} 323_{\text{four}} &= (3 \times 16) + (2 \times 4) + (3 \times 1) \\ &= 48 + 8 + 3 \\ &= 59 \end{aligned}$$

Individualizing Instruction

• *More capable pupils* can attempt to work exercises in bases greater than base ten. For example, they may be asked to express each of the following with a base-ten numeral:

36_{twelve}; 189_{eleven}; $2\text{TE}_{\text{twelve}}$; 578_{twenty}.

The symbols T and E represent *ten* and *eleven*.

They could be asked to record numerals for the counting numbers up to 100_{six} in base six, or 100_{nine} in base nine.

• *Slower learners* might enjoy making a game of inventing their own numeration systems, selecting the base of their choice and using their own symbols.

These pupils may also benefit from preparing a chart showing the place values (up to 4 places) for all bases from two through ten.

NOTES

* The number of digits used is equal to the number for the base.

10. Explain how the work in box D shows that both 537_{eight} and 351 name the same number. 537 eight means 5 sixty-fours + 3 eights + 7 ones, or 351

D

$$\begin{aligned} 537_{\text{eight}} &= (5 \times 64) + (3 \times 8) + (7 \times 1) \\ &= 320 + 24 + 7 \\ &= 351 \end{aligned}$$

11. To find a base-seven numeral for the number 58, we may think, *58 is equal to 1 forty-nine + 1 seven + 2 ones*, so a base-seven numeral for the number 58 is $\_?\_^{112}_{\text{seven}}$. Give a similar statement for finding a base-eight numeral for 67. 67 is equal to 1 sixty-four + 0 eights + 3 ones, so a base-eight numeral for 67 is 103_{eight}.

[W]

Express the whole numbers from 19 through 26 in

12. base four. ∨ 103,110,111,112,113,120,121,122 **13.** base seven. ∧ 25,26,30,31,32,33,34,35 **14.** base eight. ∨ 23,24,25,26,27,30,31,32

15. Copy and complete the following.

Base Ten	Base Four	Base Five	Base Seven	Base Eight
7	$\_?\_^{13}$	$\_?\_^{12}$	$\_?\_^{10}$	$\_?\_^{7}$
$\_?\_^{5}$	11	$\_?\_^{10}$	$\_?\_^{5}$	$\_?\_^{5}$
$\_?\_^{22}$	$\_?\_^{112}$	42	$\_?\_^{31}$	$\_?\_^{26}$
$\_?\_^{16}$	$\_?\_^{100}$	$\_?\_^{31}$	$\_?\_^{22}$	20
$\_?\_^{13}$	$\_?\_^{31}$	$\_?\_^{23}$	16	$\_?\_^{15}$

Express the number 50 in

16. base four. ∨ 302_{four} **17.** base five. ∨ 200_{five} **18.** base seven. ∨ 101_{seven} **19.** base eight. ∨ 62_{eight}

Ex. 20–27. Write a base-ten numeral for the number.

20. 332_{four} 62 **21.** 75_{eight} 61 **22.** 103_{five} 28 **23.** 444_{five} 124

24. 261_{seven} 141 **25.** 555_{seven} 285 **26.** 211_{five} 56 **27.** 66_{seven} 48

* How many digits are needed for expressing numbers in

28. base four? 4 **29.** base five? 5 **30.** base seven? 7 **31.** base eight? 8

♦ **Extra Examples.** Set 23.

* Most pupils will write an exponent form that expresses a power of a single number; however, if for Ex. 14 a pupil writes $2^2 \times 6^2$ it should be counted as correct. In fact, he may be commended for original thinking.

How Well Do You Remember?

Notation [W]

Ex. 1–5. Name the number first with a Roman numeral, then with a base-five numeral, and then with a base-eight numeral.

1. $23_{27\ \text{eight}}$ XXIII; 43_{five} 2. $30_{36\ \text{eight}}$ XXX; 110_{five} 3. $42_{52\ \text{eight}}$ XLII; 132_{five} 4. $64_{100\ \text{eight}}$ LXIV; 224_{five} 5. $92_{134\ \text{eight}}$ XCII; 332_{five}

Ex. 6–10. Write a base-ten numeral for the number.

6. XXXIV 34 7. CXLIX 149 8. 34_{five} 19 9. 21_{four} 9 10. 60_{seven} 42

* Ex. 11–18. Write an exponent form for the number.

11. $3 \times 3 \times 3 \times 3$ 3^4 12. 9×9 9^2 13. 25 5^2 14. 144 12^2

15. Two to the fifth power 2^5 16. 10,000 10^4 or 100^2 17. 125 5^3 18. 49 7^2

Ex. 19–22. Write the standard numeral for the number.

19. 2^5 32 20. 7^2 49 21. 8^2 64 22. $(2 \times 100) + (3 \times 10) + (7 \times 1)$ 237

Ex. 23–26. Copy and complete.

23. $16 = \_^{?}\_^{4}$ 24. $4^3 = 8^{-?-}$ 25. $64 = 2^{-?-}$ 26. $25 \times 25 = 5^{-?-}$

Try These Exercises

Enrichment [W]

XV = 15	$\overline{\text{XV}}$ = 15,000
XL = 40	$\overline{\text{XL}}$ = 40,000
CX = 110	$\overline{\text{CX}}$LIV = 110,054

You know that the Roman numeral XV names the number 15. When a bar is written above XV as shown in the box, the Roman numeral names the number 15,000. The bar indicates multiplication by 1,000. Look at the other numerals in the box.

Ex. 1–10. Find the number named by the Roman numeral.

1. $\overline{\text{XX}}$ 20,000 2. $\overline{\text{XIX}}$ 19,000 3. $\overline{\text{X}}$CDIV 10,404 4. $\overline{\text{CM}}$DVIII 900,508 5. $\overline{\text{MM}}$II 2,000,002
6. $\overline{\text{XV}}$DCXII 15,612 7. $\overline{\text{XC}}$CCVI 90,206 8. $\overline{\text{D}}$DCXLV 500,645 9. $\overline{\text{V}}$DLV 5,555 10. $\overline{\text{L}}$CDLXVII 50,467

Ex. 11–20. Write a Roman numeral for the number.

11. 12,050 $\overline{\text{X}}$IIL 12. 30,000 $\overline{\text{XXX}}$ 13. 52,645 $\overline{\text{L}}$IIDCXLV 14. 100,000 $\overline{\text{C}}$ 15. 600,500 $\overline{\text{DC}}$D
16. 10,925 $\overline{\text{X}}$CMXXV 17. 692,158 DCXC$\overline{\text{II}}$CLVIII 18. 60,323 $\overline{\text{LX}}$CCCXXIII 19. 32,547 $\overline{\text{XXX}}$IIDXLVII 20. 106,914 $\overline{\text{CV}}$ICMXIV

Teaching Page 26

Pupil's Objective

To test understanding of the various forms of notation previously studied.

Background

You may have a short review the day before the test so that any concepts not understood by some pupils can be presented again briefly. The main concepts being tested are:

(a) Changing from base-ten numerals to Roman numerals

(b) Changing from base-ten numerals to base-five numerals, and base-eight numerals

(c) Changing from a Roman numeral, a base-five numeral, a base-four numeral, and a base-seven numeral to a base-ten numeral

(d) Changing from a standard numeral to an exponent form

(e) Changing from an exponent form to a standard numeral

(f) Changing from an expanded form to a standard form

Enrichment exercises present the idea of multiplication by 1,000 with Roman numerals. Although the Roman system is additive, repetitive, and subtractive, computation within the system is very cumbersome. When a bar is written over a numeral or a group of numerals, it indicates that they are to be multiplied by 1,000. However, only this group of numerals is to be multiplied. For instance, $\overline{X}X$ means 10,010 not 20,000 which is written $\overline{XX}$.

Pre-Book Lesson

Before pupils open the book, make sure they understand how to work the test. You may wish to assign the enrichment section to *more capable pupils* at this time, or you may wish the entire class to attempt these exercises provided they have completed the test.

Using the Text Page

• Ex. 1–26. The tests should be corrected as soon as possible to reveal any areas of general weakness that might require reteaching or follow-up work going on to new work. The following table is prepared for your convenience in selecting work for reteaching or follow-up.

For errors in	*Review pages*
Ex. 1–5	13, 23–25
6–10	13, 22, 24, 25
11–18	19
19–22	18, 19
23–26	19

• Ex. 1–20. Before assigning the enrichment, have the pupils summarize the basic principles of the Roman system of numeration:

(a) The letters *I*, *X*, *C*, and *M* may be repeated, but no letter is written more than 3 times in succession.

(b) The letter *I* may be written before *V* and *X*; *X* may be written before *L* and *C*; and *C* may be written before *D* and *M* to indicate subtraction.

(c) A bar over a numeral indicates multiplication by 1,000.

• Ex. 1–10. Have the pupils interpret first the part of any numeral over which a bar is shown. They can then add to find the number named by the entire numeral.

• Ex. 11–20. To help other pupils work these exercises, you may suggest that they first rewrite them in partially expanded form:

For example:

$$60{,}323 = 60{,}000 + 323 = \overline{LX}CCCXXIII$$
$$106{,}914 = 106{,}000 + 914 = \overline{CVI}CMXIV$$

Individualizing Instruction

• Individual help may be provided to *all pupils* with the same general weakness by grouping them and assigning appropriate Extra Examples and Reteaching exercises. This is also a good time to check your teaching effectiveness and techniques and modify them where necessary.

• The exercises at the bottom of the page may be discussed with *slower learners* if there is sufficient time.

• *More capable pupils* may work Extra Activities or Supplementary Activities while you are doing remedial work with other pupils.

Teaching Page 27

Pupil's Objectives

(a) To resurvey the meaning of true and false mathematical sentences; and (b) to understand the meaning of the symbols $=$, $\neq$, $>$, and $<$.

Background

One of the most convenient methods of expressing ideas in mathematics is by the use of mathematical sentences. Among the most common expressions used in mathematical sentences are "is equal to," "is not equal to," "is greater than," and "is less than." In writing relatively simple mathematical sentences, we use symbols for numbers (0, 1, 2, 3, $\cdots$), symbols for operations on numbers ($+$, $-$, $\times$, $\div$), and symbols to express equalities and inequalities ($=$, $\neq$, $>$, $<$).

Pupils should be led to realize that proper use and interpretation of symbols in mathematics enables them to express fairly long or involved relationships concisely and to understand mathematical ideas more readily.

Every mathematical sentence is either true, false, or open. A mathematical sentence is true if it expresses a true idea and a mathematical sentence is false if it expresses a false idea. It is an open sentence if it must be completed before the truth or falsity of the idea it seeks to express can be determined. The following shows a set of true sentences, a set of false sentences, and a set of open sentences:

True	*False*
(a) $5 \times 6 > 28$	(a) $45 < 3 \times 15$
(b) $7 + 17 < 30$	(b) $2 \times 2 \times 2 = 6$
(c) $35 \div 5 = 7$	(c) $9 - 15 = 0$
(d) $25 \neq 5 + 5$	(d) $3 + 9 > 11$
(e) $15 \div 3 > 4$	(e) $28 \div 7 \neq 4$

Open

(a) $3 + 4 = \_ ? \_$
(b) $3 + \_ ? \_ = 7$
(c) $3 \_ ? \_ 4 = 7$
(d) $\_ ? \_ + 4 = 7$
(e) $36 \times \_ ? \_ = 72$

Pre-Book Lesson

• Have the pupils express each of the following as a mathematical sentence using numerals, symbols for operations, and symbols for equalities and inequalities: the sum of nine and three is equal to the sum of three and four; five is not equal to the sum of three and two; twenty divided by five is greater than nine subtracted from eleven; and the product of twelve and three is equal to thirty-six.

• Let pupils tell which are true and which are false sentences. Have the pupils write on the board true mathematical sentences using three elements from the set 4, 8, 12, 32 and

(a) $+$ and $=$	(b) $\times$ and $\neq$	(c) $\div$ and $\neq$
(d) $-$ and $\neq$	(e) $\div$ and $=$	(f) $\times$ and $=$

Sample solutions: (a) $4 + 8 = 12$ (e) $32 \div 8 = 4$

Using the Text Page

• Ex. 1. Have the pupils give more examples of true mathematical sentences.

• Ex. 2 and 4. Have the pupils indicate what changes might be made in the sentences in box A and box B to make the sentences true. Remind pupils that a change may be made in a numeral or an operational symbol.

• Ex. 6. Have the pupils write on the board false mathematical sentences using $=$, $\neq$, $>$, $<$.

Individualizing Instruction

• Ask the *more capable pupils* to indicate for each of the following whether the entire statement is true or false. (a, b, and c represent whole numbers.) They may actually substitute numerals for the letters if they find it necessary.

(a) If $a + b = c$ is true then $a + b \neq c$ is always false.

(b) If $b > a$ is true, then $a < b$ is always true.

(c) If $b > a$ is true, then $b \neq a$ is always true.

(d) If $a \neq b$ is true, then $a > b$ is always true.

• Before assigning the written work to the *slower learners*, have them practice with more examples similar to those in box B and box C.

True and False Mathematical Sentences

Resurvey [O]

1. Sentences such as those in box A are called *mathematical sentences*. If a mathematical sentence expresses a true idea, it is called a *true mathematical sentence*. Which of the sentences in box A are true mathematical sentences? a, c, d

A

a. $2 + 3 = 5$
b. $6 \div 2 = 4$
c. $2 + 7 = 8 + 1$
d. $10 \neq 5 \times 5$
e. $3 - 2 = 0$

2. If a mathematical sentence expresses a false idea, it is called a *false mathematical sentence*. Which of the sentences in box A are false sentences? b, e

3. Tell how to complete the following so that they will be true mathematical sentences.

a. $27 + \_?\_^{4} = 31$ **b.** $50 \_?\_^{\div} 5 = 10$ **c.** $\_?\_^{2} \times 13 = 26$

4. The symbol $<$ means "is less than." The sentence $2 \times 3 < 5 + 4$ is a true sentence because the number named by 2×3 is $\_?\_^{\text{less}}$ than the number named by $5 + 4$. Which of the sentences in box B are true sentences? a, c Which are false sentences? b

B

a. $24 < 5 + 25$
b. $68 < 67$
c. $60 \div 6 < 20$

5. The symbol $>$ means "is greater than." Why is the sentence $5 \times 10 > 4 \times 8$ a true sentence? The number named by 5×10 is greater than the number named by 4×8. Which of the sentences in box C are true sentences? a, b

C

a. $59 > 50 + 4$
b. $3 \times 12 > 15$
c. $2 + 3 > 5$

6. On the board, copy and complete each of Ex. a–c by writing $<$ or $>$ to make a true sentence.

a. $20 \_?\_^{<} 18 + 3$ **b.** $14 + 69 \_?\_^{>} 75$ **c.** $4 \times 5 \_?\_^{<} 3 \times 7$

[W]

* Ex. 7–12. Write *T* if the sentence is a true sentence and write *F* if it is a false sentence.

7. $2 + 7 = 3 \times 3$ T

8. $4 + 5 \neq 3 \times 3$ F

9. $16 \div 4 > 5$ F

10. $7 \div 7 \neq 3 \div 3$ F

11. $2 \times 30 = 3 \times 20$ T

12. $6 \times 8 < 49$ T

◆ **Extra Examples.** Set 24.

* Explain that the symbol $\neq$ means "is not equal to."

* Parentheses around 4 + 2 indicates that it is to be thought of as one number.

** Thinking of names for zero is an important skill in simplifying computation. For example, finding 93 + 78 using 100 + 71 involves 7 – 7 which is a name for zero.

Thinking about Properties of Addition

Resurvey [O]

1. Does $2 + 3$ name the same number as $3 + 2$? Yes Then is $2 + 3 = 3 + 2$ a true mathematical sentence? Yes This is an illustration of the *Commutative Property of Addition.* Complete the following so that each will be a true sentence.

a. $4 + 15 = 15 + \_?\_$ 4 **b.** $\_?\_ + 33 = 33 + 62$

* 2. At the board, find $(4 + 2) + 8$. $(4+2)+8=6+8=14$ Did you first perform the operation indicated within the parentheses? Yes Now find $4 + (2 + 8)$. 14 Is the following a true mathematical sentence? $(4 + 2) + 8 = 4 + (2 + 8)$ Yes This is an illustration of the *Associative Property of Addition.* Complete the following so that each will be a true mathematical sentence.

a. $(14 + 20) + 10 = 14 + (20 + \_?\_)$ 10

b. $65 + (4 + \_?\_) = (65 + 4) + 36$ 36

3. The sentence $(2 + 3) + 4 = 4 + (2 + 3)$ is an illustration of the $\_?\_$ Commutative Property of Addition. Explain. The sentence says $5+4=4+5$.

The Identity Element for Addition

Resurvey [O]

1. $5 + 0 = \_?\_$ 5, $35 + 0 = \_?\_$ 35, $0 + 15 = \_?\_$ 15, $0 + 0 = \_?\_$ 0, and $1 + 0 = \_?\_$ 1. If a and b represent two whole numbers and $a + b = a$, what number must b represent? 0

> **If one of two addends is the number zero, the sum of the two addends is the same number as the other addend. Because this is true, zero is called the *identity element for addition.***

** 2. The standard numeral for the identity element for addition is 0. Can the identity element for addition also be named $5 - 5$? Yes $13 - 13$? Yes $35 - (30 + 5)$? Yes

[W]

*** Ex. 3–6. Copy and complete to make a true sentence.

3. $0 + \_?\_ = 41$ 41 4. $41 - (1 + \_?\_) = 0$ 40

5. $0 + 9 = \_?\_ + 0$ 9 6. $2 + (0 + 4) = (\_?\_ + 0) + 4$ 2

*** An open sentence that is always true regardless of what number is substituted for the variable is called an identity. For example, $b + \square = \square + b$.

Teaching Page 28

Pupil's Objectives

To review (a) the Commutative Property and the Associative Property of Addition for whole numbers; and (b) the identity element for addition.

Background

Addition is an operation on two numbers called *addends*, to produce a unique third number called their *sum*. This sum may be the same number as one of the addends. The addends may be considered the cardinal numbers of two disjoint sets, and the sum of these numbers may be defined as the cardinal number of the union of the two disjoint sets.

The operation union on two sets is commutative. That is, $A \cup B = B \cup A$. The sum of the cardinal numbers of set A and set B is equal to the sum of the cardinal numbers of set B and set A. This may be generalized as $a + b = b + a$, where a and b represent any whole numbers. This is called the *Commutative Property of Addition.*

Addition is a *binary* operation. We can operate on only *two* numbers at a time. To find the sum of three numbers, we must find first the sum of two of the numbers, and then add that sum and the third number. However, there is a choice as to how this is to be performed. We may add the third number to the sum of the first two numbers or we may add the first number to the sum of the second and third numbers. The result will be the same either way. This may be generalized as $(a + b) + c = a + (b + c)$, and is called the *Associative Property of Addition.*

The sum of any whole number and zero is the given whole number. For this reason, zero is called the *identity element for addition.*

Pre-Book Lesson

Give the pupils a list of some activities and ask them to determine which are commutative. That is, may they be performed in any order without affecting the result? Examples of some activities are: putting on your shoes and putting on your socks; cooking a meal and eating a meal; putting on your rubbers and putting on your hat. Ask the same question about any pair of addends.

Using the Text Page

• Ex. 1. Ask the pupils if the sum of the number of boys in the room and the number of girls in the room is equal to the sum of the number of girls in the room and the number of boys in the room. From this discussion they can be lead to realize that the sum is the same regardless of the order of the addends.

• Ex. 2. The associative property may be introduced by showing three numbers on the chalkboard and asking pupils to add them in any order. Help them to see that, although the correct sum is the same in every case, not all pupils started the work by adding the same two numbers.

• Ex. 3. Use this exercise to help pupils distinguish between the Commutative and the Associative Property of Addition.

• Ex. 1–6. The purpose of these exercises is to have the pupils learn to recognize the Identity Element for Addition when expressed with various numerals, some the standard form (0), and others not.

Individualizing Instruction

• *More capable pupils* may be asked to try to determine whether the commutative property and the associative property apply for operations other than addition.

• *All pupils* can be asked to think of practical uses of the commutative and associative properties such as in checking addition and in changing the order and grouping of addends to make addition easier and quicker. For example:

$$17 + (19 + 23) = (17 + 23) + 19$$
$$17 + 42 = 40 + 19$$

Teaching Page 29

Pupil's Objectives

(a) To review mathematical sentences, including open sentences and true and false sentences; and (b) to learn the meaning of solving an open sentence.

Background

Pupils are familiar with the four kinds of sentences studied in their English classes. In mathematics we are concerned with the declarative sentence. Each of the following is a declarative sentence:

George Washington was our first president.
He is 11 years old.
The sum of 21 and 14 is 77.

The first sentence is *true*, the third sentence is *false*. The truth or falsity of the second sentence depends upon replacing the pronoun *he* by the name of a person whose age we know. Such a sentence is called an *open sentence*.

We can write open mathematical sentences using frames ($\square$, $\square$, $\bigcirc$) or letters (x, y, n) to represent the unknown numbers. These symbols are sometimes called *variables* because the numbers they stand for vary when used in different open sentences.

The number or numbers (replacements for a variable) that make a given open sentence true is often called the *solution set* of the open sentence. On occasion, the solution set may be the empty set. Below are some examples of open sentences and their solution sets: (Each letter represents a whole number.)

Open Sentence	*Solution Set*
(a) $3 + n = 17$	$\{14\}$
(b) $x + 0 = 0$	$\{0\}$
(c) $y = y + 3$	$\{\ \}$
(d) $n > 7$	$\{8, 9, 10, \cdots\}$

Remember, terminology used in the Background is not always used on the pupil's page, nor in class discussion. At times, you may wish to use material from the Background with *more capable pupils* only and other times with the entire class.

Pre-Book Lesson

- Ask for examples of sentences that are true, sentences that are false, and sentences that because of insufficient information, cannot be called true or false.
- Ask for the information necessary to make open sentences such as those given in the Background true or false.

Using the Text Page

- Ex. 1. Have the pupils realize that there is only one number that can be substituted for n in the open sentence $n = 3 + 12$ that will make it a true sentence.
- Ex. 2. In open sentences involving two letters such as $a = 8 - b$, show how the value represented by one letter depends upon the value represented by the other letter if the sentence is to be true. Point out that the solution set of the open sentence $n < 8$ depends on the restrictions placed on the set of possible replacements. In this example, the set of possible replacements should be restricted to whole numbers. Vary the restrictions to show changes in the solution set.
- Ex. 5. Ask what numbers n could represent if the given set were the set of all whole numbers.
- Ex. 6–14. You may want to use a variety of letters and frames as variables in the open sentences. You may wish to establish that for each sentence there is only one number that makes it true.

Individualizing Instruction

- *More capable pupils* can be asked for examples of open sentences whose solution set is the empty set, and examples of open sentences whose solution set is the set of all whole numbers of all counting numbers.
- *All pupils* should be given practice writing true mathematical sentences, false mathematical sentences, and open mathematical sentences. In this connection, let pupils work in pairs and exchange their work with a partner who will indicate the true sentences, indicate the false sentences, and solve the open sentences.

* Encourage pupils to use n, x, or y in mathematical sentences unless otherwise instructed.

Solving Mathematical Sentences

Resurvey [O]

* **1.** An *open mathematical sentence* is a mathematical sentence that has one or more symbols other than numerals which represent numbers. It is *open* because we cannot determine whether it is a true sentence or a false sentence until we know what numbers are represented by the symbols.

a. If n represents 10, is $n = 3 + 12$ a true sentence? No

b. If n represents 15, is $n = 3 + 12$ a true sentence? Yes

2. If a represents 2 and b represents 6, is $a = 8 - b$ a true mathematical sentence? Yes ^ If a represents 5, what must b represent for $a = 8 - b$ to be a true sentence? 3

** **3.** Suppose an open mathematical sentence using "=" contains the letter n and no other letters. The instruction *find the number represented by n* or the instruction *solve* means that we are to find the numbers that make the sentence a true sentence. At the board, solve each sentence in the box (find the number represented by n for each sentence). a. 9; b. 23; c. 10; d. 8

a. $n = 4 + 5$
b. $3 + 20 = n$
c. $n = 15 - 5$
d. $n = 27 - 19$

4. If n represents 2, then is $n < 8$ a true mathematical sentence? Yes ^ Name three other whole numbers that n might represent to make $n < 8$ a true mathematical sentence. Any three of 0, 1, 2, 3, 4, 5, 6, 7

5. Which members of $\{2, 4, 8, 10, 37\}$ make $n > 6$ a true mathematical sentence? 8, 10, 37

[W]

Ex. 6–14. Solve.

6. $n = 35 + 47$ 82

7. $n = 200 + 48$ 248

8. $n = 35 + 754$ 789

9. $n = 22 + 1{,}400$ 1,422

10. $25 + 76 = n$ 101

11. $654 + 748 = n$ 1,402

12. $n = 2{,}344 - 1{,}202$ 1,142 ^

13. $n = 57 - 17$ 40

14. $346 - 108 = n$ 238

15. List within braces the whole numbers which make $n < 7$ a true mathematical sentence. {0, 1, 2, 3, 4, 5, 6}

▲ **Reteaching.** Set 4. ♦ **Extra Examples.** Sets 25–27.

** When solving a mathematical sentence, we select from a set of numbers, either specified or assumed, those numbers which make the sentence true. The set of numbers selected is sometimes called the solution set.

* Unless otherwise indicated, reference is to the number that makes the sentence true. An interesting exercise is to ask pupils to describe sets of numbers for which each of Ex. 8-13 is false.

Addends-Sum Relationship

Resurvey [O]

A	$5 = 3 + 2$ $5 - 3 = 2$ $5 - 2 = 3$ $5 = 2 + 3$
B	$n - 12 = 25$ $n = 25 + 12$ $= 37$
C	$15 - n = 4$ $n = 15 - 4$ $= 11$

1. The four mathematical sentences in box A illustrate the *addends-sum relationship*. In each sentence, the numeral 5 names the sum and the numerals 3 and 2 name the addends. What operation is used to find the sum of two addends? Addition

2. In the sentence $n - 12 = 25$, does n represent the sum? Yes Explain the work in box B. When the addends are known, we add to find the sum. 25+12=37

3. At the board, find the number represented by n for each of Ex. a–c as in box B.

a. $n - 22 = 43$ 65 **b.** $n - 65 = 37$ 102 **c.** $n - 123 = 308$ 431

4. In the mathematical sentence $15 - n = 4$, what numeral names the sum? 15 Does n represent an addend? Yes We say that n represents the *unknown addend* and that 4 names the *known addend*.

5. Study the work in box C. What operation was used to find the unknown addend? subtraction To find the unknown addend, we subtract the known addend from the sum.

6. On the board, write $126 - n = 48$ and then write three other mathematical sentences to illustrate the addends-sum relationship. See left. For these sentences, tell whether n represents the sum or an addend and tell which operation to use to find the number represented by n. an addend Subtraction Now do the same for $x - 78 = 48$ and $126 - y = 78$. See left.

126−n=48
n=126−48
n+48=126
48+n=126

7. For each of Ex. 8–13, tell whether the letter represents the sum or an addend. 8. sum; 9. addend; 10. addend; 11. sum; 12. sum; 13. addend

x−78=48
x=48+78
x=78+48
x−48=78
x represents the sum.
Use addition.

126−y=78
y+78=126
78+y=126
y=126−78
y represents an addend.
Use subtraction.

[W]

* Ex. 8–13. Find the number represented by the letter.

8. $n - 23 = 10$ 33 **9.** $x + 12 = 30$ 18 **10.** $643 - n = 400$ 243

11. $n = 47 + 88$ 135 **12.** $56 = y - 20$ 76 **13.** $207 - 199 = n$ 8

Teaching Pages 30 and 31

Pupil's Objectives

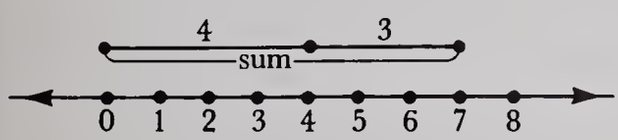
(a) To review solving open sentences; (b) to review the applying of the addends-sum relationship; and (c) to learn the meaning and use of the symbols $\leq$ and $\geq$.

Background

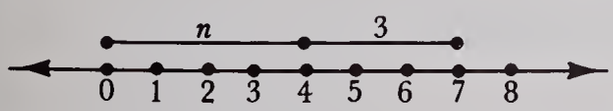
Addition is a binary operation in which two numbers, called the *addends*, are combined to result in a third number, called the *sum*. In $a + b = c$, a and b represent the addends, and c represents the sum.

The relationship between the addends and the sum is applicable to the operation subtraction. A subtraction example may be described as one in which the sum and one addend are known. The "remainder" or "difference" to be found may be defined as an unknown addend. While the unknown addend is obtained by subtracting the known addend from the sum, the reasoning employed is still: *known addend* + *unknown addend* = *sum*.

To illustrate this idea, the addends-sum relationship $4 + 3 = 7$ may be expressed by use of a number line picture:



Now let us use the number-line picture to illustrate the subtraction $7 - 3 = n$.



From this drawing, we can see that n may be considered as an unknown addend, 3 the known addend, and 7 the sum. To generalize, $a - b = c$, or sum − known addend = unknown addend.

There are four ways of expressing a single addends-sum relationship:

(1) $4 + 3 = 7$ (3) $7 - 3 = 4$
(2) $7 - 4 = 3$ (4) $3 + 4 = 7$

The symbol $<$ means "is less than," and the symbol $>$ means "is greater than." They are found in many mathematical sentences. Sometimes, we may wish to consider two numbers at the same time: those numbers less than a given number, and also the number which is equal to the given number. The symbol $\leq$ means "is less than or equal to." Such sentences as $3 \leq 15$ and $12 \leq 12$, are true. An open sentence such as $n \leq 5$ for n a whole number is true when n is replaced by 0, 1, 2, 3, 4, and also 5. However, only one of the two indicated relationships needs to be true for the sentence to be true. The symbol $\geq$ means "is greater than or equal to."

Pre-Book Lesson

Review meaning of addend and sum. Select an addends-sum relationship and express it in as many ways as possible.

Using the Text Pages

• Ex. 1. If any pupils have difficulty understanding the addends-sum relationship when expressed in the four different ways listed in box A on the pupil's page, it may help to use objects, including pupils in the class, to demonstrate this relationship. For example,

no. of pupils = no. of boys + no. of girls
no. of pupils = no. of girls + no. of boys
no. of pupils − no. of boys = no. of girls
no. of pupils − no. of girls = no. of boys

• Ex. 4–5. The use of a number-line picture may help to make this relationship clear. When the sum is unknown, it is determined by adding the addends. When an addend is unknown, it is determined by subtracting the known addend from the sum. In the first instance, we may start at zero and "jump" to the right to find the sum. In the second instance, we may start at the sum and "jump" to the left to locate the unknown addend.

• Ex. 8–13. Have the pupils use boxes B and C as guides in working these exercises.

• Ex. 14–29. While many of these sentences may be solved as presented, some of them should be rewritten as indicated in boxes B and C.

Individualizing Instruction

• *More capable pupils* can be asked to summarize in words how to solve open sentences of the type $n - 23 = 10$, in which the sum is unknown, and of the type $643 - n = 400$, in which one of the addends is unknown.

When these pupils have completed work on the addends-sum relationship, they may do the enrichment exercises. You may wish to discuss several of the exercises with them if time and conditions permit.

• *All pupils* can be given additional practice at solving open sentences by applying the addends-sum relationship, including open sentences in which the variables are various letters or frames. Pupils may also be asked to express each relationship in four different ways.

• *Slower learners* may be given very simple verbal problems and asked to write a mathematical sentence expressing the addends-sum relationship in the problem. This will serve as a preparation for the problem-solving on the next pupil's page.

NOTES

* If a compound sentence is formed with or, then it is true if either part is true. Since $x \leq 3$ is read "x is less than 3 or x is equal to 3," it is true if either $x < 3$ is true or $x = 3$ is true.

Ex. 14–29. Copy the open sentence, write *Sum* or *Addend* to indicate whether the letter represents the sum or an addend, and then solve the sentence.

14. $n = 736 + 166$ Sum; 902

15. $6{,}372 - n = 987$ Addend; 5,385

16. $9{,}215 + 5{,}024 = x$ Sum; 14,239

17. $y + 4{,}767 = 5{,}442$ Addend; 675

18. $n - 3{,}372 = 4{,}682$ Sum; 8,054

19. $x = 227 + 475$ Sum; 702

20. $x - 2{,}475 = 9{,}016$ Sum; 11,491

21. $6{,}887 + 6{,}519 = x$ Sum; 13,406

22. $8{,}685 - n = 7{,}693$ Addend; 992

23. $26{,}680 - 6{,}401 = n$ Addend; 20,279

24. $n = 785 + 916 + 397$ Sum; 2,098

25. $n = 274 + 586 + 654$ Sum; 1,514

26. $896 + 358 + 257 = y$ Sum; 1,511

27. $x = 7{,}854 + 2{,}968 + 1{,}073$ Sum; 11,895

28. $y = 19{,}975 - 3{,}100$ Addend; 16,875

29. $n = 10{,}682 + 8{,}497 + 965$ Sum; 20,144

◆ **Extra Examples.** Sets 28-32.

Try These Exercises

Enrichment [W]

* The symbol $\leq$ means "is less than or is equal to." Therefore, $5 \leq 7$ is a true sentence because 5 is less than 7. Also, $8 \leq 8$ is a true sentence because 8 is equal to 8.

The symbol $\geq$ means "is greater than or is equal to." Therefore, both $9 \geq 4$ and $6 \geq 6$ are true sentences.

Ex. 1–6. Write *T* or *F*.

1. $28 + 45 < 73$ F **2.** $28 + 45 \leq 73$ T **3.** $47 \leq 82$ T

4. $782 - 122 \geq 600$ T **5.** $46 \geq 25 + 21$ T **6.** $497 \leq 400 + 97$ T

7. List within braces the whole numbers which make $n \leq 10$ a true sentence. {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

8. List within braces the whole numbers less than 20 which make $n \geq 12$ a true sentence. {12, 13, 14, 15, 16, 17, 18, 19}

9. Which members of {3, 6, 7, 8, 10, 15} make

a. $n + 3 \geq 10$ a true sentence? 7, 8, 10, 15

b. $x - 3 \leq 5$ a true sentence? 3, 6, 7, 8

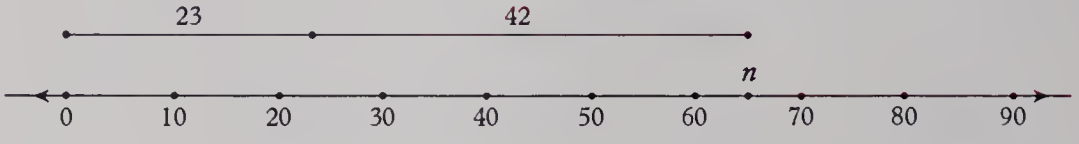
Solving Problems

Resurvey; one-step problems [O]

1. Last summer Jerry and his parents spent two days at the seashore. Jerry collected 23 shells the first day and 42 the second day. How many shells did he collect in all?

a. If n represents the number of shells that Jerry collected, explain how the mathematical sentence $n = 23 + 42$ shows the relationship expressed in the problem. Key idea: It shows the addends-sum relationship. Find the number represented by n. 65

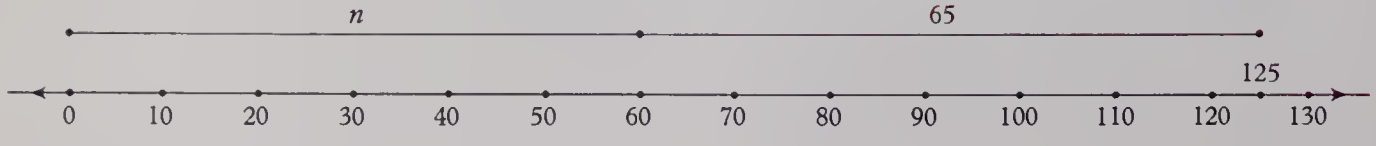
b. Explain how the number-line picture below shows the answer for the problem. Key idea: It shows the addends-sum relationship.



2. After Jerry added the 65 shells to his shell collection at home, he found he had 125 shells in all. How many shells did Jerry have before going to the seashore?

a. Explain why the mathematical sentence $n + 65 = 125$ may be used to find the answer for the problem. Key idea: It shows the addends-sum relationship. At the board, find the number represented by n. 60

b. Explain how the number-line picture below shows the answer for the problem. Key idea: It shows the addends-sum relationship.



Teaching Pages 32 and 33

Pupil's Objective

To review solving one-step problems by use of open sentences and by applying the addends-sum relationship.

Background

In solving a verbal problem, difficulty is sometimes encountered by the pupil in deciding which of the operations of arithmetic should be used to obtain the correct answer. The pupil may also have a reading problem, or may have difficulty determining which information is pertinent to the solution of the problem.

There is no single prescription that can be applied to solving all verbal problems. However, there is a general method of solution applicable to solving verbal problems in which there exists an addends-sum relationship.

In any verbal problem, the pupil should first examine the problem to determine what facts are given, which of the given facts are essential to the solution of the problem, and what number fact is to be found. He should then determine which operation is indicated by the given facts.

Next, he should express the relationship existing among the given facts in the form of an open sentence. Since there are four basic ways in which the addends-sum relationship among two addends and a sum may be expressed with an open sentence, it may be advisable to concentrate on just two forms at the beginning. If the *sum* is the unknown, let the relationship be expressed with an open sentence of the type $n = a + b$ where a and b represent the two known addends. If the unknown is an *addend*, let the relationship be expressed with an open sentence of the type $n = a - b$. In both $n = a + b$ and $n = a - b$, the solution depends upon a single addition or subtraction without first rewriting the sentence. Later, the pupils may be encouraged to express an addends-sum relationship in other ways such as $a - n = b$, and $n + a = b$.

To help pupils recognize an addends-sum relationship that may exist among the numbers in a problem, let them determine if some quantity is being increased, or if two quantities are being compared as to which is greater or which is less.

This method of solving a verbal problem may be summarized as follows:

(1) Read the problem and determine what facts are given and what fact is to be found.

(2) Determine if there exists an addends-sum relationship among the given numbers and the number to be found.

(3) Express the addends-sum relationship in the form of an open sentence.

(4) Find the solution to the open sentence by addition or subtraction.

Pupils should have an awareness of the procedure for solving verbal problems outlined above. However, they should not be required to verbalize the procedure in such a rigorous series of steps.

Pre-Book Lesson

• There is often a greater difference among pupils as to their ability to master the concept of solving verbal problems than to their ability to master other concepts in arithmetic.

• Success in solving verbal problems depends upon the pupil's ability to (a) read with understanding; (b) recognize the addends-sum relationship; (c) express this relationship as an open sentence; and (d) find the solution to the open sentence. When a pupil is having difficulty mastering the concept of solving verbal problems, it will help to determine in which of the above areas the difficulty is centered.

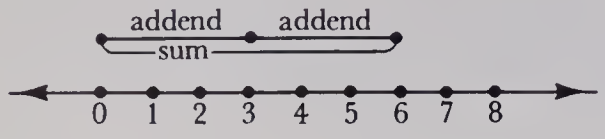
• Give the pupils practice at recognizing an addends-sum relationship in a verbal problem. Lead them to the discovery that an addends-sum relationship usually involves a quantity being increased or decreased, or the relative sizes of two quantities being compared. Devise simple problems in which a quantity is being increased or decreased, or in which two quantities are being compared, and present these to the pupils orally.

Using the Text Pages

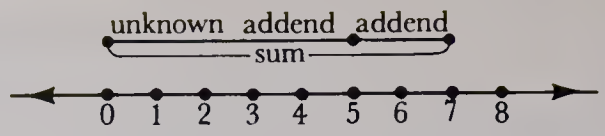
• Ex. 1–8. A quantity is being increased, a quantity is being decreased, or two quantities are

being compared to determine how much greater or how much less one is than the other. Therefore, an addends-sum relationship exists among the number facts.

• Ex. 1 and 7 are of the type in which the sum is unknown and the solution involves simply adding the known addends.



• Ex. 2, 3, 4, 5, 6, and 8 are of the type in which one addend is unknown and may be found



by subtracting the known addend from the sum.

Individualizing Instruction

• *More capable pupils* may be asked to summarize in their own words the procedure for solving verbal problems of the type involving an addends-sum relationship.

• *All pupils* can be given a set of simple open sentences such as $24 - n = 18$ and be asked to make up a verbal problem to fit each open sentence.

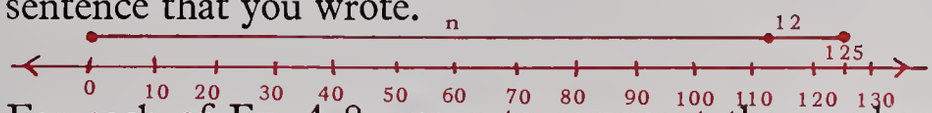
NOTES

3. When Jerry examined his collection of 125 shells, he decided that 12 of them were not worth keeping so he discarded those 12 shells. How many shells did Jerry have then?

a. On the board, write a mathematical sentence which shows the relationship expressed in the problem. n=125−12

b. Solve the sentence and give the answer for the problem. 113

c. Now draw a number-line picture for the mathematical sentence that you wrote.



[W]

For each of Ex. 4–8, use n to represent the number which answers the problem. Write a mathematical sentence involving n which shows the relationship expressed in the problem and then find the answer for the problem.

4. Jerry took 16 snapshots while at the seashore but when the film was developed, only 7 were not overexposed. How many were overexposed? n=16−7; 9 snapshots

5. When they started on their trip to the seashore, the odometer in the car registered 12,345 miles. When they arrived at the seashore, it registered 12,672 miles. How many miles was it from their home to the seashore? n=12,672−12,345; 327

6. Jerry spent $4.75 for some fishing tackle last summer. The summer before, he had spent $6.30 for fishing tackle. How much less did he spend for tackle last summer than the summer before? n=6.30−4.75; $1.55

7. In order to display his shell collection, Jerry bought a glass case for $3.25, some material to line the bottom of the case for $1.89, and a stand to set the case on for $4.60. How much did he spend in all? n=3.25+1.89+4.60; $9.74

8. Tom offered to buy Jerry's display case and stand for $12.25. At the hobby shop, Jerry saw a larger case and stand for $14.98. If he accepts Tom's offer, how much more will Jerry need to buy the larger case and stand? n=14.98−12.25; $2.73

♦ **Extra Problems.** Set 138. These sets start on page 359.

* Review use of parentheses by writing and solving several mathematical sentences for the pupils.

Solving Two-Step Problems

[O]

1. To earn money for the Science Club, its members bought 200 calendars to sell. The girls sold 122 calendars and the boys sold 65. **a.** How many calendars were sold in all? **b.** How many calendars were left?

This problem has two questions. The answer for question a is needed in order to answer question b. In the mathematical sentence $s = 122 + 65$ for question a, does s represent a sum or an addend? Sum Name the number represented by s. 187 Explain why the mathematical sentence $l = 200 - 187$ can be used to find the answer for question b. Key idea: It shows the addends-sum relationship. Below is the same problem with one question asked and the other question "hidden."

To earn money for the Science Club, its members bought 200 calendars to sell. If the girls sold 122 calendars and the boys sold 65, how many were left?

There are still two steps in the solving of the problem. The first step is to answer the hidden question, "How many calendars were sold in all?" which was the question asked in Ex. 1a.

* 2. Sometimes we can show both steps of a two-step problem by writing just one mathematical sentence using parentheses. For example, the mathematical sentence $n = 200 - (122 + 65)$ can be used to find the answer for Ex. 1b. At the board, find the number represented by n. 13 Remember to perform first the operation indicated within the parentheses.

Teaching Pages 34 and 35

Pupil's Objectives

(a) To solve two-step problems by using a single open sentence; (b) to use parentheses as an aid in solving two-step problems; and (c) to provide practice in addition and subtraction.

Background

In the open sentences previously studied, the solution depended upon a single operation performed on two numbers to obtain the answer. In every case, either two addends or a sum and one addend were given in the original problem. However, in some problems, it may be necessary to perform some operation on two or more of the numbers given in the problem before the sum or the known addend can be shown in the open sentence.

For example, consider the problem "Two boards whose lengths were about 8 inches and 12 inches respectively were sawed from a board 36 inches in length. About how long is the piece that is left?" This problem contains an addends-sum relationship of the type unknown addend = sum − known addend. The sum is 36. The known addend, however, is the sum of the measures of the two lengths sawed off. Either this sum must first be determined and then shown in the open sentence, or else the open sentence must contain provision for determining the sum of these two measures before it is subtracted from 36. This may be shown as in the following open sentence:

$$n = 36 - (8 + 12)$$

The known addend, 20, obtained by adding 8 + 12, may be subtracted from 36 to obtain the unknown addend.

A second, but similar, type of two-step problem is one in which two or more numbers used in the problem must be combined under some operation before it can be shown for the sum in the open sentence.

For example, consider the problem "On Monday Harry bought 18 marbles and on Tuesday he bought 24 marbles. Tuesday afternoon he sold 10 of his marbles to a friend. How many marbles did he then have left?" In this problem, the total number of marbles purchased must be determined first before the number can be shown as the sum in the open sentence. This may be indicated by: $n = (18 + 24) - 10$.

A second open sentence showing the sum of 18 and 24 must then be written ($n = 42 - 10$) before the known addend, 10, can be subtracted from the sum.

Pre-Book Lesson

- Have the pupils practice at "translating" verbal sentences into open mathematical sentences.
- Have the pupils work examples of mathematical phrases that contain parentheses such as 200 − (100 + 25). Be sure they understand that the operation within the parentheses is to be performed first.
- Write several sentences of the type:

$n = (15 + 7) - 12$, or $n = 45 - (30 + 6)$

on the board and have pupils ask two questions or present a two-step mathematical problem for which each given open sentence would be appropriate.

Using the Text Pages

- Ex. 2, 3, 5, and 6 are of the type in which the known addend must be shown as a sum of two numbers within parentheses in the open sentence.
- Ex. 4 is of the type in which the sum must be expressed as the sum of two numbers given in the problem before it is in the final open sentence. That is, unknown addend = sum − known addend:

$$n = (37 + 13) - 19$$

This sum must be determined before the subtraction is performed.

Be prepared to discuss the open sentence $n = (37 - 19) + 13$ if some pupil should use it in connection with Ex. 4.

- Ex. 1–6. At the bottom of the page, the pupils may verify their addition by applying the Commutative Property of Addition.

• Ex. 7–12. At the bottom of the page, the pupils can verify their subtraction by applying the addends-sum relationship.

Individualizing Instruction

• *More capable pupils* may be given more difficult verbal problems in which the open sentence would be of the type: $n = (17 + 14) - (16 + 3)$

• *Slower learners* may need assistance in reading and understanding these two-step problems. They may need to reword sentences in order to be able to select the mathematical data necessary for solving the problem. Give these pupils a few practical examples with parentheses.

NOTES

* In all computation pupils should be encouraged to check their answers by estimating either before or after they have found the exact answer.

[W]

Each of the following problems has a hidden question which must be answered before you can find the answer for the problem. For each problem, write the hidden question and a mathematical sentence using parentheses. The work in the parentheses should answer the hidden question and solving the sentence should provide the answer for the problem. Solve each problem.

3. At the end of the year, the Gold Thimble Sewing Club had $8.50 in the treasury, so its members decided to have a party. They spent $2.70 for ice cream and $1.92 for cupcakes. How much money was left in the treasury? How much did they spend in all? $n = 8.50 - (2.70 + 1.92)$; $3.88

4. On Monday, Mrs. Bishop had 37 pumpkins for sale at her roadside stand. That day she sold 19 pumpkins. Tuesday morning she put 13 more pumpkins with the unsold ones. How many pumpkins in all were for sale Tuesday morning? How mony pumpkins were left from Monday? $n = 13 + (37 - 19)$; 31

5. Three boards with lengths 16 in., 22 in., and 28 in. were sawed from a board 72 in. long. What was the length of the piece that was left? Haw much was sawed aff in all? $n = 72 - (16 + 22 + 28)$; 6 in.

6. Eric has 16 French stamps and 18 English stamps. The rest of his stamps are United States stamps. If he has 78 stamps in all, how many are United States stamps? Haw many French and English stamps daes he have? $n = 78 - (16 + 18)$; 44

♦ **Extra Problems.** Set 139.

* To Keep in Practice

[W]

Ex. 1–6. Add.

1.	2.	3.	4.	5.	6.
233	3,675	457	$14.70	$305.50	29,682
106	566	22	2.55	22.75	8,148
424	2,890	39	8.88	64.44	103,603
763	1,099	87	$26.13	102.02	2,475
	8,230	605		$494.71	143,908

Ex. 7–12. Subtract.

7.	8.	9.	10.	11.	12.
68	452	3,661	$41.53	$650.40	5,683
32	209	926	6.89	62.83	2,954
36	243	2,735	$34.64	$587.57	2,729

*Correct terminology as applied to geometry emphasizes the difference between ideas and models to represent these ideas.

Geometric Points and Line Segments

Resurvey [O]

1. A *geometric point* is an idea. However it may be thought of as a position in space. Hold a pencil in front of you. The tip of the pencil locates a position in space so we may think of the tip as locating a geometric point. Now move the pencil a few inches. Does the tip of the pencil now locate a different geometric point? Yes

*2. In geometry, a point has no *size*. However, we may think of a point in geometry as being smaller than any pencil point or any needle point or any dot. We often use a small dot as a picture, or *model*, of a point. Draw two dots of different size on the board. Which one seems to be better for picturing a point? Why? Sample answers: ●· A point has no size, so the smaller dot is the better model.

Fig. 1

3. We often use capital letters to label models of points. Name the points shown in Fig. 1. P, Q, R

Fig. 2

A, B

4. Fig. 2. Name the labeled points. How many different paths are shown from point A to point B? Copy the figure on your paper. Can you show a path which is more direct than any of the paths shown? Yes Do so. See Fig. 2

Fig. 3

**5. The most direct path from one point to another is called a *line segment*, or *segment*. A line segment is named by the two points which it connects. These two points are called the *end points* of the segment. For example, line segment MN is pictured in Fig. 3 and points M and N are its end points. Name the line segments shown in Fig. 4 and name their end points. line segment KL; K, L line segment RS; R, S

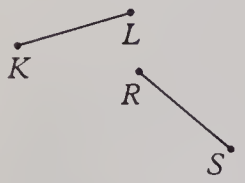


Fig. 4

6. Instead of writing *line segment* AB, we may write, $\overline{AB}$. On the board, copy Fig. 5 and then show $\overline{CE}$, $\overline{CF}$, and $\overline{CD}$. How many other segments could be shown having any two of these four points as end points? See Fig. 5. 3

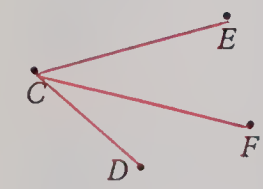
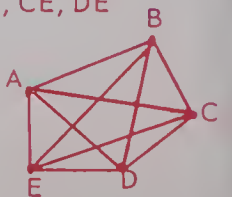


Fig. 5

7. On the board, show and label with letters five points. Show all the segments having any two of these points as end points. List the names of the segments on the board. Sample answer: $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, $\overline{AE}$, $\overline{BC}$, $\overline{BD}$, $\overline{BE}$, $\overline{CD}$, $\overline{CE}$, $\overline{DE}$

**The difference between a path, which exists in "3 dimensional" space, and a curve, which exists in a plane in space, can be intuitively developed by moving a piece of chalk through space and moving it on a chalkboard.



Teaching Pages 36 and 37

Pupil's Objectives

(a) To review the concepts of geometric point and line segment; (b) to represent these ideas with drawings; and (c) to use capital letters in naming points and segments.

Background

The word *geometry* means *land measurement*. The ancient Egyptians needed a system of land measurement to re-establish property boundaries each year after the flooding Nile receded, and we know they had some system of geometry as early as 2900 B.C. when they built the great pyramid of Gizeh.

The Egyptians were interested in the practical applications of geometry. This was not true of the ancient Greeks. By 300 B.C. the Greeks had developed geometry as a study of points, lines, and curves solely as ideas or concepts based entirely on logical reasoning. They were more interested in proving theorems than in making application of their findings.

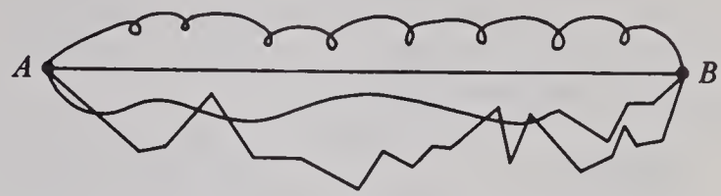
Today, the study of geometry is undergoing another change as modern terminology and concepts of sets are being applied to this subject.

Space is considered as an infinite set of points, and all geometric figures are subsets of this set. Every geometric figure may be thought of as a set of points, so the set operations of union and intersection can be applied to geometric figures.

The simplest concept in geometry is *point*. A point has no length, no width, and no thickness. It has only the property of location. Two different points must be two different locations in space, however close or far apart they may be.

A point is an idea much the same as a number is an idea. We use numerals to represent numbers, and so we use dot pictures to represent points. A dot is a drawing or model to represent a point. Capital letters are used to label models of points.

If we consider two points in space, the number of possible paths from one of the points to the other is infinite. Each path connecting two points has no width or height, only length. Regardless of whether a path in space is straight, not straight, or crosses over itself, every path is a set of points and is called a *curve*. Below is a drawing representing a few paths between point A and point B.



Between two points, there is only one shortest or most direct path. The set of points forming the most direct path between two points is a curve called a *line segment* or just *segment*. The two given points are called the *end points*. A segment may be named by drawing a small bar over the two capital letters that name the end points, such as $\overline{AB}$. The order in which the letters are written does not matter.

It is important to realize that we cannot draw a segment just as we cannot draw a point. We can draw only models or pictures to represent a point or a segment.

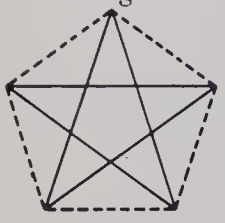
Pre-Book Lesson

• Discuss the practical uses to which the Egyptians put their knowledge of geometry: surveying and land measuring; building pyramids and other structures; geometric concepts involved in astronomy; and so on. Compare this with the Greeks' treatment of geometry as mainly ideas or concepts. However, you may wish to bring up the concept of the "Golden Rectangle" and the practical use to which it was put in their architecture, both buildings and statues. This discussion may be presented to the entire class or may be restricted to the *more capable pupils*. More information can be found in an encyclopedia such as the *Encyclopædia Britannica*.

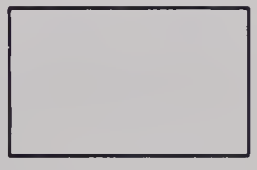
The Golden Section or Golden Ratio is a segment partitioned into proportions which are aesthetically harmonizing and thus used in architecture and other art forms. The Parthenon and Acropolis followed these proportions and they are used in modern industrial design. The architect Le Corbusier developed the Modular Scale based on the

human body whose height is taken as divided in golden section at the navel. The proportion, called the Golden Number, is an irrational number but is often replaced by artists by $\frac{5}{8}$ or 0.625. A rectangle whose sides are in the proportion $w:l = l:(w + l)$ is a Golden Rectangle. A magician's symbol for centuries, the pentagram, which is formed by the diagonals of a regular pentagon, contains 200 golden ratios.

Pentagram



Golden Rectangle



Using the Text Pages

• Ex. 1. Lead the pupils to understand that a point is not a dot, however small it may be. A point has no length, no width, no height. It is a location in space that may be represented by a dot picture.

• Ex. 4. Pupils should realize that the number of paths between two points is infinite. Have them give more examples of possible paths between two points, point A and point B at the chalkboard. Point out that each path, whether represented by a pencil mark or a chalk mark actually has no height, no width, and no thickness; it has only length. The drawings usually show width, but the path itself has no width.

• Ex. 9. Impress the pupils with the fact that any one segment may be named in two different ways by changing the order of the letters that name the two end points.

• Ex. 10. In this and subsequent exercises involving naming of segments, only one name for each segment is required. If you wish, you may request an alternate name for each segment.

• Ex. 11–12. Make it clear that $\{F, G, H\}$ and $\{L, M, N, P\}$ name sets of points and not sets of letters.

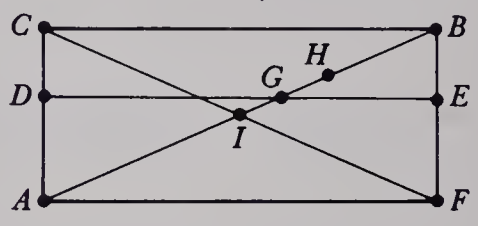
• Ex. 13–15. In drawing pictures to represent segments, a straight edge and a fine pencil should be used.

• Ex. 16. This serves as a preparation for the concept of line which is the next concept to be studied. You may wish to use this exercise in the Pre-Book Lesson for page 38.

Individualizing Instruction

• *More capable pupils* may be encouraged to apply the terminology and concepts of sets to geometry. They could be given a drawing such as that below and asked to list within braces the set of all labeled points or the set of all labeled segments. They may also be asked to picture the union or intersection of various sets of points. For example:

(a) the union of $\overline{AH}$ and $\overline{GB}$,
(b) the intersection of $\overline{AH}$ and $\overline{GB}$,
(c) the union of $\overline{DE}$ and point G,
(d) the intersection of $\overline{CF}$ and $\overline{AB}$.



• *All pupils* may write a paragraph describing the difference between a point and a dot, and between a segment and a drawing representing a segment.

• *Slower learners* may have difficulty with the idea that, unlike their representations, a point has no dimensions and a path has no width. Questions and activities such as the following may help you determine if they have actually mastered these concepts:

(a) How large is a point?
(b) What is the smallest thing you can think of?
(c) Can you cut this smallest thing in half?

Have children hold tautly a piece of thick rope, a piece of wrapping twine, and a piece of fine sewing thread and ask: "Which of these is the best model for a segment? Why?"*

* See item 9k on page xix. These items can help *slower learners* grasp geometric concepts more easily.

8. A line segment is a set of points. The set of points consists of the two end points and all the points in the most direct path connecting the end points. Do you think that you could name all the points in a segment? No Is a line segment a finite set of points or an infinite set of points? An infinite set of points

9. Three line segments are shown in Fig. 6 with their end points labeled. Name the segments. $\overline{PQ}, \overline{QR}, \overline{PR}$ Line segment RP is another name for line segment PR. Does $\overline{RQ}$ name the same segment as $\overline{QR}$? Yes

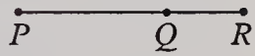


Fig. 6

[W]

10. Fig. 7. Write names for the seven line segments. $\overline{AN}, \overline{ND}, \overline{AD}, \overline{BN}, \overline{CN}, \overline{EN}, \overline{BE}$

11. How many different line segments are there with both end points in a set of labeled points $\{F, G, H\}$? 3 Name the line segments. $\overline{FG}, \overline{GH}, \overline{FH}$

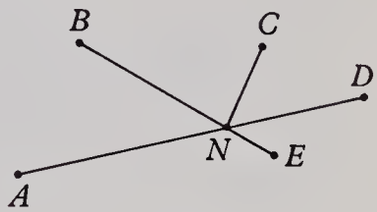
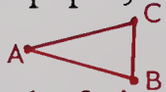


Fig. 7

12. Name the six different line segments with both end points in the set of labeled points $\{L, M, N, P\}$. $\overline{LM}, \overline{LN}, \overline{LP}, \overline{MN}, \overline{MP}, \overline{NP}$

13. On your paper, locate three points and label with letters. Picture the segments having any two of these points as end points. On your paper, record a name for each segment. Sample answer:



$\overline{AB}, \overline{AC}, \overline{BC}$

14. Fig. 8. There are a total of six labeled segments pictured. Name each segment. $\overline{IJ}, \overline{IG}, \overline{JG}, \overline{HJ}, \overline{HF}, \overline{JF}$

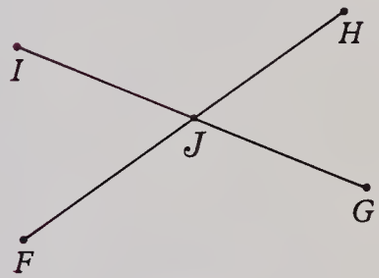


Fig. 8

15. On your paper, locate four points and label them with letters. Picture the segments having these points as end points. On your paper, record a name for each segment. Sample answer:

$\overline{AB}, \overline{AC}, \overline{AD}, \overline{BC}, \overline{BD}, \overline{CD}$

16. On your paper, locate two points. Draw a model of a line segment by drawing from one point to the other. Now, drawing in the same direction, extend your model a few inches. Is the new drawing a model of a line segment? Yes Can you think of extending the model in the opposite direction? Yes Can you think of the model being extended indefinitely in both directions? Yes

* The fact that a line has no end points and a segment has 2 end points has no influence on whether or not either is a finite set of points or an infinite set of points. All geometric figures other than the point contain an infinite set of points.

Lines

Resurvey [O]

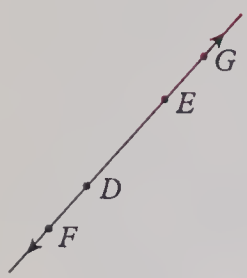


Fig. 1

* 1. Fig. 1. Think of segment DE being extended in both directions to form segment FG. Now think of it being extended indefinitely in both directions. This is a geometric idea called a *line*. A line does not have end points. Is a line a finite set of points or an infinite set of points? An infinite set of points

2. To picture a line, we draw a model with two arrowheads like Fig. 1. Why do we use arrowheads? To show that it extends indefinitely in both directions.

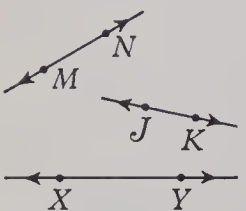


Fig. 2

3. To name a line, we use names of any two points in the line. For example, if points A and B are two points in a line, then the line may be named *line AB* or *line BA*. We may also write $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$. Name the lines shown in Fig. 2. $\overleftrightarrow{MN}$ or $\overleftrightarrow{NM}$; $\overleftrightarrow{JK}$ or $\overleftrightarrow{KJ}$; $\overleftrightarrow{XY}$ or $\overleftrightarrow{YX}$

4. Fig. 3 shows point C in line AB.

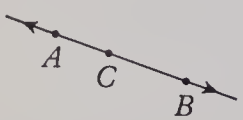


Fig. 3

a. Does $\overleftrightarrow{AB}$ name the same line as $\overleftrightarrow{AC}$? Yes as $\overleftrightarrow{CB}$? Yes

b. Do $\overline{AB}$ and $\overline{AC}$ name the same set of points? No Explain. They name different segments.

5. On the board, draw Fig. 4 and then represent lines SR and ST. Remember to use arrowheads. Then show $\overline{RT}$.

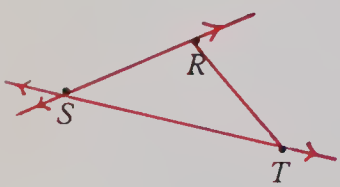
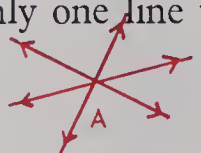
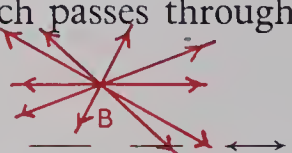


Fig. 4

6. On the board, show two points A and B. Show three different lines through point A and five different lines through point B. Is it true that many lines pass through a single point? Yes How many lines can you show which pass through both point A and point B? 1 Is there only one line which passes through two different points? Yes

Sample answer:





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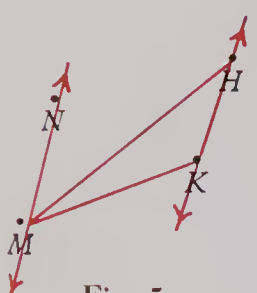


Fig. 5

7. On your paper, draw Fig. 5. Show $\overline{MH}$, $\overline{MK}$, $\overline{MN}$, and $\overleftrightarrow{KH}$.

8. Study Fig. 6.

Fig. 6

a. How many line segments are shown with their end points labeled? 3 Name them. $\overline{XY}$, $\overline{XZ}$, $\overline{YZ}$

b. How many lines are shown? 1

9. Record three names for the line pictured in Fig. 6.

Any three of these: $\overleftrightarrow{XY}$, $\overleftrightarrow{YZ}$, $\overleftrightarrow{ZX}$, $\overleftrightarrow{YX}$, $\overleftrightarrow{ZY}$, $\overleftrightarrow{XZ}$

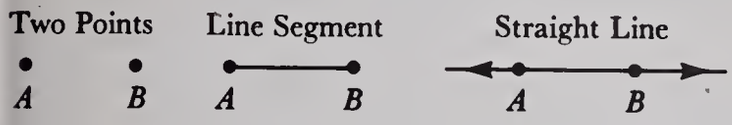
Teaching Page 38

Pupil's Objectives

(a) To review the geometric idea called a line; and (b) to represent and to name lines with models and drawings.

Background

If any two points in space are selected, such as point A and point B below, the shortest and most direct path connecting these points is the segment whose end points are the selected points. For the drawing below, this would be $\overline{AB}$. If this segment is now extended indefinitely in both directions, the result is a set of points called a *straight line* or *line*.



The arrowheads in the drawing show that the extension continues indefinitely in the directions indicated.

A line is named by showing two arrowheads over the two letters naming two distinct points in the line, such as $\overleftrightarrow{AB}$.

Pre-Book Lesson

Discuss Ex. 16 on page 37 as a preparation for the concept of a line.

Using the Text Page

- Ex. 1. Associate the ideas of infinite extension and infinite set of points in connection with a line.
- Ex. 2. The arrowheads show that the set of points extends without end in the two opposite directions indicated.
- Ex. 3. To name a line, all that is necessary is to name with capital letters any two points that are contained in that line.
- Ex. 4. Help pupils to realize that points not between the two points used in naming the line may also be in the line.
- Ex. 6. For any given point in space, there is an infinite number of lines that contain the given point. Given two points in space, there is one and only one line that contains both given points.

The concept of geometric line may be confusing to some pupils. By *line*, we always mean *straight line*. The following do not represent lines and they should not be referred to as lines. They show curves.

Individualizing Instruction

- *More capable pupils* may be encouraged to apply the terminology and concepts of sets to lines and segments. They can be given drawings of intersecting segments and lines and asked to find union or intersection of various points, segments or lines.
- Teach *all pupils* to differentiate between lines and segments by giving them a set of geometric properties and having them identify which are properties of a segment, which pertain to a line, which are properties of both a segment and a line, and which pertain to neither a segment nor a line. Some suggested properties are as follows:

(1) is a set of points
(2) is an infinite set of points
(3) is a finite set of points
(4) has no end point
(5) has two end points
(6) has one end point only
(7) extends infinitely in only one direction
(8) extends infinitely in two directions
(9) identified by naming any two of its points

- *Slower learners* can gain greater understanding of the geometric ideas of point, line, and segment by illustrating these ideas with concrete objects. The use of a flannel board with appropriate flannel models would be effective here.* Pupils should be given the opportunity and encouraged to illustrate and explain these ideas to each other.

* See item 9k on page xix. Effective use of these items depends on how well illustrative examples are related to the text page.

Teaching Page 39

Pupil's Objectives

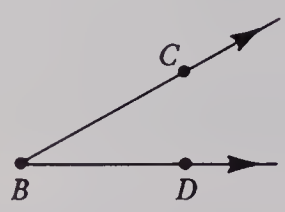
(a) To review the geometric concepts called ray and angle, and the terminology associated with them; (b) to represent these ideas with models and drawings; and (c) to name these ideas.

Background

If a line segment is thought of as extending infinitely in only one direction, the resulting set of points is called a *ray*. In naming a ray, a small arrowhead is drawn above the letter naming the end point of the ray and the letter naming any other point in the ray. The single arrowhead indicates that the ray extends indefinitely in only one direction. The order in which the two letters used in naming a ray are listed is important. The first letter names the end point and the second letter names a second point through which the line passes. Hence, the two letters indicate the direction in which the ray extends.

If two rays have a common end point and are not in the same line, the union of the two rays forms a set of points called an *angle*. The common end point is called the *vertex* of the angle.

An angle may be named by writing three letters; one letter naming a point in one ray, one letter naming the vertex of the angle, and the third letter naming a point in the other ray. In writing the three letters, the letter naming the vertex is always the middle letter. The angle pictured below may be named by writing *angle CBD*, or *angle DBC*. The symbol $\angle$ may be used in place of the word "angle," so we may also name the angle by writing $\angle CBD$ or $\angle DBC$. When there is no chance for confusion, an angle is sometimes named by a single letter indicating the vertex. The angle below may be named *angle B*, or $\angle B$.



Pre-Book Lesson

Introduce the concept of a ray by drawing and labeling a picture of a line on the chalkboard. The pupils should be familiar with the concept of indefinite extension in two directions. Use the drawing to illustrate that either of the points named and all the points in the extension in only *one* direction constitute a ray.

Using the Text Page

- Ex. 1. Emphasize that the single arrowhead indicates indefinite extension in only one direction as well as the direction in which the extension occurs in terms of the labeled picture of a ray.
- Ex. 2. Stress the importance of writing the name of the end point first when naming a ray. Emphasize that in naming a ray, the small arrowhead always lies above the name for the point in the ray other than the end point.
- Ex. 4. Point out that if the two rays are in the same line they do not form an angle.
- Ex. 6. It should be pointed out that naming an angle by naming only its vertex is permissible only when no confusion will result.

Individualizing Instruction

- *More capable pupils* may be given drawings such as Fig. 1 and asked to name as many different angles as they can.

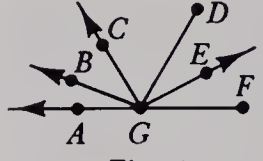


Fig. 1

Fig. 2

Fig. 3

- *All pupils* can be asked why $\overrightarrow{GF}$ and $\overrightarrow{GH}$ in Fig. 2 do not form an angle even though they have a common end point, or why $\overrightarrow{TS}$ and $\overrightarrow{TU}$ in Fig. 3 do not form an angle even though they have a common end point.
- *Slower learners* can be asked to find examples of models for angles in the classroom.

*Later, pupils will learn that the length of the models for the rays is unrelated to the size of the angle.

Rays and Angles

Resurvey [O]

1. Think of a line and a point P in that line. Now consider point P and all the points in the line on one side of point P. This is a geometric idea called a *ray*. A ray has only one end point. To picture a ray, we draw a model with one arrowhead like Fig. 1. What does the arrowhead indicate? The ray extends indefinitely in one direction.

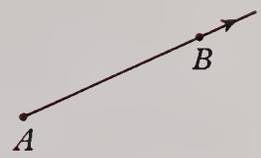


Fig. 1

2. In naming a ray, we name the end point first and then another point in the ray. For example, the ray shown in Fig. 1 may be named *ray AB*. We may also write $\overrightarrow{AB}$ to name this ray. Name the rays shown in Fig. 2. $\overrightarrow{MN}$, $\overrightarrow{TS}$, $\overrightarrow{XY}$

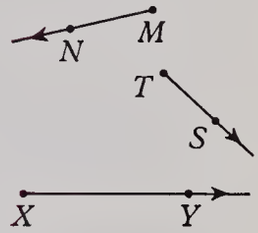


Fig. 2

3. Fig. 3. Name the three labeled points in ray CD. C, D, E Name the three labeled points in ray DC. D, C, F Are the two rays the same set of points? No Then is ray CD the same ray as ray DC? No

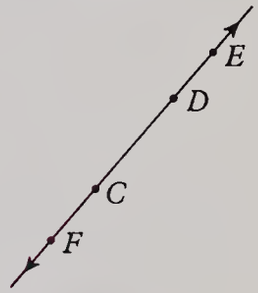


Fig. 3

* 4. Fig. 4. The geometric figure pictured may be thought of as formed by two rays, not in the same line, with a common end point. Such a set of points is called an *angle*. The two rays are the *sides* of the angle and the common end point is called the *vertex* of the angle. Name the sides and the vertex of the angle shown. sides: $\overrightarrow{HG}$, $\overrightarrow{HI}$ vertex: H

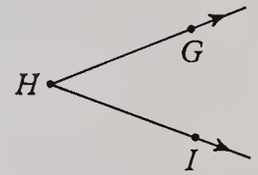


Fig. 4

5. The angle shown in Fig. 5 may be named *angle PQR* or *angle RQP*. In a name for an angle, the middle letter names the vertex. We may write, $\angle PQR$ or $\angle RQP$. Name the angles shown in Fig. 6. ∠JKL or ∠LKJ, ∠MON or ∠NOM, ∠UVW or ∠WVU

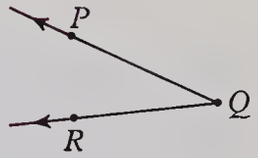


Fig. 5

6. Sometimes an angle is named by the vertex only. For example, the angle shown in Fig. 5 may be named *angle Q*. Name each of the angles shown in Fig. 6 by naming only the vertex. ∠K, ∠O, ∠V

[W]

7. Write names for the sides of an angle XYZ. $\overrightarrow{YX}$, $\overrightarrow{YZ}$

8. On your paper, show three points, not in the same line, and label them H, J, and K. Then show $\overline{HJ}$, $\overleftrightarrow{HK}$ and $\overrightarrow{JK}$.

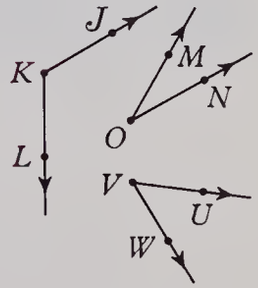
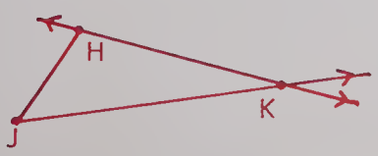


Fig. 6

● **Extra Activity.** Set 154. These sets start on page 367.

8.



* For Ex. 3, 5, 6, and 7 the sets may be described using their letter names or by listing members within braces.

To Keep in Practice

[W]

* Ex. 1–8. Complete using the sets named at the left.

$A = \{2, 3, 7\}$
$B = \{2, 6, 8, 10\}$
$C =$ The set of counting numbers
$D =$ The set of odd counting numbers

1. $A \cup B =$ _?_ {2, 3, 6, 7, 8, 10}
2. $A \cap B =$ _?_ {2}
3. $B \cap C =$ _?_ B
4. $B \cap D =$ _?_ { }
5. $C \cup D =$ _?_ C
6. $C \cap D =$ _?_ D
7. $\{\ \} \cup A =$ _?_ A
8. $\{\ \} \cap A =$ _?_ { }

Ex. 9–12. Round to the nearest ten thousand.

9. 21,562 20,000 **10.** 109,820 110,000 **11.** 35,006 40,000 **12.** 9,605 10,000

{0, 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98}

13. List within braces the multiples of 7 that are less than 100.

14. Which members of $\{2, 3, 7, 10\}$ make $n + 5 > 12$ a true sentence? 10

Ex. 15–19. Express the number first with a base-five numeral and then with a base-eight numeral.

15. 57 212_{five} 71_{eight} **16.** 70 240_{five} 106_{eight} **17.** 96 341_{five} 140_{eight} **18.** 200 1300_{five} 310_{eight} **19.** 250 2000_{five} 372_{eight}

Ex. 20–24. Write a base-ten numeral for the number.

20. 32_{five} 17 **21.** 16_{seven} 13 **22.** 100_{four} 16 **23.** 100_{eight} 64 **24.** 202_{five} 52

Remembering about Properties of Addition

[W]

a. $3 + 57 = 57 + 3$
b. $(2 + 3) + 13 = 2 + (3 + 13)$
c. $(15 + 1) + 10 = 15 + (1 + 10)$
d. $(4 + 5) + 9 = 9 + (4 + 5)$

1. Which sentences in the box illustrate the Commutative Property of Addition? a, d

2. Which sentences in the box illustrate the Associative Property of Addition? b, c

Ex. 3–8. Solve by thinking about properties of addition.

3. $n + 23 = 23 + 8$ 8
4. $456 + n = 627 + 456$ 627
5. $28 + 100 = n + 28$ 100
6. $(7 + n) + 27 = 7 + (31 + 27)$ 31
7. $n + 1{,}828 = 1{,}828 + 31$ 31
8. $(18 + 21) + n = 24 + (18 + 21)$ 24

Teaching Page 40

Pupil's Objectives

(a) To review some of the concepts and skills taught in Chapter 1; (b) to uncover weaknesses that may exist so that review or reteaching may be most effective; and (c) to recall the names of the properties of our number system.

Background

Practice is necessary to maintain skills already developed, and practice of the type presented here helps diagnose sources of difficulty of individual pupils.

The main concepts being tested are:

(a) understanding the concept of counting numbers

(b) determining the union of two sets

(c) determining the intersection of two sets

(d) understanding the concept of the empty set

(e) rounding numbers

(f) finding multiples of a number

(g) true and false mathematical sentences

(h) changing from base-ten to base-five and base-eight numerals

(i) changing from numerals in various bases to base-ten numerals

(j) Commutative Property of Addition

(k) Associative Property of Addition

Pre-Book Lesson

Briefly resurvey each of the topics listed in the Background section. Have selected pupils describe each concept either in words or by means of an example at the chalkboard. Encourage pupils to ask questions about any phase of any of the concepts which they feel they do not completely understand.

Using the Text Page

If the number of examples on this page is excessive for the *slower learners*, select only certain of the examples for them to do. Also, in connection with union and intersection of infinite sets, remind pupils of the proper use of the three raised dots in set notation.

Individualizing Instruction

• The *more capable pupils* may be given exercises pertaining to union and intersection involving more than two sets. The letters A, B, C, and D name the sets indicated at the top of the text page.

(a) $(A \cup B) \cup C$
(b) $(A \cap B) \cup C$
(c) $(B \cap C) \cap D$
(d) $(A \cup C) \cup D$
(e) $(A \cap C) \cup D$
(f) $A \cap (C \cup D)$

• Have *all pupils* restudy appropriate pages in the chapter if weaknesses are evident in certain concepts. You may suggest areas of individual weakness, but let the pupil select the material for restudy.

Teaching Page 41

Pupil's Objective

To review the geometric concept called a plane.

Background

Fig. 1 shows two lines that intersect in point P.

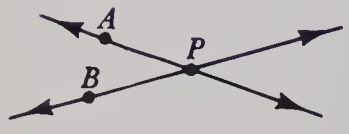


Fig. 1

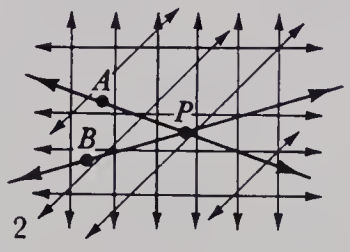


Fig. 2

Consider the set of all lines, each of which intersects both $\overleftrightarrow{AP}$ and $\overleftrightarrow{BP}$ in points other than point P as shown by Fig. 2. The set of all such lines form a set of points called a *plane of points* or a *plane*. It is somewhat more difficult to make a drawing that represents a plane than it is to make a drawing that represents a line. For this reason, most drawings and models picture only a portion of a plane. A plane may be considered as a set of points forming a perfectly flat surface having infinite length and infinite width. The surface of a chalkboard, the surface of a sheet of paper, the surface of a flat wall, or the surface of a flat desk may be considered as representing a portion of a plane.

If a pupil moves a taut piece of string over the surface of the chalkboard, he will see that *no space* may be seen between the string and the chalkboard, and that a plane is perfectly flat.

A plane may be identified by naming three points in the plane but not all in the same line. Three such points are in only one plane.

A given point, a given line, a given segment, and a given ray are each contained in an infinite number of planes. However, an angle is contained in only one plane because it contains three distinct points not all in the same line.

Pre-Book Lesson

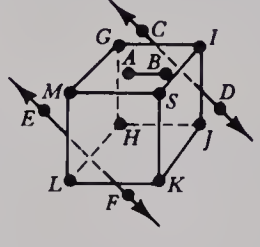
Review the fact that a point has no dimension and that a line has only one dimension, infinite length. Let pupils look at the chalkboard or one of the walls in the room and try to imagine these surfaces as stretching out in every direction. Ask pupils if such a surface would still be flat.

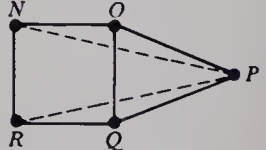
Using the Text Page

- Ex. 1. Point out that a plane has no thickness so that we are only discussing the surface of these objects as a model. Emphasize that each model represents only a very small portion of a plane.
- Ex. 2. Repeat the experiment with the taut piece of string held against the chalkboard.
- Ex. 5–10. The pupils may wonder how to name or identify a plane. You may develop the idea that any three points not all in the same line identify a plane. Therefore, a specific angle and a specific pair of intersecting lines will also identify a plane. You may wish to use one or more folded cards to illustrate the truth or falsity of a statement.

Individualizing Instruction

- *More capable pupils* may be able to comprehend the concept of a plane through the idea of intersecting lines. If they learn to identify a plane by naming three points that are in the plane but not all in the same line, they can be given drawings such as those below and asked to name as many different planes as possible, and also to identify the plane or planes in which specific lines or segments lie.





- *All pupils* may give examples of objects that can be used for models of intersecting planes, such as the floor and a wall or a door and a wall.
- *Slower learners* can be given objects such as differently shaped blocks and be asked to name what is suggested by each surface of a block (portion of a plane); each edge (a line segment); each corner (a point).

Planes

Resurvey [O]

1. Think of a flat sheet of paper as being a model of a set of points. You are thinking of a model of part of a *plane*. To think of an entire plane, think of the paper being extended indefinitely in all directions. Name some things in your classroom which may be thought of as models of parts of planes. Answers will vary.

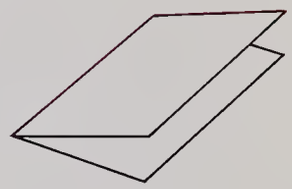
2. Think of the board as a model of part of a plane. Show a line segment on the board and label its end points A and B.

a. Does the plane contain every point of $\overline{AB}$? Yes We say that the segment *lies* in the plane.

b. Does the plane contain every point of $\overleftrightarrow{AB}$? Yes Then does line AB lie in the plane? Yes

3. A plane is a set of points. Think of the front cover of your book as suggesting a plane. Raise the cover slightly. Does the cover now suggest a different plane? Yes Do you think that there are more planes than can be counted? Yes

4. A folded card is shown at the right. Do you see that two different planes are suggested? Yes What part of the card suggests a line which lies in both planes? The fold Do you think that a line lies in two planes? Yes in more planes than can be counted? Yes Does a line segment lie in more planes than can be counted? Yes



[W]

Ex. 5–11. Write *T* or *F*.

There are more planes than can be counted which contain

5. a given point. T

6. a given line. T

7. a given ray. T

8. a given angle. F

9. One line contains only two rays. F

10. If a line segment lies in a given plane, then the line containing that segment lies in the same plane. T

11. A segment is contained in only one line. T

● **Extra Activity.** Set 155.

*Models representing geometric figures may be drawn so that they do not appear to intersect. However, in thinking of such ideas as represented by Fig. 3, it must be remembered that a line extends infinitely even though it does not appear to in the drawing.

Points of Intersection

Resurvey [O]

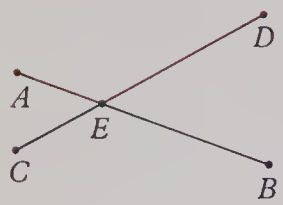


Fig. 1

1. Fig. 1. Name the point in which line segments AB and CD intersect. E Point E is called the *point of intersection* of $\overline{AB}$ and $\overline{CD}$.

a. Is point E in both segments? Yes

b. Do $\overline{AB}$ and $\overline{CD}$ intersect in only one point? Yes

c. If two segments intersect in a point, do the lines containing the segments intersect in the same point? Yes

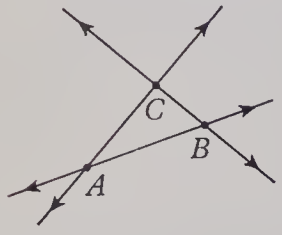


Fig. 2

2. Fig. 2. Name the three lines shown. $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{BC}$ Name the point of intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$; A of $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$; B of $\overleftrightarrow{AC}$ and $\overleftrightarrow{CB}$. C

* 3. Fig. 3 shows part of a plane and a line which does not lie in the plane. Does it appear that the line intersects the plane? Yes If a line does not lie in a plane, can the line intersect the plane in more than one point? No

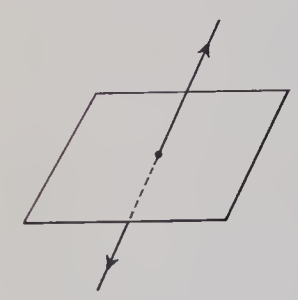


Fig. 3

4. Is the vertex of an angle a point of intersection of the sides of the angle? Yes Explain. It is in both sides (rays).

5. On the board, show two lines that intersect. Then show two lines that do not intersect.

6. If two different lines intersect, they intersect in a point. If two different planes intersect, they intersect in a line.

[W]

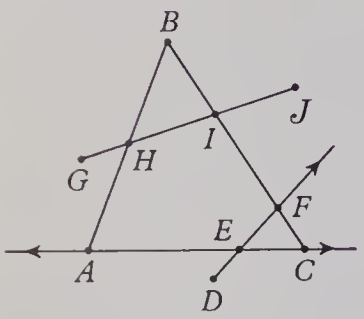


Fig. 4

7. Fig. 4. Name the point of intersection of

a. $\overline{AB}$ and $\overline{BC}$. B

b. $\overline{AB}$ and $\overleftrightarrow{EC}$. A

c. $\overrightarrow{DE}$ and $\overline{BC}$. F

d. $\overline{GJ}$ and $\overline{BC}$. I

e. $\overrightarrow{DF}$ and $\overleftrightarrow{AC}$. E

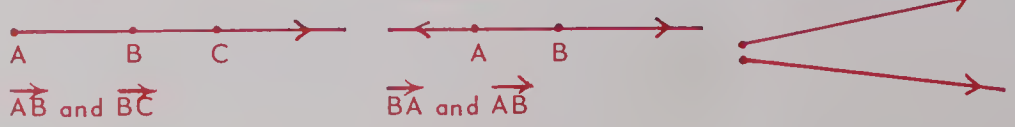
f. $\overline{BA}$ and $\overline{JG}$. H

8. Can two line segments intersect in exactly two points? No

9. Draw models of two rays whose intersection is a ray; is a segment; is the empty set. See below.

10. If two planes intersect in two points, then do they intersect in the line which contains the two points? Yes

9. Intersection: a ray; a segment; the empty set



$\overrightarrow{AB}$ and $\overrightarrow{BC}$ $\overrightarrow{BA}$ and $\overrightarrow{AB}$

Teaching Page 42

Pupil's Objectives

(a) To review the concept of intersection of sets of points; and (b) to differentiate between the intersection of lines, of planes, and of lines and planes.

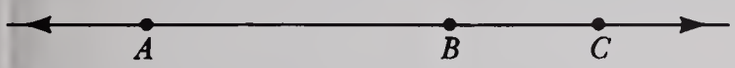
Background

Each of the geometric concepts of *point*, *segment*, *line*, *ray*, *angle*, and *plane* concerns sets of points. Set symbols, set terminology, and the set operations of union and intersection may be applied to these sets of points.

It should be pointed out that the word *intersection* as used in connection with geometric ideas has the same general meaning as the word *intersection* when used with sets of numbers, letters, or sets in general.

Pre-Book Lesson

Review the meaning of intersection as an operation on sets, and ask how this could be applied to intersection of sets of points. On the chalkboard draw and label the following picture.



Have pupils show the picture for the set of points in: $\overline{AB}$, $\overline{BC}$, $\overline{AC}$, $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{AC}$, $\overrightarrow{AB}$, $\overrightarrow{BC}$, and $\overrightarrow{AC}$. Then, working with *pairs* of the pictures suggested above, have the children tell, show, or name the set of points each pair of models or pictures has in common. It would not be difficult to progress from sets of common points to *intersection*. For example:

(a) The intersection of $\overline{AB}$ and $\overline{AC}$ is $\overline{BC}$.
(b) The intersection of $\overrightarrow{AB}$ and $\overline{AC}$ is $\overline{AC}$.
(c) The intersection of $\overrightarrow{AB}$ and $\overrightarrow{BA}$ is $\overline{AB}$.
(d) The intersection of $\overleftrightarrow{AC}$ and point A is point A.

Using the Text Page

• Ex. 1. *A point of intersection* of two geometric figures is any point that is contained in both figures or is common to both.

• Ex. 3. The intersection of a line and a plane will consist of one of the following:

(a) a point (See Fig. 3 on the pupil's page)
(b) the given line (this occurs when the given line is in the given plane)
(c) the empty set (this occurs when the line is parallel to the plane)

• Ex. 5. The intersection of two given lines will consist of one of the following:

(a) a point
(b) a line (this occurs when the two given lines are the same set of points)
(c) the empty set (either the two given lines are parallel or they are in different planes)

• Ex. 6. The intersection of two planes will consist of a line, or the empty set (this occurs when the two planes are parallel).

• Ex. 8. The intersection of two segments must consist of exactly one point, exactly one segment, or the empty set.

Individualizing Instruction

• *More capable pupils* can be given models of segments, lines, rays, angles, and planes whose intersections are sets of points or the empty set. They can list all possibilities of intersection of two segments, two rays, a ray and a segment, two lines, a line and a plane, and so forth.

• *All pupils* may be asked to picture and label as many different pairs of geometric figures whose intersection is: a point; a segment; a ray; a line; the empty set.

• *Slower learners* can be asked to find in the classroom models for points of intersection. The majority of those identified will suggest either points or segments. Start these pupils off by calling attention to corners of the room and objects within the room, edges of different objects within the room, and any flat surface within the room. Help these pupils to visualize the geometric idea suggested by places where two or more edges or surfaces "come together."

Pupil's Objectives

(a) To resurvey the meaning of right angle, acute angle, and obtuse angle; and (b) to practice computational skills involving addition and subtraction.

Background

A *right angle* is an angle whose measure is 90 in degrees. An *acute angle* is an angle whose measure is less than 90 in degrees. An *obtuse angle* is an angle whose measure is greater than 90 but less than 180 in degrees. Because pupils have not been introduced to the concept of the measure of an angle, these three angles must be described intuitively using terms with which they are already familiar. Since most pupils know what a square corner is, we may say that any angle whose model forms a square corner is called a right angle. An angle smaller than a right angle is called an acute angle and an angle that is greater than a right angle is called an obtuse angle.

If two lines intersect to form right angles, they are said to be *perpendicular*. A pair of intersecting lines are perpendicular if all four adjacent angles are congruent (the same size). Note this second definition does not depend on understanding a right angle.

Pre-Book Lesson

Have a pupil draw a model of an angle on the board. Have another pupil draw a model of a larger angle. Have a third pupil draw a model of a smaller angle. Then ask:

"Which picture shows the largest angle?"

"Which picture shows the smallest angle?"

"Does the largest angle have more points in its rays than the smallest angle?" (No, all angles have the same number of points in their rays.) In simpler terms, the measure of an angle is in no way related to the length of the sides. Inasmuch as the *sides* of all angles are rays, they contain an infinite number of points. Redraw the models of the three angles so that all share the same end point and one ray. In discussing the meaning of the size of an angle, lead the class to an intuitive realization of the following: If two angles share a common ray and a common end point, and the ray of one angle is interior to the other angle, the first angle may be said to be smaller than the second angle.

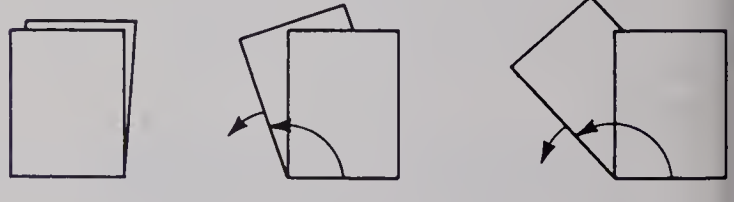
Using the Text Page

Ex. 1–5. Be careful not to discuss the meaning of right angle, acute angle, and obtuse angle in terms of the measures of the angles as the pupils have not yet been introduced to this concept. Present instead material from the Background and Pre-Book Lesson.

Individualizing Instruction

• It may not be necessary for *all pupils* to do all the examples at the bottom of the page. If a pupil has a good grasp of the skills presented, other types of activities may be more appropriate and interesting to him. The *more capable pupils* may be assigned only the more difficult examples and the *slower learners* assigned the easier examples.

• *All pupils* may get a better idea of size relationships among obtuse angles, right angles, and acute angles by pursuing the following activity. Let each start with 2 small sheets of paper held at one corner to suggest a right angle. Then start "fanning" the two sheets open as one would playing



cards. This should suggest a greater and greater obtuse angle.

To suggest acute angles, take a single sheet and cut or tear successive pieces as suggested by the dashes in the picture. This should suggest smaller and smaller acute angles.

Kinds of Angles

Resurvey [O]

1. Fig. 1. Is it true that the models of rays SR and ST form a square corner? Yes Angles such as $\angle RST$ are called *right angles.*

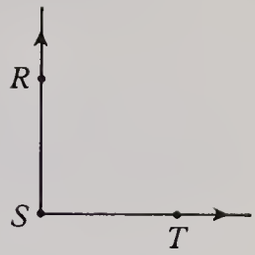
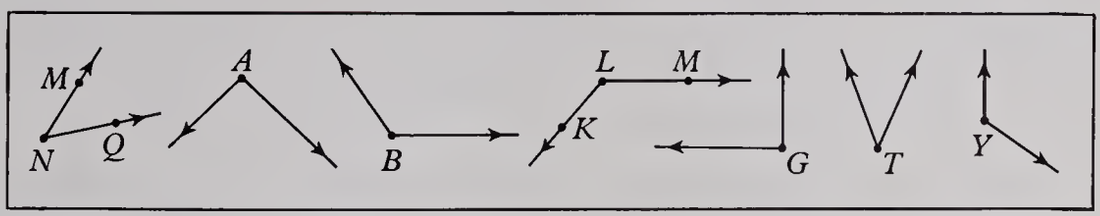


Fig. 1

2. Name the angles shown in the box below which appear to be right angles. ∠A, ∠G



3. Fig. 2 shows two *perpendicular* lines. Give a definition of perpendicular lines. Sample answer: Two lines that intersect at right angles. Name the four right angles shown. ∠AED, ∠DEB, ∠BEC, ∠CEA

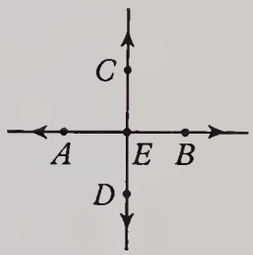


Fig. 2

4. An angle which is smaller than a right angle is called an *acute angle.* Fig. 3 shows two acute angles. Which angles shown in the box above appear to be acute angles? ∠N, ∠T

Fig. 3

5. An angle which is larger than a right angle is called an *obtuse angle.* Fig. 4 shows two obtuse angles. Which angles shown in the box above appear to be obtuse angles? ∠B, ∠L, ∠Y

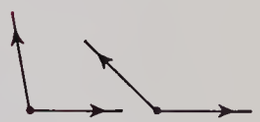


Fig. 4

◆ **Extra Examples.** Set 33. ● **Extra Activity.** Set 156.

To Keep in Practice

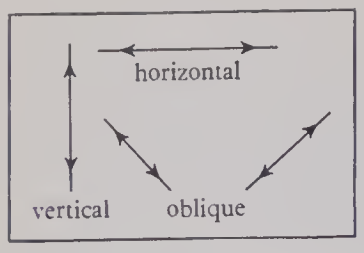
[W]

Ex. 1–12. Add.

1. 346 877 1,223	2. 998 903 1,901	3. 4,607 6,009 10,616	4. $43.67 3.50 $47.17	5. $628.22 4.98 $633.20	6. 24,687 18,595 43,282
7. 349 55 661 1,065	8. 7,808 49 38,445 46,302	9. 14,007 501 99 14,607	10. $43.50 0.49 1.98 $45.97	11. $45.67 57.97 8.88 $112.52	12. 8,062 28,973 786 37,821

Ex. 13–18. Subtract.

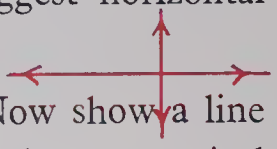
13. 984 231 753	14. 6,904 408 6,496	15. 45,987 23,775 22,212	16. $68.25 25.98 $42.27	17. $625.75 43.17 $582.58	18. 101,643 9,847 91,796



Kinds of Lines

Resurvey [O]

1. Study the kinds of lines shown in the box. Name some things in your classroom that suggest horizontal lines; vertical lines. Answers will vary.

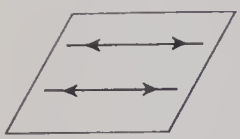


2. On the board, show a horizontal line. Now show a line perpendicular to the horizontal line. Did you show a vertical line? Yes Show a line that is neither horizontal nor vertical. Such a line is called an *oblique line*.

3. A line segment is vertical when the line in which it lies is vertical.

a. Tell when a line segment is horizontal; oblique. When the line in which it lies is harizontal / When the line in which it lies is oblique

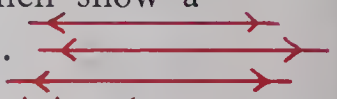
b. Tell when a ray is vertical; horizontal; oblique. When the line in which it lies is vertical; harizontal; oblique



4. The picture at the left represents a plane and two lines which lie in the plane. Do the lines appear to intersect? No

Lines which lie in the same plane and which do not intersect are called *parallel lines*.

5. On the board, show two parallel lines. Then show a third line which is parallel to each of these two lines.



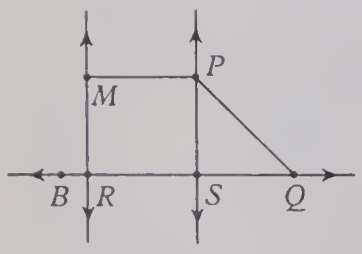
6. Line segments are parallel when the lines containing them are parallel. Tell when rays are parallel. When the lines containing them are parallel

When the lines containing them are perpendicular

7. Tell when two segments are perpendicular. Tell when two rays are perpendicular. When the lines containing them are perpendicular

[W]

Use the figure at the left for Ex. 8–12.



8. Do rays RM and PS appear to be parallel? Yes

9. Do segments MP and BS appear to be parallel? Yes

10. Name three pairs of parallel segments. Sample answers: $\overline{MR}$ and $\overline{PS}$, $\overline{MP}$ and $\overline{RS}$, $\overline{MP}$ and $\overline{RQ}$

11. If $\overleftrightarrow{BQ}$ is horizontal, name two vertical segments; $\overline{MR}$ and $\overline{PS}$ an oblique segment; a segment parallel to $\overleftrightarrow{BQ}$. $\overline{MP}$ PQ

12. Name two segments that are perpendicular to $\overleftrightarrow{RS}$. $\overline{MR}$ and $\overline{PS}$

Teaching Page 44

Pupil's Objective

To review various kinds of lines, segments and rays, including those called vertical, horizontal, oblique, or parallel.

Background

If a weight is attached to a string and held so that it is suspended above the floor, one has a *plumb line*. If the string holding the weight is considered to show part of a line, such a line may be called a *vertical line*. All vertical lines intersect at the center of the earth.

Any line perpendicular to a specified vertical line may be called a *horizontal line*. A plane containing such a line may be called a *horizontal plane*. The surface of the ceiling, the floor, and the teacher's desk each suggest a part of a horizontal plane. If a sheet of paper is laid on a horizontal desk top and lines are represented on it, each line may be thought of as lying in a horizontal plane, and is therefore a horizontal line. If lines are represented from the top of the chalkboard to the bottom, these may represent vertical lines. Models of lines drawn from one side to the other side, parallel to the floor, may suggest horizontal lines.

Pre-Book Lesson

There are many examples of objects in the classroom that suggest the geometric concepts of parallel, horizontal, vertical, and oblique. Discuss the idea of floor and ceiling as suggesting parallel planes; the lateral edges of the chalkboard as suggesting two parallel and vertical segments; the top edge and the chalk tray as suggesting parallel horizontal segments; and many others.

Using the Text Page

• Ex. 1. A vertical line is one that passes through the center of the earth. This may be represented by drawing a picture of a line from the top of the chalkboard to the bottom. Any line perpendicular to a specified vertical line is a horizontal line. While not actually true, since the earth is curved, a horizontal line may be associated with the geographic horizon.

• Ex. 4–10. To be parallel, lines must be in the same plane and must not intersect. It is possible to have two lines that do not intersect but are not parallel. Two segments contained in parallel lines are parallel as are two rays contained in parallel lines. Two planes that do not intersect are parallel as are any two portions of planes within parallel planes.

Individualizing Instruction

• *More capable pupils* may be given the definition of vertical line in terms of a plumb line. Since all vertical lines intersect at the center of the earth, theoretically, no two vertical lines are parallel. However, plumb lines are in practice used by masons and bricklayers to make certain that the walls of a building are vertical. These men would consider the distances between two opposite vertical walls as being equal throughout or, as being representative of parallel lines. The actual deviation from parallel would be so small as to be negligible.

• *All pupils* should be led to realize that vertical and horizontal are not purely geometric concepts but relate to a person's position on the surface of the earth.

• *Slower learners* could make a plumb line using fine string and a light weight to illustrate the true meaning of vertical. Pupils could use their plumb lines to test the walls of the room, legs of tables and chairs, and other similar objects to determine if they are truly vertical. They could make a simple *level* to test whether models of planes in the room were truly horizontal. A simple level could be made by filling a bottle almost completely full of water before putting a plug in it. The bottle would be laid on its side upon the surface to be tested. If the surface was truly horizontal the "bubble" in the bottle would appear at about middle position in the upper surface of the bottle, *not* at the top or bottom.

Teaching Page 45

Pupil's Objectives

(a) To review the geometric concepts of curve, closed curve, and simple closed curve; and (b) to represent various types of curves with models and drawings.

Background

The teacher may wish to refer to the Background of Teacher's Page 36, to establish a relationship between path and some forms of curves. A simple description of a curve which is intuitively acceptable is that a curve may be thought of as a set of points which can be represented by a drawing without lifting the pencil from the paper.

A curve that does not cross itself is called a *simple curve*. On the pupil's page, pictures *a*, *b*, *c*, *d*, *e*, *f*, *h*, *k*, *l*, and *m* are models of simple curves. Pictures *g*, *i*, *j*, and *n* are models of curves that intersect themselves so they are not simple curves.

A curve that has no distinct end points is called a *closed curve*. Pictures *d*, *e*, *h*, *i*, *j*, and *m* show closed curves. Pictures *a*, *b*, *c*, *f*, *g*, *k*, *l*, and *n* are not pictures of closed curves.

A closed curve that does not cross itself is called a *simple closed curve*. Pictures *d*, *e*, *h*, and *m* are models of simple closed curves.

Pre-Book Lesson

Have some pupils go to the chalkboard one at a time and draw pictures of various kinds of curves. Remind them not to lift the chalk from the surface of the chalkboard. Separate the curves into two sets, simple and nonsimple curves. Have the pupils try to determine what characteristic is common to the curves within each group. Then separate them into sets of open and closed curves, and again have the pupils try to determine the characteristic common to the curves in each group. Make sure pupils understand that segments and lines are also curves even though they do not "curve" in the colloquial sense. Have pupils try to name any simple closed curves consisting only of segments.

Using the Text Page

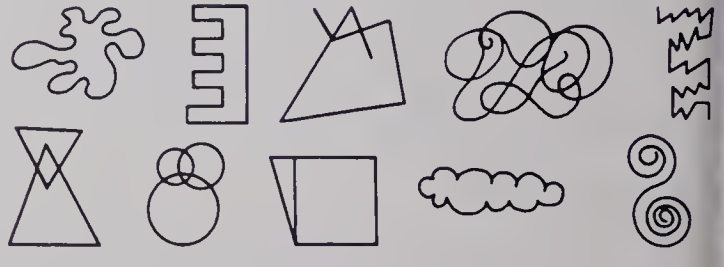
• Ex. 1. For the present, you may wish to have pupils consider a *curve* as simply a set of points which may be represented by a drawing made without lifting a pencil from the paper.

• Ex. 3. A *closed* curve is one that has no distinct end points. In other words, it has no beginning and no end. A closed curve may be simple or it may not be simple, depending upon whether it crosses itself or not.

• Ex. 6–8. The purpose of the questions pertaining to closed curves formed by line segments is to prepare pupils for the coming review of the topic of polygons.

Individualizing Instruction

• *More capable pupils* can study a series of curves such as those represented below and classify them as (a) simple; (b) not simple; (c) closed; (d) not closed.



• *All pupils* may be asked to represent simple closed curves that (a) contain no line segments; (b) contain both segments and other curves; and (c) contain line segments only.

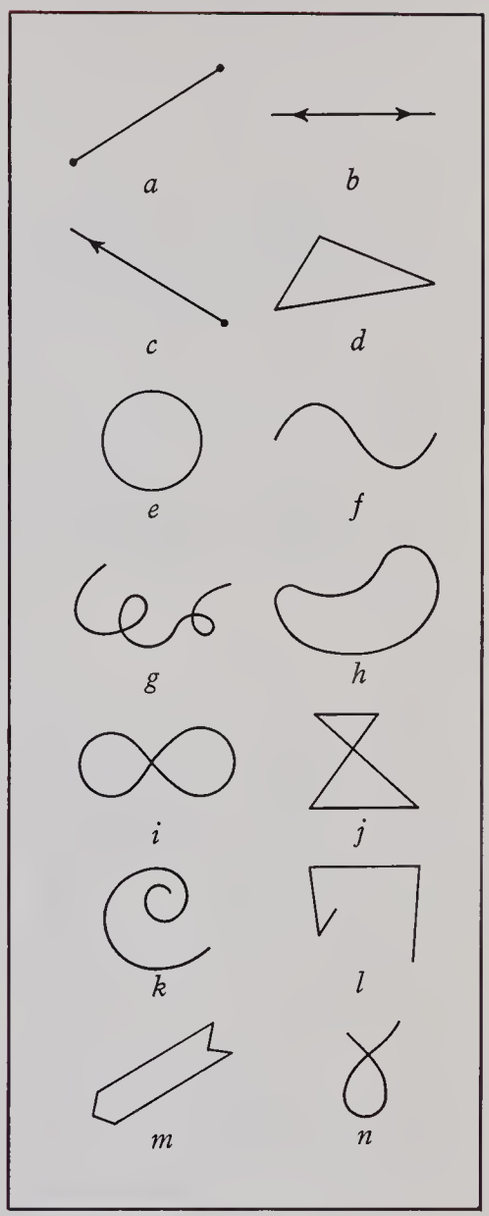
• *Slower learners* may be given the capital letters of the English alphabet and asked to classify the letters in terms of the various classifications of curves. One approach would be to have these children make four columns on a sheet of paper, label the columns *closed*, *not closed*, *simple*, *not simple*, and list each capital letter in the appropriate column. A letter may appear in more than one column.

* Geometric figures in a plane should not be confused with regions which are unions of curves and sets of points interior to them. Regions will be studied on page 90.

Curves

Resurvey [O]

* Each of the geometric figures shown in the box lies in a plane and is called a *curve*.



1. Is a curve a set of points? Yes

2. Give special names for the curves shown in Ex. a–e. a. line segment; b. line; c. ray; d. triangle; e. circle

3. Pictures d, e, h, i, j, and m are models of *closed curves*. Why do you think they are called closed curves? They start and end at the same point. Is it true that some closed curves cross themselves? Yes On the board, show a closed curve which crosses itself three times. Sample answer:

4. When a closed curve does not cross itself, it is called a *simple closed curve*. Which of the pictures in the box represent simple closed curves? d; e; h; m

5. An angle is a curve. Is it a closed curve? No Explain. It does not start and end at the same point.

6. Which of the simple closed curves shown can be thought of as being formed by segments? d; m

7. Can a simple closed curve be formed by two segments only? No

[W]

8. On your paper, show

a. a closed curve which crosses itself exactly four times. Sample answer:

b. a curve which is not a closed curve. Sample answer:

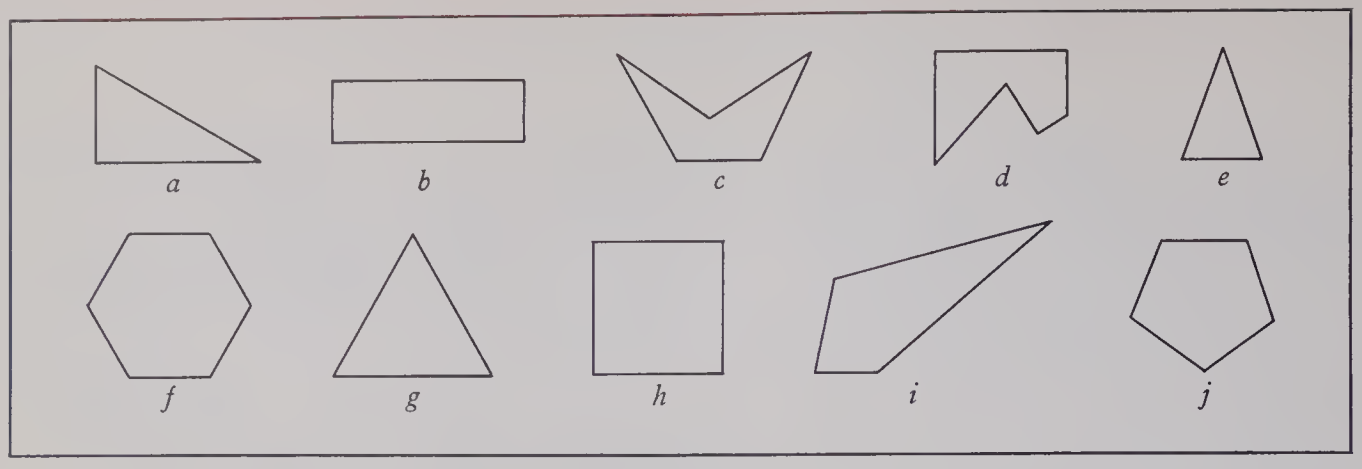
c. a simple closed curve which may be thought of as being formed by six line segments. Sample answer:

d. a curve which crosses itself three times, yet is not a closed curve. Sample answer:

9. On your paper, show a simple closed curve formed by

a. three segments. **b.** four segments. Sample answers:

* In polygons the number of sides is equal to the number of angles.



Polygons

Resurvey [O]

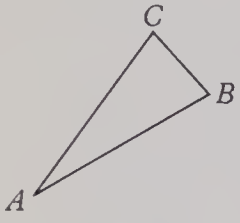


Fig. 1

* 1. Are the geometric figures shown in the box simple closed curves? Yes Can they be thought of as being formed by line segments only? Yes Such simple closed curves are called *polygons*.

A polygon is a simple closed curve that is formed by line segments only.

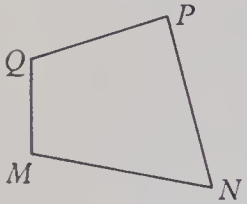


Fig. 2

2. Segments *AB*, *AC*, and *BC* are the *sides* of the polygon shown in Fig. 1. Name the sides of the polygon shown in Fig. 2. QP, PN, MN, QM

2. Give the number of sides of each polygon shown in the box. a. 3; b. 4; c. 5; d. 6; e. 3; f. 6; g. 3; h. 4; i. 4; j. 5

3. How many sides does a triangle have? 3 a quadrilateral? 4 a pentagon? 5 a hexagon? 6 Which pictures in the box represent triangles? a, e, g quadrilaterals? b, h, i pentagons? c, j hexagons? d, f

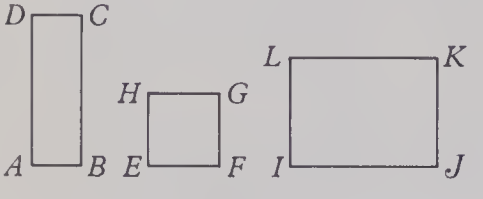


Fig. 3

4. The quadrilaterals shown in Fig. 3 are called parallelograms or rectangles? Which one is a square? EFGH

5. Name some things in your classroom that are shaped like rectangles. Answers will vary.

[W]

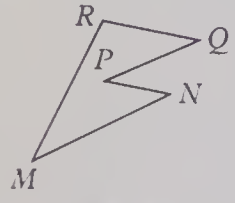
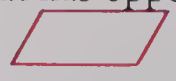


Fig. 4

6. Write names for the five sides of the polygon shown in Fig. 4. $\overline{MN}$, $\overline{NP}$, $\overline{PQ}$, $\overline{RQ}$, $\overline{MR}$

7. On your paper, show a quadrilateral that has opposite sides parallel but is not a rectangle. Sample answer: 

8. What is the least number of sides a polygon can have? 3

▲ **Reteaching.** Set 5.

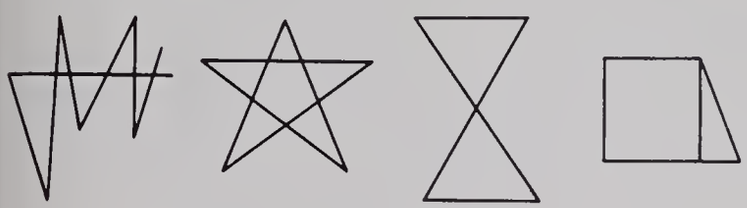
Teaching Page 46

Pupil's Objectives

(a) To review the geometric concept of polygon; (b) to both identify and draw pictures of simple polygons; and (c) to name some polygons.

Background

A simple closed curve consisting entirely of segments is called a *simple polygon*. All the polygons shown on the pupil's page are simple polygons. A closed curve consisting entirely of segments some of which cross each other is not a simple polygon. Each of the following pictures does not represent a simple polygon.



This text restricts itself to the study of simple polygons. Therefore, wherever the term *polygon* is used in the text it is to be understood to refer to a *simple polygon*.

The following table lists the names of polygons of from 3 to 10 sides:

3 sides—triangle	7 sides—septagon
4 sides—quadrilateral	8 sides—octagon
5 sides—pentagon	9 sides—nonagon
6 sides—hexagon	10 sides—decagon

The suffix *lateral* in *quadrilateral* refers to *side*, while the suffix *gon* refers to *angle*. The prefix in each name is derived from the Latin and refers to the number of sides or angles. However, it must be emphasized that while the name of a polygon may indicate a specified number of angles, there is an equal number of sides or vice versa. It is usually easier for pupils to identify a polygon in terms of the number of sides rather than the number of angles.

The point of intersection of two segments forming the sides of a polygon is called a *vertex*. A polygon may be named by using a capital letter for each of its vertices. As a rule, alphabetical order is employed in naming polygons. The pentagon in Fig. 4 on the pupil's page may be named pentagon *MNPQR*.

Pre-Book Lesson

- Have each pupil show on the chalkboard a model of a simple closed curve consisting only of segments. Number each polygon for easy identification and then group the polygons according to their number of sides.
- Guide pupils to discover the characteristics common to all the curves in each group. Also, for each polygon shown, compare the number of its sides and angles. Ask the pupils to explain what conclusions they reach as a result of these comparisons.

Using the Text Page

- Ex. 1–3. A polygon is named for the number of sides or the number of angles. You may want to lead pupils to discover that the number of sides is equal to the numbers of angles in every polygon. Also, in naming one side or more than one side of a polygon, the letters are usually listed in alphabetical order.
- Ex. 4. Each of the quadrilaterals shown in Fig. 3 is a rectangle. A square is a special rectangle all of whose sides are the same length.
- Ex. 7. This would include any parallelogram whose angles are not right angles.

Individualizing Instruction

- *More capable pupils* can be asked to look up the derivation of the prefixes *poly*, *tri*, *quad*, and the suffixes *lateral* and *gon* to see how polygons are named.
- *All pupils* should be led to understand that a polygon consists of only the set of points in the segments. Remind pupils that there are points inside (interior) and points outside (exterior) the polygon. However, the name of a specific polygon identifies only the set of points in the union of its sides (segments). A point in the interior for the polygon is not a point in the polygon itself.

Pupil's Objectives

(a) To review the geometric concepts of triangle and quadrilateral; and (b) to review the classification of triangles and quadrilaterals in terms of relationships among the sides.

Background

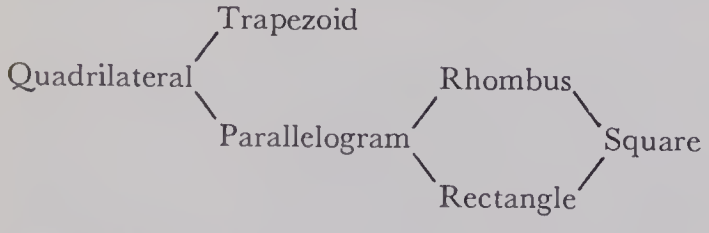
One set of classifications of triangles deals with the relationship among the sides. If no two sides of a triangle are the same length, it is called a *scalene* triangle. If two sides are the same length, it is called *isosceles*. If all three sides are the same length, it is *equilateral*. All equilateral triangles are also isosceles.

Another classification of triangles is based upon angle-size. If all three angles are acute, the triangle is called *acute* triangle. If one angle is obtuse, the triangle is called *obtuse triangle*. If one angle is a right angle, the triangle is called a *right triangle*.

Quadrilaterals may be classified also in terms of side and angle relationships. If a quadrilateral has two and only two sides parallel, it is called a *trapezoid*. When the non-parallel sides are the same length, a trapezoid is isosceles. If the opposite sides of a quadrilateral are parallel, it is called a *parallelogram*.

A parallelogram for which all angles are right angles is called a *rectangle*. A parallelogram for which all sides are the same length is called a *rhombus*. A rhombus for which all angles are right angles is called a *square*. A square may also be defined as a rectangle all of whose sides are the same length.

The diagram below shows one way of picturing the relationship among the various quadrilaterals.



Pre-Book Lesson

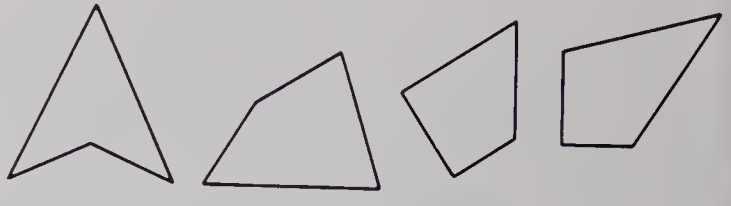
• Have a supply of various triangular shapes made from colored paper or flannel and display them on a chalkboard or flannel board, grouping them according to the number of sides that are the same length.

• Display quadrilaterals of various shapes and have the pupils group them according to a common characteristic. Suggest that the common characteristic may deal with the length or position of sides or the size of angles.

Using the Text Page

• Ex. 1–2. Introduce at this point, if desired, the classification of triangles in terms of size of sides or angles (see Background).

• Ex. 4–5. All the quadrilaterals pictured in box B are either trapezoids or parallelograms. You may wish to point out at this time that quadrilaterals representing neither trapezoids nor parallelograms have no special name. Represent a few such as the following:



• Ex. 7. Pupils should distinguish between *square* and *rectangle* and realize that all squares are rectangles, but not all rectangles are squares.

Individualizing Instruction

• *More capable pupils* can be asked to summarize the relationship among the various quadrilaterals by making a diagram similar to that above. An effective diagram would include both word names and pictures of the different classifications.

• *All pupils* can be asked to name the specific triangle or quadrilateral whose shape most closely resembles the surface of a physical object pointed out to them in the classroom. You may wish to have pupils identify specific triangular or quadrangular shapes as represented by sheets of paper that they may be instructed to fold or tear according to your directions. You may also use pictures of buildings and other structures and have pupils identify representations of specific shapes.

Triangles and Quadrilaterals

Resurvey and extension [O]

1. If two sides of a triangle are the same length, the triangle is called an *isosceles triangle*. If all three sides of a triangle are the same length, the triangle is called an *equilateral triangle*. Which pictures in box A appear to represent isosceles triangles? b, c, e, f equilateral triangles? c Are equilateral triangles also isosceles triangles? Yes

2. If no two sides of a triangle are the same length, the triangle is called a *scalene triangle*. Which pictures in box A appear to represent scalene triangles? a, d

3. If the opposite sides of a quadrilateral are parallel, the quadrilateral is called a *parallelogram*. Which pictures in box B appear to represent parallelograms? b, c, d, e

4. If a quadrilateral has exactly one pair of parallel sides, the quadrilateral is called a *trapezoid*. If the non-parallel sides of a trapezoid are the same length, it is called an *isosceles trapezoid*. Which pictures in box B appear to represent trapezoids? a, f, g, h isosceles trapezoids? h

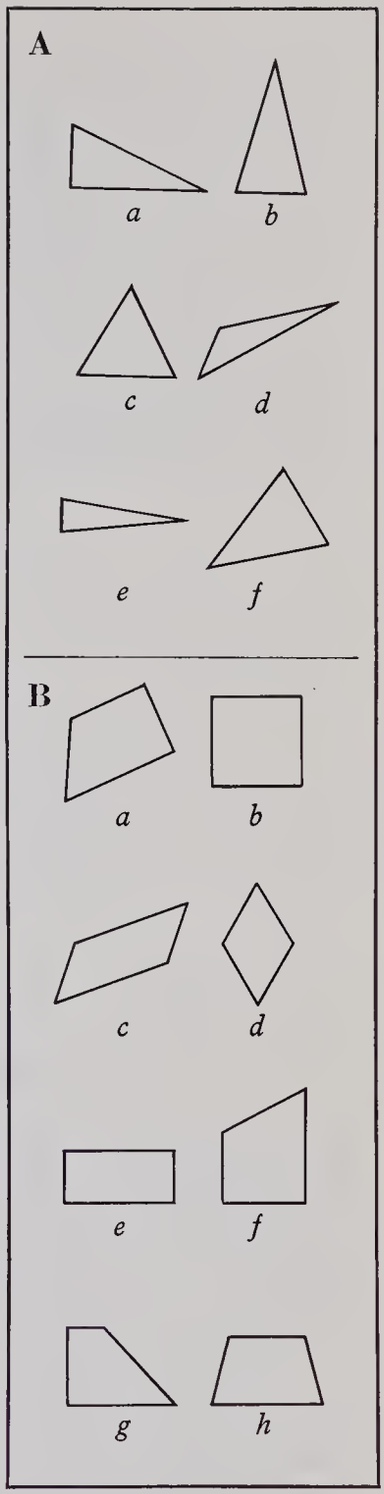
5. If all the sides of a parallelogram are the same length, the parallelogram is called a *rhombus*. Which pictures in box B appear to represent rhombuses? b, d (Sometimes, *rhombi* is used for the plural of *rhombus*.)

[W]

6. On your paper, draw a model of

a. a trapezoid that is not an isosceles trapezoid. Sample answer:

b. a quadrilateral that does not have a pair of parallel sides. Sample answer:



Ex. 7–10. Write *T* or *F*.

7. All squares are rhombuses. T

8. All rectangles are parallelograms. T

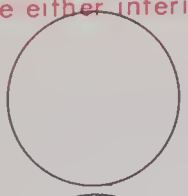
9. All rectangles are trapezoids. F

10. All parallelograms are rectangles. F

◆ **Extra Examples.** Set 34.

● **Extra Activity.** Set 157.

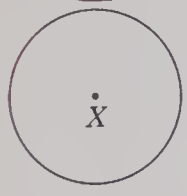
* Pupils must be impressed with the fact that only the end points of a chord or a diameter and one end point of a radius belong to the circle. All other points in these segments are either interior to or exterior to the circle.



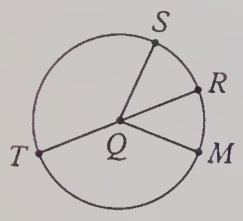
Circles

Resurvey [O]

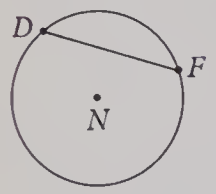
1. The picture at the left represents a figure called a _?_ circle.



2. Name the center for the circle shown at the left. X A circle may be named by the center for the circle, so the circle shown at the left may be named circle _?_ X.

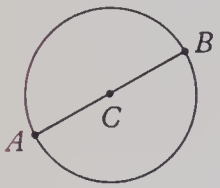


3. For a circle, a *radius* is a line segment of which one end point is the center for the circle and the other end point belongs to the circle. Name the radii (*radii* is the plural of *radius*) that are shown for circle Q. $\overline{QT}, \overline{QS}, \overline{QR}, \overline{QM}$



4. Circle N. Do the end points of $\overline{DF}$ belong to the circle? Yes $\overline{DF}$ is called a *chord* for the circle.

A chord for a circle is a line segment whose end points belong to the circle.

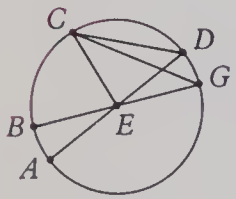


5. Circle C. Do points A and B belong to the circle? Yes Does $\overline{AB}$ contain the center for the circle? Yes Line segment AB is called a *diameter* for the circle.

* **A diameter for a circle is a chord that contains the center for the circle and whose end points belong to the circle.**

[W]

6. Is a diameter for a circle also a chord for the circle? Yes



7. Circle E. Name

a. five radii. $\overline{EA}, \overline{EB}, \overline{EC}, \overline{ED}, \overline{EG}$ b. four chords. $\overline{CD}, \overline{BG}, \overline{AD}, \overline{CG}$ c. two diameters. $\overline{BG}, \overline{AD}$

8. Copy and complete the following table.

Base Ten	Base Four	Base Five	Base Seven	Base Eight
12	_?_ 30	_?_ 22	_?_ 15	_?_ 14
_?_ 25	_?_ 121	100	_?_ 34	_?_ 31
_?_ 49	_?_ 301	_?_ 144	100	_?_ 61
_?_ 58	_?_ 322	_?_ 213	_?_ 112	72

● **Extra Activity.** Set 158.

Teaching Page 48

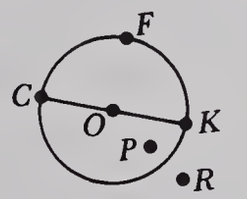
Pupil's Objectives

(a) To review the geometric concept of circle; and (b) to review the names of segments one or both of whose end points belong to the circle.

Background

A simple closed curve formed by the set of all points in a plane a specified distance from a specified point is called a *circle*. The specified point is called the *center*, and the specified distance is called the *radius*. The name *radius* is also given to any segment one of whose end points is the center for the circle and whose other end point belongs to the circle.

In the drawing below, points C, F, and K are points pictured as being contained in the set of points belonging to the circle. Points O, P, and R are not members of this set of points. This means that the center for a circle and any point in the interior or the exterior for the circle is not a part of the circle.



If the end points of a segment are in the set of points belonging to the circle, the segment is called a *chord*. Any chord that contains the center for the circle is called a *diameter* for the circle. A diameter might also be described as one of the longest chords for a given circle.

Pre-Book Lesson

Have several pupils draw on the chalkboard pictures of simple closed curves that do not contain a segment. If necessary, add a few drawings so that there will be a set of circles shown. Ask the pupils to identify those closed curves within this set that have a common characteristic and to attempt to describe this common characteristic in words.

Using the Text Page

• Ex. 2. The center for a circle is *not* a point belonging to the circle. It is a point belonging to the interior for the circle. Point out its usefulness in connection with identifying radii and diameters.

• Ex. 4–6. A *chord* is any segment whose end points are points belonging to the circle. A chord containing the center for the circle is called a *diameter*. Every diameter is a chord but every chord is not a diameter since only the longest chords are diameters. Review briefly the relationship between radius and diameter for a given circle.

Individualizing Instruction

• *More capable pupils* may study statements such as the following; tell whether they are true or false; and make such additions or corrections as will make the false statements true.

(a) A circle is a set of all points a specified distance from a specified point.

(b) Any two radii form a diameter.

(c) All radii are also chords.

(d) All chords are longer than radii.

(e) Chords are sets of points belonging to a circle.

(f) Radii are sets of points all of which are in the interior for a circle.

• *All pupils* can be asked to list as many facts as they can about the geometric concept of circle. For example, (a) it is a set of points; (b) the points form a closed curve; (c) all the points are in the same plane; and (d) a circle has only one dimension, and that is length. A circle has no width nor any thickness. A circle may be likened to a hoop, or a ring, not the cover of a tin can or a target.

• *Slower learners* may be asked to prepare posters picturing circles of various sizes and the line segments associated with a circle: chord, radius, and diameter.

They may also prepare lists of objects which suggest one or more circles such as the following: tin cans; glasses; watch dial; ice cream cone; some cereal boxes; and jars.

Teaching Pages 49–51

Pupil's Objectives

To take the regular end-of-chapter tests, including (a) Test of Information and Meaning 1; (b) Diagnostic Test 1; (c) Problem Test 1; (d) Computation Test 1; and (e) to evaluate the extent to which the skills and understandings presented in Chapter 1 have been mastered.

Background

These four tests, beginning on page 49 and extending through pages 50 and 51, constitute a well rounded testing of the skills and understandings presented in Chapter 1. An analysis of the results of these tests should reveal where individual weaknesses exist. These weaknesses should be diagnosed and corrective measures taken before proceeding with the work in the next chapter. These corrective measures may take the form of reteaching, where the number of pupils involved indicates the need for it, and also the individual assignment of exercises from appropriate sets of Extra Examples as follow-up to the reteaching. The time and effort invested at this point in overcoming weaknesses should pay big dividends in the future.

Pre-Book Lesson

• A general review may be held a day prior to the test and major concepts re-established. Vocabulary and reading skills should be examined and a step-by-step procedure for problem-solving established.

• Explain to the pupils the purposes of these tests in evaluating their understandings of the concepts and skills covered in the chapter. Indicate the value to the individual of these tests, both as a means of showing what he has learned and what he should have learned but did not learn. Try to promote a favorable attitude toward these tests, and tests in general, particularly among pupils who tend to become nervous or upset by testing, and those who do not realize the importance of the tests. Emphasize the positive aspect of tests as an opportunity for the pupil to prove to himself and to the teacher that he has mastered his work.

Using the Text Page

• The tests may be given over a period of two or more days in any order. No more than two tests should be given in any one day.

• Before each test, have the pupils scan the page and read the directions to be sure they understand what they are to do. Explain the function of the Diagnostic Test 1, and the use of the Practice Sets.

• Have the pupils check their addition by the use of the Commutative Property of Addition and check their subtraction by use of addition.

• Before taking Problem Test 1, remind pupils to: (a) determine the mathematical relationship in the problem; (b) express this relationship as a mathematical sentence; (c) estimate the answer; (d) solve the mathematical sentence and compare the answer to the estimate.

Individualizing Instruction

• After the tests have been completed, *all pupils* should correct their mistakes and then work appropriate Extra Examples Sets to strengthen weak areas. They may also work Supplementary Activities found at the end of the chapter if they complete the test early.

An error-analysis chart showing for each example the number of pupils making errors in that example will help you visualize areas of individual and class weaknesses. Any concept in which there is widespread pupil weakness should be retaught. As a follow-up to the reteaching, you may wish to assign appropriate exercises from the Extra Examples Sets provided.

• *More capable pupils* may help *slower learners* with Extra Examples Sets.

Discuss the mathematics program with your pupils. From such a discussion, you may determine what they enjoy learning and what topics they would like to explore further. If there are topics they find very difficult or do not enjoy, discuss this also. Encourage in every way possible the development of proper attitudes toward the study of mathematics, and an appreciation of not only the practical value but also the future value of this tremendously useful subject.

Do You Understand?

Test of Information and Meaning 1

1. For the numeral 345,719, write the digit that is in

a. ten's place. 1 **b.** thousand's place. 5 **c.** hundred-thousand's place. 3

2. In two different ways, write the meaning of 435. Sample answers: $400+30+5$, $(4\times10^2)+(3\times10)+5$

3. Round to the nearest hundred: **a.** 637 600 **b.** 1,292 1,300 **c.** 99 100

Ex. 4–7. List within braces the members of the set. $A = \{0, 3, 5, 7\}$

4. $A \cup B$ $\{0, 2, 3, 4, 5, 7\}$ 5. $A \cap B$ $\{3\}$ 6. $A \cap C$ $\{\ \}$ $B = \{2, 3, 4\}$

7. The intersection of set A and the set of multiples of 5 $\{0, 5\}$ $C = \{6, 9, 10, 11\}$

8. Rename with Roman numerals: **a.** 33 XXXIII **b.** 45 XLV **c.** 99 XCIX

9. Rename with base-five numerals: **a.** 6 11_{five} **b.** 27 102_{five} **c.** 81 311_{five}

10. Rename with base-ten numerals: **a.** LIX 59 **b.** 42_{five} 22 **c.** 23_{four} 11

11. What operation would you use to solve $n - 23 = 57$? Addition

12. Using 5 and 8 for the addends and n to represent the sum, write four mathematical sentences which illustrate the addends-sum relationship. $n=5+8$; $n-8=5$; $n-5=8$; $n=8+5$

13. Draw a number-line picture which shows the relationship expressed by the sentence $4 + n = 9$. See below.

14. Rename with exponent forms: **a.** 49 7^2 **b.** 125 5^3 **c.** 8 2^3

15. Fig. 1. Name a line; a ray; a segment; an angle. Sample answers: $\overline{AB}$ $\overline{EC}$ $\overline{DE}$ $\angle DEC$

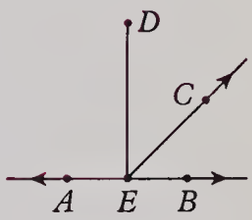


Fig. 1

16. How many end points does a line segment have? 2 a ray? 1

17. Name the sides of the polygon shown in Fig. 2. $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AD}$

18. Does a diameter for a circle contain the center for the circle? Yes

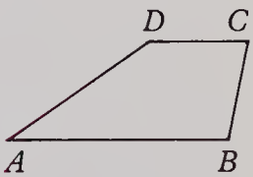


Fig. 2

On your paper, show

19. an acute angle. Sample answer:

20. two parallel segments.

21. two simple closed curves.

22. a scalene triangle.

23. Write a mathematical sentence which illustrates the Associative Property of Addition. Sample answer: $(1+2)+3=1+(2+3)$

13.

Do You Make Mistakes?

Diagnostic Test 1

Copy and work the examples in rows 1–3. If you make mistakes in any row, work the practice sets for that row.

	a	b	c	d	e	Practice: Use Sets
1. Add	27 98 125	539 63 602	7,809 3,445 11,254	$72.63 5.88 $78.51	$320.05 17.95 $338.00	25–26
2. Add	438 9 38 485	58 472 8,550 9,080	3,837 556 7,779 12,172	$55.69 0.74 33.88 $90.31	$360.90 4.05 125.55 $490.50	28–29
3. Subtract	297 36 261	304 85 219	407 287 120	3,798 866 2,932	$315.00 37.98 $277.02	30–31

Can You Solve Problems?

Problem Test 1

Let n represent the number which will answer the question in the problem. Write a mathematical sentence for the problem and then solve to find the answer for the problem.

1. Jack is saving his money to buy a bicycle which costs $37.95. He has saved $18.40. How much more does he need? $18.40+n=37.95$; $19.55

2. 2,378 is how much greater than 1,450? $2,378-n=1,450$; 928

3. 478 is how much greater than 375? $478-n=375$; 103

4. Jack earned $8.75 during a 3-day period raking leaves. He earned $2.25 the first day and $3.80 the second day. How much did he earn the third day? $n=8.75-(2.25+3.80)$; $2.70

5. Mr. Brown owns a 21-acre farm in Orange County and a 137-acre farm in Austin County. That is how many acres? $n=21+137$; 158

6. 456 is how much less than the sum of 109, 87, and 556? $n=(109+87+556)-456$; 296

You may wish to keep the scores for end-of-chapter tests on 3″ × 5″ cards. The sample card below, if kept up to date, will tell at a glance the level of a pupil's achievement in different areas. In addition, it will indicate whether a given pupil is improving or not in the three areas tested.

Harry Green		Grade 6
Chapter	*Per Cent Right Computation Tests*	*Per Cent Right Problem Tests*
1	77	88
2		
3		
4		
5		
6		
7		

The table of per cents shown below will enable you to determine quickly the per cent for each pupil's score. For instance, Harry's Problem-Test score for Chapter 1 was 7 so, from the table, his per cent correct is found to be 88. His Computation-Test score was 20, so his per cent correct is 77.

Supplementary Activities

The end-of-chapter tests will indicate which pupils need additional help or review before proceeding to the work of the next chapter. Those pupils who do not need additional review may be assigned some of the Supplementary Activities while the teacher is engaged in conducting remedial work and review.

The suggested activities that follow may be used for review or enrichment at any time throughout the year, including assignment to pupils who finish end-of-chapter tests early. They may be used with *all pupils* at the discretion of the teacher.

- For each of the sets described in Ex. 7–11 on page 3, have the pupils name another set which is equivalent to the set described. (These would be any sets with the same number of members as the sets in Ex. 7–11.)
- For each of the sets described below, have the pupils describe another set which will have a one-to-one correspondence with the given set:

(a) the set of all states in the United States (any other set with 50 members),

(b) the set consisting of the first 26 counting numbers,

(c) the set of all countries in the United Nations Organization (This will vary from year to year, so it should be checked in a recent encyclopedia or almanac.),

(d) the set of all the pupils in the class,

(e) the set of all whole numbers (This set is infinite so the one-to-one correspondence will have to be described in words. One such correspondence is between each whole number and a whole number 1 greater. That is, the set of counting numbers. Another correspondence is between each whole number and a number twice as great. That is, the set of all even numbers.)

Table of Per Cents for Chapter 1 Scores

Problem Test		Computation Test					
Score	Per Cent	Score	Per Cent	Score	Per Cent	Score	Per Cent
1	13	1	4	10	39	19	73
2	25	2	8	11	42	20	77
3	38	3	12	12	46	21	81
4	50	4	15	13	50	22	85
5	63	5	19	14	54	23	88
6	75	6	23	15	58	24	92
7	88	7	27	16	62	25	96
8	100	8	31	17	65	26	100
		9	35	18	69		

• Draw the following diagrams on the board, explaining that circles A, B, and C represent set A, set B, and set C respectively.

Fig. 1

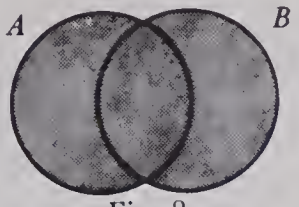


Fig. 2

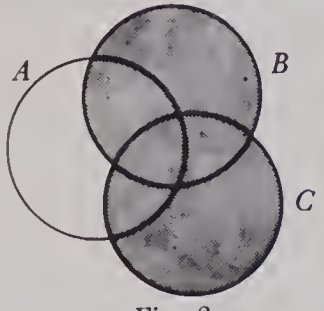


Fig. 3

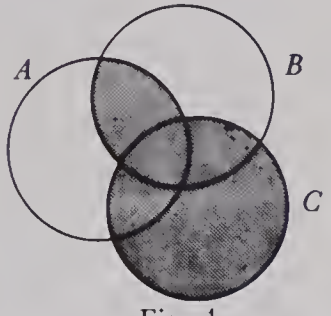


Fig. 4

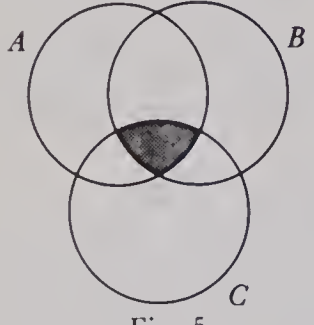


Fig. 5

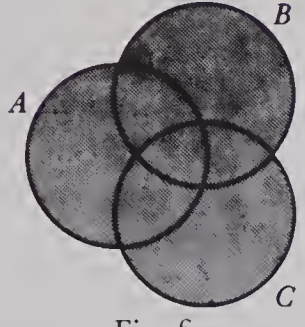


Fig. 6

Ask *more capable pupils* to indicate which region shown with shading represents each of these sets:

a. $A \cup B$ (Fig. 2)
b. $A \cap B$ (Fig. 1)
c. $(A \cap B) \cup C$ (Fig. 4)
d. $(A \cap B) \cap C$ (Fig. 5)
e. $B \cup C$ (Fig. 3)
f. $(A \cup B) \cup C$ (Fig. 6)

• In Ex. 5–10 on page 9, have pupils find:

(a) $(D \cup E) \cup F$
(b) $(E \cup F) \cap G$
(c) $(D \cap F) \cap G$
(d) $(D \cap F) \cup G$
(e) $(D \cap E) \cap F$
(f) $(D \cup E) \cup G$

• Ex. 21–23 on page 11, round to the nearest (a) hundred (b) thousand (c) million.

• Ex. 24–27 on page 11, round to the nearest ten cents.

• At the bottom of page 12. If the notation "$n(A)$" represents the number of elements in set A, which of the following represent true sentences?
(a) $n(A \cup B) = n(A) + n(B) - n(A \cup B)$ (False)
(b) $n(A \cup B) = n(A) - n(B) + n(A \cup B)$ (False)
(c) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ (True)

• Page 13, Ex. 5–19. Write an Egyptian numeral for each of these Roman or Arabic numerals.

• Have *all pupils* try the following:

(a) Count by twos through twenty using base-two numerals. (10; 100; 110; 1000; 1010; 1100; 1110; 10000; 10010; 10100)

(b) How can you recognize an even number named with a base-two numeral? (the digit in the one's place is 0)

(c) Count by threes through 40 using base-three numerals. (10; 20; 100; 110; 120; 200; 210; 220; 1000; 1010; 1020; 1100; 1110)

(d) How can you recognize a multiple of three when written using base-three numerals? (the digit in the one's place is 0)

(e) Without performing any calculations, but using the answers to questions a–d as a guide, count by fives through 80 in base five. (10; 20; 30; 40; 100; 110; 120; 130; 140; 200; 210; 220; 230; 240; 300; 310)

7. At Midville Garden Supply, Mr. Thomas bought some grass seed for $3.98 and a garden hose for $2.69. How much change did he receive from $10.00? n = 10.00 − (3.98 + 2.69); $3.33

8. In constructing a new pen for his dog, Mr. Thomas used some lumber which cost $7.85 and some fencing material which cost $6.40. The material for the old pen had only cost $8.75. How much more did the material for the new pen cost than the material for the old pen?

n = (7.85 + 6.40) − 8.75; $5.50

How Well Can You Compute?

Computation Test 1

Ex. 1–5. Copy and add.

1.	2.	3.	4.	5.
438	707	3,678	37,890	$19.98
229	639	447	4,333	5.98
667	1,346	4,125	42,223	$ 25.96

Ex. 6–10. Copy and subtract.

6.	7.	8.	9.	10.
374	$4.89	$360.00	5,677	345,762
209	2.95	18.45	289	44,837
165	$ 1.94	$ 341.55	5,388	300,925

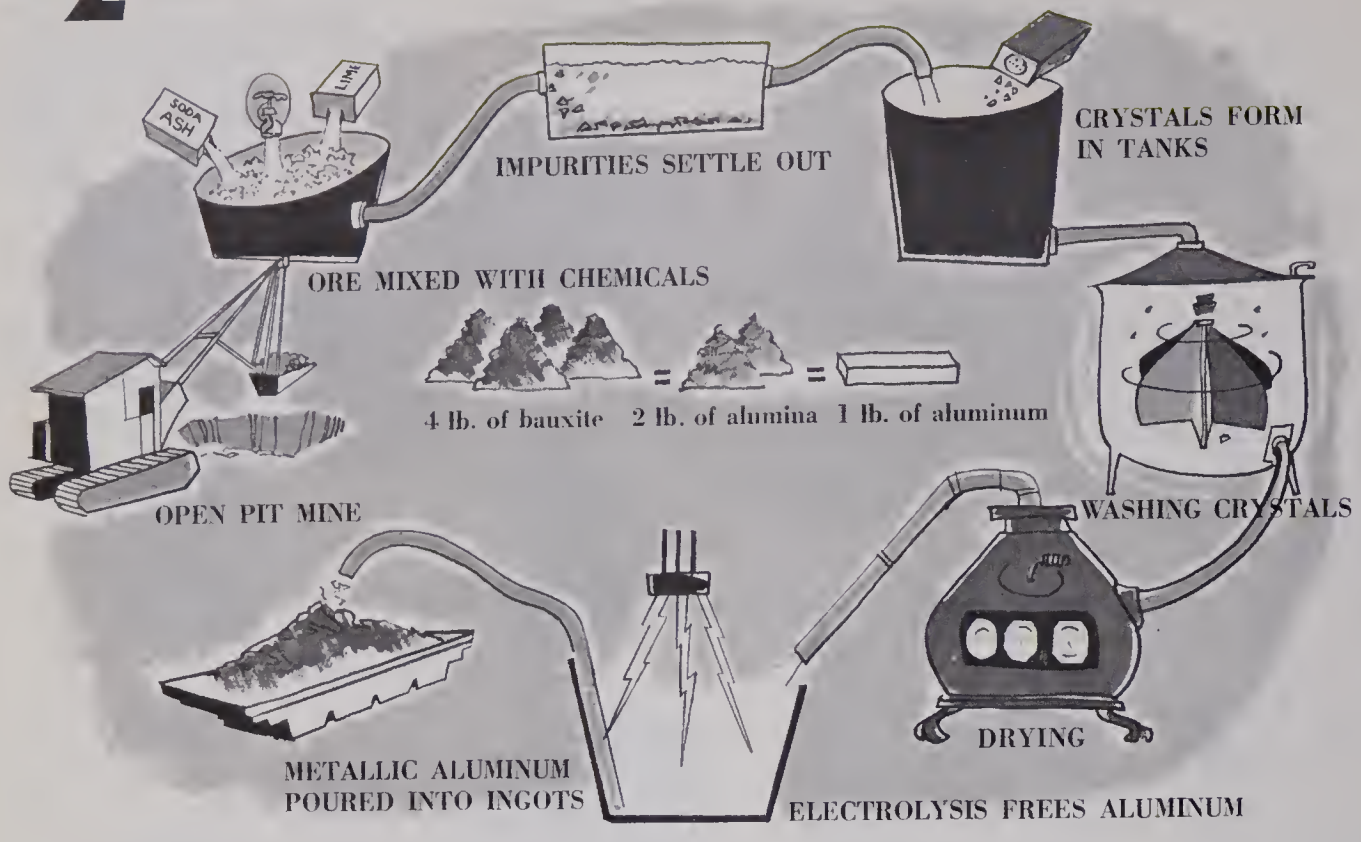
Ex. 11–22. Find the number represented by the letter.

11. $n = 387 + 439$ 826
12. $n = 674 - 448$ 226
13. $n - 47 = 798$ 845
14. $694 - n = 268$ 426
15. $x + 480 = 567$ 87
16. $873 - 39 = y$ 834
17. $57 + n = 780$ 723
18. $200 + x = 398$ 198
19. $n - 2{,}983 = 4{,}886$ 7,869
20. $29{,}445 - 3{,}628 = x$ 25,817
21. $29 + n = 8{,}963$ 8,934
22. $x + 3{,}820 = 6{,}000$ 2,180

Ex. 23–26. Find the standard numeral for the number represented by n.

23. $n = 356 - (46 + 97)$ 213
24. $n = 17 + (498 - 301)$ 214
25. $n = 63 + 26 + 147 + 98$ 334
26. $n = 400 - (17 + 27 + 154)$ 202

2



Remembering about Multiplication and Division

Number-line pictures [O]

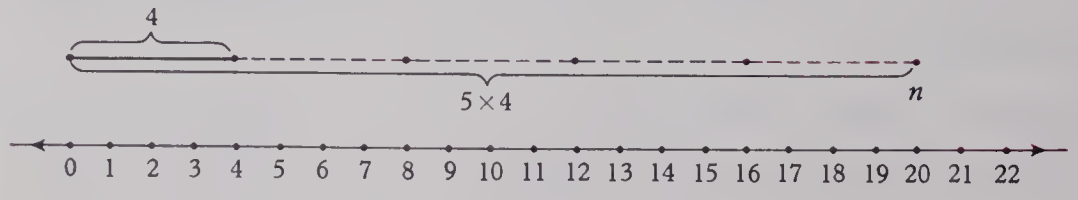
An ore called bauxite is the chief source of aluminum. A white substance called alumina is refined from bauxite and then metallic aluminum is produced from alumina.

1. How many ounces of aluminum can be extracted from 5 lb. of bauxite if 1 lb. of bauxite yields 4 oz. of aluminum?

a. To find the answer for the problem, we multiply 4 by _?_. 5

b. Explain how the mathematical sentence $n = 5 \times 4$ shows the relationship expressed in the problem. Give the number represented by n. Key idea: It expresses the factors-product relationship. 20

c. Explain the number-line picture below. 5 pounds of bauxite yields _?_ ounces of aluminum. 20 Key idea: It shows the factors-product relationship.



Overview - Chapter 2

• Chapter 2 continues to provide extensions from the strands of modern topics and introduces new material concerning these topics. Since much of this chapter is a resurvey of multiplication and division with whole numbers, presented in earlier grades, it is important to judge properly the rate at which the material should be covered so that no pupils will become bored and lose interest, and no pupils, especially the slower learners, will develop a feeling of frustration.

• **Resurvey.** Ideas concerning properties of the four operations and prime and composite numbers; the factors-product relationship; sets in geometry including planes, simple closed curves, and interiors for circles and angles; use of and understanding of the four operations; using number-line pictures; and use of zeros in division are some of the topics resurveyed in this chapter.

• **Extension.** Set operations and terminology previously presented as a unique topic are presently applied to geometry, subsets in geometry, and the study of regions. Other topics extended include factors, prime and composite numbers, prime factors and products of primes including those expressed in exponent form; the identity element for multiplication and quicker ways to divide; and an introduction to statistics in the form of mean, mode, and median.

• **Problem-Solving.** The use of mathematical sentences to structure systematic procedures for solution of problems involving a great variety of topics is stressed. The factors-product relationship; differentiating between addition and multiplication; and differentiating between multiplication and division are presented. Such areas as the following are covered: geography; science; money problems; farming and household situations; problems involving social situations, school, and averages; and 2-step problems using the four operations and choosing the appropriate operation.

• **Maintenance.** Pages involving maintenance include such topics as the four operations and their properties; inequalities and true and false mathematical sentences; multiplying by tens and greater numbers; standard numerals and different forms for operations; use of set symbols and terminology, operations on sets, and use of sets in geometry; the addends-sum and factors-product relationship and showing remainders; and a brief introduction to statistics involving the mean, mode, and median.

• **Enrichment.** On page 63 is an enrichment. topic pertaining to the fact that every counting number is either a power of 2 or is a sum of powers of 2. Operations with exercises showing missing numerals are covered, and counting numbers expressed as sums of numbers which are specified members of a set are presented.

Judicious use of the enrichment topics can help challenge the *more capable pupils* during the resurveys presented in this chapter.

• **Tests.** The chapter is closed with the four basic chapter tests that provide you with a very good evaluation of each pupil's readiness to understand the work of the next chapter. Occasionally, sets of exercises within the chapter can be used as testing material particularly the "How Well Do You Remember?" and the "To Keep in Practice" exercises. Topics covered include using and understanding the four operations and their properties; use of factor trees and factors in mathematical sentences; prime and composite numbers and exponent form; sets of numbers and sets in geometry; problem-solving and computation; finding averages and using measurements.

Teaching Pages 52 and 53

Pupil's Objectives

(a) To develop an understanding of the relationship between multiplication and division by use of number-line pictures; and (b) to solve problems involving the factors-product relationship.

Background

Division is the inverse operation of multiplication and can be illustrated by analyzing the following problem:

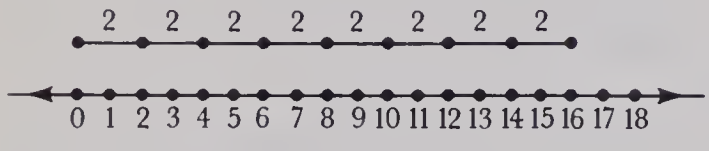
If air mail rate is 8¢ per letter, how many letters can be mailed using 16 four-cent stamps?

(a) There are 16 four-cent stamps.

(b) It takes 2 stamps to mail one letter.

(c) The problem is to determine the number of sets of 2 stamps each there are in a set of 16 four-cent stamps.

(d) The number-line picture which can be used as a model for the problem is shown below.



In this interpretation, the product is 16, the known factor is 2 (thc number of elements in each set), and the missing factor is n (the number of equivalent sets). This relationship can be represented by $n = 16 \div 2$. From the number-line picture we can see that the number of equivalent sets of 2 four-cent stamps each, in a set of 16 four-cent stamps, is 8. $n = 8$

You may refer to a gcneral chemistry text or an encyclopedia for information about the aluminum-refining process in order to be able to discuss it more fully with pupils.

Pre-Book Lesson

• Use objects in the room to rcview the idea of division as involving a known total number of elements and a known numbcr of equivalent sets when the number of elements in each of the equivalent sets is to be found. For example, "If there are 72 books in the room and 36 pupils, how many books are to be given to each pupil?"

• Use objects in the room to review the idea of division as involving a known total number of elements and the known number of elements in each equivalent set when the number of equivalent sets is to be found. For example, "If there are 120 sheets of paper to be distributed so that each pupil is to receive 5 sheets, how many pupils can be supplied with paper?"

• Discuss the problem from the second paragraph in the Background. Have pupils do the work at the board with the help of number-line pictures.

Using the Text Pages

• Ex. 1. The relation between ounces of aluminum and pounds of bauxite might be best brought out by asking, "If 1 pound of bauxite yields 4 ounces of aluminum, 2 pounds will yield how much?" and proceeding from there. Pounds do not have to be converted to ounces, and vice versa in this particular problem.

• Ex. 3a. Some pupils may mark off on the number-line picture two congruent segments each 7 units in length, while other pupils may mark off 7 congruent segments each 2 units in length. Since multiplication is commutative, either method will serve as a model for the multiplication.

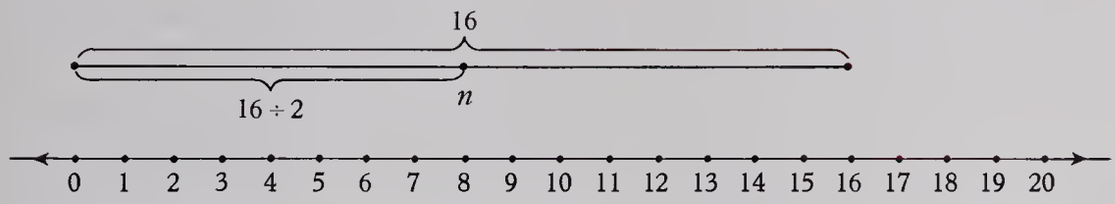
Individualizing Instruction

Slower learners will benefit from a thorough discussion of the aluminum-refining process involving the terms *ore*, *bauxite*, *castings*, *kilowatt-hours*, *aluminum*, *alumina*, *substance*, *refined*, and *extracted*. Any reading problems should be clarified during the oral discussion. These pupils may need to use models and number-line pictures with the written work.

2. If 1 pound of alumina can be refined from 2 pounds of bauxite, how many pounds of alumina can be refined from 16 pounds of bauxite?

a. To find the answer for the problem, we divide 16 by 2.

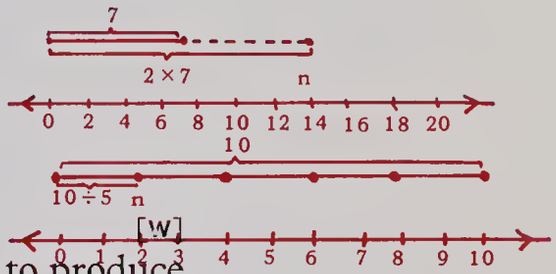
b. On the board, write a mathematical sentence to show the relationship expressed in the problem. $n = 16 \div 2$ Then explain the number-line picture below. Key idea: It shows the factors-product relationship.



c. Give the answer for the problem. 8 pounds

3. On the board, draw a number-line picture for

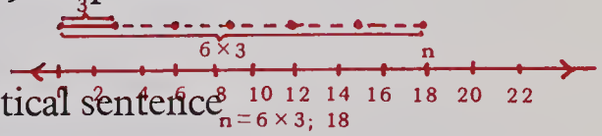
a. $n = 2 \times 7$. **b.** $n = 10 \div 5$.



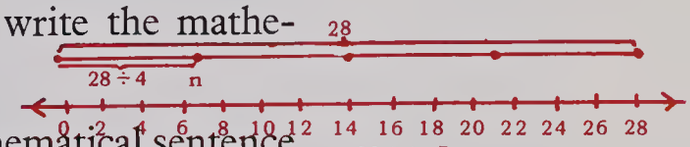
4. If it takes about 10 kilowatt-hours of electricity to produce a pound of aluminum, about how many kilowatt-hours of electricity will be needed to produce 75 pounds of aluminum? 750

5. One year, 2,117,929 tons of aluminum were produced in the United States. The next year, 2,312,528 tons were produced. That was an increase of how many tons? 194,599

6. Some aluminum castings weigh as much as 8,000 pounds. That is how many tons? 4



7. Draw a number-line picture for the mathematical sentence $n = 6 \times 3$. Below the number-line picture, write the mathematical sentence and show the number for n. $n = 6 \times 3$; 18



8. Draw a number-line picture for the mathematical sentence $n = 28 \div 4$. Write the mathematical sentence and then solve. $n = 28 \div 4$; 7

9. Name three things which are made from aluminum. Read more about the production of aluminum. Write three problems like Ex. 4–6 for the information you read. Answers will vary.

* This is an illustration of the Commutative Property of Multiplication because, thinking of 4×5 as a name for the number 20, the sentence is simply $20 \times 9 = 9 \times 20$.

Thinking about Properties of Multiplication

Resurvey [O]

1. Does 3×7 name the same number as 7×3? Yes Is $3 \times 7 = 7 \times 3$ a true mathematical sentence? Yes This is an illustration of the *Commutative Property of Multiplication*. Complete the following so that each will be a true sentence.

a. $5 \times 12 = 12 \times \_?\_^{5}$ **b.** $\_?\_^{10} \times 32 = 32 \times 10$

2. At the board, find $(3 \times 2) \times 5$. 30 Did you first perform the operation indicated within the parentheses? Yes Now find $3 \times (2 \times 5)$. 30 Is $(3 \times 2) \times 5 = 3 \times (2 \times 5)$ a true mathematical sentence? Yes This is an illustration of the *Associative Property of Multiplication*. Complete the following so that each will be a true sentence.

a. $(17 \times 5) \times 20 = 17 \times (5 \times \_?\_^{20})$

b. $9 \times (31 \times \_?\_^{50}) = (9 \times 31) \times 50$

* 3. The sentence $(4 \times 5) \times 9 = 9 \times (4 \times 5)$ is an illustration of the $\_?\_^{\text{Commutative}}$ Property of Multiplication.

4. The sentence $(21 + 4) + 8 = 21 + (4 + 8)$ is an illustration of the Associative Property of $\_?\_^{\text{Addition}}$.

5. To find the number represented by n in $n = 4 \times 5 \times 7$, we can find $(4 \times 5) \times 7$, which is $\_?\_^{20} \times 7$, or we can find $4 \times (5 \times 7)$, which is $4 \times \_?\_^{35}$. Find in two ways the number represented by n. 140

6. Without using paper and pencil, name the number represented by n in $49 \times n = 57 \times 49$. 57 Explain your reasoning.

From the Commutative Property of Multiplication we know that $49 \times 57 = 57 \times 49$.

[W]

Ex. 7–18. Solve.

7. $n = 4 \times 8 \times 5$ 160
8. $n = 20 \times 2 \times 3$ 120
9. $n = 9 \times 4 \times 5$ 180
10. $5 \times y = 5 \times 27$ 27
11. $13 \times 75 = 75 \times n$ 13
12. $5 \times 2 \times 8 = x$ 80
13. $n = 7 \times 1 \times 8$ 56
14. $26 \times 4 \times 25 = n$ 2,600
15. $65 \times 125 = n$ 8,125
16. $n = 50 \times 4 \times 49$ 9,800
17. $84 + 16 + 218 = n$ 318
18. $y \times 35 = 5 \times (7 \times 18)$ 18

Teaching Page 54

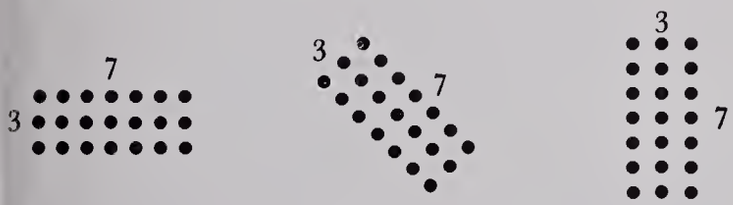
Pupil's Objective

To review the Commutative and Associative Properties of Multiplication.

Background

Multiplication is a binary operation on two numbers, called factors, to produce a third number called the product. If the factors are whole numbers, the multiplication may be thought of in terms of repeated addition in which all addends are the same number. Another way to find the name for the product 3×7 is through the use of the addition $7 + 7 + 7$.

Multiplication is commutative; the factors may occur in any order without affecting the produet. That is, $3 \times 7 = 7 \times 3$, and may be verified by using repeated addition, $7 + 7 + 7 = 3 + 3 + 3 + 3 + 3 + 3 + 3$. The Commutative Property of Multiplication may be illustrated by the use of an array:



An array of 3 rows of 7 dots each can represent the multiplication 3×7. Then, by a simple 90° rotation this can be changed to an array of 7 rows of 3 dots each, representing the multiplication 7×3. The total number of dots remains unchanged.

To find the product of three numbers, either the third number is multiplied by the product of the first two numbers, or the produet of the seeond and third numbers is multiplied by the first. The final product is the same either way. This idea may be represented by $(a \times b) \times c = a \times (b \times c)$, and it is called the Associative Property of Multiplication.

Pre-Book Lesson

Discuss how you can find the result for a multiplication using repeated addition. Picture an array of 3 rows with 7 dots in each row. Lead pupils to discover that the total number of dots may be found by multiplying 3 times 7 or we may add the number of dots (7) in each of the 3 rows. Then have pupils picture an array of 7 rows with 3 dots in each row. From this array, encourage pupils to discuss how they might find the total number of dots using 7×3 or $3 + 3 + 3 + 3 + 3 + 3 + 3$. From experiences with arrays pupils should discover $3 \times 7 = 7 \times 3$ and $7 + 7 + 7 = 3 + 3 + 3 + 3 + 3 + 3 + 3$.

Using the Text Page

- Ex. 1–3. In Ex. 3, the order of the factors as well as their grouping is changed so it is an illustration of both properties of multiplication. Refer to the Background and use arrays on cardboard to illustrate the properties.
- Ex. 4. This is an example of the Associative Property of Addition.
- Ex. 5. When multiplying 3 numbers, it is usually easier to first multiply two numbers whose product is a multiple of 10. This makes the subsequent multiplication simpler. For example $(4 \times 5) \times 7 = 20 \times 7$ which is certainly easier than $4 \times (5 \times 7) = 4 \times 35$.
- In Ex. 7, 8, 9, 12, 14, 16, and 17, first mentally perform the multiplication or addition that results in a multiple of 10.

Individualizing Instruction

- Present to *more capable pupils* the problem of multiplying one chair by one chair, in order to bring out the concept that multiplication is an operation on numbers; it is not an operation on sets. Have these pupils make up more complicated exercises with 4 or more factors and illustrate the associative and commutative properties using these exercises.
- *All pupils* can be given exercises in multiplication and be told to verify or check their multiplication by applying the Commutative Property of Multiplication.*

* See items 9R and 9S on page xix. These items can be used as manipulative materials to illustrate the properties.

Teaching Page 55

Pupil's Objectives

(a) To review the Distributive Property of Multiplication over Addition; and (b) to apply this property to the solution of mathematical sentences.

Background

There is a relationship between the operation of multiplication and the operation of addition. This relationship is expressed by the Distributive Property of Multiplication over Addition:

$$a \times (b + c) = (a \times b) + (a \times c)$$

At first, this may seem like a mathematical concept too abstract for sixth-grade arithmetic. However, mastering this concept is essential for understanding the algorithms of multiplication using the vertical form.

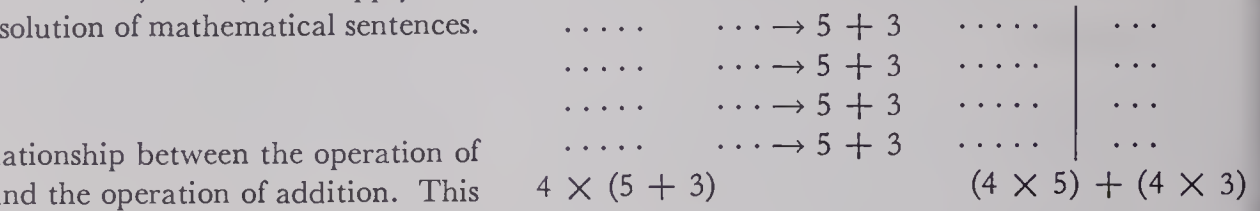
If a pupil is given the mathematical sentence $n = 4 \times (5 + 3)$, he may solve this by first performing the addition within the parentheses and then performing the multiplication. However, this can also be solved by applying the idea that the result of a multiplication may be found by performing repeated addition. The result of $4 \times (5 + 3)$ can therefore be found by performing $(5 + 3) + (5 + 3) + (5 + 3) + (5 + 3)$. Since addition is commutative, the order of the addends may be changed so that like addends are grouped together: $(5 + 5 + 5 + 5) + (3 + 3 + 3 + 3)$. Since we can use repeated addition to find the result of a multiplication, we can also think about using multiplication to find the result of a repeated addition. Then $(5 + 5 + 5 + 5) = (4 \times 5)$ and $(3 + 3 + 3 + 3) = (4 \times 3)$. To summarize:

$$\begin{aligned} 4 \times (5+3) &= (5+3)+(5+3)+(5+3)+(5+3) \\ &= (5+5+5+5)+(3+3+3+3) \\ &= (4 \times 5)+(4 \times 3) \end{aligned}$$

When presented in this manner, the distributive property is well within the ability of 6th-graders to understand.

Pre-Book Lesson

As a preparation for the discussion of the distributive property, show a 4-by-8 array on the chalkboard and separate it in such a manner as to demonstrate that $4 \times (5 + 3)$ is the same as $(4 \times 5) + (4 \times 3)$:



Using the Text Page

• Ex. 1. If the pupils have difficulty in understanding the reasoning behind the distributive property, insert two additional steps into box B so that it reads:

$$\begin{aligned} n &= 4 \times (5 + 3) \\ &= (5 + 3) + (5 + 3) + (5 + 3) + (5 + 3) \\ &= (5 + 5 + 5 + 5) + (3 + 3 + 3 + 3) \\ &= (4 \times 5) + (4 \times 3) \end{aligned}$$

• Ex. 3–4. The distributive property may also be expressed as $(b + c) \times a = (b \times a) + (c \times a)$ because multiplication is commutative.

• Ex. 5–16. The pupils will probably find it easier to solve these mathematical sentences by using the method shown in box A. However, it is essential that they also solve them using the method in box B, applying the distributive property, as a knowledge of this method is necessary to master the concept of performing multiplication efficiently.

Individualizing Instruction

• *More capable pupils* may be asked if multiplication is distributive over subtraction and if it is distributive over division. Multiplication is distributive over subtraction but not over division. This can be demonstrated by the following:

$$\begin{aligned} 3 \times (12 \div 4) &\stackrel{?}{=} (3 \times 12) \div (3 \times 4) \\ 3 \times 3 &\stackrel{?}{=} 36 \div 12 \\ 9 &\stackrel{?}{=} 3 \end{aligned}$$

• *Slower learners* can be given objects such as blocks or toothpicks to arrange in an array as shown in the Background section, and then the array may be partitioned to demonstrate the distributive property.

* When more than three lines are used, as in box B, it is better to write "n" again.
** A different form of the distributive property is $(4 + 5) \times 2 = (4 \times 2) + (5 \times 2)$.

The Distributive Property

Resurvey [O]

1. Study the work in boxes A and B. Is it true that both $4 \times (5 + 3)$ and $(4 \times 5) + (4 \times 3)$ are names for the number 32? Yes Is the following a true sentence? $4 \times (5 + 3) = (4 \times 5) + (4 \times 3)$ Yes This is an illustration of the *Distributive Property of Multiplication over Addition*, or simply, the *Distributive Property*. Complete the following so that each will be a true sentence.

A

$n = 4 \times (5 + 3)$
$= 4 \times 8$
$= 32$

B *

$n = 4 \times (5 + 3)$
$= (4 \times 5) + (4 \times 3)$
$= 20 + 12$
$n = 32$

a. $7 \times (5 + 13) = (\_? \overset{7}{} \times 5) + (\_? \overset{7}{} \times 13)$

b. $9 \times (3 + 7) = (9 \times 3) + (9 \times \_? \overset{7}{})$

c. $(3 \times 41) + (3 \times 9) = \_? \overset{3}{} \times (41 + 9)$

** **2.** At the board, solve each of Ex. a–f as in box B. Check your work by solving each sentence as in box A.

a. $n = 2 \times (4 + 5)$ 18

b. $n = 7 \times (1 + 9)$ 70

c. $n = 3 \times (11 + 9)$ 60

d. $n = 15 \times (10 + 2)$ 180

e. $n = 9 \times (50 + 5)$ 495

f. $n = 20 \times (3 + 7)$ 200

3. Study the work in box C. At the board, solve $n = (20 + 5) \times 4$ as in box C. To check your work, $(20 + 5) \times 4 = 25 \times 4$, or $\_? \overset{100}{}$.

n = (20+5)×4 = (20×4)+(5×4) = 80+20 = 100

C

$n = (3 + 2) \times 6$
$= (3 \times 6) + (2 \times 6)$
$= 18 + 12$
$n = 30$

4. Do $(3 + 2) \times 6$ and $6 \times (3 + 2)$ name the same number? Yes What property does this illustrate? At the board, find the standard numeral for the number named by $6 \times (3 + 2)$ as in box B.

Commutative Property of Multiplication

n = 6×(3+2) = 18+12
= (6×3)+(6×2) n = 30

[W]

Ex. 5–16. Solve first as in box A and then as in box B.

5. $n = 9 \times (21 + 9)$ 270

6. $n = 10 \times (10 + 20)$ 300

7. $n = 8 \times (1 + 1)$ 16

8. $n = 30 \times (2 + 5)$ 210

9. $n = 11 \times (10 + 1)$ 121

10. $n = 1 \times (8 + 15)$ 23

11. $n = 6 \times (0 + 12)$ 72

12. $n = 24 \times (8 + 7)$ 360

13. $n = 6 \times (50 + 2)$ 312

14. $n = 12 \times (10 + 5)$ 180

15. $n = 7 \times (40 + 9)$ 343

16. $n = 4 \times (73 + 27)$ 400

◆ **Extra Examples.** Set 35.

* Notice that for Ex. 1-4 the set of numbers which a and b can represent is very precisely stated.

Identity Elements

Resurvey and extension [O]

*1. $3 \times 1 = \_?\_$ 3, $8 \times 1 = \_?\_$ 8, $15 \times 1 = \_?\_$ 15 and $1{,}254 \times 1 = \_?\_$ 1,254 If a represents any whole number, is $a \times 1 = a$ a true sentence? Yes

2. If a and b represent any two counting numbers and $a \times b = a$, what number must b represent? 1

3. If b represents *any* whole number, is $0 \times b = 0$ a true sentence? Yes Explain. The product of 0 and ony whole number is 0.

4. If a represents any whole number, is $1 \times a = a$ a true sentence? Yes Give two illustrations. Sample onswers: $1 \times 2 = 2$ $1 \times 356 = 356$

When two whole numbers are multiplied and one of the numbers is 1, the product is the other number. Because this is true, 1 is called the *identity element for multiplication.*

5. Name the identity element for addition. 0

6. Sample answers:
$1 + 0 = 1$
$263 + 0 = 263$
$0 + 56 = 0$
$1 \times 0 = 0$
$0 \times 263 = 0$
$0 \times 56 = 0$
$1 \times 0 = 0$
$263 \times 1 = 263$
$56 \times 1 = 56$

6. On the board, write three mathematical sentences which illustrate the result of adding a number to 0; multiplying a number by 0; multiplying a number by 1.

7. The number 1, the identity element for multiplication, is often referred to as the *multiplicative identity*. The number 0 is often referred to as the *additive identity*. Is $5 - 5$ another name for the additive identity? Yes Is $17 - (15 + 1)$ another name for the multiplicative identity? Yes Explain. 5–5 is onother name for 0. 17–(15+1) is onother nome for 1.

[W]

Ex. 8–13. Find the number represented by n without using paper and pencil. Record the answer.

8. $n = 0 + 537$ 537

9. $n = 29 \times (27 - 26)$ 29

10. $1 \times n = 78$ 78

11. $0 + n = 440$ 440

12. $n = 1 \times (1 \times 1)$ 1

13. $n = (15 + 3) - 18$ 0

a. $29 - (4 \times 7)$
b. $1 \times 1 \times 1$
c. $3 \times (5 - 5)$
d. $1 \times (0 + 0)$
e. $447 - 446$
f. 0×1

14. Which of the expressions in the box name the additive identity? c, d, f the multiplicative identity? o, b, e

Teaching Page 56

Pupil's Objectives

(a) To review the Identity Element for Multiplication; and (b) to review the special use of zero in multiplication.

Background

On Page 28 the concept of zero being the identity element for addition was reviewed. There is also an identity element for multiplication. 1 is the only whole number that can be combined with any whole number n under multiplication such that the product is the given whole number n. That is, $n \times 1 = n$ is true for any whole number n. The number 1 is called the *identity element for multiplication*, or the *multiplicative identity*.

The number zero has a special property in multiplication. The product of any whole number and zero is always zero. If n represents any whole number, $n \times 0 = 0$.

The identity element for multiplication and the role of zero in multiplication can be described in terms of an array. A 1-by-n array consists of one row having n elements, so the array will have a total of exactly n elements. Therefore, $1 \times n = n$. Multiplication is commutative so it follows that $n \times 1 = n$.

A 5-by-0 array will consist of five rows each with zero elements. Since the array will contain zero elements, $5 \times 0 = 0$. An n-by-0 array will consist of n rows each containing zero elements. The array will contain zero elements so therefore $n \times 0 = 0$. Multiplication is commutative so it follows that $0 \times n = 0$.

The role of zero in multiplication can also be described by interpreting multiplication of whole numbers using repeated addition:

$$5 \times 0 = 0 + 0 + 0 + 0 + 0$$
$$= 0$$

Pre-Book Lesson

Review the identity element for addition and have pupils work some exercises involving this element.

Ask which number of the set of whole numbers could be an identity element for multiplication. Discuss what characteristics this number must have.

Using the Text Page

- Ex. 1. You may wish to raise the question "How do you *know* that $3 \times 1 = 3$ and $8 \times 1 = 8$?". After pupils have memorized a multiplication table, they sometimes forget how they determined the products in the table in the first place.
- Ex. 2. If $a \times b = a$, then b must represent the number 1 if a and b both represent *counting* numbers. However, if a and b can represent *whole* numbers, then a and b may both represent zero and the sentence $a \times b = a$ will be true. This is why it is necessary to restrict a and b to counting numbers.
- Ex. 3. The sentence $0 \times b = 0$ is always true, regardless of what number b represents. You may ask if $0 \times b = b$, what number must b represent? In this example, b could only represent zero.
- Ex. 14c, d, and f are numerals for the number zero so they name the additive identity.
- Ex. 15a, b, and e are numerals for the number 1, so they name the multiplicative identity.

Individualizing Instruction

More capable pupils can be given the addition and multiplication tables for the system of numeration used by the inhabitants of the planet "Xeropides."

+	@	#	%
@	@	#	%
#	#	%	#@
%	%	#@	##

×	@	#	%
@	@	@	@
#	@	#	%
%	@	%	##

They can then be asked to determine whether or not this system of numeration contains an identity element for addition, an identity element for multiplication, and the special property of an element under multiplication that zero has in our system of numeration. The additive identity is @, the multiplicative identity is #, and $n \times @ = @$.

Pupil's Objectives

(a) To resurvey differentiating between addition and multiplication as operations to use in solving problems; and (b) to gain more experience solving problems by use of mathematical sentences.

Background

In earlier grades, pupils have been taught that finding the number for a total is involved when either addition or multiplication is used, and that multiplication is the more appropriate operation when the problem deals with equivalent sets.

In solving problems of the type on the text page, the pupil must first recognize whether the problem involves like addends or unlike addends. Finding a sum involving unlike addends requires addition. If finding a sum involving like addends, then multiplication may be substituted for the operation addition.

Since multiplication is used in solving problems involving a number of equivalent sets, the product is found by multiplying the number of one set by the number of equivalent sets.

This does not mean that the results of all multiplications can be found through addition of like addends. The multiplication $^{-}3 \times {}^{-}4$ is such an example. Finding the product of two negative integers requires reasoning involving the application of the relationship between multiplication and addition which is described by the distributive property.

Pre-Book Lesson

- Use sets of objects in the classroom which are not equivalent sets to demonstrate finding a sum when the sets show unlike addends.
- Use equivalent sets of objects in the classroom to demonstrate finding a total when these sets can be thought of as representing like addends and then show how the operation multiplication can be used. If the room has five rows of six desks each, then the total number of desks can be found by multiplying the number of the set of desks in one row by the number of such equivalent sets of desks.
- Use examples of finding a sum involving like addends where addition is impractical and where multiplication is obviously the better operation to use by thinking of equivalent sets. "What would be the cost of 49 tickets if each cost $1.55?"*

Using the Text Page

- Ex. 1–7. Select a pupil to read each problem. Have the pupil tell whether the problem involves equivalent sets or sets that are not equivalent, whether the sum involves like addends or unlike addends, and what operation he would use and why.
- Before beginning the written work, have *slower learners* name the addends in each problem and indicate whether the addends are like or unlike.

Individualizing Instruction

- Ask *all pupils* for the name of the property that describes the relationship between multiplication and addition (distributive property).
- Have *slower learners* suggest problems which require addition and some problems which require multiplication, with other pupils telling what operation should be used in solving the problem.
- Have *more capable pupils* suggest two-step problems which require both addition and multiplication such as "What is the product resulting from multiplying 36 by the sum of 11 and 14?"

* See items 8 and 9R, page xix. By interchanging the cards and using them with sets of pegs, pupils may more easily visualize a relation between addition and multiplication.

* Addition could be used, but when the addends are equal it is better to use multiplication.
** Encourage pupils to write "about" when answering questions such as those in Ex. 2, 4, and 6.

* Add or Multiply?

Resurvey [O]

For each of Ex. 1–7, tell what operation to use to find the answer for the problem. Give the reason for your choice. Key idea: for each problem, the addends are equal.

1. Mr. Canty received $7.25 for each of 24 wood carvings. How much did he receive in all? Multiplication; $n=7.25\times24$; $174.00

** 2. Mr. Canty has a four-shelf bookcase. Each shelf holds about 26 books. About how many books does the bookcase hold? Multiplication; $n=4\times26$; about 104

3. Eric and his five friends each spent 45¢ for lunch while at the Zoo. How much did they spend in all for their lunches? Multiplication; $n=6\times0.45$; $2.70

4. Each year for five years the Science Club has sold about 275 calendars. About how many calendars were sold during the five-year period? Multiplication; $n=5\times275$; about 1,375

5. The membership of the Science Club is 3 times that of the Art Club. The Art Club has 29 members. The Science Club has how many members? Multiplication; $n=3\times29$; 87

6. If Mr. Brown uses about 25 gallons of gasoline each week, about how many gallons would he use in 1 year (52 weeks)? Multiplication; $n=25\times52$; about 1,300

7. If gasoline costs 31¢ per gallon, how much would 15 gallons cost? Multiplication; $n=15\times31$; $4.65

[W]

Ex. 1–7. Write and solve a mathematical sentence for each.

* There is only one addends-sum relationship given two addends and a sum. There are 4 mathematical sentences which express this relationship.

Factors-Product Relationship

Resurvey [O]

A
$6 = 3 \times 2$
$6 \div 3 = 2$
$6 \div 2 = 3$
$6 = 2 \times 3$

* 1. The four mathematical sentences in box A illustrate the *factors-product relationship*. In each sentence, the numeral 6 names the product and the numerals 3 and 2 name the factors. What operation is used to find the product of two factors? Multiplication

2. On the board, write $28 \div 7 = 4$ and then, using 28, 7, and 4, write three other mathematical sentences to illustrate the factors-product relationship. $28 = 4 \times 7$; $28 = 7 \times 4$; $28 \div 4 = 7$

3. In the sentence $5 \times 2 = n$, does n represent the product or a factor? Give the standard numeral for the number represented by n. 10

B
$n \div 8 = 4$
$n = 8 \times 4$
$= 32$

4. In the sentence $n \div 8 = 4$, does n represent the product? Yes Explain the work in box B. Were the factors multiplied to find the product? Yes Key idea; $n \div 8 = 4$ can be shown as $n = 8 \times 4$.

5. As in box B, find the number represented by n for each of Ex. a–f.

a. $n \div 8 = 10$ 80 **b.** $n \div 3 = 11$ 33 **c.** $n \div 25 = 12$ 300

d. $18 = n \div 65$ 1,170 **e.** $n \div 7 = 92$ 644 **f.** $102 = n \div 3$ 306

C
$42 \div n = 7$
$n = 42 \div 7$
$= 6$

6. In the sentence $42 \div n = 7$, what numeral names the product? Does n represent a factor? We say that n represents the *unknown factor* and that 7 names the *known factor*. Study the work in box C. What operation was used to find the number represented by n? Division

7. At the board, find the number represented by n for each of Ex. a–f as in box C.

a. $20 \div n = 5$ 4 **b.** $33 \div n = 3$ 11 **c.** $63 \div n = 7$ 9

d. $14 = 98 \div n$ 7 **e.** $306 \div n = 9$ 34 **f.** $11 = 121 \div n$ 11

8. If the factors of a product are known, we multiply to find the product. If a product of two factors is known and one of the two factors is known, we divide to find the other factor.

Teaching Pages 58 and 59

Pupil's Objective

To review solving mathematical sentences involving the factors-product relationship.

Background

When a division is expressed by $12\overline{)36}$, it is readily apparent that you are trying to find a factor which when multiplied by 12 will result in the product 36. However, when the division is shown by $36 \div 12 = n$ it may not be so readily apparent that 36 is the product of the known factor 12 and the unknown factor n. The pupil will probably have little difficulty in understanding that if $5 \times 4 = 20$ then $5\overline{)20}$ is 4 and $4\overline{)20}$ is 5. He may have more difficulty when this same idea is expressed by use of mathematical sentences: if $5 \times 4 = 20$, then $20 \div 5 = 4$, and $20 \div 4 = 5$. This is particularly true when the relationship is expressed as "If $a \times n = b$, then $b \div n = a$ and $b \div a = n$." It may make more sense to the pupil if words are used instead of the letters:

(first factor) $\times$ (second factor) = (product)
(product) $\div$ (first factor) = (second factor)
(product) $\div$ (second faetor) = (first factor)

There is one restriction in this relationship, and that is division by zero is undefined and not permitted. Hence, a multiplication for which one of the factors is zero cannot be associated with a division by that zero factor.

Pre-Book Lesson

• Review meaning of *factor* and *product*. Select two factors and a product, and express the factors-product relationship among these three numbers in as many ways as possible.

For example, $3 \times 4 = 12$ $\quad$ $12 \div 4 = 3$
$4 \times 3 = 12$ $\quad$ $12 \div 3 = 4$

• Write several mathematical sentences and show n for an unknown addend, sum, factor, or product in several different positions. Have pupils show n in a position for solving the mathematical sentence and identify the number for n.

Using the Text Pages

• Ex. 1. If pupils have difficulty understanding the factors-product relationship, it may help to use physical objects or even the pupils themselves to demonstrate this relationship. For example, if 12 pupils are arranged in 4 groups of 3 pupils each, (number of groups) times (number in each group) equals (total number of pupils); (total number of pupils) divided by (number of groups) equals (number in each group); and (total number of pupils) divided by (number in each group) equals (number of groups).

• Ex. 8 summarizes one of the basic principles of the factors-product relationship. It is important that the pupils understand this principle.

• Ex. 9. Be sure pupils know the correct form for the division. When the product and one factor are known, the *product* is divided by the *known factor* to find the *unknown factor*. Be sure pupils do not divide the *known factor* by the *product*. Division is not commutative, so the order of the product and known factor is important.

Individualizing Instruction

• *More capable pupils* may be asked to use the idea of number of groups and number in each group to try to explain why an example like $16 \div 0 = n$ has no meaning while $0 \div 16 = n$ has the solution 0.

• *All pupils* may create simple word problems for which one of Ex. 12–23 would express the factors-product relationship for the numbers in the problem.

• *Slower learners* may need to use manipulative materials to use the factors-product relationship in multiplication and division. You may wish to have these pupils group together sets having the same numbers of objects to show multiplication, or separate a set to illustrate division.*

* See 12 and 13, page xix. Cubical counting blocks and plastic discs are especially useful where work with the addends-sum and factors-product relationship require that pupils be able to visualize manipulation of sets of concrete objects.

NOTES

9. Box D. Do both sentences show the same relationship? Yes The product and one factor are known so we divide to find the other factor. On the board, write a mathematical sentence that shows which operation to use to find the number represented by n. Then solve the sentence. $n = 54 \div 6$; 9

D

$6 \times n = 54$

$n \times 6 = 54$

10. Box E. Name the pairs of sentences which show the same relationship. For example, sentences a and f show the same relationship. b and c; d and g; e and h

E

a. $n \div 2 = 6$	**b.** $24 \div n = 6$	**c.** $6 \times n = 24$	**d.** $n \div 24 = 6$
e. $2 \times n = 6$	**f.** $2 = n \div 6$	**g.** $24 \times 6 = n$	**h.** $n = 6 \div 2$

11. For each of Ex. 12–29, tell whether n represents the product or a factor and tell what operation to use to solve the sentence.

[W]

Ex. 12–29. Find the number represented by n.

factor; Division;
12. $64 \div 8 = n$ 8

product; Multiplication;
13. $n = 22 \times 40$ 880

factor; Division;
14. $81 \div n = 3$ 27

product; Multiplication;
15. $n \div 5 = 9$ 45

factor; Division;
16. $100 \div n = 50$ 2

product; Multiplication;
17. $34 \times 15 = n$ 510

product; Multiplication;
18. $n \div 15 = 10$ 150

factor; Division;
19. $n \times 4 = 100$ 25

product; Multiplication;
20. $29 \times 0 = n$ 0

factor; Division;
21. $n \times 2 = 62$ 31

product; Multiplication;
22. $18 = n \div 2$ 36

factor; Division;
23. $45 = 3 \times n$ 15

factor; Division;
24. $225 \div n = 45$ 5

product; Multiplication;
25. $n = 125 \times 6$ 750

product; Multiplication;
26. $n \div 9 = 18$ 162

product; Multiplication;
27. $n = 22 \times (16 + 4)$ 440

product; Multiplication;
28. $(50 + 6) \times 8 = n$ 448

product; Multiplication;
29. $109 \times 16 = n$ 1,744

Ex. 30–37. Copy and complete by writing $=$ or $<$ or $>$ to make a true sentence.

30. $7 \times 10 \;\_?\_\; 160 \div 2$ $\;\;<$

31. $5 + 33 \;\_?\_\; 5 \times 8$ $\;\;>$

32. $6 \times (2 + 3) \;\_?\_\; 52 - 22$ $\;\;=$

33. $81 \div 9 \;\_?\_\; 2 \times 4$ $\;\;>$

34. $27 \div 3 \;\_?\_\; 9 - (1 \times 0)$ $\;\;=$

35. $45 \times 33 \;\_?\_\; 33 \times 45$ $\;\;=$

36. $77 \div 7 \;\_?\_\; 36 \div 3$ $\;\;<$

37. $185 \;\_?\_\; 4 \times 45$ $\;\;>$

* Give pupils every opportunity to discover this "quick way" on their own.

Multiplying by Multiples of Ten

Resurvey [O]

* 1. Tell a quick way to find the product of two counting numbers if one of the numbers is 10; 100; 1,000; 10,000. By showing 0's to the right, move the digits naming the one factor to the left as many places as there are 0's in the numeral naming 10; 100; 1,000; 10,000.

2. For each of Ex. a–d, give the standard numeral for the product without using paper and pencil.

a. 7×10 70 b. 10×25 250 c. 78×100 7,800 d. 265×100 26,500

3. To find 20×7, we can think of 20 as _?_ 2 tens. $20 \times 7 = 2$ tens $\times 7 =$ _?_ 14 tens, or _?_ 140.

A
$n = 20 \times 7$
$= (10 \times 2) \times 7$
$= 10 \times (2 \times 7)$
$= 10 \times 14$
$n = 140$

4. Box A shows another way to find 20×7. Explain each step. What property was used? Associative Property of Multiplication

5. At the board, find 40×8 as in box A.

5.
n = 40 × 8
= (10 × 4) × 8
= 10 × (4 × 8)
= 10 × 32
n = 320

6. For each of Ex. a–d, give the standard numeral for the product without using paper and pencil.

a. 50×7 350 b. 30×9 270 c. 60×3 180 d. 7×30 210

7. To solve $n = 300 \times 4$, we can think of 300 as _?_ 3 hundreds. $300 \times 4 =$ _?_ 3 hundreds $\times 4 =$ _?_ 12 hundreds, or _?_ 1,200. Use this type of thinking to solve $n = 2{,}000 \times 9$. 18,000

8. For each of Ex. a–d, give the standard numeral for the number without using paper and pencil.

a. 600×7 4,200 b. $3{,}000 \times 5$ 15,000 c. 8×700 5,600 d. $30{,}000 \times 2$ 60,000

B
$n = 30 \times 70$
$= (3 \times 10) \times (7 \times 10)$
$= (3 \times 7) \times (10 \times 10)$
$= 21 \times 100$
$n = 2{,}100$

9. Box B. Explain the work shown to solve $n = 30 \times 70$. Can we think of 30×70 as 3 tens $\times$ 7 tens, or 21 hundreds? Yes

10. Give the standard numeral for Ex. a–c.

a. 40×20 800 b. 50×90 4,500 c. 500×500 250,000

[W]

Ex. 11–18. Solve.

11. $n = 20 \times 8$ 160 12. $n = 500 \times 7$ 3,500 13. $4{,}000 \times 8 = n$ 32,000 ∧ 14. $600 \times 50 = x$ 30,000 ∧

15. $n = 60 \times 20$ 1,200 ∨ 16. $300 \times 900 = x$ 270,000 ∨ 17. $n = 10{,}000 \times 45$ 450,000 ∨ 18. $3{,}000 \times 50 = n$ 150,000 ∨

◆ **Extra Examples.** Set 36.

Teaching Page 60

Pupil's Objectives

(a) To review multiplying by one or more tens; and (b) to apply the associative and commutative properties of multiplication when multiplying by tens.

Background

Multiplying by a power of ten (10; 100; 1,000; 10,000; 100,000; · · ·) you can show the product by the numeral for the other factor with a zero for each power of ten at its right. Another way of expressing this is to say that the product is always shown by the numeral for one factor with one more zero shown for each zero in the name of the power of ten. For example, to find the product of $27 \times 1{,}000$, show three zeros to the right of 27 to give 27,000. To find the product of 130×100, show two zeros to the right of 130 to give 13,000.

This short cut, together with the Associative Property of Multiplication, can be used in performing multiplications by one or more tens. For example,

$n = 60 \times 9$
$= (10 \times 6) \times 9$
$= 10 \times (6 \times 9)$ Associative Property of Multiplication
$= 10 \times 54$
$n = 540$

When both factors are one or more tens, the Commutative Property of Multiplication may also be applied in performing the multiplication. For example,

$n = 50 \times 70$
$= (5 \times 10) \times (7 \times 10)$
$= 5 \times [10 \times (7 \times 10)]$ Associative
$= 5 \times [(7 \times 10) \times 10]$ Commutative
$= 5 \times [7 \times (10 \times 10)]$ Associative
$= (5 \times 7) \times (10 \times 10)$ Associative
$= 35 \times 100$
$n = 3{,}500$

Pre-Book Lesson

• Review briefly our place-value system and ask how knowledge of the system could be a help in multiplication by powers of 10. Some pupils may already be familiar with the short cut and be able to bring out its use in a class discussion.

• Review the associative and commutative properties of multiplication by using arrays.

Using the Text Page

• Ex. 1. Ask pupils *why*, when one factor is a power of ten, the product is always shown by the numeral for the other factor with as many zeros to the right as zeros in the numeral for the power of ten. One way to explain this is in terms of place value. When performing the multiplication $7 \times 1{,}000$, the product will be shown as 7 with 3 zeros to the right. The 7 means thousands, not ones.

• Ex. 2–6. Emphasize that application is being made of the Associative Property of Multiplication.

• Ex. 9. Emphasize the application of the Commutative Property of Multiplication.

Individualizing Instruction

• *More capable pupils* may be asked why the vertical form for the multiplication

$$\begin{array}{r} 237 \\ \times\ 200 \\ \hline 47{,}400 \end{array}$$

is just a shorter way for $200 \times 237 = 2 \times (100 \times 237)$
$= 2 \times 23{,}700$
$= 47{,}400$

• *All pupils* can try their skill at multiplying by one or more tens by trying to find the answer without using pencil and paper for Ex. 11–18.

• *Slower learners* may have to use the horizontal form if they have serious difficulty, such as for

$$\begin{array}{r} 30 \\ \times\ 50 \\ \hline 1{,}500 \end{array}$$

using the mathematical sentences and the short cut for performing these multiplications.

Teaching Page 61

Pupil's Objective

To review use of the vertical form for multiplication by applying the Distributive Property of Multiplication.

Background

The multiplication 4×56 is performed by applying the distributive property, regardless of whether the horizontal form or the shorter vertical form is used. 56 is considered as $6 + 50$, then the distributive property is applied:

$$\begin{aligned} 4 \times 56 &= 4 \times (6 + 50) \\ &= (4 \times 6) + (4 \times 50) \\ &= 24 + 200 \\ &= 224 \end{aligned}$$

In vertical form, this would be written:

$$\begin{array}{r} 56 \\ \times\ 4 \\ \hline 24 \\ 200 \\ \hline 224 \end{array}$$

When using the shorter vertical form, the 56 is considered as 6 ones and 5 tens. 4×6 is equal to 24 or two tens and 4 ones. The 4 is written in the one's place in the numeral for the product, and the 2 tens are remembered. 4×5 tens is equal to 20 tens, or 2 hundreds. A 2 is written in the ten's place for the two tens that were to be remembered, and a 2 is written in the hundred's place of the numeral for the product.

$$\begin{array}{r} 56 \\ \times\ 4 \\ \hline 224 \end{array}$$

These principles are extended to apply to examples where both factors are shown by two-place numerals.

$$\begin{aligned} 24 \times 35 &= (20 \times 35) + (4 \times 35) \\ &= 700 + 140 \\ &= 840 \end{aligned}$$

$$\begin{array}{r} 35 \\ \times\ 24 \\ \hline 140 \\ 700 \\ \hline 840 \end{array} \quad \text{or} \quad \begin{array}{r} 35 \\ \times\ 24 \\ \hline 140 \\ 70 \\ \hline 840 \end{array}$$

Pre-Book Lesson

Review the Distributive Property of Multiplication as shown by both forms below:

$$a \times (b + c) = (a \times b) + (a \times c)$$
$$(b + c) \times a = (b \times a) + (c \times a)$$

Use arrays to show that $5 \times (20 + 6) = (5 \times 20) + (5 \times 6)$.

Using the Text Page

- Ex. 1. Point out that this is an application of the Distributive Property of Multiplication.
- Ex. 2. The term "remember 1 ten" is preferable to the traditional term "carry 1" since it encourages more work to be done mentally. The term "carry" refers to the process of "carrying" one digit from one column to another and is useful in earlier grades to develop the mental process of renaming. Pupils should now be able to do work mentally.
- Ex. 4. Point out that this is an application of the distributive property. Explain that the 0 is usually not shown for 450 in step 2, although there is nothing mathematically wrong with showing it.

Individualizing Instruction

- *More capable pupils* can be asked to perform the multiplications in exercises 6–12 by the method shown in box A.
- *All pupils* can be separated into two groups and given a set of exercises to multiply. One group is to use the vertical form and the other group is to use the horizontal form. Try to determine which is the faster method and which is the most accurate method when showing multiplication using these two forms.
- *Slower learners* may benefit by thinking of Ex. 6–12 in terms of multiplying ones, then tens, then finding the sum of the two. They may wish to label each step.

* Explain to pupils why the "0" of "450" is not recorded in step (2).

** Commuting factors means interchanging the factors for an indicated multiplication.

Using Vertical Forms for Multiplication

Resurvey [O]

1. To find 3×45, we may record our work as in box A or box B. 45 may be renamed $40 + 5$, or $5 + 40$, so $3 \times 45 = 3 \times (5 + 40) = (3 \times 5) + (3 \times 40)$. Thus 3×45 may be found by thinking of the sum of 3×5 and 3×40. In box A, which step shows 3×5? (1) 3×40? (2) Do you see that step (3) shows the sum of these two numbers? Yes

2. Box B shows a shorter vertical form for multiplication. Think of 45 as 5 ones + 4 tens.

Multiply ones: $3 \times 5 = \_?$ 15 Rename 15 ones as 1 ten + 5 ones. Write a 5 in one's place in the numeral for the product. Remember 1 ten.

Multiply tens: $3 \times 4 = \_?$ 12 12 tens + 1 ten = $\_?$ 13 tens. Rename 13 tens as 1 hundred + 3 tens. Write a 3 in ten's place and a 1 in hundred's place in the numeral for the product.

3. At the board, find 6×24 first as in box A and then as in box B. 144

4. Is $13 \times 45 = (10 + 3) \times 45$ a true mathematical sentence? Yes Is $(10 + 3) \times 45 = (10 \times 45) + (3 \times 45)$ a true sentence? Yes Then to find 13×45, we can find the sum of $10 \times \_?$ 45 and $3 \times \_?$ 45

In box C, which step shows 3×45? (1) 10×45? (2) Do you see that step (3) shows the sum of these two numbers? Yes

** 5. Find 23×64 as in box C. Check your work by commuting the factors and multiplying again. 1,472

A Multiply

45	
3	
15	(1)
120	(2)
135	(3)

B Multiply

45	
3	
135	

C Multiply

45	
13	
135	(1)
45	* (2)
585	(3)

[W]

Ex. 6–8. Multiply as in box A and then as in box B.

Ex. 9–12. Multiply as in box C.

6.	7.	8.	9.	10.	11.	12.
38	88	32	66	72	81	63
7	4	5	27	67	55	78
266	352	160	1,782	4,824	4,455	4,914

◆ **Extra Examples.** Sets 37–38.

* A good illustration of the practical application of the Associative Property of Multiplication is in rationalization of the algorithm for multiplication of greater numbers.

Multiplying Greater Numbers

Resurvey [O]

A Multiply

```
  427
  235
-----
2 135   (1)
12 81   (2)
85 4    (3)
-------
100,345 (4)
```

B Multiply

```
  235
  427
-----
1 645
4 70
?4 0
-------
???,?45
100 3
```

* 1. Box A shows a vertical form for finding 235×427.

a. We can think of 235 as $200 + 30 + \_?\_5\_$.

b. Then $235 \times 427 = (200 + 30 + 5) \times \_?\_427\_$.

c. We can think of $(200 + 30 + 5) \times 427$ as $(200 \times 427) + (30 \times 427) + (5 \times \_?\_427\_)$.

d. Thus, 235×427 may be thought of as the sum of 200×427, $\_?\_(30 \times 427)$, and $\_?\_(5 \times 427)$.

e. Which step in box A shows 5×427? (1) 30×427? (2) 200×427? (3) Does step (4) show the sum of these three numbers? Yes

2. Box B. The work in box A may be checked by commuting the factors and multiplying again as shown in box B. Tell how to complete the work. See box B.

3. At the board, find 462×843 as in box A. Check by commuting the factors and multiplying again. 389,466

4. At the board, solve each of Ex. a–c.

a. $n = 3 \times 467$ 1,401 **b.** $n = 64 \times 575$ 36,800 **c.** $135 \times 288 = n$ 38,880

[W]

Ex. 5–16. Multiply.

5. 64 × 46 = 2,944	6. 351 × 30 = 10,530	7. 481 × 300 = 144,300	8. 1,357 × 23 = 31,211	9. $70.25 × 20 = $1,405.00	10. $312.55 × 12 = $3,750.60
11. 1,604 × 18 = 28,872	12. 447 × 239 = 106,833	13. 590 × 400 = 236,000	14. 666 × 281 = 187,146	15. $4.15 × 50 = $207.50	16. 1,467 × 223 = 327,141

Ex. 17–25. Find the number represented by n.

17. $133 \times 29 = n$ 3,857	20. $n = 62 \times 372$ 23,064	23. $n \div 500 = 442$ 221,000
18. $n = 1{,}694 \times 87$ 147,378	21. $n = 44 \times 880$ 38,720	24. $n = 23 \times (456 + 243)$ 16,077
19. $n \div 19 = 672$ 12,768	22. $n \div 23 = 460$ 10,580	25. $n = 81 \times 461 \times 23$ 858,843

◆ **Extra Examples.** Sets 39–41.

Teaching Page 62

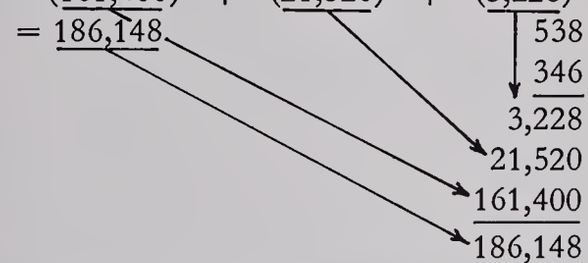
Pupil's Objective

To review multiplication of greater numbers using the vertical form.

Background

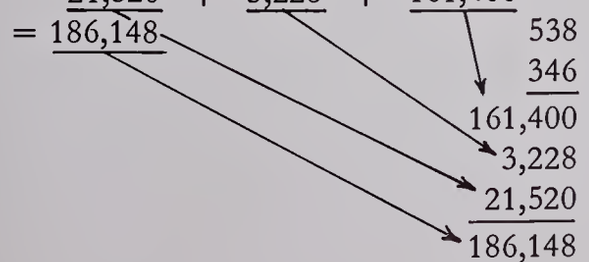
Multiplication of numbers named by numerals with more than two digits is a simple extension of the concept of multiplication of numbers named by numerals with one or two digits. The concept is developed through application of the distributive property, and the vertical form is used to write the work in performing the multiplication. For example,

$$346 \times 538 = (300 + 40 + 6) \times 538$$
$$= (300 \times 538) + (40 \times 538) + (6 \times 538)$$
$$= (161{,}400) + (21{,}520) + (3{,}228)$$
$$= 186{,}148$$



In writing the work for this multiplication in vertical form, it is common not to show the zero in 21,520 and the zeros in 161,400. However, there are exercises in multiplication in which these zeros should be shown. For example, when the vertical form is used, 538 is first multiplied by 6, then 40, and then 300. The multiplication by first using the number named by the righthand digit is only by common convention. The multiplication can begin using any one of the three digits, because addition is commutative and the order in which the addition is performed will not affect the final sum.

$$346 \times 538 = (40 + 6 + 300) \times 538$$
$$= (40 \times 538) + (6 \times 538) + (300 \times 538)$$
$$= 21{,}520 + 3{,}228 + 161{,}400$$
$$= 186{,}148$$



This concept has application in algebra, where multiplication is performed using numbers named from left to right in the algebraic expression.

Pre-Book Lesson

- Have pupils show simple multiplication exercises in vertical and horizontal forms on the board.
- Diagram a simple multiplication exercise using the form in the Background then have pupils do the same using other examples.
- Ask pupils what property allows them to multiply numbers named in an order other than least to greatest.
- Ask the pupils why the zero need not be shown in step 2 in box C on page 61.

Using the Text Page

It may be well to have the pupils first show the work in the horizontal form and then in the vertical form as shown in the Background. The vertical form is fast and efficient but tends to be so highly mechanical that later the pupil is unable to justify each step in the procedure and is unable to apply the multiplication concept in more advanced mathematics courses such as algebra.

Individualizing Instruction

- *More capable pupils* may be asked about working 235×427 by first multiplying by 400, then 20, and then 7; or first multiplying by 20, then by 400, and then by 7. These pupils may be assigned to do library research on other forms for showing multiplication, such as the use of Napier's Bones.
- *All pupils* might enjoy some simple exercises involving multiplication in which a letter is used to represent a digit and the pupil is to determine what digit the letter represents. Below are three such exercises with answers:

(a)	ICC	144
	IN	12
	NTT	288
	ICC	1 44
	IANT	1,728

(b)	INU	125
	NU	25
	LNU	625
	NUS	2 50
	OINU	3,125

Help pupils get started by telling them that N = 2 in each exercise.

Pupil's Objectives

(a) To satisfactorily complete the diagnostic test; and (b) to learn to express certain numbers as a sum of powers of 2.

Background

A short review may be held prior to the test so that any concepts not understood by some pupils can be presented again. The main concepts being tested are:

(a) Commutative Property of Addition
(b) Commutative Property of Multiplication
(c) Associative Property of Addition
(d) Associative Property of Multiplication
(e) determining which operation to use in solving a problem involving the factors-product relationship
(f) Identity Element for Addition
(g) Identity Element for Multiplication

The enrichment exercises at the bottom of the page are based on the concept that any counting number which is not a power of 2 can be expressed as a sum of powers of 2 without repeating an addend. The reasoning behind this concept is based on the following:

(a) Every counting number can be expressed with a base-two numeral.

(b) The place values of base-two numerals are powers of 2. 1 is defined as 2^0, that is, 1 is the zero power of 2.

(c) The only digits in base two are 1 and 0.

(d) If a number is a power of 2, its base-two numeral will contain one "1" and the rest of the digits will be "0."

(e) If a number is not a power of 2, its base-two numeral will contain two or more "1's" and the rest of the digits will be "0's."

Therefore, any counting number which is not a power of 2 can be expressed as a sum of powers of 2, and no addend will be repeated.

The pupil may wonder if these exercises have any practical applications. The procedure is used when a great number of multiplications must be performed when each of the multiplications contains the same factor. The table the pupil is to make for Ex. 10–13 can be used in finding the product in any of the multiplications

$\{(1 \times 19), (2 \times 19), (3 \times 19), \cdots, (127 \times 19)\}$.

By calculating only 7 products, any one of the 127 products can be found by simply adding certain of the 7 products and by using the distributive property.

Pre-Book Lesson

- Review may be held the day of the test or the day prior to the test. Topics examined in the test are listed in the Background.
- You may wish to make provisions before turning to the text page for *all pupils* to work pre-selected remedial exercises or Extra Examples Sets to help correct weaknesses revealed by the test. *More capable pupils* can work the enrichment exercises.

Using the Text Page

- Ex. 1–10 (top). All pupils should understand what is expected of them before they start the test. Concepts for which pupils show weakness should be retaught before proceeding further.
- Ex. 1–13 (bottom). While the enrichment exercises are intended for those who have finished the test early and who may not need to work exercises designed for remedying weaknesses, you may wish to discuss with the entire class the exercises and the nature of the underlying mathematical principles as presented in the Background.

Individualizing Instruction

- *All pupils* will benefit from individual help in areas which are weak. Pupils with the same weakness may be grouped and given Extra Practice Sets provided in the back of the book.
- *More capable pupils* may be interested in exploring the possibility of expressing as a sum of powers of 3 any counting number which is not a power of 3. They will find that the numbers cannot be expressed as sums of powers of 3 without repeating addends. However, they may discover interesting patterns about the tripling of consecutive products of numbers when one set of factors is consecutive powers of 3.

* You may want pupils to name the property of addition or multiplication they used to find the answer.

How Well Do You Remember?

[W]

*Ex. 1–4. Copy and complete by thinking about properties of addition and multiplication.

1. $7 \times (63 + 87) = 7 \times (87 + \underset{63}{\_?\_})$
2. $(60 \times 31) \times 47 = 60 \times (31 \times \underset{47}{\_?\_})$
3. $(37 + 64) + 9 = 37 + (\underset{64}{\_?\_} + 9)$
4. $9 \times (13 + 62) = (9 \times 13) + (9 \times \underset{62}{\_?\_})$

Ex. 5–10. Tell whether the letter represents the product or a factor and then solve.

5. $n \times 13 = 312$ factor; 24
6. $78 \div x = 6$ factor; 13
7. $17 = y \div 23$ product; 391
8. $n = 356 \times 27$ product; 9,612
9. $n \div 19 = 135$ product; 2,565
10. $n = 43 \times (13 + 68)$ product; 3,483

Try These Exercises

Enrichment [W]

Every counting number which is not a member of $\{1, 2, 4, 8, 16, \cdots\}$ can be expressed as a sum of members of that set without using any member more than once. For example, $15 = 8 + 4 + 2 + 1$ and $12 = 8 + 4$. Using this fact and the table in box A, we can find 12×17 as in box B.

A

$1 \times 17 = 17$
$2 \times 17 = 34$
$4 \times 17 = 68$
$8 \times 17 = 136$
$16 \times 17 = 272$

B

$12 \times 17 = (8 + 4) \times 17 = (8 \times 17) + (4 \times 17) = 136 + 68 = 204$

Ex. 1–5. Express as a sum of members of $\{1, 2, 4, 8, \cdots\}$.

1. 5 4+1
2. 11 8+2+1
3. 14 8+4+2
4. 26 16+8+2
5. 30 16+8+4+2

Ex. 6–9. Find the standard numeral as in box B. See below.

6. 5×17
7. 14×17
8. 26×17
9. 28×17

Ex. 10–13. Make a table as in box A showing 1×19, 2×19, 4×19, 8×19, and so on through 64×19 and then find standard numerals for Ex. 10–13 as in box B. See Teacher's Page 65.

10. 14×19
11. 20×19
12. 36×19
13. 83×19

$1 \times 19 = 19$
$2 \times 19 = 38$
$4 \times 19 = 76$
$8 \times 19 = 152$
$16 \times 19 = 304$
$32 \times 19 = 608$
$64 \times 19 = 1{,}216$

6. $5 \times 17 = (4 + 1) \times 17 = (4 \times 17) + (1 \times 17) = 68 + 17 = 85$
7. $14 \times 17 = (8 + 4 + 2) \times 17 = (8 \times 17) + (4 \times 17) + (2 \times 17) = 136 + 68 + 34 = 238$
8. $26 \times 17 = (16 + 8 + 2) \times 17 = (16 \times 17) + (8 \times 17) + (2 \times 17) = 272 + 136 + 34 = 442$
9. $28 \times 17 = (16 + 8 + 4) \times 17 = (16 \times 17) + (8 \times 17) + (4 \times 17) = 272 + 136 + 68 = 476$

Two-Step Problems

[o]

1. To buy a certain new television set, Mr. Appleby agreed to pay $30 down and then 12 monthly payments of $18 each. How much did he agree to pay in all for the set?

a. Does the problem have a hidden question? Yes ^ What is it? v
What was the total of the 12 monthly payments?

b. Explain how the following mathematical sentence shows the relationship expressed in the problem. $n = 30 + (12 \times 18)$ v
(12 x 18) gives the total number of dollars for the 12 monthly payments.

c. At the board, solve the mathematical sentence and then give the answer for the problem. 246; $246

[w]

Ex. 2–8. Write and solve a sentence to find the answer.

2. Mr. Hill earns $94.20 per week. Mr. Fox earns $5,400 per year (52 weeks). How much more does Mr. Fox earn per year than Mr. Hill? n = 5,400 – (94.20 x 52); $501.60

3. A 121-acre farm has 35 acres in corn, 28 acres in alfalfa, and 16 acres in sugar beets. The remaining acreage is in pasture land. How many acres are in pasture land? n = 121 – (35 + 28 + 16); 42

4. A tank holds 175 gallons of oil. When there were 50 gallons of oil in the tank, it was filled again. At 18¢ per gallon, what was the cost of filling the tank? n = (175 – 50) x 0.18; $22.50

Teaching Pages 64 and 65

Pupil's Objectives

(a) To review solving two-step problems involving the addends-sum and factors-product relationship; and (b) to practice computational skills.

Background

The main purpose of teaching a pupil the operations on numbers is to enable him to solve meaningful problems. In solving two-step verbal problems, a pupil must learn to recognize what operations must be performed and the order of performing them. When a mathematical sentence expresses a relationship involving more than one operation, care must be taken to use parentheses properly so that these operations are performed in the correct order. For example, the mathematical sentence

$$n = 3 + 4 \times 5$$

is ambiguous unless the order of operations is specified by the use of parentheses, as

$$n = 3 + (4 \times 5).$$

In solving the problems on this page, the pupil must decide whether to add or multiply in order to obtain the information requested. A number that can be named by multiplication may also be named by a repeated addition in which all the addends are the same number. If a problem on the text page involves a number of things or a number of groups of things each having the same value or the same number of elements, and the pupil is to find the total value or total number of elements, multiplication is the operation to use.

If a problem involves a number of things or a number of groups of things having various values or various numbers of elements and the pupil is asked to find the total value or the total number of elements, then addition is usually the operation to use.

Pre-Book Lesson

• Review solving mathematical sentences involving the addends-sum relationship, and mathematical sentences involving the factors-product relationship.

• Review the use of parentheses to impress upon the pupils the importance of their use when two or more operations are involved, or when a non-associative operation (subtraction and division) is performed more than once.

For example, $5 + (4 \times 6) \neq (5 + 4) \times 6$
$(7 \times 10) \div 5 \neq 7 \times (10 \div 5)$
$(6 - 4) \div 2 \neq 6 - (4 \div 2)$
$(12 \times 6) - 3 \neq 12 \times (6 - 3)$
$36 - (12 - 8) \neq (36 - 12) - 8$
$(36 \div 6) \div 3 \neq 36 \div (6 \div 3)$

For associative operations (addition and multiplication), the use of parentheses when the operation is performed more than once is immaterial, but helpful for some pupils.

$$(3 + 4) + 5 = 3 + (4 + 5)$$
$$(2 \times 3) \times 4 = 2 \times (3 \times 4)$$

• Give instructions for use of the practice exercises before pupils begin work. These exercises may be used as a test, or you may wish that *more capable pupils* work only selected exercises and work Extra Activities when they have finished with page 65.

Using the Text Pages

• First discuss Ex. 1–8 orally and write mathematical sentences on the board. Then erase these and have pupils write the mathematical sentences on their own and solve each problem.

The pupils may suggest more than one way to express a relationship in a problem. For example, the pupils may suggest for Ex. 2, any of the following:

$$n = 5{,}400 - (5 \times 94.20)$$
$$n = 5{,}400 - (94.20 \times 5)$$
$$n + (94.20 \times 5) = 5{,}400$$

It may be well to encourage pupils to discover more than one way of expressing the relationship in all the problems, particularly if they can explain why the expressions they choose are equivalent to the accepted expression, in terms of properties of the set of whole numbers.

• Ex. 3 involves groups that do not contain the same number of elements. The pupil is asked to find the total number of elements, so addition is the operation to use.

• Ex. 1, 2, and 4–8 each involve a number of things or a number of groups of things with the same value or the same number of elements. The pupil is asked to find information involving the total value or total number of elements, so multiplication is one of the main operations to use.

Have the pupils try to explain why they decided on a particular set of operations to use in solving each problem.

Individualizing Instruction

• *More capable pupils* can be asked to write their own two-step problems and have the other pupils attempt to solve them.

• *All pupils* may be given sample two-step problems together with a mathematical sentence to be used in solving the problem, with some of these sentences expressing the relationship in the problem incorrectly. The pupils are to determine which mathematical sentences are incorrect and why they are incorrect.

• *Slower learners* may be given a simple mathematical sentence such as $n = 4 + (2 \times 3)$ and be asked to make up a problem for which the mathematical sentence is an expression for the relationship among the numbers in the problem. These pupils may need help reading the problem, selecting the important data, and expressing it correctly in a mathematical sentence. Read through some problems with them and help them select pertinent data so that they may express it meaningfully using a mathematical sentence.

ANSWERS NOT SHOWN ON PAGE 63

10. $14 \times 19 = (8 + 4 + 2) \times 19 = (8 \times 19) + (4 \times 19) + (2 \times 19) = 152 + 76 + 38 = 266$
11. $20 \times 19 = (16 + 4) \times 19 = (16 \times 19) + (4 \times 19) = 304 + 76 = 380$
12. $36 \times 19 = (32 + 4) \times 19 = (32 \times 19) + (4 \times 19) = 608 + 76 = 684$
13. $83 \times 19 = (64 + 16 + 2 + 1) \times 19 = (64 \times 19) + (16 \times 19) + (2 \times 19) + (1 \times 19) =$
    $1{,}216 + 304 + 38 + 19 = 1{,}577$

NOTES

5. For shoveling snow for 5 hours, Jerry received 75¢ per hour. He deposited his earnings in the bank with the $12.85 that was already there. How much did he then have in the bank? $n = (5 \times 0.75) + 12.85$; $16.60

6. At a coin shop, David bought 18 Indian-head pennies at 21¢ per penny. How much change did he receive from $5? $n = 5.00 - (18 \times 0.21)$; $1.22

7. During July, Mr. Hall drove his car 783 miles on company business. During August, he drove it 967 miles. The company Mr. Hall works for pays 7¢ per mile for car expenses. How much did the company pay Mr. Hall for car expenses during that 2-month period? $n = 0.07 \times (783 + 967)$; $122.50

8. The old high-school building seated 550 pupils. The new high-school building has 24 classrooms and each classroom can seat 34 pupils. The new building can seat how many more pupils than the old building? $n = (24 \times 34) - 550$; 266

To Keep in Practice

[W]

Ex. 1–6. Copy and add.

1.	2.	3.	4.	5.	6.
379	3,009	27,888	$23.89	$376.99	192,068
455	428	8,402	3.68	38.79	74,932
834	3,437	36,290	$27.57	402.68	18,648
				$818.46	285,648

Ex. 7–12. Copy and subtract.

7.	8.	9.	10.	11.	12.
488	7,003	56,880	37,900	$237.89	20,653
239	407	9,206	13,885	156.77	9,758
249	6,596	47,674	24,015	$ 81.12	10,895

Ex. 13–18. Copy and multiply.

13.	14.	15.	16.	17.	18.
37	69	638	793	$32.89	6,049
22	73	75	284	35	187
814	5,037	47,850	225,212	$1,151.15	1,131,163

Ex. 19–30. Solve.

19. $n = 37 \times 677$ 25,049
20. $x \div 28 = 57$ 1,596
21. $n - 68 = 17$ 85
22. $798 - 537 = y$ 261
23. $n = 367 - (276 + 39)$ 52
24. $n = 289 \times (35 + 18)$ 15,317
25. $n = 27 + 389 + 84$ 500
26. $y = 35 \times 28 \times 18$ 17,640
27. $x \div 128 = 64$ 8,192
28. $19{,}682 = n + 9{,}680 + 476$ 9,526
29. $x = 1{,}632 \times 78$ 127,296
30. $n = 24 \times (162 + 93)$ 6,120

* Although the order of listing numbers of a set is irrelevant, the "natural order" is preferred.

Finding the Factors of a Number

Resurvey [O]

1. What is the remainder when 14 is divided by 1? 0 by 2? 0 by 7? 0 by 14? 0 Are 1, 2, 7, and 14 *factors* of 14? Yes How do you know? When 14 is divided by each of these numbers, the remainder is 0. Can you find another whole number which is a factor of 14? No

*2. When we speak of the factors of a number, we mean all the *whole-number factors* of that number. Thus the set of factors of 14 is {_?1_, _?2_, _?7_, _?14_}.

3. Is a counting number a whole number? Yes Is every counting number a factor of itself? Yes

4. Is 1 a factor of every counting number? Yes

5. Every counting number greater than 1 has at least two factors, the number itself and _?1_.

6. Does 20 have factors other than 1 and 20? Yes Give the factors of 20. 1, 2, 4, 5, 10, 20

7. Does 13 have factors other than itself and 1? No How can you show that 1 and 13 are the only factors of 13? Only 13 ÷ 1 and 13 ÷ 13 have a remainder of zero.

8. On the board, list within braces the factors of the number.

a. 15 {1, 3, 5, 15} b. 18 {1, 2, 3, 6, 9, 18} c. 19 {1, 19} d. 25 {1, 5, 25} e. 36 {1, 2, 3, 4, 6, 9, 12, 18, 36} f. 37 {1, 37}

9. {1, 2, 3, 6} is the set of factors of 6 and {1, 2, 5, 10} is the set of factors of 10. Name the set of numbers that are factors of both 6 and 10. {1, 2} Is the set you named the *intersection* or the *union* of the two sets? Name the members of the intersection of the sets for Ex. 8, a and b; 1, 3 c and d; 1 e and f. 1

[W]

10. Express 36 as a product of a pair of factors using as many pairs as possible. 1 x 36, 2 x 18, 3 x 12, 4 x 9, 6 x 6

Ex. 11–16. List within braces the factors of the number.

11. 16 {1, 2, 4, 8, 16} 12. 21 {1, 3, 7, 21} 13. 23 {1, 23} 14. 28 {1, 2, 4, 7, 14, 28} 15. 100 {1, 2, 4, 5, 10, 20, 25, 50, 100} 16. 48 {1, 2, 3, 4, 6, 8, 12, 16, 24, 48}

17. List within braces the members of the intersection of the sets for Ex. 11 and 12; {1} Ex. 13 and 14; {1} Ex. 15 and 16. {1, 2, 4}

Teaching Page 66

Pupil's Objective

To review finding factors of a number.

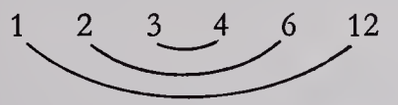
Background

There are various ways of defining *factor*. One way involves the use of the term *multiple*. If n is a counting number and k is a whole number, then $k \times n$ is a *multiple* of n. The set of all multiples of any counting number n is obtained by multiplying n by each whole number in turn, beginning with 0. For example, the set of all multiples of 3 is $\{0, 3, 6, 9, 12, \ldots\}$.

A counting number f is called a *factor* of a number n if n is a multiple of f. That is, 4 is a factor of 12 because 12 is a multiple of 4.

Another way of stating this is to say that a counting number f is a factor of the number n if n is divisible by f. 12 is divisible by 4, so therefore 4 is a factor of 12. 12 is also divisible by 1, 2, 6, and 12. The set of factors of 12 is $\{1, 2, 3, 4, 6, 12\}$.

To show all possible product expressions involving exactly two factors for a counting number, name all the factors of the number in order of increasing value and then pair the first factor with the last one, the second with the second from the last, and so on.



$1 \times 12, \quad 2 \times 6, \quad 3 \times 4$

These are all the possible product expressions involving exactly two factors of 12.

Some of the important points concerning factors are as follows:

(a) The term *factor* as used in this text always refers to a counting number.

(b) Every counting number is a factor of itself.

(c) 1 is a factor of every counting number.

Every counting number greater than 1 has a set of factors consisting of at least two factors, itself and 1.

Pre-Book Lesson

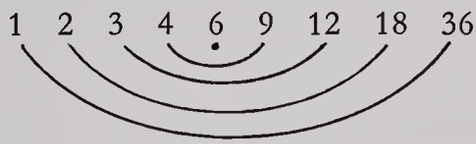
Review the principle that a member of a set is never listed twice. If the pupil is asked for the set of factors of 1, the correct answer is $\{1\}$ and *not* $\{1, 1\}$. Review union and intersection of sets since they are referred to in some of the exercises.

Using the Text Page

• Ex. 5. The concept that every counting number greater than 1 has at least two factors, the number itself and 1, is very important and is essential for understanding the next topic of prime and composite numbers.

• Ex. 8. The pupils may want some systematic method for determining all the factors of a given number. One method is to test each counting number from 2 to the number that is at least one-half as great as the given number, to determine which number can be used as a divisor of the given number, and give a remainder of 0. To determine all the factors of 15, each of the numbers 2, 3, 4, 5, 6, 7, and 8 are tested as divisors to see by which 15 is divisible. It is divisible by 3 and by 5. Therefore, the set of factors of 15 is $\{1, 3, 5, 15\}$.

• Ex. 10. Show all the factors of 36 in order of increasing value. Pair the first with the last, the second with the next to the last, and so forth.



All possible pairs of factors of 36 are 1×36; 2×18; 3×12; 4×9; 6×6.

Individualizing Instruction

• *More capable pupils* can study common factors of two numbers. If A is the set of factors of 36, and B is the set of factors of 42, then

$$A = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$$
$$B = \{1, 2, 3, 6, 7, 14, 21, 42\}$$
$$A \cap B = \{1, 2, 3, 6\}$$

The least common factor of any two counting numbers is always 1. The *greatest common factor* (G.C.F.) of 36 and 42 is 6.

• *Slower learners* can review how to determine by inspection whether a number has 2 as a factor and whether a number has 5 for a factor.

Teaching Page 67

Pupil's Objective

To review the meaning of prime numbers and the meaning of composite numbers.

Background

The set of factors of 13 is {1, 13}. A counting number greater than 1 which has only two factors, itself and 1, is called a *prime number* or a *prime*. This means that the number 1 is not prime and the number zero is not prime.

Any counting number which has more than two factors is called a *composite number*. 6, 18, and 24 are each a composite number since each has more than two factors. The number 1 is not composite.

The set of whole numbers is the union of the following four disjoint sets:

(1) the set of all prime numbers,
(2) the set of all composite numbers,
(3) the set having 0 as its only member,
(4) the set having 1 as its only member.

A scheme for finding all primes from 1 to any given number is called *the sieve of Eratosthenes*. To find all the primes between 1 and 50, all the numbers from 1 to 50 are shown in proper order.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~

The number 1 is not prime so the numeral for 1 is struck out. The next number shown is 2. All numerals for multiples of 2 greater than 2 are struck out. The next number shown on the list is 3. All numerals for multiples of 3 greater than 3 are struck out. The same process is repeated with the numbers 5 and 7. The numerals remaining name all the primes between 1 and 50. This system can be used for finding the set of all primes between 1 and any given counting number.

Pre-Book Lesson

Have the pupils determine the number of factors each number from 2 to 13 has. How many of these have exactly 5 factors? 4 factors? 3 factors? 2 factors? Allow pupils to test for factors by dividing.

Using the Text Page

- Ex. 1–3. Pupils sometime receive the mistaken impression that any whole number is either prime or it is composite. This is not true. The number 1 and the number 0 are neither prime nor composite. Emphasize this point.
- Ex. 7. Ask if some pupil can devise a systematic method for determining all the primes less than 25. Ask them why all the multiples of 2 greater than 2 and all the multiples of 5 greater than 5 cannot be prime.
- Ex. 8, 9, and 12. If they understand that all multiples of 2 greater than 2 and all multiples of 5 greater than 5 cannot be primes, they should be able to recognize immediately that 26, 32, and 50 are composite numbers.

Individualizing Instruction

- *More capable pupils* can be asked to construct a sieve of Eratosthenes to determine all the primes between 1 and 100. Ask them to explain how they know when they have finished.

Some of these pupils may volunteer to do outside reading about primes and report to the class.

- *All pupils* can be asked to list in order as many primes as they are able and then see if there is any pattern to them.

* Since 2 is a factor of every even counting number, all even numbers except the number 2 are composite numbers.

Prime and Composite Numbers

Resurvey [O]

1. On the board, list vertically the counting numbers from 2 through 15. Beside each numeral, list within braces the factors of that number. See right. Which of the numbers have only two factors? 2, 3, 5, 7, 11, 13

1. 2 {1, 2}
3 {1, 3}
4 {1, 2, 4}
5 {1, 5}
6 {1, 2, 3, 6}
7 {1, 7}
8 {1, 2, 4, 8}
9 {1, 3, 9}
10 {1, 2, 5, 10}
11 {1, 11}
12 {1, 2, 3, 4, 6, 12}
13 {1, 13}
14 {1, 2, 7, 14}
15 {1, 3, 5, 15}

A counting number greater than 1 that has only two factors, itself and 1, is called a *prime number*, or *prime*. The number 1 is not considered a prime number.

*2. Name the factors of 2. 1, 2 Is 2 a prime number? Yes Explain. 2 has only two factors, itself and 1. Name the prime numbers between 1 and 16. 2, 3, 5, 7, 11, 13 Name the next two prime numbers. 17, 19

A counting number which has more than two factors is called a *composite number*.

3. Name the composite numbers between 1 and 16. 4, 6, 8, 9, 10, 12, 14, 15

4. Which counting number is neither a prime number nor a composite number? 1 Explain. Prime numbers are greater than 1. Composite numbers have more than two factors. 1 has only one factor.

5. A composite number can be expressed as a product of two factors other than 1 and the number itself. For example, 6 can be expressed as 2×3. Express Ex. a–e in this way.

Sample answers:

a. 20 2 x 10 **b.** 25 5 x 5 **c.** 100 4 x 25 **d.** 22 2 x 11 **e.** 39 3 x 13

[W]

6. List vertically the counting numbers from 15 through 25. Beside each numeral, list within braces the factors of that number. Underline the numerals which name prime numbers.

6. 15 {1, 3, 5, 15}
16 {1, 2, 4, 8, 16}
17 {1, 17}
18 {1, 2, 3, 6, 9, 18}
19 {1, 19}
20 {1, 2, 4, 5, 10, 20}
21 {1, 3, 7, 21}
22 {1, 2, 11, 22}
23 {1, 23}
24 {1, 2, 3, 4, 6, 8, 12, 24}
25 {1, 5, 25}

7. List within braces the prime numbers less than 25. {2, 3, 5, 7, 11, 13, 17, 19, 23}

Ex. 8–19. Write *C* for composite or *P* for prime.

8. 26 C **9.** 32 C **10.** 17 P **11.** 37 P **12.** 50 C **13.** 51 C

14. 63 C **15.** 99 C **16.** 91 C **17.** 102 C **18.** 131 P **19.** 129 C

Ex. 20–25. Express each number as a product of two factors other than 1 and the number itself. Sample answers:

20. 18 3 x 6 **21.** 75 5 x 15 **22.** 50 2 x 25 **23.** 39 3 x 13 **24.** 42 2 x 21 **25.** 121 11 x 11

* This is called the fundamental theorem of arithmetic. Disregarding the order of factors, a composite number can be expressed as a product of prime factors in only one way.

Prime Factors and Products of Primes

{1, 2, 4, 7, 14, 28} *Resurvey and extension* [O]

1. On the board, list within braces the factors of 28. Then list just those factors of 28 which are primes. 1, 2, 7 Using only these *prime factors*, write a product expression for 28. 2 × 2 × 7 You may use a prime factor more than once. For example, a product expression for 12 using primes is $2 \times 2 \times 3$.

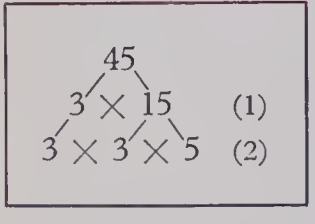
{1, 2, 4, 5, 10, 20, 25, 50, 100}

2. On the board, list within braces the factors of 100. Then list the prime factors of 100. 2, 5 Using only these prime factors, write a product expression for 100. 2 × 2 × 5 × 5

> When a number is expressed as a product of primes, each prime factor of that number will appear at least once in the product expression.

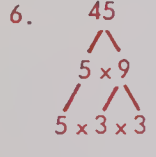
3. Is $2 \times 3 \times 15$ a product expression for 90? Yes Is it a product of primes? No Why not? 15 is not a prime number.

4. $90 = 2 \times 3 \times 15$ and $15 = 3 \times$ _?5_, so 90 expressed as a product of primes is $2 \times 3 \times$ _?3_ $\times$ _?5_.



5. The box at the left shows a factor tree for expressing 45 as a product of primes. What is the product of the factors in step (1)? 45 step (2)? 45

6. On the board, make a factor tree for 45 as in the box, but start by writing 5×9 in step (1). See left. Except for the order, does your factor tree show the same primes in step (2) as the factor tree in the box? Yes

6. 

> * A composite number can be expressed as a product of primes in only one way if the order in which the factors appear is not considered.

[W]

Ex. 7–12. Express the number as a product of primes. Make a factor tree if it will help you.

7. 24 — 2 × 2 × 2 × 3
8. 50 — 2 × 5 × 5
9. 375 — 3 × 5 × 5 × 5
10. 69 — 3 × 23
11. 180 — 2 × 2 × 3 × 3 × 5
12. 135 — 3 × 3 × 3 × 5

▲ **Reteaching.** Set 6. ◆ **Extra Examples.** Set 42.

Teaching Page 68

Pupil's Objectives

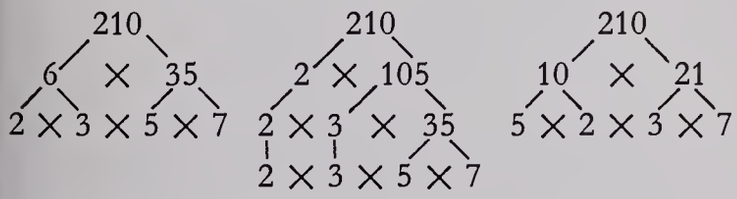
(a) To review naming a number as a product of primes; and (b) to review the use of the factor tree.

Background

Every counting number greater than 1 is either a prime or a composite. If it is a prime, it can be expressed as a product of counting numbers in only one way, if the order of naming the factors is not considered. If it is a composite number, it can be expressed as a product of counting numbers in more than one way. Some possible factorizations of 24 are: (a) 2×12; (b) 3×8; (c) $2 \times 3 \times 4$; (d) $2 \times 2 \times 2 \times 3$.

The factorization $2 \times 2 \times 2 \times 3$ consists entirely of primes. Every composite number can be expressed as a product of primes. The reason for this is that every composite number can be expressed as a product of factors less than itself. If any of these factors are composite, they can be expressed as a product of factors less than themselves. This continues until the result is a product expression containing only factors that cannot be factored further. Therefore, every factor is a prime.

A factor tree may be used for expressing a counting number as a product of primes. Below are three ways of using a factor tree for performing the prime factorization of 210:



The primes are always the same except for the order in which they are listed. Any composite number can be expressed as a product of prime factors in only one way if the order of naming the factors is not considered.

Pre-Book Lesson

Review the primes less than 20 and the multiples of these primes less than 100. This will help in recognizing primes and multiples of primes when making a factor tree.

Using the Text Page

Ex. 1–6. The purpose of these exercises is to give the pupil the opportunity to develop a correct method for factoring a composite number into a set of prime factors. The use of the factor tree is a simple and efficient method. However, the pupil may discover other methods. He may prefer to write his work horizontally:

$$\begin{aligned} 210 &= 6 \times 35 \\ &= 2 \times 3 \times 5 \times 7 \end{aligned}$$

Pupils using this method will probably soon discover that it is simply a variation of the factor-tree method.

Individualizing Instruction

• *More capable pupils* can be taught how to find the prime factors of a number such as 1,001, which is somewhat difficult using the factor-tree method. When difficulty is encountered with the factor-tree method, have the pupils test each of the primes as a divisor beginning with the least prime number that gives zero as the remainder. Then divide the number for the quotient by the least possible prime, beginning with the prime last used as a divisor, to get zero as the remainder. Continue this until a prime results as the number for the quotient.

$$\begin{array}{r} 7\overline{)1{,}001} \\ 11\overline{)143} \\ 13 \end{array} \qquad 1{,}001 = 7 \times 11 \times 13$$

$$\begin{array}{r} 2\overline{)646} \\ 17\overline{)323} \\ 19 \end{array} \qquad 646 = 2 \times 17 \times 19$$

• *All pupils* can select one of Ex. 7–12 and make as many different factor trees as possible so they will be convinced of the truthfulness of the idea that a composite number can be expressed as a product of primes in only one way if the order of the factors is not considered.

Teaching Page 69

Pupil's Objective

To learn that the exponent form may be used when expressing a number as a product of primes.

Background

Any composite number can be expressed as the product of prime factors in only one way if the order of naming the factors is not considered. Since multiplication is commutative, the factors may be rearranged into any convenient order.

It is often convenient to arrange the factors in order of increasing value. With this arrangement, the factorization $2 \times 5 \times 2 \times 2 \times 5$ would be shown as $2 \times 2 \times 2 \times 5 \times 5$. This may also be written in exponent form as $2^3 \times 5^2$.

Some of the things we know about primes are very interesting. Euclid proved that there is no greatest prime. No matter how great a prime we discover, there is always a greater one. Another interesting fact is that between any counting number (other than 1) and its double, there is at least one prime.

There are also some things which we do not know about primes. We have never been able to develop a formula that can be used to derive all primes. The formula $n^2 - n + 41$ results in a prime whenever any counting number less than 41 is substituted for n. However, when 41 is substituted for n, the result is 41^2 which is not prime.

A pair of odd primes whose difference is 2 is called *twin primes*. (3,5), (5,7), (11,13), and (17,19) are examples of twin primes. No one has ever been able to determine if there is a last pair of twin primes.

Probably the most famous unanswered question concerning primes is called Goldbach's conjecture. This conjecture is that every even number greater than 2 is the sum of two primes. No one has ever been able to prove it true, and yet no one has ever been able to find an even number that is not the sum of two primes.

Pre-Book Lesson

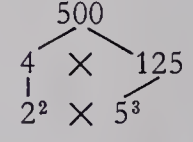
Review exponent form of a numeral. Have pupils apply the commutative property to show the product of prime factors in many different ways.

Using the Text Page

- Ex. 3. Discuss the advantages of using exponent form when showing the prime factorization of a number.
- Ex. 5. Have pupils apply the commutative property and express the products in as many different ways as possible.
- Ex. 7–18. Some pupils may find it helpful to use factor trees before expressing the product using exponent form.

Individualizing Instruction

- *More capable pupils* can be taught to recognize the first few powers of the primes 2, 3, and 5 so that they can perform prime factorizations faster. If they memorize $2^2 = 4$; $2^3 = 8$; $2^4 = 16$; $3^2 = 9$; $3^3 = 27$; $5^2 = 25$; $5^3 = 125$, they can perform the factorization of 500 as follows:



- *All pupils* can discuss the following:

(a) There is no greatest prime.

(b) Between any number (except 1) and its double there is at least one prime.

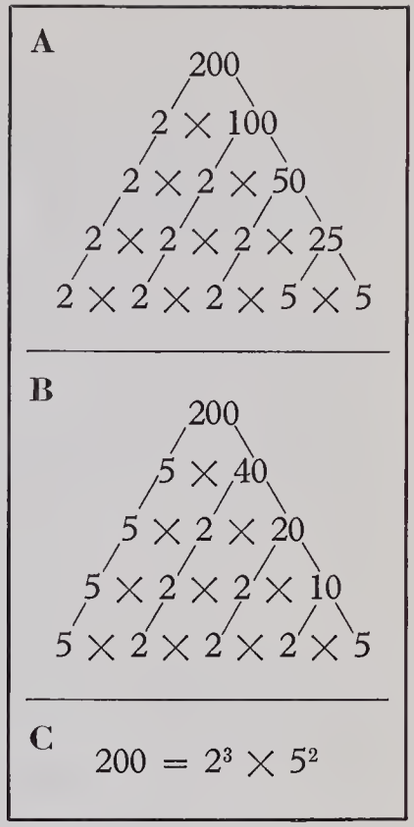
(c) No one has ever developed a formula for deriving all primes.

(d) No one has determined if there is a last pair of twin primes. Twin primes are a pair of odd primes whose difference is 2. Examples of twin primes are (3,5), (5,7), (11,13), and (17,19).

(e) No one has been able to prove or disprove that every even number greater than 2 is the sum of two primes.

Products of Primes

Extension; using exponent form [O]



1. Box A shows one way to find a product expression for 200 so that each factor of the product expression is a prime. Explain each step. Key idea: Another factor is shown in each step.

2. In box B we start with the product expression $5 \times$ _?40_. Explain each step. Is it true that

$5 \times 2 \times 2 \times 2 \times 5 = 2 \times 2 \times 2 \times 5 \times 5$? Yes

3. An exponent form for $2 \times 2 \times 2$ is 2^3. An exponent form for 5×5 is _?5^2_. Using exponent forms, $2 \times 2 \times 2 \times 5 \times 5$ may be expressed as _?2^3 × _?5^2.

4. The sentence in box C is read, *200 is equal to 2 to the* third *power times 5 to the* second *power.*

5. On the board, use exponent forms as in box C to express each of the following:

a. $2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7$ $2^4 \times 7^3$ **b.** $3 \times 5 \times 5 \times 7 \times 7$ $3 \times 5^2 \times 7^2$

c. $5 \times 7 \times 5 \times 13 \times 13 \times 7 \times 13$ $5^2 \times 7^2 \times 13^3$ **d.** $7 \times 7 \times 5 \times 7 \times 7$ 5×7^4

6. Expressed as a product of primes, 61,875 is $5 \times 5 \times 3 \times 11 \times 3 \times 5 \times 5$. On the board, express 61,875 as a product of primes showing equal factors together. $3 \times 3 \times 5 \times 5 \times 5 \times 11$ Then express 61,875 using exponent forms. $3^2 \times 5^4 \times 11$

[W]

Ex. 7–18. Express the number as a product of primes using exponent forms.

7. 42 $2 \times 3 \times 7$ **8.** 48 $2^4 \times 3$ **9.** 72 $2^3 \times 3^2$ **10.** 60 $2^2 \times 3 \times 5$ **11.** 150 $2 \times 3 \times 5^2$ **12.** 36 $2^2 \times 3^2$

13. 64 2^6 **14.** 1,000 $2^3 \times 5^3$ **15.** 225 $3^2 \times 5^2$ **16.** 144 $2^4 \times 3^2$ **17.** 90 $2 \times 3^2 \times 5$ **18.** 84 $2^2 \times 3 \times 7$

Ex. 19–24. Find the standard numeral for the number.

19. $2^2 \times 3 \times 5^2$ 300 **20.** $2^3 \times 3^2 \times 5$ 360 **21.** $2 \times 3 \times 5 \times 7 \times 11$ 2,310

22. $7^2 \times 11$ 539 **23.** $5^3 \times 13$ 1,625 **24.** $2^3 \times 5^2 \times 7$ 1,400

♦ **Extra Examples.** Set 43.

* Point out that the notation for an infinite set is three raised dots. This can be thought of as "and so on according to the pattern."

To Keep in Practice

[W]

Ex. 1–4. Write *T* or *F*.

1. $23 + 0 = 23 \times 0$ F
2. $(32 + 17) \times 1 = 32 + 17$ T
3. $7 \times (5 + 58) = (7 \times 5) + 58$ F
4. $18 - (6 + 2) = (18 - 6) + 2$ F

Ex. 5–9. Write a base-five numeral for the number.

5. 8 13_{five}
6. 12 22_{five}
7. 15 30_{five}
8. 25 100_{five}
9. 56 211_{five}

Ex. 10–14. Write a base-ten numeral for the number.

10. 24_{five} 14
11. 30_{four} 12
12. 21_{eight} 17
13. 26_{seven} 20
14. 111_{five} 31

Ex. 15–19. Round to the nearest hundred.

15. 821 800
16. 3,099 3,100
17. 1,499 1,500
18. 345,567 345,600
19. 1,200,079 1,200,100

Ex. 20–23. Copy and complete by writing $=$ or $<$ or $>$ to make a true sentence.

20. $80 \div 4$ _?_ 17
21. $18 \div 6$ _?_ $347 - (4 \times 86)$
22. $24 + 45$ _?_ 70
23. 64×18 _?_ $16 \times (70 + 4)$

24. Using 6 and 7 as factors and n to represent the product, write four mathematical sentences which illustrate the factors-product relationship. $n = 6 \times 7$; $n = 7 \times 6$; $n \div 6 = 7$; $n \div 7 = 6$

* Remembering about Sets

Resurvey [W]

Ex. 1–3. Write *F* if the set is finite and write *I* if it is infinite.

1. The set of counting numbers greater than 1,000 I
2. The intersection of $\{3, 5\}$ and the set of whole numbers F
3. The union of $\{3, 5\}$ and the set of whole numbers I

$A = \{0, 3, 7\}$
$B = \{0, 2, 4, 6, \cdots\}$
$C = \{1, 3, 5, 7, \cdots\}$
$D = \{5, 6, 7, 8, 9\}$

Ex. 4–9. Copy and complete.

4. $A \cap B =$ _?_ $\{0\}$
5. $A \cap C =$ _?_ $\{3, 7\}$
6. $B \cap C =$ _?_ $\{\ \}$
7. $A \cup D =$ _?_ $\{0, 3, 5, 6, 7, 8, 9\}$
8. $A \cap D =$ _?_ $\{7\}$
9. $C \cap D =$ _?_ $\{5, 7, 9\}$

Teaching Page 70

Pupil's Objectives

(a) To review some of the concepts and skills in Chapters 1 and 2; and (b) to find where weaknesses exist so that they can be corrected by remedial work.

Background

These practice exercises will help maintain skills already developed and will help diagnose sources of difficulty of individual pupils.

Knowledge of the following concepts is being assessed in these exercises:

(a) Identity Element for Multiplication
(b) Identity Element for Addition
(c) property of zero in multiplication
(d) distributive property
(e) renaming numbers expressed in bases other than ten with base-ten numerals
(f) rounding numbers
(g) use of the symbols "=," "<," and ">"
(h) factors-product relationship
(i) addends-sum relationship
(j) finite and infinite sets
(k) union of sets
(l) intersection of sets.

The teacher may wish to refer to Background material on pages presenting each of these topics in order to have a more complete discussion of areas which may need review or remedial work.

Pre-Book Lesson

- Mention each of the topics listed in the Background section, having selected pupils describe the main points of each of the concepts listed. Encourage pupils to ask questions if there is any phase of a concept they do not understand.
- You may wish to have pupils refer to their mathematics notes (these notes were recommended in Chapter 1) to clarify concepts and check to see if particular topics are included. Vocabulary should also be clarified and terminology re-established.

Using the Text Page

- After the Pre-Book Lesson, have pupils open their books and proceed with the written work.
- If the number of examples on this page is excessive for slower learners, they may be assigned over a period of two days if you prefer.

Individualizing Instruction

- Have *all pupils* re-study appropriate pages in the chapters if weaknesses are evident in certain concepts.
- *More capable pupils* may work Extra Activities Sets at the end of the text or Supplementary Activities at the end of the chapter if they finish the work early.

NOTES

Pupil's Objective

To review division, using the vertical form.

Background

On pages 58 and 59, division is presented in terms of finding an unknown factor when one factor and the product are known. In the mathematical sentence $948 \div 4 = n$, 948 names the product, 4 names the known factor, and n represents the unknown factor. This division may be shown in the vertical form:

$$\begin{array}{rrl} & n & \textit{unknown factor} \\ \textit{known factor} & 4\overline{)948} & \textit{product} \end{array}$$

The product, the known factor, and the unknown factor may be called the *dividend*, the *divisor*, and the *quotient* respectively:

$$\begin{array}{rrl} & n & \textit{quotient} \\ \textit{divisor} & 4\overline{)948} & \textit{dividend} \end{array}$$

When the remainder is 0, the quotient is the unknown factor. Division is the most troublesome of the arithmetic operations. Not only does it involve both multiplication and subtraction, but it is the only operation which involves estimating.

Another difficulty is that we have presented division in terms of finding an unknown factor, yet when two whole numbers such as $947 \div 4$ are to be divided, there may be no whole number such that its product with the known factor equals the given product. There is no whole number n such that $n \times 4 = 947$. Until rational numbers are introduced as the unknown factor, do not call such examples as $947 \div 4$ "division" and consider the answer 236, R3.

Several different methods have been developed to minimize or overcome the difficulties inherent in teaching division. The method presented in this text is mathematically sound, quite teachable, and is efficient in terms of time and effort necessary to obtain the correct answer. It also lends itself to a more thorough understanding of the mathematical concepts involved. There may be some pupils who have learned other methods, and who may need to work more closely with the teacher until they understand the methods used in this text. The pupil should understand that there are many mathematically sound approaches to the operation division, and that some methods have distinct advantages over other methods.

Pre-Book Lesson

Have the pupils work some mathematical sentences involving the factors-product relationship. Review the procedure for finding a quotient and remainder.*

Using the Text Page

• Ex. 1–3. There is no whole number which when multiplied by 4 gives the product 947. However, when given the whole numbers 947 and 4, 947 can be expressed as a multiple of 236 plus a remainder of 3. $947 = (4 \times 236) + 3$ When performing this division, you are finding the greatest multiple of 4 which is less than or equal to 947 and if the remainder is greater than zero.

For any two whole numbers, p and f, there is a unique pair of whole numbers q and r such that $p = (f \times q) + r$ where $r < f$. We can say that q is the quotient and r is the remainder when the number p is divided by the number f. This is a mathematical sentence used to verify the quotient and remainder obtained.

• Ex. 5–16. You may wish to have pupils express these examples with mathematical sentences.

Individualizing Instruction

• *More capable pupils* can be presented with the division problems $0\overline{)0}$ and $0\overline{)21}$ and be led into a discussion as to why division by zero is undefined and not permitted.

• *Slower learners* may need more drill work with their multiplication tables. Division requires rapid and accurate recall of memorized multiplication facts. The pupils who have difficulty with division often have not mastered the multiplication facts.

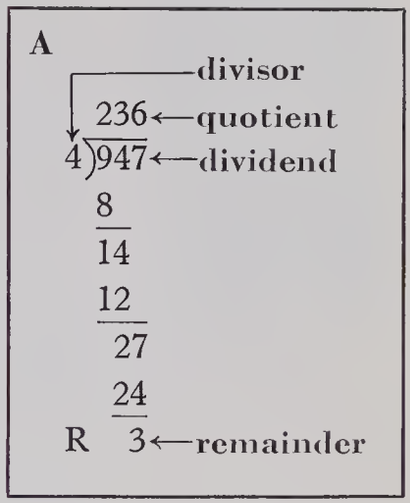
* See items 9J and 26, page xix. These may be very beneficial as review aids.

* Every division has a remainder; it is either 0 or greater than 0.

Using a Vertical Form for Division

Resurvey; terms [O]

1. Box A shows a vertical form for finding 947 ÷ 4. Name the dividend; the divisor; the quotient; the remainder.



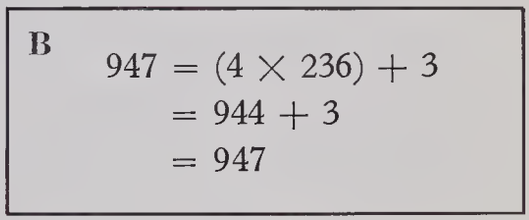
2. Box A. To divide, we think of 947 as 9 hundreds + 4 tens + 7 ones.

a. *Divide hundreds:* What is the greatest multiple of 4 that can be subtracted from 9? $8 = 4 \times 2$, so we write a ?2 in hundred's place in the numeral for the quotient. How many hundreds are left to be divided? 1 How is this shown? As a remainder We bring down the digit 4 to show 4 tens and think of 1 hundred + 4 tens as 14 tens.

b. Explain the steps and the thinking for dividing tens.

c. Explain the steps and the thinking for dividing ones.

* 3. Box B shows a checking sentence for the work in box A. Explain each step. Is the sentence a true sentence? Yes Is 4 a factor of 947? No Explain. 947 ÷ 4 has a remainder of three.



4. At the board, find 723 ÷ 3 as in box A and then write and test a checking sentence for your work. Is 3 a factor of 723? Yes Explain. 723 ÷ 3 has a remainder of zero.

241
3)723
6
12
12
3
3
0

723 = (3 × 241) + 0
= 723 + 0
= 723

[W]

Ex. 5–16. Work as in box A and then write and test a checking sentence for your work.

5. 6)859 — 143, R1
6. 3)1,281 — 427, R0
7. 5)760 — 152, R0
8. 8)7,692 — 961, R4
9. 4)1,044 — 261, R0
10. 7)4,865 — 695, R0
11. 9)8,092 — 899, R1
12. 8)5,061 — 632, R5
13. 3)862 — 287, R1
14. 7)3,918 — 559, R5
15. 6)4,593 — 765, R3
16. 5)872 — 174, R2

Ex. 17–22. Write *T* or *F*.

17. 640 = (9 × 71) + 3 F
18. 5,136 = (8 × 642) + 6 F
19. 128 = (7 × 16) + 6 F
20. 1,618 = (7 × 231) + 1 T
21. 564 = (5 × 132) + 4 F
22. 1,566 = (9 × 174) + 2 F

* If pupils are not familiar with the multiples of 12, they can estimate by thinking of multiples of 10.

Divisors Greater Than Ten

Resurvey [O]

```
    31
12)372
   36
   12
   12
R   0
```

1. For $372 \div 12$, compare the dividend with 12×10 and 12×100. Will the quotient be greater than 10? Yes less than 100? Yes Then will the quotient be represented by a 2-place numeral? Yes Can we think of 372 as 37 tens + 2 ones? Yes Refer to the work in the box at the left as the following steps are discussed.

* **a.** *Divide tens:* What is the greatest multiple of 12 that can be subtracted from 37? 36 Before trying any multiples, estimate by thinking, *37 ÷ 10 =* about 3 ? . What is the first multiple to try? 36 In which place in the numeral for the quotient is a 3 written? Ten's place

b. *Divide ones:* $12 \div 12 =$? 1 Where do we write a 1? One's place What is the remainder? 0

c. For $372 \div 12$, the quotient is ? 31 and the remainder is ? 0. On the board, write and test a checking sentence.

2. At the board, show and explain how to find $365 \div 18$. Write and test a checking sentence for your work. $372 = (12 \times 31) + 0$ Is 18 a factor of 365? No See left.

2.
```
    20
18)365
   36
    5
    0
R   5
```

$365 = (18 \times 20) + 5$
$= 360 + 5$
$= 365$

[W]

Ex. 3–17. Divide and check.

3. $63\overline{)962}$ 15, R17
4. $57\overline{)803}$ 14, R5
5. $33\overline{)1{,}069}$ 32, R13
6. $11\overline{)157}$ 14, R3
7. $21\overline{)876}$ 41, R15
8. $44\overline{)546}$ 12, R18
9. $39\overline{)700}$ 17, R37
10. $31\overline{)410}$ 13, R7
11. $87\overline{)982}$ 11, R25
12. $77\overline{)956}$ 12, R32
13. $23\overline{)550}$ 23, R21
14. $59\overline{)840}$ 14, R14
15. $17\overline{)455}$ 26, R13
16. $82\overline{)973}$ 11, R71
17. $25\overline{)783}$ 31, R8

Ex. 18–26. Find the number represented by n.

18. $n \div 63 = 49$ 3,087
19. $1{,}629 \div n = 9$ 181
20. $n \times 45 = 630$ 14
21. $n = 264 \div 12$ 22
22. $n = 756 \div 36$ 21
23. $364 = n \times 13$ 28
24. $n = 350 \div (5 + 9)$ 25
25. $n = 297 \div (55 \div 5)$ 27
26. $n = 253 + (280 \div 20)$ 267

27. Mr. Whitten has 235 strawberry plants. If he wishes to have the same number of plants in each of 16 rows, what is the greatest number of plants that he can place in each row? 14

▲ **Reteaching.** Set 7.

Pupil's Objective

To review division in which the divisor is greater than 10.

Background

It is important to develop the meaning of the position of the digits in the numeral for the quotient when teaching division. In performing the division $32\overline{)736}$ the pupil must first determine how many digits there will be in the quotient name. Since $1 \times 32 = 32$; $10 \times 32 = 320$; and $100 \times 32 = 3{,}200$; the quotient will be a number between 10 and 100. It will be named by a two-place numeral.

The next step is to estimate the number to be shown by the ten's digit in the quotient, and then test it to see if it is correct. 736 can be considered as 73 tens and 6 ones, and then the greatest multiple of 32 that does not exceed 73 can be determined. It may help to mentally use the number of tens instead of 32 and determine the greatest multiple of 30 that does not exceed 73. The answer is 2. Therefore, we have

$$\begin{array}{r} 2 \\ 32\overline{)736} \\ \underline{64} \\ 96 \end{array}$$

To determine the one's digit for the quotient, the greatest multiple of 32 that does not exceed 96 must be determined. Again, the number of tens may be used instead of 32 for the purpose of estimating. The greatest multiple of 30 that does not exceed 96 is 90. So, try 32×3.

$$\begin{array}{rr} & 23 \\ 32\overline{)736} & \\ & \underline{64} \\ & 96 \\ & \underline{96} \\ \text{R} & 0 \end{array}$$

To summarize, the pupil should first determine how many digits there will be in the numeral for the quotient. He then considers 736 as being 73 tens and 6 ones, and estimates the number shown by the ten's digit in the quotient name by estimating the greatest multiple of 32 that does not exceed 73, mentally using 30 instead of 32 if necessary in making this estimation. The one's digit is then determined by estimating the greatest multiple of 32 that does not exceed 96. Then verify the accuracy of the estimate.

Pre-Book Lesson

Review multiplying by 10, 100, and 1,000. Give the pupils one factor and a product and have them determine how many digits will be in the numeral for the other factor.

Using the Text Page

- Ex. 1. Be sure the pupil understands how he can tell, when estimating any number in the quotient, whether his estimate is too great or not great enough.
- Ex. 18–23 are based on the factors-product relationship that (a) if two factors are known, they are multiplied to obtain the product; (b) if one factor and the product are known, the product is divided by the known factor to obtain the other factor.
- Ex. 24–26. Division is not associative, so the work indicated within the parentheses must be performed first.

Individualizing Instruction

- *More capable pupils* may be asked questions of the type "Find the least number that must be added to 381 to make it divisible by 12."
- *Slower learners* may find it helpful to label or diagram divisions such as those for Ex. 3–26 to show ones, tens, and hundreds. They should be encouraged to remember the subtraction in the vertical form for the division.

Pupil's Objective

To review division in which the quotient is greater than 100.

Background

When performing divisions in which the quotient is greater than 100, the procedure tends to become rote so that little thought is given to the actual mathematics being performed and the properties being utilized. To better understand the mathematics of the division procedure, consider

$$\begin{array}{r} 137 \\ 56\overline{)7{,}675} \\ \underline{5\,6} \\ 2\,07 \\ \underline{1\,68} \\ 395 \\ \underline{392} \\ 3 \end{array} \quad \text{written as} \quad \begin{array}{r} \left.\begin{array}{r} 7 \\ 30 \\ 100 \end{array}\right\} 137 \\ 56\overline{)7{,}675} \\ \underline{5{,}600} \\ 2{,}075 \\ \underline{1{,}680} \\ 395 \\ \underline{392} \\ 3 \end{array}$$

This can be analyzed as follows:

$$\begin{aligned} 7{,}675 &= (56 \times 100) + 2{,}075 \\ &= (56 \times 100) + (56 \times 30) + 395 \\ &= (56 \times 100) + (56 \times 30) + (56 \times 7) + 3 \\ &= 56 \times (100 + 30 + 7) + 3 \\ &= (56 \times 137) + 3 \end{aligned}$$

Pupils would not ordinarily be given this explanation, but it does show what is actually happening when division is performed using the vertical form.

It is necessary that pupils work with the multiples of 10, 100, and 1,000 so that they are placing the digit in the quotient name by true understanding rather than mere guessing.

Pre-Book Lesson

Discuss how to recognize a division problem when a dividend and divisor are given and a quotient and remainder are to be determined. Many times such a problem involves a separation of objects into groups.

Using the Text Page

- Ex. 1. There is a total of 11,622 trees arranged in 78 rows, each with the same number of trees. The 11,622 is the product of the number of rows, which is 78, and the number of trees in each row which is to be determined. Therefore, this is a problem using division.
- Ex. 5–14. Pupils may be asked to show all the work for these divisions using estimated and rounded numbers.

Individualizing Instruction

- *More capable pupils* can give to the class some hints as to how they recognize a division problem, and how they estimate for the quotient.
- *All pupils* may be asked to make up word problems for which one of the divisions in Ex. 4–14 would result in the solution to the problem.
- If any of the *slower learners* experience difficulty in performing division using divisors shown by 2-place numerals, have them round the divisor mentally to the lesser multiple of 10 when estimating the next digit in the quotient. Have them practice estimating multiples for numbers between 10 and 100.

* Emphasize that merely writing the checking sentence is not a check of the division. The "check" is finding if it is or is not a true sentence.

Greater Quotients

Resurvey; divisors greater than 10 [O]

1. In the orange grove pictured above there are 11,622 trees planted in 78 rows. If each row contains the same number of trees, how many trees are there in each row? Can we find the answer by dividing 11,622 by 78? Yes ^ Explain. The quotient is the number of trees in each of the 78 rows.

* **2.** Explain the work in the box. How many trees are in each row? 149 ^ On the board, write and test a checking sentence for the work shown. $11{,}622 = (78 \times 149) + 0$, $= 11{,}622 + 0$, $= 11{,}622$

3. When dividing 7,324 by 3, will the quotient be named by a 3-place numeral or by a 4-place numeral? A 4-place numeral ^ Explain. ∨ At the board, find 7,324 ÷ 3. 2,441, R0 ^ Check your work. ∨ See below box. Key idea: The quotient will be greater than 1,000.

4. For each of Ex. a–d, give the number of places shown by the numeral for the quotient. On the board, work each of Ex. a–d. Check your work.

a. 8)1,273 159, R1 — 3 places

b. 12)4,944 412, R0 — 3 places

c. 3)4,371 1,457, R0 — 4 places

d. 11)567,754 51,614, R0 — 5 places [W]

	149
78)11,622	
	78
	382
	312
	702
	702
R	0

3. $7{,}324 = (3 \times 2{,}441) + 1$, $= 7{,}323 + 1$, $= 7{,}324$

Ex. 5–14. Divide and check.

5. 2)3,513 1,756, R1

6. 13)27,911 2,147, R0

7. 42)17,400 414, R12

8. 18)11,538 641, R0

9. 8)52,893 6,611, R5

10. 26)8,871 341, R5

11. 7)9,303 1,329, R0

12. 25)77,850 3,114, R0

13. 15)46,711 3,114, R1

14. 96)78,602 818, R74

◆ **Extra Examples.** Sets 44–45.

* In a division, whenever a remainder is zero, the divisor and quotient are both factors of the dividend.

Divisors Named by 3-Place Numerals

Resurvey [O]

1. Box A. $18 \div 6 =$ _?3_, so $180 \div 60 =$ _?3_ and $1{,}800 \div 600 =$ _?3_. At the board, find $1{,}905 \div 600$. See below.

2. Without using paper and pencil, give the quotient for each of Ex. a–c.

a. $500\overline{)1{,}500}$ 3 b. $700\overline{)5{,}600}$ 8 c. $900\overline{)8{,}100}$ 9

* 3. Explain the work in box B. Is 231 a factor of 15,477? Yes How do you know? The remainder is 0.

4. At the board, find $2{,}230 \div 104$. Explain how you estimate the number of hundreds, tens, and ones in the quotient when doing the work. See below.

5. A nursery sold 120 shrubs to the City Park System for $255.60. If the shrubs each sold for the same price, what was the price of each shrub?

a. We do not need to show the symbol "$" in a mathematical sentence or when doing the work since it is evident from the problem that we are referring to a number of dollars. Is $n = 255.60 \div 120$ a mathematical sentence for the problem? Yes

b. Box C. Explain each step shown.

c. What was the price of each shrub? $2.13

A

```
      3
600)1,800
  1 800
R     0
```

B

```
      67
231)15,477
  13 86
   1 617
   1 617
R      0
```

C

```
      2.13
120)255.60
  240
   15 6
   12 0
    3 60
    3 60
R       0
```

[W]

Ex. 6–13. Divide and check.

6. $142\overline{)540}$ 3, R114
7. $330\overline{)8{,}700}$ 26, R120
8. $112\overline{)50{,}512}$ 451, R0
9. $162\overline{)17{,}982}$ 111, R0
10. $301\overline{)6{,}341}$ 21, R20
11. $200\overline{)32{,}426}$ 162, R26
12. $250\overline{)8{,}000}$ 32, R0
13. $446\overline{)1{,}400}$ 3, R62

Ex. 14–16. Find the number represented by n.

14. $n = 11{,}128 \div 214$ 52
15. $n = 112{,}400 \div 400$ 281
16. $n \times 125 = 4{,}125$ 33

17. Mr. Thomas sold 750 bu. of wheat for $1,597.50. What was the price per bushel? $2.13

◆ **Extra Examples.** Sets 46–47.

1.
```
      3
600)1,905
  1 800
R   105
```

4.
```
      21
104)2,230
  2 08
    150
    104
R    46
```

Teaching Page 74

Pupil's Objective

To review division in which the divisor is named by a 3-place numeral.

Background

There is a relationship among a product and its factors which is very useful. This relationship, sometimes referred to as "compensation," may be stated as "If the product and known factor are both multiplied or divided by the same non-zero number, the unknown factor remains unchanged."

For example, given $36 \div 9 = 4$, multiply product and known factor by

2: $72 \div 18 = 4$; 5: $180 \div 45 = 4$;
10: $360 \div 90 = 4$; 100: $3{,}600 \div 900 = 4$;
1,000: $36{,}000 \div 9{,}000 = 4$.

If a pupil knows that $36 \div 9 = 4$, then he should be able to conclude that each of $360 \div 90$, $3{,}600 \div 900$, and $36{,}000 \div 9{,}000$ are also 4.

This concept can be used in estimating for the quotient and is also useful in work with ratio, per cent, and division with decimals.

Pre-Book Lesson

• Review estimating quotients in division. Write several examples on the board involving division with zeros, and divisors named by 2-place numerals.

• Have pupils practice using a short-cut method of multiplying by multiples of ten and powers of ten. Ask pupils if they can think of a method of dividing by multiples or powers of 10. Apply answers to Ex. 1.

Using the Text Page

• Ex. 1. Be certain pupils realize that a dividend and divisor may be multiplied or divided by the same non-zero number without changing the answer. Since $19 \div 6$ is approximately 3, then $1{,}900 \div 600$ will be approximately 3.

• Ex. 5. Point out that division is an operation on numbers, not on sets. A dollar cannot be divided by a shrub any more than a dollar can be multiplied by a shrub. When performing any division, it is only the numbers that are engaged in the operation.

Individualizing Instruction

• *More capable pupils* can be told that the quotient of two numbers is 36 and asked to determine how the quotient will change if

(a) the known factor were doubled.
(b) the known factor were halved.
(c) the product were tripled.
(d) the product were halved.
(e) the known factor and product were both halved.
(f) the known factor and product were both doubled.

• *All pupils* could be given the division

$$133\overline{)1{,}862}\quad 14$$

and be asked how the quotient would be changed if

(a) the known factor were doubled.
(b) the product were doubled.
(c) the product were increased by 133.
(d) the product were decreased by 266.
(e) the product were multiplied by 10.

• *Slower learners* may need to work Ex. 6–20 with your assistance. Have them "think aloud" each step and write their work beside the example. Establish the idea of using multiples of the known factor and of repeated subtraction of greatest multiples of the known factor from the product.

Pupil's Objective

To review division in which there is a 0 in the numeral for the quotient.

Background

When performing the division $9\overline{)1,836}$, the number of digits in the numeral for the quotient must be determined first. Since $9 \times 10 = 90$, $9 \times 100 = 900$, and $9 \times 1{,}000 = 9{,}000$, the quotient is between 100 and 1,000; and therefore, it is named by a three-place numeral.

To determine the hundred's digit for the quotient, we determine the greatest multiple of 9 that does not exceed 18. This is 2 (step a).

To determine the ten's digit, we determine the greatest multiple of 9 that does not exceed 3. What we are really doing is determining the greatest number of tens that can be multiplied by 9 whose product does not exceed 3 tens. Only 0 tens multiplied by 9 will give a product that does not exceed 3 tens as shown in step b.

To determine the one's digit for the quotient, we determine the greatest multiple of 9 that does not exceed 36. This is 4 (step c).

(a)
$$\begin{array}{r} 2 \\ 9\overline{)1,836} \\ \underline{1\,8} \\ 3 \end{array}$$

(b)
$$\begin{array}{r} 20 \\ 9\overline{)1,836} \\ \underline{1\,8} \\ 3 \\ \underline{0} \\ 36 \end{array}$$

(c)
$$\begin{array}{r} 204 \\ 9\overline{)1,836} \\ \underline{1\,8} \\ 3 \\ \underline{0} \\ 36 \\ \underline{36} \\ \text{R}\quad 0 \end{array}$$

Pre-Book Lesson

• Present an incorrect division such as

$$7\overline{)1,421}\,23$$

to the class and ask the pupils to use a mathematical sentence to verify the answer. When they realize the quotient is incorrect, have them analyze the work to find that a 0 should have been written between the 2 and the 3 for the quotient.

Using the Text Page

• Ex. 1. When the work has reached the point shown in step a below, pupils should realize that they are now trying to determine the greatest number of tens that can be multiplied by 9 so that the product does not exceed 2 tens. The only number of tens that 9 can be multiplied by so that the product does not exceed 2 tens is 0 tens as shown in step b.

(a)
$$\begin{array}{r} 2 \\ 8\overline{)1,624} \\ \underline{1\,6} \\ 2 \end{array}$$

(b)
$$\begin{array}{r} 20 \\ 8\overline{)1,624} \\ \underline{1\,6} \\ 2 \\ \underline{0} \\ 24 \end{array}$$

The number of ones that can be multiplied by 8 to give a product of 24 is 3, so

$$\begin{array}{r} 203 \\ 8\overline{)1,624} \\ \underline{1\,6} \\ 2 \\ \underline{0} \\ 24 \\ \underline{24} \\ \text{R}\quad 0 \end{array}$$

or, more simply

$$\begin{array}{r} 203 \\ 8\overline{)1,624} \\ \underline{1\,6} \\ 24 \\ \underline{24} \\ \text{R}\quad 0 \end{array}$$

• Ex. 1–12. Pupils may need to diagram the divisions showing tens, hundreds, and thousands.

Individualizing Instruction

• *More capable pupils* may be given exercises such as the following and asked to find the digits that the letters represent:

(a)
$$\begin{array}{r} \text{ADC} \\ 3\text{A}\overline{)3\text{BBC}} \\ \underline{3\text{A}} \\ \text{ABC} \\ \underline{\text{ABC}} \end{array}$$

(b)
$$\begin{array}{r} 5\text{E} \\ 4\text{E}\overline{)\text{B}47\text{E}} \\ \underline{225} \\ 22\text{E} \\ \underline{22\text{E}} \end{array}$$

(c)
$$\begin{array}{r} \text{EE} \\ \text{BB}\overline{)\text{ABAO}} \\ \underline{\text{AAO}} \\ \text{AAO} \\ \underline{\text{AAO}} \end{array}$$

Answers: A = 1, B = 2, C = 4, D = 0, and E = 5

0's in the Numeral for the Quotient

Resurvey [O]

1. Box A. Why is there a 0 in the numeral for the quotient? To show 0 tens. Explain each step shown.

2. At the board, find 24,038 ÷ 24. 1,001, R14 How many 0's appear in the numeral for the quotient? 2 Check your work.

3. For her son's birthday party, Mrs. Higgins bought 18 plastic models, each costing the same. The total cost was $3.60. Each model cost _?_ 30 cents.

a. Box B. We can express $3.60 as _?_ 360 ¢. We often think of $0.20 as 20 cents, but it really means _?_ 20 hundredths of a dollar.

b. Explain the work in box C. What did each model cost? $0.20 or 20¢

A

$$\begin{array}{r} 203 \\ 8\overline{)1{,}624} \\ \underline{16} \\ 24 \\ \underline{24} \\ \text{R}\ \ 0 \end{array}$$

B

$$\begin{array}{r} 20 \\ 18\overline{)360} \\ \underline{360} \\ \text{R}\ \ 0 \end{array}$$

C

$$\begin{array}{r} 0.20 \\ 18\overline{)3.60} \\ \underline{3\,60} \\ \text{R}\ \ 0 \end{array}$$

[W]

Ex. 4–12. Divide and check.

4. 9)1,818 — 202, R0
5. 27)29,781 — 1,103, R0
6. 35)28,092 — 802, R22
7. 404)16,160 — 40, R0
8. 37)44,474 — 1,202, R0
9. 12)2,412 — 201, R0
10. 37)18,835 — 509, R2
11. 234)2,405 — 10, R65
12. 75)105,115 — 1,401, R40

Practice in Finding Quotients and Remainders

[W]

Ex. 1–20. Divide.

1. 23)48 — 2, R2
2. 49)346 — 7, R3
3. 9)1,567 — 174, R1
4. 37)445 — 12, R1
5. 68)5,779 — 84, R67
6. 368)400 — 1, R32
7. 203)5,687 — 28, R3
8. 440)8,888 — 20, R88
9. 109)110,456 — 1,013, R39
10. 36)36,000 — 1,000, R0
11. 450)78,950 — 175, R200
12. 19)567,827 — 29,885, R12
13. 400)82,436 — 206, R36
14. 99)10,938 — 110, R48
15. 25)2,550 — 102, R0
16. 47)35,682 — 759, R9
17. 18)21,947 — 1,219, R5
18. 29)54,633 — 1,883, R26
19. 87)85,782 — 986, R0
20. 39)27,496 — 705, R1

Ex. 21–26. Write *T* or *F*.

21. $(932 + 118) \div 42 = 25$ T
22. $348 \div 34 > 11$ F
23. $439 = (19 \times 23) + 2$ T
24. $510 = (7 \times 70) + 12$ F
25. $476 \div 68 > 168 \div 28$ T
26. $1{,}600 \div (6 \times 16) < 19$ T

* Most pupils should be able to divide by divisors less than 10 without showing the work.

How Well Do You Remember?

[W]

1. Write a mathematical sentence which illustrates
   Sample answers:
   a. the Commutative Property of Multiplication. $7 \times 4 = 4 \times 7$
   b. the Associative Property of Multiplication. $(2 \times 3) \times 4 = 2 \times (3 \times 4)$
   c. the Distributive Property. $2 \times (3 + 4) = (2 \times 3) + (2 \times 4)$

Ex. 2–6. List within braces the factors of the number.

2. 5 $\{1, 5\}$ 3. 22 $\{1, 2, 11, 22\}$ 4. 17 $\{1, 17\}$ 5. 81 $\{1, 3, 9, 27, 81\}$ 6. 105 $\{1, 3, 5, 7, 15, 21, 35, 105\}$

Ex. 7–11. Express the number as a product of primes.

7. 24 $2^3 \times 3$ or $2 \times 2 \times 2 \times 3$ 8. 50 2×5^2 or $2 \times 5 \times 5$ 9. 84 $2^2 \times 3 \times 7$ or $2 \times 2 \times 3 \times 7$ 10. 102 $2 \times 3 \times 17$ 11. 1,000 $2^3 \times 5^3$ or $2 \times 2 \times 2 \times 5 \times 5 \times 5$

Ex. 12–16. Write *P* for prime or *C* for composite.

12. 17 P 13. 31 P 14. 57 C 15. 87 C 16. 23,688 C

Ex. 17–19. Does *n* represent the product or a factor?

17. $n \div 24 = 49$ The product 18. $368 \times n = 6{,}256$ A factor 19. $264 \div n = 12$ A factor

Ex. 20–25. Find the number represented by the letter.

20. $n = 234 \times 38$ 8,892 21. $x = 910 \div 65$ 14 22. $y = 297 \div (44 \div 4)$ 27

23. $x \div 72 = 105$ 7,560 24. $33{,}000 \div n = 11$ 3,000 25. $53 = x \div 400$ 21,200

* Try These Exercises

[W]

A

$$8\overline{)1{,}944}\quad \begin{matrix} 3\,2 \\ 243 \end{matrix}$$

B

$$8\overline{)1{,}944}\quad 243$$

Study the work in box A. Multiples of 8 are subtracted without showing the work. Think, *19 − (8 × 2) = 3, so there are 3 hundreds left to be divided.* This is shown by the small raised 3. Then there are 34 tens to be divided, and so on.

Box B. Some pupils divide the shorter way without writing the small numerals shown in box A.

Ex. 1–8. Copy and divide as in box A or box B.

1. $9\overline{)423}$ 47, R0 2. $7\overline{)1{,}764}$ 252, R0 3. $8\overline{)4{,}072}$ 509, R0 4. $3\overline{)12{,}768}$ 4,256, R0

5. $7\overline{)7{,}994}$ 1,142, R0 6. $5\overline{)7{,}685}$ 1,537, R0 7. $5\overline{)346{,}725}$ 69,345, R0 8. $6\overline{)4{,}734}$ 789, R0

Teaching Page 76

Pupil's Objectives

(a) To review some of the main ideas taught in Chapters 1 and 2; and (b) to find a shorter way to divide by multiples of a number.

Background

A short review may be held prior to the test so that concepts not understood by some pupils can be presented again. The main concepts being tested are:

(a) Commutative Property of Multiplication
(b) Associative Property of Multiplication
(c) distributive property
(d) set of factors of a number
(e) prime factorization of a number
(f) prime and composite numbers
(g) factors-product relationship
(h) division involving a remainder

Dividing using a shorter way by thinking of the greatest multiple of the divisor which can be subtracted from the dividend is an extension of divisions where work for each step is shown. The mental processes in either case are the same, as is shown below.

```
  243
8)1,944   19 hundreds − (8 × 2) = 3 hundreds
 16                    or 30 tens
  34      34 tens − (8 × 4) = 2 tens or 20 ones
  32
  24      24 ones − (8 × 3) = 0 ones
  24
   0
```

Pre-Book Lesson

The brief review suggested in the Background may be held on the day of the test. If some pupils need more work, review may be held a day prior to the test. Pupils should not open their books to the page until they are ready to work the test.

Using the Text Page

• Ex. 1–25. Concepts for which the pupils show weakness should be retaught before going on to new work. Do remedial work with groups or individuals who have difficulty with any of the exercises. Refer to the appropriate pages in Chapters 1 and 2, and use the practice sets and reteaching sets at the end of the text.

• Ex. 1–8. The exercises at the bottom of the page show how to perform division without showing the work when the known factor is represented by a 1-place numeral. Several examples of this type may be worked at the board with all work shown. Pupils may then attempt to work the same examples the shorter way, explaining how they "think" their work and write only the answer. These examples should be worked by the entire class, but *slower learners* may need extra help from the teacher.

Individualizing Instruction

• Even though this page is designated for written work, *slower learners* may experience difficulty. You may wish to have a brief oral review with these pupils on the properties of multiplication, factoring, and prime and composite numbers.

• Individual help can be provided to *all pupils* according to their errors on the test. Pupils with the same weakness may be grouped and given Extra Practice Sets or work together with the teacher on Reteaching Sets provided at the end of the text.

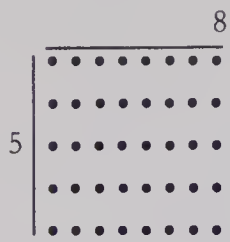
• While the exercises at the bottom of the page may be worked with least difficulty by *more capable pupils*, members of the entire class who finish the test early should be permitted to work the exercises on their own. After everyone has finished the test, the exercises may be explained by the teacher for the benefit of *all pupils*.

Pupil's Objective

To review how to recognize whether to use multiplication or division in solving a verbal problem.

Background

Very often an array is helpful in teaching analysis of problems that require multiplication or division for their solution. Consider an array of 5 sets each with 8 members.



The total number of members is equal to the product of the number of sets and the number of members in each set. $5 \times 8 = 40$ and $8 \times 5 = 40$. If we substitute the number 40 for the dots in the above array we have $5\overline{)40}^{\,8}$ which immediately suggests the relationship among the three numbers:

$$5\overline{)40}^{\,8}$$

If the number of sets and the number of members in each set are given, and the total number of members is to be found, we have $? \div 5 = 8$ which suggests that the requested information can be determined by finding the product of 5 and 8.

If the total number of members is given, and either the number of sets or the number of members in a set is given, and the other factor is to be determined, $40 \div 5 = ?$ or $40 \div ? = 8$ suggests division.

Pre-Book Lesson

- From pages 64 and 65, review how to recognize when a problem requires addition or multiplication for its solution. Make up some problems, read them to the class, and have them tell whether to use addition or multiplication for their solution.
- Review expressing the mathematical relationships in a verbal problem by means of a number sentence.

Using the Text Page

- Ex. 1. If the total number of sets and the number of members in each set is given, and the total number of members is to be determined, then multiplication is usually required for solution of the problem. If the number of sets and the total number of members is given, and the number of members in each set is to be determined, then division is usually required for solution of the problem. Division is also usually required when the total number of members and the number of members in each set is given, and the number of sets is to be determined.
- Ex. 2–6. Pupils may use a mathematical sentence to express the relationships among the numbers in the problem. They may label the factors and the product and show their work in both horizontal and vertical forms.

Individualizing Instruction

- *Slower learners* may use an array to analyze the problems if necessary. The teacher may also wish to read some of the problems with pupils to provide for reading difficulties.
- *More capable pupils* may make up arrays and have other pupils make up verbal problems which the arrays suggest.

Multiply or Divide?

Resurvey [O]

For each of Ex. 1–6, tell what operation to use to find the answer for the problem.

1. For use in games to be played at her daughter's party, Mrs. Fox separated 288 slips of paper by placing the same number of slips of paper in each of 24 packages. How many slips of paper were there in each package? Division; 12

2. For one game, each of the 24 girls used 15 paper clips. That was how many paper clips? Multiplication; 360

3. The party lasted 210 minutes. That was 3 hours and how many minutes? Division; 30

4. Seven games were played and each of the 7 winners received a book of puzzles. If each book cost 39¢, what was the total cost of the 7 books? Multiplication; $2.73

5. How many 39¢-books could Mrs. Fox have purchased for $4.68? (Hint: Think of $4.68 as 468¢.) Division; 12

6. For a prize, each of the 24 girls made 3 quick sketches of Mrs. Fox's Siamese cat. That was how many sketches in all? Multiplication; 72 [W]

Now work each of Ex. 1–6.

* It might be helpful for some pupils to write out the hidden question separately for each example and to express it in a separate mathematical sentence.

Problems Involving Two Operations

Resurvey; two-step problems [O]

1. For raking leaves one Saturday morning, Tom and his friend earned $2.80 which they shared equally. That afternoon, Tom worked alone and earned $1.25. How much did Tom earn in all that Saturday?

a. Does the problem have a hidden question? Yes ^ What is it? ^ How much did Tom earn that Saturday morning? v

b. On the board, write and solve a mathematical sentence for the problem using parentheses so that the work within the parentheses answers the hidden question. See left.

(2.80 ÷ 2) + 1.25 = n
1.40 + 1.25 = n
2.65 = n

c. How much did Tom earn in all that Saturday? $2.65

* Ex. 2–7. Write a mathematical sentence for the problem. Do not solve the sentence yet.

2. The cost of 13 pictures for the school at $3.75 per picture was shared equally by 25 people. How much did each person pay? n = (13 × 3.75) ÷ 25; $1.95

3. A printer cuts 72 small cards from each large sheet. If each sheet costs $2.25, what would be the cost of enough sheets to make 2,232 small cards? n = 2.25 × (2,232 ÷ 72); $69.75

4. The total cost of Mr. Dow's new car, including finance charges, was $2,664. He paid $664 down and the remainder in 25 equal payments. How much was each of the 25 payments? v n = (2,664 − 664) ÷ 25; $80

5. Two years after buying the car, Mr. Dow paid $82.40 in all for a battery and 4 tires. The battery cost $15.20. What was the cost of each tire? n = (82.40 − 15.20) ÷ 4; $16.80

6. One week, Mr. Jordan sold 1,440 eggs at 45¢ per dozen. How much did he receive in all? n = 0.45 × (1,440 ÷ 12); $54.00

7. For every 26 tomato plants ordered, the Garden Supply Company gives away 3 pepper plants. Mr. Jordan ordered 416 tomato plants. How many pepper plants did he receive? v n = 3 × (416 ÷ 26); 48 [W]

Now work each of Ex. 2–7.

♦ **Extra Problems.** Set 140.

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Pupil's Objective

To study problems that require two operations for their solution.

Background

There is no simple formula that will show pupils how to solve verbal problems. However, if a pupil is having difficulty, the steps below may be of some help:

(1) Determine what information is given.

(2) Determine what information is being asked for in the problem.

(3) Determine the relationship (addends-sum or factors-product) between the information given and the information being requested.

(4) Express the relationships in the form of a mathematical sentence, using parentheses to indicate clearly the correct order of operations.

(5) Solve the mathematical sentence.

Pre-Book Lesson

• Review how to recognize addends-sum relationships and factors-product relationships by making up a few simple problems. Clearly indicate where and why the two steps and two operations occur in the problem.

• Discuss material from the Background in order to establish clearly in the pupils' minds a specific procedure to follow for problem-solving. This will facilitate work and make it less liable to error.

Using the Text Page

• Ex. 1–7. Pupils who have difficulty solving verbal problems very often simply do not understand the situation described in the problem. This may be due to reading difficulties or to vocabulary difficulties.

To minimize reading and vocabulary difficulties, each problem should be read aloud by the teacher or by a pupil. Any words that may be unfamiliar to any of the pupils should be discussed.

When all the pupils understand the situation described in each problem, have them explain what information is given and what information is being requested. Have them point out the addends-sum relationship and the factors-product relationship. Then have them express the relationship for the information given and the information requested in the form of a mathematical sentence.

After they have done this for each problem, they are to go back and solve the mathematical sentence for each problem, checking their answer with the information given in the problem to be sure the answer is reasonable.

• Ex. 4. Some pupils may not understand what finance charges are. Discuss the situation described in each problem so that the entire problem seems reasonable to the pupil.

• Ex. 7. Have a pupil explain why the Garden Supply Company gives away 3 pepper plants for every 26 tomato plants ordered.

Individualizing Instruction

• *More capable pupils* may be given practice estimating answers to problems. Give them a series of problems and have them estimate the answer without doing any work with pencil and paper. Then have them solve the problem and compare their answer with the estimate they made.

• *All pupils* may be given problems in which they are to determine whether there is too little, too much, or just the right amount of information given them to solve. If a problem contains too little information, the pupil is to indicate what additional information is needed.

• The teacher may need to work orally with *slower learners* who have reading and vocabulary difficulties so that they understand the mathematics involved in the problem. This might be a good procedure to follow on any other text page that involves verbal problems where there might be a possibility of reading difficulties.

Pupil's Objectives

(a) To review the concept of arithmetic mean; and (b) to review the method of calculating an arithmetic mean.

Background

One method of summarizing a set of data is to refer to its averages. An *average* is a measure of central tendency. In statistics, there are three different types of averages: the *mode*, the *median*, and the *arithmetic mean*.

The mean, or arithmetic mean, is the number obtained by dividing the *sum* of all the numbers in the set of data by the *number* of members in the set of data. This quotient is then used to represent all the numbers in the set of data. The numbers are thought of as measures when finding the mean.

Any method of summarizing data must be used with caution. Any average, including the mean, may not be truly representative of the data. This can be clearly presented to the pupils by the use of a somewhat ridiculous situation such as a man who has half his body in an oven at 158°F. and the other half of his body in an adjoining refrigerator at 38°F. His average body temperature is a quite normal 98°F.

A more realistic example is that of measuring the intelligence quotients of two groups of pupils as indicated in the tables below:

IQ Group A	*IQ Group B*
88	103
89	103
91	104
92	104
118	106
119	106
121	107
122	107
8)840	8)840
105	105

The mean of the I.Q.'s of each group is 105, yet the two groups are clearly quite different. In statistics, this difference is indicated by the use of the *average deviation*. The positive difference between a number in a set of data and the mean of the data is called the deviation of that number. The mean of the deviations of all numbers in the set is called the *average deviation*.

Group A has a mean I.Q. of 105 with an average deviation of 15. Group B has a mean I.Q. of 105 with an average deviation of 1.5. Group B is much more homogeneous than Group A, even though the mean of the I.Q.'s of each group is the same.

Pre-Book Lesson

- Discuss the material from the Background using the examples cited.
- Discuss some common uses of the word *average*.
- Bring in some examples from other books, newspapers, or magazines where information is distorted by the use of statistics.

Using the Text Page

- Ex. 1–3. Point out that the concept of arithmetic mean can be expressed by use of the factors-product relationship in any one of the following ways:

(a) (sum of measures) ÷ (number of measures) = mean

(b) (number of measures) × (mean) = (sum of measures)

(c) (sum of measures) ÷ (mean) = (number of measures)

- Ex. 4–7. Have pupils describe situations where the given measures would be applicable.

Individualizing Instruction

- *More capable pupils* can be asked to point out some disadvantages of using the mean, and to think of some situations where its use might lead to erroneous conclusions. Lead them to realize the need for additional information such as the nature of the data as a whole, or the average deviation for the data.
- It can be pointed out to *all pupils* that the word *average* is often used when the word *mean* is more appropriate, as there are several different kinds of averages.

* Mean, arithmetic mean, and mean average are all terms referring to the same measure of central tendency.

Arithmetic Mean

Resurvey [O]

1. At a cookout, the numbers of hamburgers cooked by four boys were: Don, 14; Jim, 18; Bob, 15; Dick, 21. To find an average for these numbers, or measures, we add 14, 18, 15 and 21 and then divide by _?4_. Explain the work in the box for finding this average.

14	17
18	4)68
15	4
21	28
68	28
	R 0

* 2. The average found by adding and then dividing as for Ex. 1 is called the *arithmetic mean*. We often refer to this as the *mean*. At the board, find the mean for these measures: 2, 12, 8, 5, 11, 4.

To find the arithmetic mean, we use the numbers as addends and find the sum. Then we divide the sum by the number of addends.

2
12
8
5
11
4
42

7
6)42

3. To attend the cookout, the mean number of miles traveled by the boys doing the cooking was 7. To find the total number of miles traveled by the four boys, we _?multiply_ 7 by _?4_. Why? Key Idea: 7 and 4 are factors. Multiply to get their product. What was the total number of miles traveled by the four boys? 28

[W]

Ex. 4–7. Find the arithmetic mean for the measures.

4. 22, 27, 35 28

5. 14, 31, 10, 51, 4 22

6. 127, 223, 338, 360 262

7. 1,651, 1,004, 1,224 1,293

8. Find the sum of 23 measures if the mean is 297. 6,831

♦ **Extra Examples.** Set 48.

Computing Arithmetic Means

Resurvey [O]

For each of Ex. 1–9, give a mathematical sentence which may be used to find the answer for the problem. For Ex. 1, the sentence could be $n = (12 + 14 + 9 + 17) \div 4$.

1. On a fishing trip, Joe caught 12 fish, Don caught 14, Pete caught 9, and Bill caught 17. What was the mean number of fish caught? $n = (12 + 14 + 9 + 17) \div 4$; 13

2. The numbers of people attending the last 4 weekly meetings of the Science Club were 42, 50, 33, and 47. What was the mean number of people attending? $n = (42 + 50 + 33 + 47) \div 4$; 43

3. Joe rode his bicycle 6 miles the first hour, 8 miles the second hour, and 7 miles the third hour. What was the mean number of miles traveled per hour? $n = (6 + 8 + 7) \div 3$; 7

4. In 6 days, Mr. Robinson drove 1,488 miles. What was the mean number of miles traveled per day? $n = 1{,}488 \div 6$; 248

5. At a mean average of 45 miles per hour, how many hours will it take Mr. Todd to drive 315 miles? $n = 315 \div 45$; 7

6. At Elm Elementary School, 105 books were borrowed one week by the 35 pupils in the sixth grade. What was the mean number of books borrowed per pupil? $n = 105 \div 35$; 3

7. During a period of 7 days, the mean number of pumpkins sold at Kane's Market per day was 78. How many pumpkins were sold during that 7-day period? $n = 7 \times 78$; 546

8. Mr. Kane sold 288 loaves of bread during a 16-day period. What was the mean number of loaves sold per day? $n = 288 \div 16$; 18

9. During a 5-month period, Jane made the following deposits in her savings account: \$12.25, \$14.50, \$13.75, \$9.80, and \$14.95. What was the mean number of dollars deposited?

$n = (12.25 + 14.50 + 13.75 + 9.80 + 14.95) \div 5$; \$13.05 [W]

Now work each of Ex. 1–9.

♦ **Extra Problems.** Set 141.

Pupil's Objectives

(a) To review computing the arithmetic mean; and (b) to apply the factors-product and addends-sum relationships when computing arithmetic means.

Background

Computing the arithmetic mean by finding the sum of the data and dividing this sum by the number of numbers in the data can become quite mechanical. It is a satisfactory procedure when the measures are given and the pupil is simply asked to calculate the mean. However, there are three instances where the factors-product relationship in the problem must be determined before it can be solved:

	given	*to be found*
(a)	mean and number of measures	sum of measures
(b)	sum of measures and number of measures	mean
(c)	sum of measures and mean	number of measures

factors-product relationship

(a) sum of measures = mean × number of measures
(b) mean = sum of measures ÷ number of measures
(c) number of measures = sum of measures ÷ mean

If the pupil realizes that he is finding the mean by use of these relationships, then he should have no difficulty in finding the solution for a problem.

Pre-Book Lesson

• Review the factors-product relationship, including how any one item in the relationship can be expressed in terms of the other two items of the relationship.

• Review the meaning and use of the term *average*. Newspapers and magazines may be used in conjunction with the discussion.

Using the Text Page

• Ex. 1–3 and 9. The pupil is asked to calculate the mean by the use of mathematical sentences. He may wonder why he is being asked to use this horizontal form instead of the more convenient vertical form. This is to enable him to see the factors-product relationship so that he can apply this relationship to the other problems on the page. The relationship in these problems is: mean = (sum of measures) ÷ (number of measures) in which the measure of each member is given and the mean is to be determined. Use the material from the Background to point out other forms for this relationship. Pupils may wish to label the data in the problems so they can more easily write a mathematical sentence for the data.

• Ex. 4, 6, and 8. The relationship is the same as in Ex. 1–3, except the sum of the measures is given. Make sure pupils realize that the terms *mean number*, *arithmetic mean*, and *mean average* all refer to the same number and have the same meaning.

• Ex. 5. The mean and the sum of the measures are given, and the number of measures is to be found. The relationship may be expressed as (number of measures) = (sum of measures) ÷ (mean).

• Ex. 7. The mean and the number of measures are given and the sum of the measures is to be found. The relationship may be expressed as (sum of measures) = (mean) × (number of measures).

Individualizing Instruction

• *More capable pupils* can do library research on arithmetic mean with such books as *How to Lie With Statistics* and report to the class.

• *All pupils* can engage in data-gathering activities such as determining the height of each pupil in the class, or determining the age of each pupil, and then finding the arithmetic mean of the data they have collected.

Pupil's Objective

To learn the meaning and use of the terms *mode* and *median*.

Background

In most sets of data, the measures will tend to concentrate somewhere midway between the extremes. This is what is meant by *central tendency*. The *mode* indicates the measure which occurs most frequently in the data.

In some applications, it is possible for there to be two modes in a set of data. In the data: 39, 43, 43, 43, 43, 49, 53, 53, 53, 53, 59, and 60, there are two modes, 43 and 53. If the greatest frequency is common to three or more numbers, these numbers lose significance and we say there is no mode.

When the measures in a set of data are arranged in order of increasing or decreasing value, the number in the middle is called the *median*. The median is a measure of central tendency that selects the middle number as being representative of the set of data. If there is an even number of items in the data, there will be no middle number. In such a case, the median is the arithmetic mean of the two middle numbers.

A decision as to which best represents a set of data, the median, the mode, or the mean, must be based on each individual set of data. For example, the annual salaries received by ten employees are: $5,600; $5,000; $5,600; $5,200; $5,400; $5,300; $5,100; $5,600; $8,100; and $12,100. The mean is $6,300; the median is $5,500; and the mode is $5,600. In this example, the median best describes the typical salary of these employees, and the mean is probably the poorest indication of the typical salary of these employees.

Pre-Book Lesson

Give the pupils sets of data for which the mean is not a good indication of central tendency, but for which the most frequent measure or middle measure would be a better indication of central tendency. Good examples are data in which there are a few extreme scores, such as, on a test in which 20 pupils scored between 40 and 50 and 5 scored 100. Use newspapers and magazines having statistical data showing measures of central tendency other than the mean.

Using the Text Page

- Ex. 1–3. Give the pupils other examples of data in which there are two modes or in which the mode is a poor indication of central tendency such as 1, 2, 4, 5, 6, 8, 17, 17. Have pupils make up examples where the mode is a good indication of central tendency, and have them explain why it is a good measure of central tendency. For example, a clothing manufacturer might keep a record of how much men pay for a suit. The price that occurs most frequently will help him to decide what kind of suit to manufacture in the largest quantity.
- Ex. 4–6. Give the pupils some examples of sets of data where the median is a poor indication of central tendency such as 10, 50, 60, 89, 90, 91, 92.

Individualizing Instruction

- Give the *more capable pupils* a set of data and ask them to find the mode, median, and mean; and then decide which of these three averages is the best indication of central tendency for that particular set of data. Have them determine which average would be affected the most if the greatest number in a given set of data is reduced by more than one half.
- *All pupils* can engage in data-gathering activities such as finding out how much time each pupil spends a day watching television or playing outdoors, and then they can determine the mode, median, and mean of their data.

* A set of data can have two modes if two numbers which occur the same number of times also occur most frequently in the data.

Mode and Median

Extension; statistics [O]

Statistics is a branch of mathematics which is concerned with collecting and interpreting sets of numbers, or measures, called *data*. When interpreting data, a statistician often computes the *arithmetic mean*, the *mode*, or the *median*.

Telephone Calls per day
1
4
6
3
4
2
3
8
3
5
5
44

1. We already know how to find the arithmetic mean for data. At the board, find this mean for the data shown in the box. 4

$$11\overline{)44}\quad 4$$

2. How many times does the measure 3 occur in the data shown in the box? 3 Does the measure 3 occur more times than any other measure? Yes For these data, 3 is called the *mode*.

The *mode* is the measure which occurs most frequently in the data.

* **3.** For each of Ex. a–d, give the mode for the measures.

a. 2, 7, 4, 3, 3, 4, 4 4 **b.** 5, 2, 1, 3, 4, 5, 2, 5, 5 5

c. 23, 25, 26, 25 25 **d.** 31, 37, 38, 33, 38, 30 38

4. For a set of measures, the middle measure when they are shown in order is called the *median*. For example, the measures shown in the box can be arranged in order as follows: 1, 2, 3, 3, 3, 4, 4, 5, 5, 6, 8. Since there are 11 measures, the middle measure is the 6th measure. The middle measure, or the median, for these data is _? 4.

5. Find the median for each of Ex. 3a and 3b. a. 4; b. 4

6. If there are an even number of measures, then the median for the data is the mean of the two middle measures when they are arranged in order. For example, the median for 1, 2, 4, 7 is $(2 + 4) \div 2$, or _? 3. Find the median for Ex. 3c and 3d. c. 25; d. 35

[W]

Ex. 7–12. Find the mean, mode, and median.

7. 7, 2, 2, 8, 1, 6, 2 4; 2; 2 **8.** 2, 8, 8, 6 6; 8; 7 **9.** 21, 22, 22, 23 22; 22; 22

10. 86, 92, 100, 92, 90 92; 92; 92 **11.** 18, 36, 36, 30 30; 36; 33 **12.** 112, 112, 136, 132 123; 112; 122

♦ **Extra Examples.** Set 49. ● **Extra Activity.** Set 159.

* It is customary to list the powers in decreasing order, but other arrangements are not wrong.

To Keep in Practice

Computation [W]

Ex. 1–6. Copy and add.

1.	2.	3.	4.	5.	6.
389	3,784	56,708	$300.85	$48.88	24,348
307	4,773	308,994	28.77	0.67	17,732
696	8,557	365,702	$329.62	$49.55	42,080

Ex. 7–12. Copy and subtract.

7.	8.	9.	10.	11.	12.
506	$37.08	23,836	$500.00	$30.20	61,480
89	1.99	3,788	84.99	22.88	19,772
417	$35.09	20,048	$415.01	$7.32	41,708

Ex. 13–18. Copy and multiply.

13.	14.	15.	16.	17.	18.
38	381	3,897	793	$3,892.50	6,803
23	77	9	207	45	102
874	29,337	35,073	164,151	$175,162.50	693,906

Ex. 19–23. Copy and divide.

19.	20.	21.	22.	23.
$23\overline{)478}$	$9\overline{)4,987}$	$30\overline{)3,788}$	$324\overline{)16,988}$	$7\overline{)645,057}$
20, R 18	554, R1	126, R8	52, R140	92, 151 R0

Ex. 24–27. Find the mean, the mode, and the median.

24. 5, 7, 3, 8, 3, 9, 10, 3 — 6; 3; 6

25. 23, 65, 45, 52, 55, 45, 44 — 47; 45; 45

26. 104, 107, 102, 109, 102, 100 — 104; 102; 103

27. 17, 17, 16, 17, 16, 20, 14, 19 — 17; 17; 17

Try These Exercises

Enrichment [W]

Ex. 1–4. Copy and complete.

1. Add	2. Add	3. Subtract	4. Divide
926	3,57?	3,9?7	1,?04
7?8	?,623	694	$7\overline{)9,82?}$ 8
1,004	5,??6 (19)	3,293	

5. List within braces the numbers which make $n \times n = n$ a true sentence. {0, 1}

*Ex. 6–11. Express the number as a sum of powers of 2. For example, $22 = 2^4 + 2^2 + 2$.

6.	7.	8.	9.	10.	11.
12	28	40	14	82	116
$2^3 + 2^2$	$2^4 + 2^3 + 2^2$	$2^5 + 2^3$	$2^3 + 2^2 + 2$	$2^6 + 2^4 + 2$	$2^6 + 2^5 + 2^4 + 2^2$

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Pupil's Objectives

(a) To review and practice mixed computational skills; and (b) to find missing numbers in various mathematical situations.

Background

These exercises give the pupils mixed computational practice with whole numbers, plus practice at solving mathematical sentences and finding averages. Each of the exercises involves only whole numbers.

Although the enrichment material at the bottom of the page is probably best suited for the *more capable pupils*, you may wish to permit *all pupils* to try these exercises. Even pupils who do not have high ability in mathematics may find these exercises interesting, challenging, and fun.

Pre-Book Lesson

• Practically no review should be necessary for these exercises, except perhaps the meaning of mean, median, and mode which should be reviewed quickly before assigning the written work. However, you may wish to work a few simple practice exercises at the board involving work similar to that on the page.

• Before pupils open the book, establish whether or not you wish to have *all pupils* attempt the enrichment exercises or whether these are intended only for *more capable pupils* at this time. Others who finish early may work Extra Activities or Supplementary Activities.

Using the Text Page

• Ex. 1–27. These exercises may be used simply as practice exercises for diagnosing weaknesses in computation skills or as a test for marking purposes. If the number of exercises is excessive for one-day completion for *slower learners*, the work may be assigned over a two-day period.

• Ex. 1–4. These exercises in the enrichment material at the bottom of the page have the characteristics of a puzzle. You may wish to have *all pupils* attempt them or restrict them to *more capable pupils*. Besides being challenging and fun, the exercises may stimulate the pupils' thinking about the nature of each of the operations in arithmetic and patterns of powers of numbers.

• Ex. 5. This example is interesting in that there is a set of numbers consisting of more than one number which will make the sentence true. As an even greater challenge, you may wish to put on the board one or more of the following and have pupils describe the set of numbers which, when substituted for the letter, would make the sentence true:

$$\begin{array}{ll} \text{a. } n + n = 2n & \text{c. } y \div y = 1 \\ \text{b. } 2x - x = x & \text{d. } m = m + 2 \end{array}$$

$$\text{e. } m - m = 0$$

Any number will make sentences a, b, and e true. Any number except zero will make sentence c true, and there is no number that will make sentence d true.

• In Ex. 6–11, pupils should be led to realize that one way of doing these exercises is to first express the number with a base-two numeral because the place values in base two are powers of 2.

$$\begin{aligned} 116 &= 1110100_{\text{two}} \\ &= 2^6 + 2^5 + 2^4 + 2^2 \end{aligned}$$

Individualizing Instruction

• Have *all pupils* restudy appropriate pages in earlier chapters and do appropriate practice exercises and Extra Examples if certain exercises are missed consistently.

• After remedial work, *more capable pupils* may work with the teacher at the board on examples similar to those in the enrichment section. Let other pupils complete this section on their own if they have not already attempted them.

Pupil's Objectives

(a) To review the meaning of subset; (b) to review the concept that every set is a subset of itself; and (c) to review the concept that the empty set is a subset of every set.

Background

The set with the greatest number of members under consideration is called the *universal set* or the *universe*. Members of the universal set may be grouped in various ways to form other different sets. These new sets are called subsets of the universal set.

If every member of set X is also a member of set Y, then set X is called a subset of set Y. This can be indicated by use of the symbols $X \subseteq Y$, which is read, "X is a subset of Y." The negation of the subset symbol $\subseteq$ is $\nsubseteq$. $A \nsubseteq B$ is read "set A is not a subset of set B."

Every set is considered a subset of itself. Therefore, for every set (X), $X \subseteq X$.

The empty set is considered a subset of every set. Therefore, for every set (X), $\{\ \} \subseteq X$.

The total number of subsets of a set consisting of one member is 2 (the set itself and the empty set). A set consisting of two members will have 4 subsets. A set consisting of 3 members will have 8 subsets, and so on. An examination of the pattern shows that the total number of subsets of a set consisting of n members is 2^n. A set consisting of 5 members will have 2^5 or 32 subsets. If a class consists of 20 pupils, there are 2^{20} or 1,048,576 subsets of the set consisting of the members of the class.

When we say that set X is a subset of set Y, it is possible that set X and set Y have the same members. However, if set X is a subset of set Y, and if set Y contains at least one member which is not in set X, then set X can be called a *proper subset* of set Y. This is indicated by the symbols $X \subset Y$, which is read, "set X is a proper subset of set Y."

From examination of the above paragraph, it should be apparent that all subsets of a given set are proper subsets except the set itself. For example, if $A = \{1, 2, 3\}$, then the proper subsets of A are $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$, and $\{\ \}$. Since the number of all subsets is 2^n, the number of proper subsets must be $2^n - 1$.

Pre-Book Lesson

Review meaning of set, use of braces in designating a set, and meaning of the empty set.

Using the Text Page

- Ex. 1–4. Have the pupils give examples of subsets of the set of pupils in the class, including the set itself and the empty set.
- Ex. 5. The pupils may wonder why the empty set is a subset of every set. According to our definition of subset, if X is a subset of Y, every member of X is a member of Y. This implies that if X is *not* a subset of Y, then it must contain at least one member that is not a member of Y. But the empty set contains no such member. Therefore, the empty set is always a subset of every set.

Individualizing Instruction

- *More capable pupils* can be asked to find the total number of subsets of a set of 1 member, of a set of 2 members, of a set of 3 members, and of a set of 4 members. Then have them examine the pattern and encourage them to try to determine how many subsets a set of n members would have. The answer is 2^n as explained under Background.
- *Slower learners* can make a list of sets of which they are a subset.

* Remember that while { } names the empty set, {0} is a set having one member, zero.
** An n-element set has 2^n subsets.

Subsets

Resurvey [O]

1. Study the sets described in box A.

a. R is the set of whole numbers greater than _?2 and less than _?6.

b. S is the set of whole numbers greater than _?1 and less than _?7.

A
$R = \{3, 4, 5\}$
$S = \{2, 3, 4, 5, 6\}$

2. Box A. Is every member of R also a member of S? Yes Set R is called a *subset* of set S.

If every member of a set X is also a member of a set Y, then set X is called a *subset* of set Y.

3. Is $\{2, 5, 6\}$ a subset of S? Yes Why? Every member of {2, 5, 6} is also a member of S.

4. Is every member of S also a member of S? Yes Then is S a subset of itself? Yes Is R a subset of itself? Yes

Any set is a subset of itself.

* 5. The symbol { } names the empty _?_ set.

The empty set is a subset of every set.

B
$G = \{2, 7, 8, 12\}$
$H = \{2, 8\}$ Yes
$I = \{3, 12\}$ No
*** $J = \{\ \}$ Yes
$K = \{7, 8, 9\}$ No
$L = \{2, 7, 8, 12\}$ Yes

6. Box B. For each of sets H–L, tell whether or not the set is a subset of set G.

[W]

** 7. List all the subsets of $\{1, 2, 3\}$ which have exactly one member; {1}, {2}, {3} exactly two members; {1,2}, {1,3}, {2,3} three members; {1,2,3} no members. { } How many subsets did you list? 8 Are there any subsets that you did not list? No

8. List all the subsets of $\{1, 3, 6, 8\}$ which have exactly four members; {1, 3, 6, 8} three members; {1, 3, 6}, {1, 6, 8}, {1, 3, 8}, {3, 6, 8} two members; {1, 3}, {1, 6}, {1, 8}, {3, 6}, {3, 8}, {6, 8} one member; {1}, {3}, {6}, {8} no members. { } How many subsets did you list? 16

Ex 9–10. Write T or F.

9. $\{2, 4, 8, 10\}$ is a subset of $\{2, 4, 8\}$. F

10. $\{0, 3, 5, 9\}$ is a subset of the set of counting numbers. F

● **Extra Activity.** Set 160.

*** If pupils have difficulty with this, ask the question, "Are there any members of the empty set which are not members of G?"

Subsets in Geometry

Resurvey and extension [O]

Fig. 1

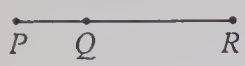


Fig. 2

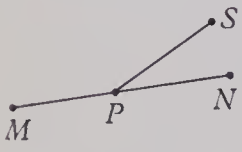


Fig. 3

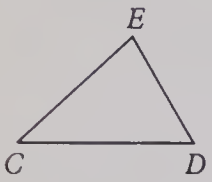


Fig. 4

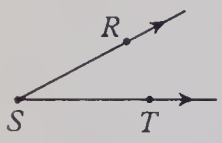


Fig. 5

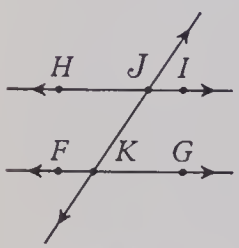


Fig. 6

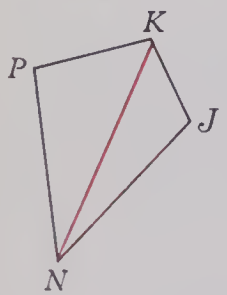


Fig. 7

1. Fig. 1. Name the line segment shown. $\overline{AB}$ Is a line segment a set of points? Yes Does a line segment contain its end points? Yes Then $\overline{AB}$ contains points A and B.

2. The set of end points of line segment AB may be described as $\{A, B\}$. Is $\{A, B\}$ a subset of $\overline{AB}$? Yes Why? Every member of {A, B} is also a member of $\overline{AB}$.

3. Is $\{A\}$ a subset of $\overline{AB}$? Yes Is $\{B\}$ a subset of $\overline{AB}$? Yes Is the empty set a subset of $\overline{AB}$? Yes

4. You know that every set is a subset of itself. Then is $\overline{AB}$ a subset of $\overline{AB}$? Yes Describe the two subsets of $\{A\}$. {A}, { }

5. Fig. 2. Name the three labeled segments. $\overline{PQ}$, $\overline{PR}$, $\overline{QR}$ Is every point in $\overline{PQ}$ also in $\overline{PR}$? Yes Then $\overline{PQ}$ is a subset of $\overline{PR}$. Every member of $\overline{PS}$ is not a member of $\overline{MN}$

6. Fig. 3. Is $\overline{PS}$ a subset of $\overline{MN}$? No Explain. $\overline{MN}$, $\overline{MP}$, $\overline{PN}$ Name three labeled segments that are subsets of $\overline{MN}$.

7. Fig. 4. Name the sides of triangle CDE. $\overline{CD}$, $\overline{DE}$, $\overline{CE}$ You know that a triangle is a set of points. Is $\overline{CD}$ a subset of triangle CDE? Yes Why? Every member of $\overline{CD}$ is also a member of triangle CDE.

8. Fig. 5. Name two rays that are subsets of angle RST. $\overrightarrow{SR}$, $\overrightarrow{ST}$ Is $\{R\}$ a subset of $\overrightarrow{ST}$? No Explain. R is not a member of $\overrightarrow{ST}$.

[W]

Ex. 9–11. Refer to Fig. 6.

9. Name three sets each containing exactly one point which are subsets of $\overline{HI}$; $\overline{FG}$. {H}, {I}, {J}; {F}, {K}, {G}

10. Name three labeled segments which are subsets of $\overline{HI}$; $\overline{FG}$. $\overline{HI}$, $\overline{HJ}$, $\overline{JI}$; $\overline{FK}$, $\overline{FG}$, $\overline{KG}$

11. Name two labeled rays that are subsets of angle GKJ. $\overrightarrow{KJ}$, $\overrightarrow{KG}$

12. Name four labeled segments that are subsets of the quadrilateral shown in Fig. 7. $\overline{PK}$, $\overline{KJ}$, $\overline{JN}$, $\overline{NP}$ Copy Fig. 7 and show $\overline{KN}$. Is $\overline{KN}$ a subset of polygon $NJKP$? No

♦ **Extra Examples.** Set 50. ● **Extra Activity.** Set 161.

Teaching Page 84

Pupil's Objective

To learn what application the concept of subsets has in geometry.

Background

According to the modern view of geometry, space may be thought of as consisting entirely of a set of points. Every geometric concept such as point, line, plane, ray, and segment is a subset of this set of points. Also, every one of these subsets has subsets. For example, a point is a subset of the set of all points, and yet this set has two subsets, itself and the empty set. The set consisting of the point A has the subsets $\{A\}$ and $\{\ \}$. It is important to realize that it is the point that is a member of the set, and not the letter A. The use of the letters is necessary to identify points, but the letters themselves are not members of the set.

Pre-Book Lesson

Review the geometric concepts of point, segment, line, ray, and angle, and methods of naming them.

Using the Text Page

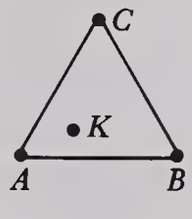
• Ex. 1–4. It may be necessary to restate the principle that a line segment is a set of points. Any point or points contained in the segment form a subset of that segment. It may be best to read $\{A\}$ as "the set whose member is point A." This will place emphasis on the idea that the set consists of a point and not of a letter.

The principles that every set is a subset of itself and that the empty set is a subset of every set apply to all sets, including sets of points. Some pupils may be a little hesitant to accept the idea of the empty set being a subset of a set of points, but it must be accepted in order to be consistent with set concepts and definitions.

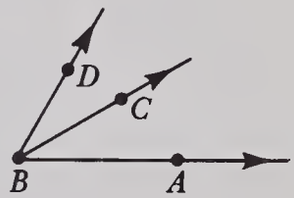
• Ex. 5. Some pupils may feel that there are more than three segments shown in the drawing. $\overline{PR}$ and $\overline{RP}$ each name the same segment; $\overline{QR}$ and $\overline{RQ}$ name the same segment; $\overline{PQ}$ and $\overline{QP}$ name the same segment. Therefore, there are only three different segments shown in the drawing.

Individualizing Instruction

• *More capable pupils* can be given drawings and questions such as the following to test their mastery of the concept of geometric figures such as triangles, angles, and other polygons.



Is point K a subset of triangle ABC? (*No*)



Is angle ABC a subset of angle ABD? (*No*)

• *Slower learners* may use bits of clay on a string to represent points in a line and, after making models of segments, angles, triangles, and other geometric figures, they may determine more easily which sets of points are subsets of the given set.

Teaching Page 85

Pupil's Objective

To learn to apply the idea of intersection to sets of points in geometry.

Background

The intersection of set A and set B is the set of all members belonging to both A and B. In order for a member to be in the intersection, it must belong to both A and B.

The operation of intersection can be performed on any two sets, including sets of points in geometry.

Intersection is commutative and also associative. Therefore, the set $\triangle DEH \cap \overline{DE}$ is the same as the set $\overline{DE} \cap \triangle DEH$, and $(\overline{AD} \cap \overline{AC}) \cap \overline{BC}$ is the same as $\overline{AD} \cap (\overline{AC} \cap \overline{BC})$.

Pre-Book Lesson

• Review intersection of sets by using sets of concrete objects in the room and by referring to sets of numbers.

• Using models of geometric figures, ask pupils if they can name various parts of each figure or different sets of points that are part of each figure. Ask if points belonging to a particular segment also belong to the line containing that segment.

Using the Text Page

• Ex. 3. The intersection of two segments is a segment, a point, or the empty set. If two segments are not part of the same line, their intersection is either one point or the empty set.

You may wish to review briefly the concepts taught on page 42.

• Ex. 4. $\overline{JK}$ is not a subset of circle C. The only points that are in both $\overline{JK}$ and circle C are the point J and the point K. The other points of $\overline{JK}$ are not a part of the circle but are part of the interior for the circle. The intersection of a circle and any one of its chords, including its diameter, consists of just the two end points of the chord.

• Ex. 5. All diameters of a circle are chords containing the point that is the center for the circle. Therefore all diameters of a circle intersect at the center but only at the center. The point of intersection of any two diameters of a circle is the center for the circle.

Individualizing Instruction

• *More capable pupils* can be given exercises involving intersection of sets of points to demonstrate that intersection is commutative and associative.

• *All pupils* can be given a statement involving intersection of sets of points such as $\overline{AB} \cap \overline{DF} = \overline{BD}$ and be asked to make a drawing for the two sets of points that will satisfy the conditions of the statement.

• *Slower learners* may benefit from working the examples with the aid of models using strips of cardboard to represent lines and segments, and the clay on a string to represent other figures.*

* See item 19, page xix.

* The intersection of two sets is a set. The intersection of $\overline{AC}$ and $\overline{CB}$ is the set which contains point C. Although we often say that two segments (lines, etc.) intersect in a point, their intersection is a set containing a point.

Intersection of Sets in Geometry

Extension [O]

* **1.** Fig. 1. How many points belong to both $\overline{AC}$ and $\overline{CB}$. 1 We write, $\overline{AC} \cap \overline{CB}$ which is read, *the* intersection *of segments AC and CB.* $\overline{AC} \cap \overline{CB} = \{C\}$

Fig. 1

{P}

**2. Fig. 2. Describe the intersection of $\overline{MP}$ and $\overline{PN}$; $\overline{MQ}$ and $\overline{PN}$; $\overline{MN}$ and $\overline{PQ}$; $\overline{MN}$ and $\overline{PQ}$; $\overline{QS}$ and $\overline{MP}$; $\overline{QS}$. { } {Q} Does it seem that the intersection of two line segments may be a segment or a set containing just one point or the empty set? Yes

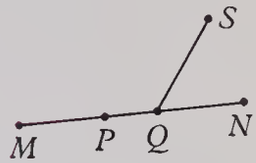


Fig. 2

3. On the board, show two segments which lie in the same line and whose intersection is { }. Sample answer: $\overline{AB} \cap \overline{CD} = \{\ \}$

4. Fig. 3. Circle C is pictured. Line segment JK is called a diameter for the circle. Is $\overline{JK}$ a subset of circle C? No Explain. Every member of JK is not also a member of circle C. The intersection of circle C and $\overline{JK}$ is {J, K}. Is the intersection of a circle and a diameter for the circle always a set which contains exactly two points? Yes Explain. Only 2 points of a diameter are also members of the circle.

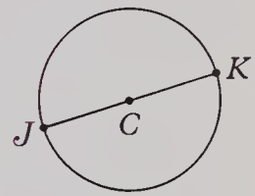


Fig. 3

5. Fig. 4. The end points of line segments TU and RS belong to the circle shown. $\overline{TU} \cap \overline{RS} = \{M\}$ If $\overline{TU}$ and $\overline{RS}$ are diameters for the circle, what is point M? The center for the circle

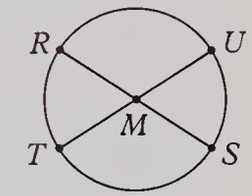


Fig. 4

[W]

6. Fig. 5. Find the intersection of

a. the circle and diameter MN. {M,N} **b.** the circle and chord $\overline{BN}$. {B, N}

c. diameter MN and radius CN. $\overline{CN}$ **d.** the circle and $\{C\}$. { }

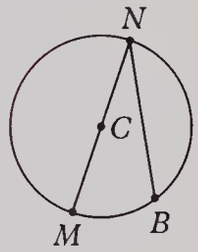


Fig. 5

Ex. 7–15. Refer to Fig. 6. Name the set of points described. ($\triangle DEH$ means *triangle DEH.*)

7. $\overline{DE} \cap \overline{EG}$ {E}
8. $\overline{FH} \cap \overline{EH}$ $\overline{EH}$
9. $\overline{DG} \cap \overline{FH}$ {E}
10. $\triangle DEH \cap \overline{DE}$ $\overline{DE}$
11. $\overline{DH} \cap \overline{FG}$ { }
12. $\triangle DEH \cap \triangle EFG$ {E}
13. $\overline{FH} \cap \triangle EFG$ $\overline{EF}$
14. $\overline{FH} \cap \overline{DH}$ {H}
15. $\overline{DG} \cap \overline{GD}$ $\overline{DG}$ (or $\overline{GD}$)

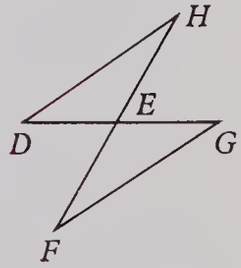


Fig. 6

16. If two chords for a circle both contain the center for the circle, what do you know about the lengths of the two chords? They are the same.

▲ **Reteaching.** Set 8.

** $\overline{PQ}$ is correct notation for the answer. $\overline{PQ}$ names a segment—a set of points; $\{\overline{PQ}\}$ does not name a set of points—it names a set of segments.

Union of Sets in Geometry

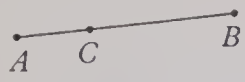


Fig. 1

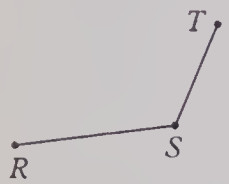


Fig. 2

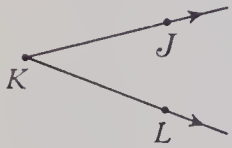


Fig. 3

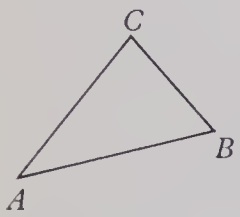


Fig. 4

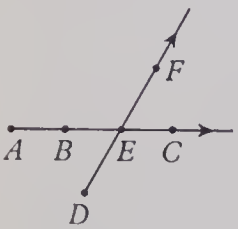


Fig. 5

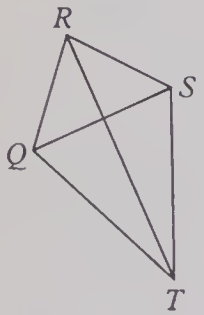


Fig. 6

Extension [O]

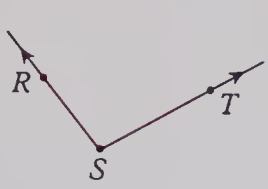
1. Fig. 1. The set of all points that belong to $\overline{AC}$ or to $\overline{CB}$ or to both $\overline{AC}$ and $\overline{CB}$ is called the union of $\overline{AC}$ and $\overline{CB}$. We write, $\overline{AC} \cup \overline{CB}$ which is read, *the union of segments AC and CB*. Is the following a true sentence? $\overline{AC} \cup \overline{CB} = \overline{AB}$ Yes

2. Fig. 2. Does every point shown belong to $\overline{RS}$ or to $\overline{ST}$? Yes Then can we think of this figure as the union of $\overline{RS}$ and $\overline{ST}$? Yes Is the union of two segments always a segment? No

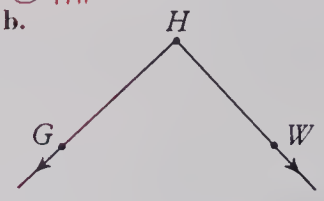
3. Fig. 3. You know that $\angle JKL$ may be thought of as formed by rays KJ and KL. Can it also be thought of as the union of these two rays? Yes We write, $\overrightarrow{KJ} \cup \overrightarrow{KL} = \angle JKL$.

4. On the board, express each of the following angles as the union of two rays, using the symbol "$\cup$."

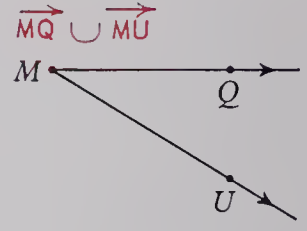
a. $\overrightarrow{SR} \cup \overrightarrow{ST}$



b. $\overrightarrow{HG} \cup \overrightarrow{HW}$



c. $\overrightarrow{MQ} \cup \overrightarrow{MU}$



5. Fig. 4. You know that a polygon is a simple closed curve which may be thought of as formed by line segments only. Can a polygon also be thought of as the union of line segments? Yes Is triangle ABC the union of segments AB, BC, and AC? Yes

[W]

Ex. 6–14. Refer to Fig. 5. Name the set of points described.

6. $\overline{AB} \cup \overline{BE}$ $\overline{AE}$
7. $\overline{AC} \cap \overline{DF}$ $\{E\}$
8. $\overrightarrow{EF} \cup \overrightarrow{EC}$ $\angle FEC$
9. $\overrightarrow{CA} \cap \overrightarrow{CB}$ $\overrightarrow{CA}$ $(\overrightarrow{CE}, \overrightarrow{CB})$
10. $\overrightarrow{BC} \cap \overrightarrow{AE}$ $\overrightarrow{BC}$ $(\overrightarrow{BE})$
11. $\overrightarrow{EF} \cup \overrightarrow{DF}$ $\overrightarrow{DE}$ $(\overrightarrow{DF})$
12. $\overline{AB} \cap \overline{EC}$ $\{\}$
13. $\overline{AE} \cup \overline{BC}$ $\overline{AC}$
14. $\overrightarrow{BC} \cup \overline{AB}$ $\overrightarrow{AC}$ $(\overrightarrow{AE}, \overrightarrow{AB})$

Ex. 15–16. Refer to Fig. 6. Copy end complete the sentence.

15. The union of segments QR, RS, and QS is triangle QRS.
16. Triangle QRT is the union of $\overline{QR}$, $\overline{QT}$, and $\overline{RT}$.

▲ **Reteaching.** Set 9.

Pupil's Objectives

To learn to apply the concept of union to sets of points in geometry.

Background

The union of set A and set B, denoted by the symbols $A \cup B$, is the set of all members belonging to A or to B or to both. If an element is a member of at least one of the sets, it is a member of the union set.

The set operation of union can be performed on any two sets, including sets of points in geometry.

One application of union in geometry is that several concepts can be defined in terms of the union of two or more sets of points. A polygon can be defined as a simple closed curve formed by the union of three or more line segments. An angle can be defined as the set of points formed by the union of two rays that share a eommon end point.

Union is commutative and also it is associative. These operations apply to all sets, including sets of points in geometry. Therefore, the set described by $\triangle ABC \cup \overline{DE}$ is the same as the set described by $\overline{DE} \cup \triangle ABC$, and the set described by $(\overline{AB} \cup \overline{BC}) \cup \overline{CD}$ is the same as the set described by $\overline{AB} \cup (\overline{BC} \cup \overline{CD})$.

In operations on sets, there are two distributive properties. Union is distributive over intersection and intersection is distributive over union:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

These distributive properties may also be applied to sets of points in geometry.

Pre-Book Lesson

• Review union of sets, including the commutative and associative properties as applied to union. Use objects in the room, sets of numbers, and the pupils themselves to illustrate these properties.

• Using strips of cardboard labeled with letters to represent points, have pupils show the intersection of various figures. Using the same figures, have them list sets of points whose union forms these figures.

Using the Text Page

• Ex. 1. Have some pupils name various unions, and have other pupils show segments and lines corresponding to these unions.

• Ex. 6–16. These examples may be supplemented with exercises involving the union of three sets of points or the interseetion of three sets of points to demonstrate the associative property of union. Other supplementary exercises could involve both union and intersection, such as $(\overline{QS} \cup \overline{ST}) \cap \overline{RT}$ for the drawing in Fig. 6.

Individualizing Instruction

• *More capable pupils* may be given exercises that demonstrate the property that union distributes over intersection, and also that intersection distributes over union. For the drawing in Fig. 5: $\overline{AB} \cup (\overline{BC} \cap \overline{DF}) = (\overline{AB} \cup \overline{BC}) \cap (\overline{AB} \cup \overline{DF})$, and $\overline{AB} \cap (\overline{BC} \cup \overline{DF}) = (\overline{AB} \cap \overline{BC}) \cup (\overline{AB} \cap \overline{DF})$

• *All pupils* can be asked to describe or define as many geometric concepts as they can in terms of the union of sets of points. For instance, the union of rays not in the same line, to form angles, or the union of two rays with a common end point to form a line.

• *Slower learners* may be asked to draw pictures of as many geometric figures as they can that could be formed by the union of 2 segments; of 3 segments; of 4 segments.

Pupil's Objectives

(a) To check understanding of important ideas studied to this point; (b) to review the meaning of one-to-one correspondence; and (c) to do remedial work in areas where weaknesses are revealed.

Background

The exercises on this page may be used as a progress test of important ideas which have been taught, and for maintaining the ideas and skills which have been developed. Once a concept or skill has been developed by the pupil, it must be reviewed periodically to maintain it.

The main concepts being tested by these exercises are:

(a) meaning of a prime factor
(b) idea of a ray
(c) subsets
(d) prime number
(e) parallel lines
(f) union and intersection of sets
(g) idea of a circle
(h) prime factorization using a factor tree
(i) geometric figures formed by the union of sets and by the intersection of sets

The exercises at the bottom of the page are a review of some of the ideas pertaining to one-to-one correspondence. Two of the main understandings being reviewed are:

(a) If the elements in two sets can be matched in one way to show one-to-one correspondence, they can be matched in all possible ways.

(b) If a one-to-one correspondence exists between the elements of one set and the elements of another set, the two sets are equivalent.

If two sets each have n elements, they can be matched in $n!$ (n-factorial) different ways.

Number of elements in each set	*Number of different ways of matching the two sets*
1	1
2	2×1, or 2
3	$3 \times 2 \times 1$, or 6
4	$4 \times 3 \times 2 \times 1$, or 24
5	$5 \times 4 \times 3 \times 2 \times 1$, or 120
n	$n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$ This is n-factorial ($n!$).

Pre-Book Lesson

You may wish to have a brief review of topics on the page and then allow pupils to do the written work as a maintenance exercise or as a test on their own.

Using the Text Page

- Ex. 1–15. After pupils have completed the written exercises at the top of the page, discuss each example and have pupils explain their work. Do remedial work with groups or individuals who have difficulty with any of the exercises.
- Ex. 1–3. These examples at the bottom of the page should be worked by the pupils and then discussed with the teacher.

Individualizing Instruction

- Remind *all pupils* that they are now nearing the end of the chapter and the regular end-of-chapter tests will be coming up soon. If there is any aspect of these exercises that they do not understand, they should ask questions now and do remedial work as necessary.
- *More capable pupils* may work Extra Activities Sets or Supplementary Activities if they finish the work early.

* If a one-to-one correspondence exists between two sets, then the sets are equivalent. If the members of both sets are the same, then the sets are equal.

How Well Do You Remember?

[W]

Ex. 1–7. Write *T* or *F*.

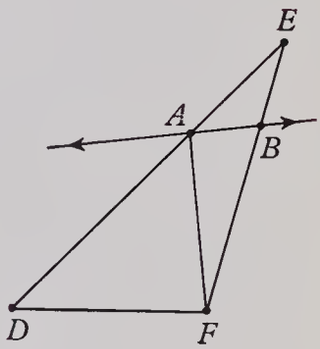
1. 15 is a prime factor of 45. F
2. A ray has two end points. F
3. $\{0\}$ is a subset of $\{\ \}$. F
4. 23 is a prime number. T
5. Two parallel lines never intersect. T
6. $\{0, 1\}$ is a subset of $\{0, 1, 3\}$. T
7. The intersection of $\{3, 6, 8\}$ and $\{5, 6, 8, 9\}$ is a subset of the union of $\{2, 5, 6\}$ and $\{5, 8, 10\}$. T

8. Make a factor tree for 244.

8. 244 / 2 × 122 / 2 × 2 × 61

9. List all the subsets of $\{5, 6, 7\}$. $\{5,6,7\}, \{5,6\}, \{5,7\}, \{6,7\}$ $\{5\}, \{6\}, \{7\}, \{\ \}$

Ex. 10–15. Refer to the figure at the right and name the set of points described.

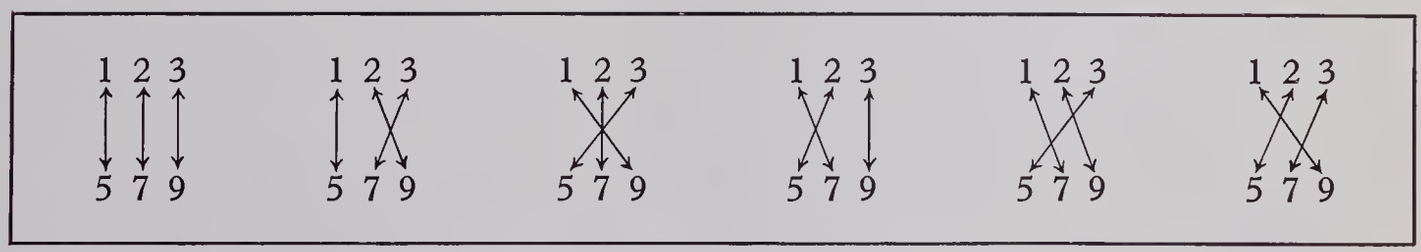


10. $\overleftrightarrow{AB} \cap \overline{DF}$ { }
11. $\overleftrightarrow{AB} \cap \triangle FAE$ {A, B}
12. $\overleftrightarrow{AB} \cap \triangle DEF$ {A, B}
13. $\overleftrightarrow{AB} \cap \overline{FE}$ {B}
14. $\triangle DAF \cap \triangle FAE$ $\overline{AF}$
15. $\overline{AB} \cup \triangle FAB$ $\triangle FAB$

Remembering about One-To-One Correspondence

Resurvey [O]

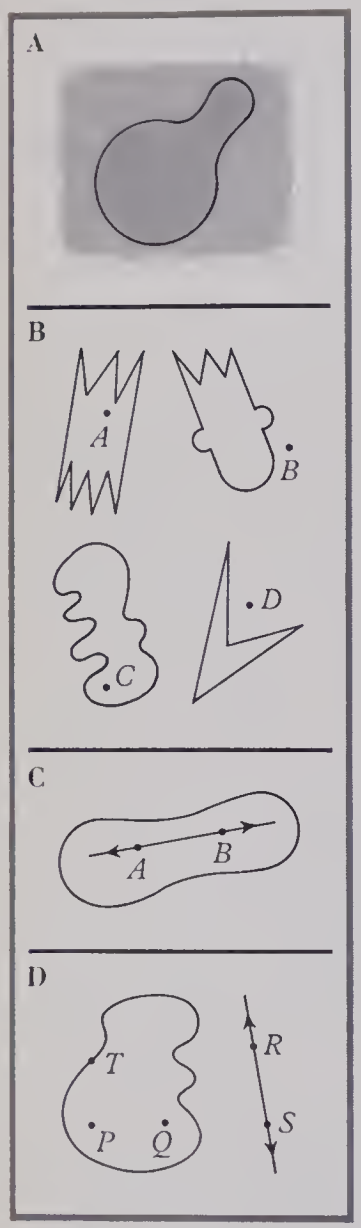
* In the box, the members of $\{1, 2, 3\}$ are shown matched with the members of $\{5, 7, 9\}$ in six ways, each way showing that a one-to-one correspondence exists between the two sets.



1. Are $\{1, 2, 3\}$ and $\{5, 7, 9\}$ equivalent sets? Yes
2. Are $\{0, 7, 9, 10\}$ and $\{17, 20, 38\}$ equivalent sets? No
3. Match the members of $\{4, 5, 6, 7\}$ with the members of $\{0, 2, 8, 9\}$ in three different ways so that each way shows that a one-to-one correspondence exists between the two sets. Sample answers:

Simple Closed Curves

Resurvey; partitioning [W]



The simple closed curve shown in box A lies in one plane. The plane is *partitioned*, or separated, into three distinct sets of points.

(1) The set of points belonging to the curve
(2) The set of points in the interior for the curve
(3) The set of points in the exterior for the curve

1. Box A. What color is used to show the interior for the curve? Blue What color is used to show the exterior for the curve? Gray

2. Box B. For each closed curve, tell whether the labeled point shown is in the interior for the curve or in the exterior for the curve. A, interior; B, exterior; C, interior; D, exterior

3. Box C. Are points A and B and $\overline{AB}$ in the interior for the closed curve? Yes Does $\overline{AB}$ intersect the curve? No Does $\overleftrightarrow{AB}$ intersect the curve? Yes Explain. Key idea: Two points of AB are also members of the curve.

4. Box D. Which labeled points are in the interior for the closed curve? P, Q Which are in its exterior? R, S Which belong to the closed curve? T Does $\overleftrightarrow{RS}$ intersect the curve? No

5. On your paper, make a large copy of Fig. 1. Lightly shade to show the interior for the curve and then show three different lines which lie in the exterior for the curve and which do not intersect the curve. Remember to use arrowheads when drawing models of lines. See Fig. 1 for sample answer.

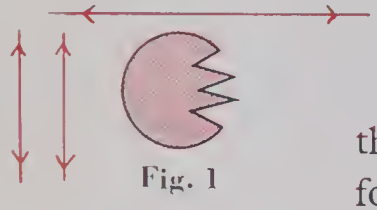


Fig. 1

6. Make a large copy of Fig. 2 and then lightly shade to show the intersection of the interior for the triangle and the interior for the circle. See Fig. 2

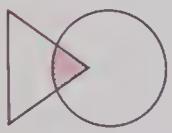
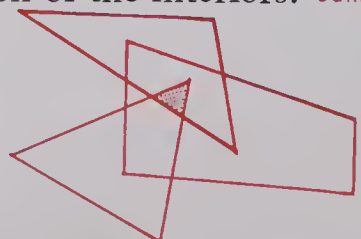


Fig. 2

7. Make a drawing of three simple closed curves that intersect. Shade to show the intersection of the interiors. Sample answer:



● **Extra Activity.** Set 162.

Pupil's Objectives

(a) To review the meaning of simple closed curve; (b) to learn that a simple closed curve partitions a plane into three distinct sets of points; and (c) to learn to identify the interior and exterior for a simple closed curve.

Background

A set of points whose picture can be drawn without lifting the pencil from the paper is called a *curve*. It is a path in a plane. A curve that does not cross itself is called a *simple curve*. A curve that has no distinct end points is called a *closed curve*. If a curve has no distinct end points and also does not cross itself, it is called a *simple closed curve*.

A simple closed curve partitions the remaining points of the plane into just two distinct sets of points, the set of points interior to the curve, and the set of points exterior to the curve. It is usually intuitively obvious which of the two sets of points lies in the inside or interior and which lies in the outside or exterior. A precise definition of interior and of exterior for a simple closed curve may be beyond the level of this course. It probably is adequate to say that the interior for a simple closed curve is the set of all points in the plane that are bounded by the curve.

Pre-Book Lesson

• Review meaning of *curve*, *closed curve*, *simple curve*, and *simple closed curve*. Have the pupils make drawings of each. You may wish to review pages 45 and 46.

• Draw models of some simple closed curves on the board and have pupils find the "outside" and "inside" for these figures.

Using the Text Page

• Ex. 2. Ask the pupils if each of the drawings in box B is a picture of a simple closed curve. Some pupils may hesitate to identify a curve consisting of only line segments as a curve. Emphasize that a line segment is a curve.

• Ex. 3. Any two points in the interior (or any two points in the exterior) for a simple closed curve may be joined by a curve that does not intersect the simple closed curve.

• Ex. 4–5. If a set of points contains a line that does not intersect the simple closed curve, then the set of points is in the exterior for the simple closed curve.

Individualizing Instruction

• *More capable pupils* may try to develop a definition of interior for a simple closed curve, and find some way of determining whether a given point is in the exterior for the curve. For example, "If there exists a ray (whose end point is the given point A) in the plane of the simple closed curve that does not intersect the given closed curve, then point A is in the exterior for the curve."

These pupils may label sets of points pictured on models of geometric figures such that their union or intersection causes them to lie in the exterior or interior for a curve. In Fig. 1 below, $\overline{AB} \cup \overline{BC}$ causes the end point A to lie in the interior for the curve, while the end point C lies in the exterior for the curve.

• *All pupils* may be given a drawing such as Fig. 2 and be asked to identify as many simple closed curves as possible, and for each such curve, to identify a point in each of the following sets:

(a) the exterior for the simple closed curve.

(b) the interior for the simple closed curve.

(c) the simple closed curve.

• The *slower learners* may be given a drawing such as Fig. 3 and be asked to identify a point that meets the following conditions:

(a) is in the interior for curve l and in the exterior for curve m.

(b) is in the interior for curve m and in the interior for curve n.

(c) is in the interior for curve l, in the interior for curve m, and in the interior for curve n.

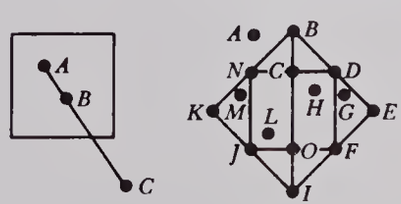


Fig. 1

Fig. 2

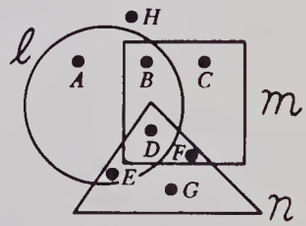


Fig. 3

Pupil's Objectives

(a) To review the meaning of interior and exterior for circles and angles; and (b) to learn to determine whether a given point is in the interior or in the exterior for a given circle or a given angle.

Background

A circle is a simple closed curve, so it has an interior and an exterior. All points, except the end points, of every chord of the circle are in the interior for the circle. The end points of each chord of the circle are a part of the circle so they are not in the interior for the circle.

The interior for a circle may be thought of as the set of all points in the plane of the circle whose distance from the center for the circle is shorter than the radius of the circle.

An angle, just like a simple closed curve, partitions a plane into three distinct sets of points:

(a) the set of all points forming the angle.

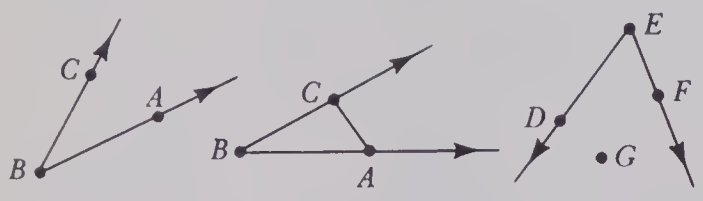
(b) the set of all points in the interior for the angle.

(c) the set of all points in the exterior for the angle.

To the pupils, it probably is obvious which of the three sets of points form the interior for an angle. However, the following are two descriptions of interior for an angle:

(1) The interior for $\angle ABC$ is the set of points which contains the line segment whose end points are point A and point C (neither of which is the vertex).

(2) The point G is in the interior for $\angle DEF$ if it is on the same side of $\overleftrightarrow{EF}$ as point D and on the same side of $\overleftrightarrow{ED}$ as point F.



Pre-Book Lesson

• Review the concepts of circle and angle using material from Chapter 1 where it is appropriate.

• Ask the pupils if any geometric figure other than a simple closed curve partitions the plane into three distinct sets of points. This can then lead to a discussion as to whether an angle has an interior and an exterior.

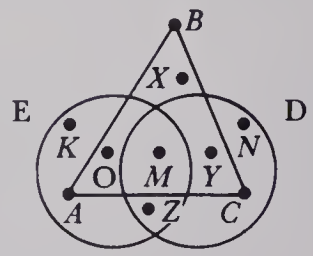
Using the Text Page

• Ex. 1, 2, 7, and 8. The pupils should first have clearly in mind the definition of a circle as the set of all points in a plane a given distance from a fixed point called the center. The distance from the center to each point in the interior for the circle will therefore be less than the length of the radius of the circle. With this in mind, they should have little difficulty understanding that all points in a chord, except its end points, will be in the interior for the circle. The one point in the radius which is not in the interior for the circle will be the end point that is a part of the circle.

• Ex. 3. The interior for a simple closed curve is bounded by the curve itself. However, the interior for an angle is not bounded entirely by the angle itself since the sides of the angle are rays.

Individualizing Instruction

All pupils may be given the drawing below



and be asked to identify a point that is in the

(a) interior for $\triangle ABC$ and interior for circle D.

(b) exterior for $\triangle ABC$ and interior for circle E.

(c) interior for the intersection of the following simple closed curves: $E \cap B \cap D$; $E \cap B$; $D \cap B$; $E \cap D$.

(d) exterior for curves E and D and the interior for curve B.

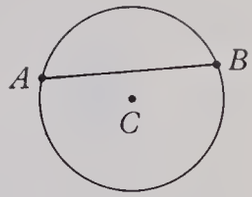
(e) exterior for the intersection of the following simple closed curves: $E \cap B \cap D$; $E \cap B$; $D \cap B$; $E \cap D$, and so on.

* The center for a circle does not belong to the curve. It is in the set of points in the interior for the curve.

Interiors for Circles and Angles

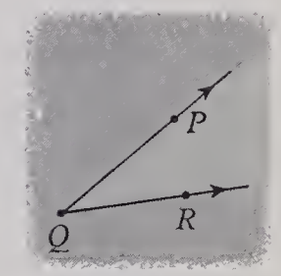
Resurvey [O]

1. Fig. 1. Points A and B belong to circle C. Line segment AB is called a _?_ (chord) for circle C. Do all points of chord AB except its end points lie in the interior for circle C? Yes

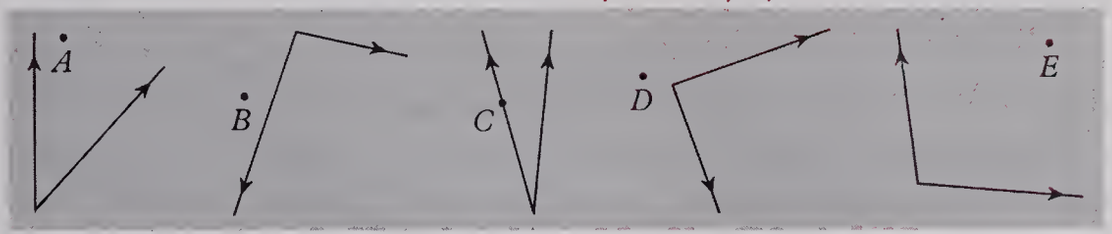


* 2. Does the center for a circle always lie in the interior for the circle? Yes

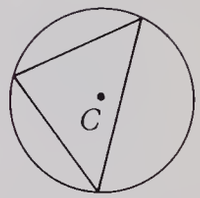
3. Since rays go on and on, we can think of an angle as partitioning a plane so that there are three distinct sets of points: the angle itself, the interior for the angle, and the exterior for the angle. For angle PQR shown at the right, the interior is shown with the color gray. What color shows the exterior for the angle? Blue



4. For each angle shown in the box below, tell whether the labeled point belongs to the angle, is in the interior for the angle, or is in the exterior for the angle. A, interior; B, exterior; C, belongs; D, exterior; E, interior

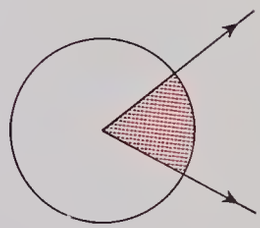


5. At the right, a triangle is shown whose vertices belong to circle C. Is the interior for the triangle a subset of the interior for the circle? Yes



[W]

6. Make a large copy of the figure at the right and then lightly shade to show the intersection of the interior for the angle and the interior for the circle.



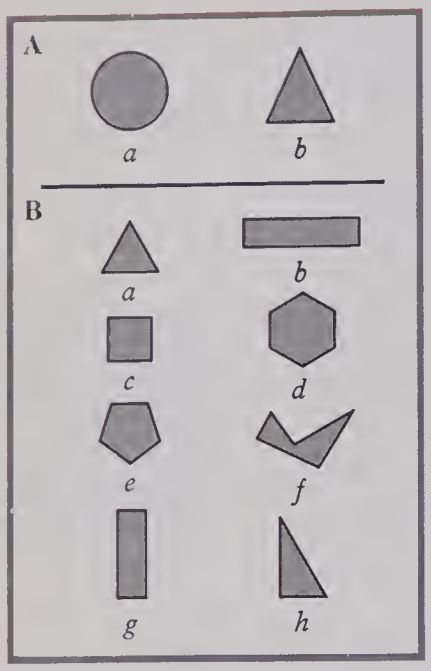
7. How many points in a radius for a circle do not belong to the interior for the circle? 1

8. How many points in a diameter for a circle do not belong to the interior for the circle? 2

* Polygonal is pronounced pəlig´ə · nəl (Barnhart).
** Quadrangular is pronounced Kwod · rang´gū lər (Barnhart).

Regions

Resurvey and extension [O]



1. The union of a simple closed curve and its interior is called a *region.* Such a region is often named according to the shape of the simple closed curve. For example, region *a* shown in box A is a *circular region.* Region *b* is a triangular region.

* 2. Of the *polygonal* regions shown in box B, which appear to be

a. triangular regions? a, h

b. rectangular regions? b, c, g

c. square regions? c

d. pentagonal regions? e, f

e. hexagonal regions? d

** 3. A region which is the union of a quadrilateral and its interior is called a *quadrangular region.* On the board, show and lightly shade a quadrangular region which is not a rectangular region. Sample answer:

4. Name some models of polygonal regions in your classroom. Whenever possible, give a more specific name for the region such as triangular region, rectangular region, and so on. Answers will vary.

5. A STOP sign such as the one shown at the left has the shape of an octagonal region.

[W]

6. On your paper, draw and lightly shade a model of

a. a circular region.

b. a square region.

c. a region that is neither a circular region nor a polygonal region. Sample answer:

7. Are all rectangular regions quadrangular regions? Yes

8. Are all quadrangular regions rectangular regions? No

● **Extra Activity.** Set 163.

Pupil's Objectives

(a) To review the meaning of region; and (b) to learn to name regions according to the geometric figure by which they are bounded.

Background

The union of a simple closed curve and its interior is called a *region*. A region is usually named according to the name of the geometric figure bounding it.

If the region is bounded by a	*then the region is called*
circle	circular
polygon	polygonal
triangle	triangular
quadrilateral	quadrangular
trapezoid	trapezoidal
rectangle	rectangular
square	square
pentagon	pentagonal
hexagon	hexagonal
octagon	octagonal

Before studying this topic of regions, the pupils may have called a triangular shaped piece of paper a triangle. Such a piece of paper is not a model for a triangle because a triangle, being the union of three segments, has no dimensions other than length. The triangular shaped piece of paper is a model of a *triangular region*.

This difference between a polygon and a polygonal region and the difference between a circle and a circular region has application in finding areas. For example, if a pupil were asked to find the area of a square each of whose sides is 3 inches in length, the correct answer would be zero square inches. A *square* has no area. However, a *square region* has area.

Pupils should understand that most people use the term *circle* when they really mean *circular region*, and the term *square* when they mean *square region*.

Pre-Book Lesson

- Review naming of polygons according to the number of sides from triangle through octagon.
- Review the ideas of union of sets of points in geometric figures and the concepts of interior and exterior for a curve.

Using the Text Page

- Ex. 1. Differentiate between the mathematical definition of *circle* and the common use of the word *circle* such as when a person is asked to cut a circle from a piece of paper.
- Ex. 4. Compare the correct name for the polygonal regions with the terms the pupils may have been using in the past to name these models.

Individualizing Instruction

- *More capable pupils* may be given a drawing such as Fig. 1 below.

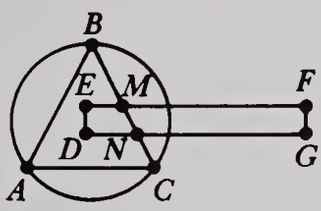


Fig. 1

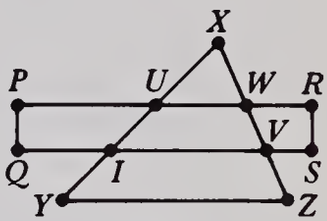


Fig. 2

Ask pupils to shade to show the union or intersection of various regions. For example, region *ABC* $\cap$ region *DEFG*.

- *All pupils* may be given a drawing such as Fig. 2 and asked to identify as many different regions as possible and to name each region according to the polygon bounding the region.

Teaching Pages 91, 92, and 93

Pupil's Objectives

To take the regular end-of-chapter tests including (a) Test of Information and Meaning 2; (b) Diagnostic Test 2; (c) Problem Test 2; and (d) Computation Test 2.

Background

These tests are similar in nature to the end-of-chapter tests given for the previous chapter. They are designed not only to measure the pupil's ability, but also to diagnose weaknesses which should be corrected before proceeding with the work in the next chapter. Extra Practice Sets and individual instruction should be used carefully to these ends.

Pre-Book Lesson

Explain to pupils that the understandings and skills being tested are not confined to those in Chapter 2, but include any concepts presented this far in the course. In this respect, the tests are cumulative, and the pupils are always responsible during testing for any concepts previously presented.

Using the Text Pages

- The tests should be spaced so as to minimize fatigue. They may be given in any order. No more than two tests should be given at one time.
- Before each test, have pupils scan the tests to be sure they understand the directions. Help those pupils who have reading and vocabulary difficulties to be certain they understand the meaning of each question, particularly the problems in Problem Test 2. Make sure they check their answers whenever possible and follow a step-by-step procedure on the problem-solving test.
- An item analysis of errors will help in planning the remedial work for individuals and for groups of pupils who had errors in common. For pupils making errors in Diagnostic Test 2, explain the use of the practice sets listed in the text.

Individualizing Instruction

- The purpose of the testing program at the end of each chapter is not merely to award grades, but to measure the pupil's progress in skills and understandings, and to evaluate your teaching. You should be constantly alert to strengths and weaknesses of individuals and of the class as a whole, helping each pupil overcome his weaknesses and trying to determine new ways of improving your teaching techniques.
- After the tests have been completed and the papers returned, all pupils should explain the underlying reasoning for their responses and correct their mistakes. They should then work appropriate Extra Examples Sets or Reteaching Sets.
- *More capable pupils* may work Supplementary Activities at the end of the chapter if they complete the work ahead of time.
- You may use the tables below to find the per cent scores for the two tests. The scores may be entered on individual record cards.

Table of Per Cents for Chapter 2 Scores

Problem Test		Computation Test					
Score	Per Cent	Score	Per Cent	Score	Per Cent	Score	Per Cent
1	14	1	4	10	39	19	73
2	29	2	8	11	42	20	77
3	43	3	12	12	46	21	81
4	57	4	15	13	50	22	85
5	71	5	19	14	54	23	88
6	86	6	23	15	58	24	92
7	100	7	27	16	62	25	96
		8	31	17	65	26	100
		9	35	18	69		

Do You Understand?

Test of Information and Meaning 2

1. What operation is used to find the product when the factors are known? Multiplication

2. What operation is used to find the unknown factor when the product and one of the two factors are known? Division

Ex. 3–5. Tell whether the letter represents the product or a factor and then solve.

3. $n \div 13 = 1{,}326$ Product; 17,238

4. $56 = x \times 8$ Factor; 7

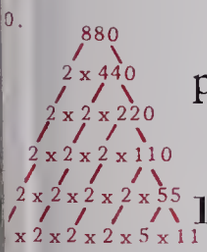
5. $207 \div y = 9$ Factor; 23

6. Using 17 and 21 for the factors and n to represent the product, write four mathematical sentences which illustrate the factors-product relationship. $n = 17 \times 21$; $n = 21 \times 17$; $n \div 21 = 17$; $n \div 17 = 21$

7. List within braces the factors of **a.** 18. $\{1, 2, 3, 6, 9, 18\}$ **b.** 52. $\{1, 2, 4, 13, 26, 52\}$

8. List within braces the prime factors of **a.** 120. $\{2, 3, 5\}$ **b.** 210. $\{2, 3, 5, 7\}$

9. Express 200 as a product of primes. $2 \times 2 \times 2 \times 5 \times 5$



10. Make a factor tree for 880 and then express 880 as a product of primes using exponent forms. $2^4 \times 5 \times 11$

Find the standard numeral for

11. $3 \times 3 \times 5$. 45 12. $2 \times 2 \times 2 \times 7 \times 7$. 392 13. $2^4 \times 3^2 \times 5^2$. 3,600

14. Which set is a subset of every set? $\{\}$, the empty set.

15. There are how many subsets of $\{10, 20, 30\}$? 8

16. Fig. 1. Name three segments which are subsets of $\overrightarrow{RS}$. $\overline{RS}$, $\overline{ST}$, $\overline{RT}$

17. Make large copies of Fig. 2–4 and lightly shade to show the interiors for the curves.

18. On your paper, show two segments whose intersection is Sample answers: **a.** $\{\}$. **b.** a set containing only one point. **c.** a segment.

19. Write a mathematical sentence which illustrates the Distributive Property. Sample answer: $2 \times (3 + 4) = (2 \times 3) + (2 \times 4)$

20. How many points are there in the intersection of two sides of a triangle? 1

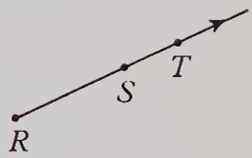


Fig. 1

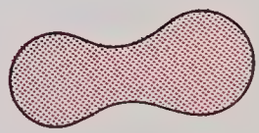


Fig. 2

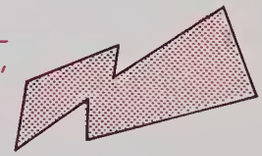


Fig. 3

Fig. 4

Do You Make Mistakes?

Diagnostic Test 2

	a	b	c	d	Study Pages	Practice: Use Sets
1. Multiply	37 × 8 = 296	507 × 9 = 4,563	83 × 24 = 1,992	58 × 75 = 4,350	61	37–38
2. Multiply	237 × 62 = 14,694	1,062 × 55 = 58,410	738 × 516 = 380,808	1,421 × 149 = 211,729	62	39–40
3.	23, R0	951, R5	24, R0	20, R19	71–73	44–45
	7)161	9)8,564	18)432	83)1,679		
4.	8, R0	34, R33	36, R0	28, R95	74–75	46–47
	117)936	101)3,467	250)9,000	304)8,607		

Can You Solve Problems?

Problem Test 2

Let n represent the number which will answer the question in the problem. Write a mathematical sentence for the problem and then solve to find the answer for the problem.

1. Mr. Cooper drove 2,295 miles in 17 days. That was a mean average of how many miles per day? $n = 2{,}295 \div 17$; 135

2. At a mean average of 725 miles per hour, how far can a transcontinental jet aircraft travel in 3 hours? $n = 3 \times 725$; 2,175 miles

3. 467 times 38 is how much greater than 1,285? $n = (467 \times 38) - 1{,}285$; 16,461

4. The sum of 379 and 455 is how much less than the product of 528 and 477? $n = (528 \times 477) - (379 + 455)$; 251,022

5. During a recent sale, the Camera Shop sold 19 cameras at a special sale price of $37.85 each. That was how much money? $n = 19 \times 37.85$; $719.15

Supplementary Activities

The suggested activities that follow may be used for review or enrichment at any time throughout the year, including assignments to pupils who finish end-of-chapter tests early. They may be used with *all pupils* at the discretion of the teacher.

• In 1845 the French mathematician Bertrand postulated that between any counting number other than 1, and its double, there is at least one prime. (Actually, there is a prime between any counting number n and $(2n - 2)$ when n is greater than 3. However, the simpler presentation is sufficient for this text.) Have *all pupils* test this postulate for the set of primes $\{2, 3, 5, \cdots, 47\}$. (The postulate checks.)

• Does the formula $n^2 - n + 41$ yield a prime number no matter what counting number is substituted for n? (No) Test it with any whole number less than 41. (Yes) Test it with the number 41. (No. The result is $41^2 - 41 + 41 = 41^2$ which is not prime.)

• Twin primes are a pair of primes whose difference is 2. The pair 3 and 5 is an example of twin primes. Name other twin primes (5,7; 11,13; 17,19; 29,31).

• Given each of the three equations

$@ \times \# = *$ $\quad ! = @ + \#$ $\quad * \times ! = \blacksquare$

as being true, (sample values: $@ = 2, \# = 3, * = 6, ! = 5; \blacksquare = 30$) fill in the blank in each of the following to form a true mathematical sentence:

1. $* \div \# = \underline{(@)}$
2. $! - @ = \underline{(\#)}$
3. $* \div \underline{(\#)} = @$
4. $\underline{(\blacksquare)} \div ! = *$
5. $\underline{(*)} \div @ = \#$
6. $\blacksquare \div * = \underline{(!)}$
7. $! - \# = \underline{(@)}$
8. $\blacksquare \div \underline{(!)} = *$

• Consider the set $\{1, 3, 5, 7, \ldots\}$. The first element in the set is the second power of what number? (1). The sum of the first two elements is the second power of what number? (2). The sum of the first three elements is the second power of what number? (3). What is the pattern of these sums? (The sum of the first n elements is n^2.) Test this pattern for the first four elements; five elements; six elements.

• Below is the addition table and the multiplication table for a system of numeration whose only single-digit numerals are c, f and i.

+	c	f	i
c	c	f	i
f	f	i	c
i	i	c	f

×	c	f	i
c	c	c	c
f	c	f	i
i	c	i	f

In this system of numeration,

(a) is addition commutative? (yes)

(b) is addition associative? (yes)

(c) is there an identity element for addition? (c)

(d) is multiplication commutative? (yes)

(e) is multiplication associative? (yes)

(f) is there an identity element for multiplication? (f)

(g) is multiplication distributive over addition? (yes)

• By substituting whole numbers for the letters in each of the following, determine which of the statements are always true and which are not always true:

1. $a \times (b - c) = (a \times b) - (a \times c)$ (true)
2. $(b - c) \times a = (a \times b) - (a \times c)$ (true)
3. $a \div (b + c) = (a \div b) + (a \div c)$

[This is not always true because $36 \div (9 + 4) \neq (36 \div 9) + (36 \div 4)$.]

4. $(b + c) \div a = (b \div a) + (c \div a)$ (true)
5. $(b - c) \div a = (b \div a) - (c \div a)$ (true)

• Complete the addition table and the multiplication table in the base-five system of numeration (answers in parentheses):

+	0	1	2	3	4
0	(0)	(1)	(2)	(3)	(4)
1	(1)	(2)	(3)	(4)	(10)
2	(2)	(3)	(4)	(10)	(11)
3	(3)	(4)	(10)	(11)	(12)
4	(4)	(10)	(11)	(12)	(13)

×	0	1	2	3	4
0	(0)	(0)	(0)	(0)	(0)
1	(0)	(1)	(2)	(3)	(4)
2	(0)	(2)	(4)	(11)	(13)
3	(0)	(3)	(11)	(14)	(22)
4	(0)	(4)	(13)	(22)	(31)

Teacher's Page 92

Fill in each blank with the correct base-five numeral by using the tables on Teacher's Page 92.

1. $3_{five} + 3_{five} + 3_{five} = \_?\_$ (14_{five})
2. $31_{five} \div 4_{five} = \_?\_$ (4_{five})
3. $22_{five} \div \_?\_ = 4_{five}$ (3_{five})
4. $11_{five} \div \_?\_ = 2_{five}$ (3_{five})
5. $\_?\_ \div 2_{five} = 4_{five}$ (13_{five})
6. $13_{five} - 4_{five} = \_?\_$ (4_{five})
7. $\_?\_ - 4_{five} = 3_{five}$ (12_{five})
8. $12_{five} - 3_{five} = \_?\_$ (4_{five})

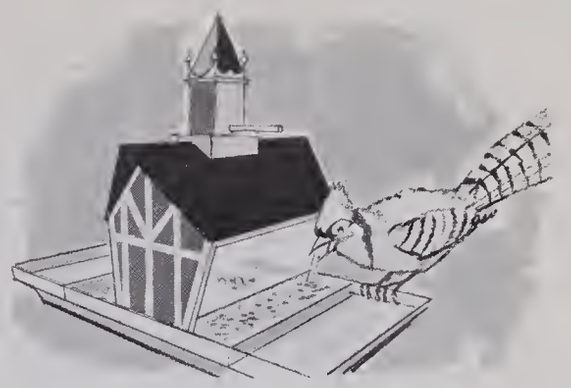
• On page 63, the pupils were introduced to the concept that every counting number is either a power of 2 or a sum of powers of 2 in which no addend is repeated. This concept can be applied to the development of a very interesting game.

Prepare five cards as shown below. Card "A" shows all the whole numbers less than 64 whose base-two numeral has a "1" in the first place whose place value is 1. Card "B" shows all the whole numbers less than 64 whose base-two numeral has a "1" in the second place whose place value is 2. Card "C" shows all the whole numbers less than 64 whose base-two numeral has a "1" in the third place whose place value is 4. Card "D" shows all the whole numbers less than 64 whose base-two numeral has a "1" in the fourth place whose place value is 8. The numbers shown on cards "E" and "F" follow this same pattern.

Have a pupil select, without telling you, any counting number less than 64, and tell you on which cards this number is shown. By telling you on which cards the number is shown, he is actually telling you the base-two numeral for the number. You can then mentally rename the number with a base-ten numeral and tell him what the number is. For example, if he said, "Card 'A,' card 'C,' and card 'E,' " you would know that the number he is thinking of is named by the numeral 10101_{two}. 10101_{two} names the number 21. Notice that 21 is the only number that is named on each of cards "A," "B," and "C" and only on those cards. Pupils can then be taught the game and practice it on each other.

Card "A"	*Card "B"*	*Card "C"*	*Card "D"*	*Card "E"*	*Card "F"*
1, 3, 5	2, 3, 6	4, 5, 6	8, 9, 10	16, 17, 18	32, 33, 34
7, 9	7, 10	7, 12	11, 12	19, 20, 21	35, 36, 37
11, 13	11, 14	13, 14	13, 14	22, 23, 24	38, 39, 40
15, 17	15, 18	15, 20	15, 24	25, 26, 27	41, 42, 43
19, 21	19, 22	21, 22	25, 26	28, 29, 30	44, 45, 46
23, 25	23, 26	23, 28	27, 28	31	47, 48, 49
27, 29	27, 30	29, 30	29, 30	48, 49, 50	50, 51, 52
31	31	31	31	51, 52, 53	53, 54, 55
33	34, 35	36, 37, 38, 39	40, 41	54, 55, 56	56, 57, 58
35	38, 39	44, 45, 46, 47	42, 43	57, 58, 59	59, 60, 61
37	42, 43	52, 53, 54, 55	44, 45	60, 61, 62	62, 63
39	46, 47	60, 61, 62, 63	46, 47	63	
41	50, 51		56, 57		
43	54, 55		58, 59		
45	58, 59		60, 61		
47	62, 63		62, 63		
49					
51					
53					
55					
57					
59					
61					
63					

6. The Oakdale Youth Club maintains 18 bird feeders during the winter months. Each feeder cost $14.75. What was the total cost of the 18 feeders? n = 18 x 14.75; $265.50



7. How much is saved by buying four 10-lb. bags of bird seed at $2.35 per bag instead of eight 5-lb. bags at $1.39 each?
n = (8 x 1.39) − (4 x 2.35); $1.72

How Well Can You Compute?

Computation Test 2

Ex. 1–6. Copy and multiply.

1.	2.	3.	4.	5.	6.
437	607	$45.89	$750.35	7,899	925
× 66	× 121	× 23	× 12	× 738	× 874
28,842	73,447	$1,055.47	$9,004.20	5,829,462	808,450

Ex. 7–11. Copy and divide.

7. $23\overline{)4{,}763}$ 207, R2 8. $18\overline{)19{,}873}$ 1,104, R1 9. $6\overline{)\$41.04}$ $ 6.84, R0 10. $27\overline{)\$835.98}$ $ 30.96, R6¢ 11. $4\overline{)546{,}003}$ 136,500, R3

Find the mean, mode, and median for

12. 24, 29, 21, 23, 27, 22, 23, 25, 22, 23, 24, 23, and 26. 24, 23, 23

13. 1,065, 1,068, 1,062, 1,065, 1,067, and 1,069. 1,066, 1,065, 1,066

14. Twenty-four metal rods were measured and the mean length was found to be 27 inches. If the rods were placed end-to-end in a straight line, the over-all measurement would be

a. how many inches? 648 **b.** how many feet? 54

Ex. 15–26. Find the number represented by the letter.

15. $n = 345 + 608 + 200$ 1,153

16. $n = 378 - 344$ 34

17. $n - 4{,}876 = 3{,}777$ 8,653

18. $n \div 46 = 400$ 18,400

19. $x = 35 - (456 \div 76)$ 29

20. $x = 45 \times (234 + 500)$ 33,030

21. $y = 34 \times 26 \times 18$ 15,912

22. $x = 3^3 + 6^2$ 63

23. $n = 7 \times 7 \times 7 \times 8 \times 9$ 24,696

24. $n = 20 \times (7 + 40 + 68)$ 2,300

25. $x - 45 = 23 \times 67$ 1,586

26. $n = (154 + 297) \div 11$ 41

3

Rational Numbers

Resurvey; fractions [O]

1. In the picture above, how many boys are shown? 5 How many are wearing caps? 3 Is it true that *three fifths* of the boys shown are wearing caps? Yes We write, "$\frac{3}{5}$."

2. Numerals such as $\frac{3}{5}$ are called *fractions* and the numbers they name are called *rational numbers*. Is it true that two fifths of the boys shown are wearing glasses? Yes On the board, write a fraction for two fifths. $\frac{2}{5}$

A

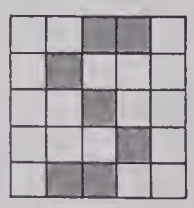
$\frac{3}{5}$ ← 3: numeral for the numerator; 5: numeral for the denominator

3. Box A. Which numeral names the numerator of the fraction? 3 Which numeral names the denominator? 5

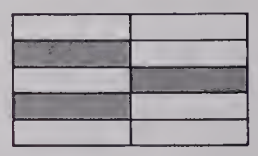
In a fraction, the symbol above the bar represents a number called the *numerator* and the symbol below the bar represents a number called the *denominator*.

4. On the board, write a fraction which tells what part of the diagram is colored for each of Ex. a–d.

a $\frac{2}{7}$

b $\frac{7}{25}$ 

c $\frac{3}{5}$

d $\frac{3}{10}$ 

Overview--Chapter 3

The major portion of Chapter 3 is devoted to resurveying and extending the ideas of addition and subtraction of rational numbers to include simplest and mixed forms. Using decimals to name rational numbers is restricted to reading and writing decimals, renaming from a fraction or a mixed form to a decimal, renaming from a decimal to a fraction or a mixed form, and rounding with decimals and fractions.

The presentations require a higher level of maturity than in previous grades and each of the extensions of the ideas will probably be challenging to all the pupils. New ideas presented include renaming rational numbers using the G.C.F.; number-line pictures for ordering rational numbers; the L.C.M. and L.C.D. for rational numbers; the addends-sum relationship and properties of addition as applied to rational numbers; vertical form for adding rational numbers and addition of 3 rational numbers; measuring models of segments and angles using standard and nonstandard units and a protractor; measuring the angles of a triangle and quadrilateral; regular polygons; and congruent angles and triangles.

• **Problem-Solving.** Pupils are given experience solving problems involving the addends-sum relationship in social situations using rational numbers expressed with fractions and mixed forms. On pages 118 and 119 are problems involving the addends-sum relationship requiring two operations. Also included are estimating and rounding numbers and problems involving measurements, household situations, and geometry.

• **Maintenance.** Sets of practice exercises are on pages 102, 108, 109, and 117. Sets of summary exercises are on pages 102 and 127. These exercises are not only designed to help maintain skills and ideas presented in Chapter 3, but to help maintain skills and ideas presented in earlier chapters and earlier grades. They cover the topics resurveyed and new ideas.

• **Enrichment.** The meaning of *relatively prime*, that is, when the G.C.F. of two numbers is 1, is presented on page 108. Other enrichment topics are found in each lesson plan, in the Extra Activities at the back of the book, and in Supplementary Activities at the end of the chapter.

• **Tests.** The summary exercises on pages 102 and 127, and the practice exercises on page 117 may be used as tests of material presented in the chapter. The practice exercises on pages 102, 108, and 109 may be used to test ideas and skills taught in earlier chapters. The entire set of exercises or just selected exercises may be used for testing purposes.

Topics included in the test are rounding and inequalities with fractions and decimals; mixed and simplest forms; congruent figures and the measurement of angles; addition and subtraction of rational numbers; the four operations; problems in social situations involving rational numbers and measurements.

Teaching Pages 94 and 95

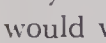
Pupil's Objective

To review the meaning of rational numbers.

Background

A rational number is one which may be expressed in the form $\frac{a}{b}$ where a represents a whole number, and b represents a counting number. Since division by zero is not defined, b cannot be zero. The rational numbers studied in this text are non-negative rational numbers and, like other numbers, are ideas. Several names may be used for expressing them. They may be represented by fractions or decimals. Thus, $\frac{3}{4}$, $\frac{16}{16}$, $\frac{103}{7}$, $\frac{0}{2}$, 3, and 0.021 all represent rational numbers. The numerals 3 and 0.021 name rational numbers because these same rational numbers can be shown in the form $\frac{3}{1}$ and $\frac{21}{1,000}$.

If a rational number is thought of as physically representing part of a whole, the number $\frac{2}{7}$ may be interpreted as representing 2 of the 7 equal parts of a whole. In the physical representation, the denominator (7) tells how many parts of the same size are in the whole, and the numerator (2) tells how many of these are being considered.

The ancient Egyptians had a system of representing rational numbers consisting almost entirely of unit fractions, which in our system would mean fractions whose numerator is 1. For their fractions, they would write the symbol and beneath it would write the numeral for the number of equal parts into which the whole had been subdivided.  was the Egyptian numeral for three so was their numeral for one third, or one of three equal parts of the whole. Similarly,

= one fifth

= one tenth

= one twelfth

The one exception to this was the symbol which represented the non-unit fraction, two thirds. One of the main limitations of this system was the difficulty of trying to write an expression for any non-unit fraction. An example of this is found in the famous Rhind-Papyrus written or transcribed by the Egyptian priest Ahmes in which the fraction $\frac{2}{43}$ is written as

which means $\frac{1}{42} + \frac{1}{86} + \frac{1}{129} + \frac{1}{301}$

The Hindus originated the vertical notation $\begin{matrix}3\\4\end{matrix}$ to represent "3 of the 4 equal parts into which the whole has been divided," and the Arabs later introduced the use of the bar to separate the numerator and the denominator: $\frac{3}{4}$.

Pre-Book Lesson

• To establish clearly the need for fractions, have the pupils do a few problems for which the answers are not whole numbers. Try to have pupils refer to a part of an object such as half a pie without using a fraction.

• By referring to $\frac{1}{4}$ of several objects, try to have pupils develop the concept that a rational number is an idea. Each time a fourth is referred to, the same idea comes to mind.

Using the Text Pages

• Ex. 1. Point out that a fraction is composed of two numerals, one which indicates into how many parts of the same size the whole has been divided. This is called the denominator. The other numeral, called the numerator, indicates how many of these equal parts are being considered. The fraction $\frac{3}{5}$ usually represents "3 of 5 equal parts." In this exercise, the "whole" is a set of boys subdivided into three subsets of the same size, each subset consisting of one boy.

• Ex. 2. Fractions are numerals that name rational numbers. Rational numbers may be positive, negative, or zero, but only the non-negative rationals are considered in this course. That is, zero and all rational numbers greater than zero.

• Ex. 6–9. For every rational number, there is an infinite set of fractions naming that number. For example, a set of fractions that names the number two fifths is $\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \cdots\}$.

• Ex. 10. Every whole number a can be expressed by a fraction in the form $\frac{a}{1}$, so every whole number is a rational number. The set of counting numbers is a subset of the set of whole numbers, so every counting number is a rational number.

Individualizing Instruction

• *More capable pupils* can be shown the fraction symbols used by the ancient Egyptians and the system used by the Romans, and then be asked to point out disadvantages of these systems compared to the Hindu-Arabic system.

• *All pupils* can be led in a discussion as to why $\frac{0}{3}$ is a meaningful numeral, but $\frac{3}{0}$ is meaningless. These pupils should understand how the difference between *numeral* and *number* applies to *fraction* and *rational number*.

• *Slower learners* may be given blocks of wood or pieces of cardboard of the same size to master the ideas of fractions on these pages.

* Do not confuse $\frac{0}{1}$ with $\frac{1}{0}$. $\frac{1}{0}$ does not name a number. Division by zero is not allowed.

5. Box B. Which fractions show the same numerator? d, e, f Which show the same denominator? d, e, f On the board, write standard numerals for the numerator and the denominator of each fraction. Which of the fractions in box B name the same number? c, d, e, f

B		
a.	$\frac{12-12}{3\times 3}$	$\frac{0}{9}$
b.	$\frac{4-1}{7-2}$	$\frac{3}{5}$
c.	$\frac{5+3}{11+1}$	$\frac{8}{12}$
d.	$\frac{2\div 1}{3}$	$\frac{2}{3}$
e.	$\frac{2}{3}$	$\frac{2}{3}$
f.	$\frac{8\div 4}{6\div 2}$	$\frac{2}{3}$

6. Rational numbers have many names. Explain how the diagram in Fig. 1 suggests that $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ all name the same rational number? The shading covers $\frac{1}{2}$, $\frac{2}{4}$, or $\frac{4}{8}$ of the diagram.

7. Study the diagram in Fig. 2. Do $\frac{2}{3}$ and $\frac{4}{6}$ both name the same rational number? Yes We write, "$\frac{2}{3} = \frac{4}{6}$" to indicate that these two fractions name the same number.

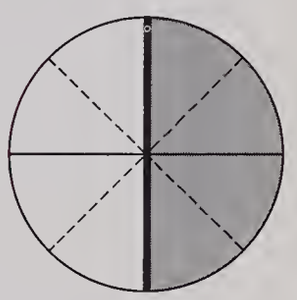


Fig. 1

8. On the board, draw a diagram which suggests that $\frac{1}{2}$ and $\frac{6}{12}$ both name the same rational number. See below. $\frac{1}{2} = \frac{6}{?}$ 12

* 9. The whole number 0 may be named with the fraction $\frac{0}{1}$. The whole number 1 may be named with the fraction $\frac{?}{1}$ 1. On the board, write fractions for the whole numbers from 2 through 9. $\frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}, \frac{6}{1}, \frac{7}{1}, \frac{8}{1}, \frac{9}{1}$

10. The whole numbers 0, 1, 2, and so on may be named with the fractions $\frac{0}{11}$, $\frac{11}{11}$, $\frac{4}{2}$ and so on. Such fractions name rational numbers, therefore whole numbers must be rational numbers.

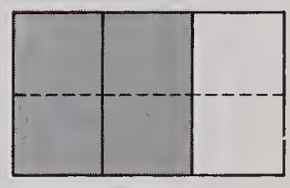


Fig. 2

11. Is each counting number a whole number? Yes Is the set of counting numbers a subset of the set of rational numbers? Yes

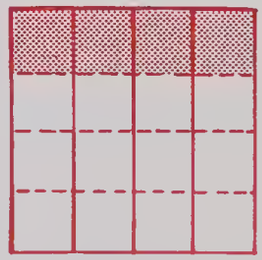
[W]

12. Which rational number is a whole number but is not a counting number? 0

13. Draw a diagram which suggests that $\frac{1}{3} = \frac{5}{15}$. Sample answer:

14. Draw a diagram which suggests that $\frac{1}{4} = \frac{4}{16}$. See right.

14. Sample answer.



Ex. 15–20. Copy and complete. Make a diagram if you need help.

15. $\frac{1}{2} = \frac{?}{8}$ 4

16. $\frac{?}{6} = \frac{2}{3}$ 4

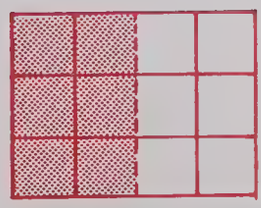
17. $3 = \frac{?}{1}$ 3

18. $\frac{1}{4} = \frac{2}{?} = \frac{?}{12}$ 8, 3

19. $\frac{1}{3} = \frac{?}{6} = \frac{4}{12}$ 2

20. $\frac{4}{12} = \frac{1}{?} = \frac{2}{?}$ 3, 6

8. Sample answer:



* It will be pointed out later that we are, in fact, multiplying by the identity element for multiplication, but pupils have not yet studied multiplying by a number named with a fraction.

Renaming Rational Numbers

Resurvey [O]

A	$\frac{2}{3} = \frac{2 \times 2}{2 \times 3} = \frac{4}{6}$
B	$\frac{5}{7} = \frac{4 \times 5}{4 \times 7} = \frac{20}{28}$

* 1. Box A. We can obtain the numerator and the denominator of $\frac{4}{6}$ by multiplying both the numerator and denominator of $\frac{2}{3}$ by $\_?\underline{2}$. Do $\frac{2}{3}$ and $\frac{4}{6}$ name the same rational number? Yes

If, for the fraction $\frac{a}{b}$, we multiply both the numerator and the denominator by a counting number, we obtain another fraction which also names the rational number named by $\frac{a}{b}$. Key idea: Both numerator and denominator are multiplied by 4.

2. Box B. Explain the work shown. Do $\frac{5}{7}$ and $\frac{20}{28}$ both name the same rational number? Yes

3. At the board, find another fraction for the number named by $\frac{6}{7}$ by multiplying both the numerator and the denominator of $\frac{6}{7}$ by 2; by 5; by 10. $\frac{12}{14}, \frac{30}{35}, \frac{60}{70}$

4. To find a fraction showing denominator 12 for the rational number $\frac{2}{3}$, we multiply both the numerator and the denominator of $\frac{2}{3}$ by $\_?\underline{4}$. $\frac{2}{3} = \frac{\overset{4}{?} \times 2}{4 \times 3} = \frac{\overset{8}{?}}{12}$

5. On the board, copy and complete the following:

a. $\frac{2}{5} = \frac{\overset{4}{?}}{10}$ **b.** $\frac{4}{5} = \frac{\overset{16}{?}}{20}$ **c.** $\frac{3}{8} = \frac{6}{\underset{16}{?}}$ **d.** $5 = \frac{5}{1} = \frac{\overset{15}{?}}{3}$

6. At the board, find four other fractions for the number named by $\frac{5}{8}$. Sample answers: $\frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \frac{25}{40}$

7. The fraction $\frac{0}{1}$ names the number $\_?\underline{0}$. If we multiply both the numerator and the denominator of $\frac{0}{1}$ by 2, we obtain $\frac{\overset{0}{?}}{2}$. Then is $\frac{0}{2}$ another name for the number 0? Yes

[W]

Ex. 8–14. Write two other fractions for the number.

Sample answers: $\frac{2}{12}, \frac{3}{18}$

8. $\frac{1}{2}$ $\frac{2}{4}, \frac{3}{6}$ **9.** $\frac{1}{6}$ **10.** $\frac{2}{3}$ $\frac{4}{6}, \frac{6}{9}$ **11.** $\frac{3}{5}$ $\frac{6}{10}, \frac{9}{15}$ **12.** $\frac{4}{9}$ $\frac{8}{18}, \frac{12}{27}$ **13.** $\frac{3}{11}$ $\frac{6}{22}, \frac{9}{33}$ **14.** $\frac{10}{11}$ $\frac{20}{22}, \frac{30}{33}$

Ex 15–18. Copy and complete.

15. $\frac{2}{5} = \frac{\overset{6}{?}}{15}$ **16.** $\frac{3}{4} = \frac{\overset{12}{?}}{16}$ **17.** $\frac{3}{5} = \frac{\overset{12}{?}}{20}$ **18.** $\frac{2}{7} = \frac{6}{\underset{21}{?}}$

◆ **Extra Examples.** Set 51.

Pupil's Objective

To review the basis for renaming rational numbers.

Background

For every fraction that names a rational number, there are countless other fractions that name the same rational number. The set of fractions that names the rational number one half, is $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\}$

Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ name the same rational number if and only if $(a \times d) = (b \times c)$. This is sometimes called the *cross-products test*. If the product of the numerator shown by the first fraction and the denominator named by the second fraction equals the product of the denominator named by the first fraction and the numerator shown by the second fraction, then the two fractions name the same rational number.

If the numerator and the denominator of $\frac{a}{b}$ are both multiplied by the same counting number c in the form $\frac{c}{c}$, so that c is the identity element for multiplication, we find the resulting fraction is $\frac{c \times a}{c \times b}$. Applying the cross-products test, we find that $\frac{a}{b} = \frac{c \times a}{c \times b}$ because $(a \times c \times b) = (b \times c \times a)$. This shows that if the numerator and the denominator are both multiplied by the same counting number in the form $\frac{c}{c}$, the resulting fraction names the same rational number as the given fraction.

Pre-Book Lesson

Review the concept of fraction and rational number as presented on the previous page. Make sure pupils understand that a rational number, just like any other number, has many names.*

Using the Text Page

• Ex. 1. Emphasize that the numerator and denominator shown by the fraction are being multiplied by the same *counting* number. If they were multiplied by zero, the result would be a number whose denominator is zero, and there is no such number in our numeration system. Make sure pupils understand that the same counting number is used to multiply the numerator and denominator.

• Ex. 5. For $\frac{2}{5} = \frac{?}{10}$, pupils should think, "for what number multiplied by 5 is the product 10?" This number should be multiplied by 2 to give the other numerator. Pupils may think of this as a factors-product relationship such as $5 \times n = 10$, and use this concept in solving other problems of this type. Since the denominator 5 is multiplied by 2, the numerator must be multiplied by 2.

• Ex. 6–7. Point out that the set of fractions that name the rational number five eighths and the set of fractions that name the rational number zero are each an infinite set.

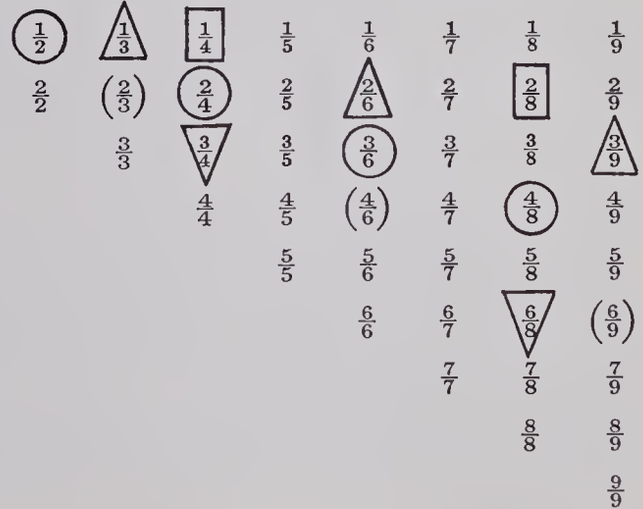
Individualizing Instruction

• *More capable pupils* can be asked to compare the ancient Egyptian, Roman, and Hindu-Arabic fraction notation to find if they have more than one fraction to name the same rational number.

• *All pupils* may be given sets of five fractions such as $\{\frac{3}{5}, \frac{6}{10}, \frac{12}{25}, \frac{21}{35}, \frac{30}{50}\}$ and be asked to determine which of the five fractions does not name the same rational number as the other four fractions.

• *Slower learners* may be given more work with geometric figures such as circles to demonstrate the relationship among different fractions that name the same rational number.

These pupils may also make a chart showing different names for the same rational number such as the one below.



* See items 1 and 6 on page xix. The number line and fractional parts could be used to illustrate the concept presented here.

Pupil's Objective

To review fractions in simplest form.

Background

On the previous page we stated that if $\frac{a}{b}$ is any fraction and c is any counting number in the form $\frac{c}{c}$, then $\frac{a}{b} = \frac{c \times a}{c \times b}$.

Reversing this mathematical sentence we have $\frac{(c \times a)}{(c \times b)} = \frac{a}{b}$. This can be explained not only in terms of the cross-products test but also in terms of "undoing" the multiplication by dividing the numerator and denominator by the counting number c. $\frac{(c \times a) \div c}{(c \times b) \div c} = \frac{a}{b}$. From this we can see that if the numerator and the denominator are both divided by the common factor c, the resulting fraction names the same number as the given fraction.

If the numerator and the denominator named by a fraction have no common factor greater than 1, then the fraction is said to be in simplest form. To find a fraction in simplest form, the numerator and denominator shown must be divided by the greatest common factor.

A fraction in simplest form may also be found by dividing the numerator and the denominator by any common factor and continuing to divide the resulting numerator and denominator by a common factor until the only common factor of the numerator and denominator is 1.

Pre-Book Lesson

Review the meaning of common factor and the meaning of greatest common factor. Have pupils practice finding several common factors of two numbers by using the idea of intersection of sets. Review using the completely factored form for finding the common factor.

Using the Text Page

• Ex. 2. When a common factor of the numerator and denominator is found, we again have the identity element in the form $\frac{c}{c}$. Dividing a number by 1 does not change its value.

• Ex. 3d. The numerator for the fraction is a greater number than the denominator. Point out that this is a fraction because it names a rational number. Some pupils may be hesitant to accept $\frac{9}{7}$ as a fraction. Any numeral in the form $\frac{a}{b}$, where a is a whole number and b is a counting number is a fraction. Also, $\frac{9}{7}$ is in simplest form. Some pupils may feel that it should be written in the mixed form $1\frac{2}{7}$ to be called simplest form. This is not true. The numerator and the denominator of the number $\frac{9}{7}$ have no common factor greater than 1 so it is therefore in simplest form.

• Ex. 5. Point out that if a numerator and denominator are both divided by any common factor, and the resulting numerator and denominator are both divided by any common factor, and if the procedure is continued, the fraction in simplest form will eventually result.

Individualizing Instruction

• *More capable pupils* may be asked to devise the most efficient method of finding a fraction in simplest form. This may lead to the idea of dividing the numerator and denominator by the greatest common factor.

• *All pupils* can be led to the understanding that if a number is to be divided by a second number and the resulting quotient divided by a third number, the result is the same regardless of which division is performed first. For example:

$$(36 \div 9) \div 4 = (36 \div 4) \div 9$$

They may also be led to the understanding that to divide by the counting number a and then divide the quotient by the counting number b will have the same result as dividing the given number by the number $(a \times b)$: $(36 \div 9) \div 2 = 36 \div 18$.

• Finding a fraction in simplest form may be easier for *slower learners* if they are told to first try the prime numbers 2, 3, 5, and 7 when trying to find common factors of the numerator and the denominator.

These pupils may show many names for the numbers in Ex. 7–18. They should start with the simplest form, then multiply by such numbers as $\frac{7}{7}$, $\frac{2}{2}$, $\frac{5}{5}$, and so on.

* Notice that even though the numerator is greater than the denominator, $\frac{7}{4}$ is in simplest form since the numerator and denominator have no common factor greater than 1.

Simplest Form

Resurvey [O]

1. Box A. Do the fractions shown all name the same rational number? Yes Give four other fractions for this number. Sample answers: $\frac{5}{10}, \frac{6}{12}, \frac{7}{14}, \frac{8}{16}$

A		
	$\frac{1}{2}$	$\frac{2}{4}$
	$\frac{3}{6}$	$\frac{4}{8}$

2. The *greatest common factor* of the numerator and the denominator of $\frac{4}{8}$ is _?4_; of $\frac{3}{6}$ is _?3_; of $\frac{2}{4}$ is _?2_; of $\frac{1}{2}$ is _?1_.

* 3. Each of the fractions $\frac{1}{2}$, $\frac{3}{5}$, and $\frac{7}{4}$ show a numerator and a denominator which have no common factor greater than 1. Such fractions are said to be in *simplest form*. Which of the following fractions are in simplest form?

a. $\frac{3}{4}$ **b.** $\frac{4}{6}$ **c.** $\frac{5}{10}$ **d.** $\frac{9}{7}$ **e.** $\frac{10}{12}$ **f.** $\frac{15}{20}$

4. Do the numerator and the denominator of $\frac{6}{10}$ have a common factor greater than 1? Yes Is the fraction $\frac{6}{10}$ in simplest form? No On the board, write a fraction in simplest form for the number named by $\frac{6}{10}$. $\frac{3}{5}$

5. Box B. Is 3 a factor of both the numerator and the denominator of $\frac{12}{18}$? Yes To find a fraction in simpler form for the number named by $\frac{12}{18}$, we may divide both the numerator and the denominator of $\frac{12}{18}$ by 3. $12 \div 3 =$ _?4_ and $18 \div 3 =$ _?6_. Is the fraction $\frac{4}{6}$ in simplest form? No Why not? They have a common factor, 2, which is greater than 1. Explain the other work in box B. Both the numerator and denominator of $\frac{4}{6}$ are divided by 2. Give a fraction in simplest form for the number named by $\frac{12}{18}$. $\frac{2}{3}$

B

$$\frac{12}{18} = \frac{12 \div 3}{18 \div 3} = \frac{4}{6}$$

$$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

[W]

6. Find a fraction in simplest form for the number named by $\frac{18}{30}$ by dividing first by 3 and then by 2 as in box B. $\frac{18}{30} = \frac{18 \div 3}{30 \div 3} = \frac{6}{10}$ $\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$

Ex. 7–18. Find a fraction in simplest form for the number.

7. $\frac{5}{10}$ $\frac{1}{2}$ 8. $\frac{6}{18}$ $\frac{1}{3}$ 9. $\frac{22}{33}$ $\frac{2}{3}$ 10. $\frac{30}{45}$ $\frac{2}{3}$ 11. $\frac{15}{20}$ $\frac{3}{4}$ 12. $\frac{9}{12}$ $\frac{3}{4}$

13. $\frac{14}{6}$ $\frac{7}{3}$ 14. $\frac{25}{60}$ $\frac{5}{12}$ 15. $\frac{150}{300}$ $\frac{1}{2}$ 16. $\frac{14}{16}$ $\frac{7}{8}$ 17. $\frac{20}{25}$ $\frac{4}{5}$ 18. $\frac{13}{39}$ $\frac{1}{3}$

Ex. 19–21. Write *T* or *F*.

19. $\frac{10}{12} = \frac{5}{6}$ T 20. $\frac{1}{2} = \frac{24}{48}$ T 21. $\frac{18}{27} = \frac{3}{4}$ F

Using the Greatest Common Factor

Writing fractions in simplest form [O]

A
$$\frac{12}{18} = \frac{12 \div 6}{18 \div 6} = \frac{2}{3}$$

B
$A = \{1, 2, 3, 4, 6, 12\}$
$B = \{1, 2, 3, 6, 9, 18\}$
$A \cap B = \{1, 2, 3, 6\}$

1. An easy way to find a fraction in simplest form for the number named by $\frac{12}{18}$ is: Divide both the numerator and denominator of $\frac{12}{18}$ by their greatest common factor. Explain the work in box A. The numerator and denominator of $\frac{12}{18}$ are divided by 6, their greatest common factor.

2. Box B shows how to use sets to find the greatest common factor of 12 and 18.

a. Is set A the set of all factors of 12? Yes ∧ Set B is the set of all factors of _?18.

b. $A \cap B$ is the intersection of sets _?A and _?B. Is it the set of all common factors of 12 and 18? Yes

c. The greatest number in $A \cap B$ is _?6. Is this the greatest common factor of 12 and 18? Yes

3. At the board, find the greatest common factor of 24 and 30 by using sets as in box B. Then find a fraction in simplest form for the number named by $\frac{24}{30}$. A={1, 2, 3, 4, 6, 8, 12, 24} B={1, 2, 3, 5, 6, 10, 15, 30}

A∩B={1, 2, 3, 6} $\frac{24}{30}$ $\frac{24 \div 6}{30 \div 6}$ $\frac{4}{5}$ [W]

Ex. 4–11. List the two sets of factors for the numbers and then list the intersection of the two sets. Then name the greatest common factor of the two numbers. See left.

4. 15, 20
5. 9, 24
6. 30, 45
7. 60, 100
8. 18, 24
9. 20, 40
10. 8, 12
11. 5, 8

Ex. 12–26. Find a fraction in simplest form for the number.

12. $\frac{22}{48}$ $\frac{11}{24}$
13. $\frac{15}{90}$ $\frac{1}{6}$
14. $\frac{5}{15}$ $\frac{1}{3}$
15. $\frac{20}{32}$ $\frac{5}{8}$
16. $\frac{26}{52}$ $\frac{1}{2}$
17. $\frac{22}{33}$ $\frac{2}{3}$
18. $\frac{14}{10}$ $\frac{7}{5}$
19. $\frac{4}{48}$ $\frac{1}{12}$
20. $\frac{30}{45}$ $\frac{2}{3}$
21. $\frac{60}{45}$ $\frac{4}{3}$
22. $\frac{68}{88}$ $\frac{17}{22}$
23. $\frac{8}{18}$ $\frac{4}{9}$
24. $\frac{27}{33}$ $\frac{9}{11}$
25. $\frac{100}{30}$ $\frac{10}{3}$
26. $\frac{14}{21}$ $\frac{2}{3}$

27. If both the numerator and the denominator shown in a fraction are even numbers, is the fraction in simplest form? No

♦ **Extra Examples.** Set 52.

4. {1, 3, 5, 15}; {1, 2, 4, 5, 10, 20}; {1, 5}; 5
5. {1, 3, 9}; {1, 2, 3, 4, 6, 8, 12, 24}; {1, 3}; 3
6. {1, 2, 3, 5, 6, 10, 15, 30}; {1, 3, 5, 9, 15, 45}; {1, 3, 5, 15}; 15
7. {1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60}; {1, 2, 4, 5, 10, 20, 25, 50, 100}; {1, 2, 4, 5, 10, 20}; 20
8. {1, 2, 3, 6, 9, 18}; {1, 2, 3, 4, 6, 8, 12, 24}; {1, 2, 3, 6}; 6
9. {1, 2, 4, 5, 10, 20}; {1, 2, 4, 5, 8, 10, 20, 40}; {1, 2, 4, 5, 10, 20}; 20
10. {1, 2, 4, 8}; {1, 2, 3, 4, 6, 12}; {1, 2, 4}; 4
11. {1, 5}; {1, 2, 4, 8}; {1}; 1

Teaching Page 98

Pupil's Objective

To learn to find a fraction in simplest form for a rational number by dividing the numerator and denominator by their greatest common factor.

Background

A fraction in simplest form is found by dividing the numerator and denominator by their greatest common factor. One way of determining their greatest common factor is to show the set of all the factors of the numerator and the set of all the factors of the denominator. The intersection of these two sets will be the set of all factors common to both the numerator and denominator. The greatest factor in this set will be the greatest common factor.

This method is satisfactory when the number of factors is not great. To find a fraction in simplest form for the number $\frac{24}{36}$, (set A is the set of factors for 24, set B is the set of factors for 36)

$$A = \{1, 2, 3, 4, 6, 8, 12, 24\}$$
$$B = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$$
$$A \cap B = \{1, 2, 3, 4, 6, 12\}.$$

The greatest common factor is 12. Therefore,

$$\frac{24}{36} = \frac{24 \div 12}{36 \div 12} = \frac{2}{3}.$$

There is another method of finding the greatest common factor which is more suitable for use when the number of factors of the numerator and denominator is quite great. This method is based on the concept that the greatest common factor is always equal to the product of the prime factors common in the prime factorization of the numerator and the denominator. For example, to find the greatest common factor of 140 and 1,155:

$$140 = 2 \times 2 \times (5 \times 7)$$
$$1{,}155 = 3 \times (5 \times 7) \times 11.$$

5×7 is common to both factorizations so 35 is the greatest common factor of the two numbers.

Pre-Book Lesson

Review factoring a number. Review the meaning of intersection of two sets. Have pupils find the G.C.F. of two numbers by using the idea of intersection of sets, and also by finding the product of the prime factors common to the sets. Pupils may use factor trees to find the prime factors.

Using the Text Page

• Ex. 1–3. These exercises demonstrate that "sets" is not an isolated topic in mathematics, but is a unifying concept that has many applications.

Whenever practical, have the pupils use correct mathematical language when they describe the operations they are performing. When determining a fraction in simplest form for the number $\frac{12}{18}$, the pupils should not say "6 goes into 12 twice and 6 goes into 18 three times." *Goes into* is not a mathematical operation. The correct way of describing the operation is "12 divided by 6 is 2, and 18 divided by 6 is 3."

• Ex. 27. If both the numerator and denominator of a fraction are even numbers, they have the common factor 2. Therefore, the fraction is not in simplest form.

Individualizing Instruction

• *More capable pupils* may be taught the method of determining the greatest common factor of two numbers by finding the product of the factors common to the prime factorizations of the two numbers. To find the greatest common factor of 36 and 48:

$$36 = (2 \times 2 \times 3) \times 3$$
$$48 = 2 \times 2 \times (2 \times 2 \times 3).$$

The greatest common factor is $2 \times 2 \times 3$ or 12.

• *All pupils* may be given three numbers and be asked to find the greatest common factor. These pupils may also work Ex. 4–11 using factor trees.

• For the *slower learners*, practice in factoring a number may be made into a game by having them find two numbers less than 30 such that the sum of all the factors of each of the numbers is the number itself. The answer is 6 and 28.

These pupils may name the rational numbers shown in Ex. 12–26 in other ways by using multiples of numbers named by the fraction in simplest form.

Pupil's Objective

To learn to represent rational numbers using a number-line picture.

Background

A number-line picture may be constructed by having a number correspond to each point represented. Point *A* may be selected to represent 0. Other points, represented at equal intervals, may correspond to consecutive whole numbers. This picture gives us a representation of the set of whole numbers.

To represent rational numbers by this picture, each interval may be marked off into various parts of the same size as shown on the text page, and each mark labeled with a fraction to name the rational number represented by that point. The denominator for the fraction indicates the number of parts of the same size into which each interval between two whole numbers has been separated, and the numerator indicates the number of parts the fraction is from the whole number named on the left. At this time we are only interested in rationals which can be pictured between 0 and 1. When a single number-line picture is used to lay off various sets of intervals, it can be readily seen that any point in the number line can be labeled with an infinite number of fractions, all naming the same rational number.

Pre-Book Lesson

Review the use of a number-line picture to represent the set of whole numbers. Have a pupil draw a picture on the board explaining how he would locate equidistant dots to represent various whole numbers from left to right in order of increasing value.

Using The Text Page

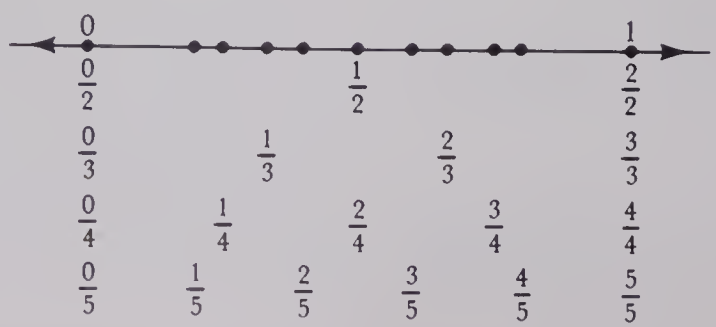
Ex. 1–8. The process of subdividing a unit length in the number line continues without end. Between any two points in the number line, the number of points that represent rational numbers is infinite.

The arrowhead pointing to the left in the number-line picture indicates that there are points to the left of zero that represent negative numbers. However, all work with rational numbers in this course is confined to the non-negative rationals.

Individualizing Instruction

• *More capable pupils* may be asked to think of some method for naming the set of fractions that represent the same point in the number line. This set of fractions may be named by multiplying the numerator and denominator for the fraction in simplest form by consecutive whole numbers beginning with 2.

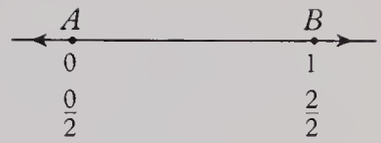
• *All pupils* should realize that the point in the number line representing the fraction $\frac{3}{1}$ also represents the whole number 3. The fraction $\frac{3}{1}$ is in simplest form because the numerator and denominator have no common factor other than 1. Whenever a rational number can be named with a whole number, it is usually more convenient to use the whole number name rather than the fraction to name it. Pupils should draw a number-line picture such as that for Ex. 4 and locate fractions in proper positions.



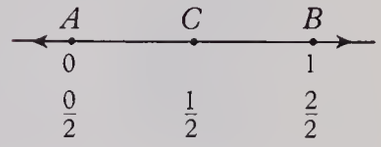
Picturing Rational Numbers

The number line [O]

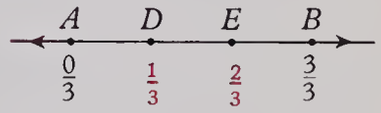
1. In the number-line picture at the right, the end points of $\overline{AB}$ represent the numbers 0 and ? 1. What fraction is shown for the number 0? $\frac{0}{2}$ for the number 1? $\frac{2}{2}$ Give some other fractions for these two numbers.
Sample answers: $\frac{0}{3}, \frac{0}{4}, \frac{0}{5}, \cdots; \frac{3}{3}, \frac{4}{4}, \frac{5}{5}, \cdots$



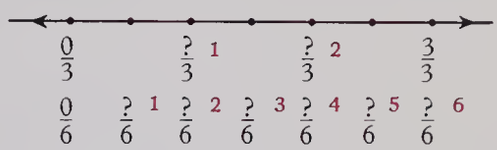
2. At the right, $\overline{AB}$ is shown partitioned into how many segments of the same length? 2 Then is $\overline{AB}$ shown partitioned into halves? Yes What fraction is shown for the number represented by point C? $\frac{1}{2}$



3. At the right, $\overline{AB}$ is shown partitioned into thirds. Give two fractions showing denominator 3 for the numbers represented by points D and E.



4. On the board, copy and complete the diagram at the right.



5. On the board, make a number-line picture and then show $\frac{0}{8}, \frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}$, and $\frac{8}{8}$.

[W5]

Ex. 6–7. Copy and complete.

6. In a number-line picture, the point representing the number $\frac{1}{2}$ can be labeled $\frac{1}{2}$ or $\frac{2}{?}$ or $\frac{3}{?}$ or $\frac{4}{?}$.

7. In a number-line picture, the point representing the number $\frac{1}{3}$ can be labeled $\frac{1}{3}$ or $\frac{2}{?}$ or $\frac{3}{?}$ or $\frac{4}{?}$.

8. Draw a number-line picture showing 12 segments of the same length. Label the points for Show number line as for Ex. 4 with appropriate points labeled.

a. $\frac{0}{2}, \frac{1}{2}$, and $\frac{2}{2}$.

b. $\frac{0}{3}, \frac{1}{3}, \frac{2}{3}$, and $\frac{3}{3}$.

c. $\frac{0}{4}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}$, and $\frac{4}{4}$.

d. $\frac{0}{6}, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}$, and $\frac{6}{6}$.

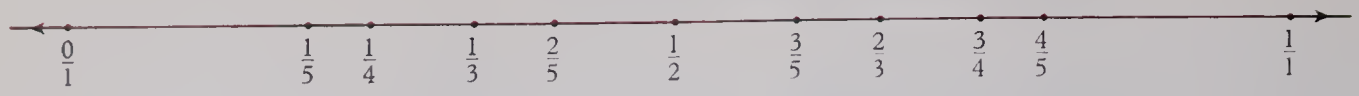
e. $\frac{0}{12}, \frac{1}{12}, \frac{2}{12}, \frac{3}{12}, \frac{4}{12}, \frac{5}{12}, \frac{6}{12}, \frac{7}{12}, \frac{8}{12}, \frac{9}{12}, \frac{10}{12}, \frac{11}{12}$, and $\frac{12}{12}$.

9. List all the subsets of {10, 100, 1,000}. { }, {10}, {100}, {1,000}, {10,000}, {10, 1,000}, {100, 1,000}, {10, 100, 1,000}

Ex. 10–13. Write an exponent form for the number.

10. 6×6 6^2 **11.** $3 \times 3 \times 3$ 3^3 **12.** $10 \times 10 \times 10$ 10^3 **13.** 144 12^2

* "Number-line picture" means the typical horizontal number-line picture used throughout the text.



Is Less Than or Is Greater Than?

Ordering rational numbers [O]

1. Study the diagram at the left. What part of the diagram is colored gray? $\frac{1}{5}$ What part is colored blue? $\frac{2}{5}$ How does the diagram suggest that $\frac{1}{5}$ is less than $\frac{2}{5}$? We write, "$\frac{1}{5} < \frac{2}{5}$."
The gray part is smaller than the blue part.

2. Look at the number-line picture above. Is the point representing $\frac{1}{5}$ shown to the left of the point representing $\frac{2}{5}$? Yes

> * In a number-line picture, if the point shown for one number is to the left of the point shown for a second number, then the first number is less than the second number.

3. Give a similar statement which relates points in a number line when one number is greater than a second number.
Key idea: The greater number will be shown to the right of the other number.

4. Refer to the number-line picture above.

a. Is $\frac{2}{3}$ shown to the left of $\frac{4}{5}$? Yes Then is $\frac{2}{3}$ less than $\frac{4}{5}$? Yes

b. Is $\frac{2}{3}$ shown to the right of $\frac{3}{5}$? Yes Then is $\frac{2}{3}$ greater than $\frac{3}{5}$? Yes

5. Refer to the number-line picture above to determine which of the following are true and which are false. Give reasons for your answers. On the number line picture above:

a. $\frac{1}{5} < \frac{1}{4}$ T b. $\frac{3}{5} > \frac{1}{2}$ T c. $\frac{2}{3} < \frac{3}{5}$ F d. $\frac{1}{3} < \frac{2}{5}$ T e. $\frac{1}{3} < \frac{1}{4}$ F

$\frac{1}{5}$ is to the left of $\frac{1}{4}$; $\frac{3}{5}$ is to the right of $\frac{1}{2}$; $\frac{2}{3}$ is to the right of $\frac{3}{5}$; $\frac{1}{3}$ is to the left of $\frac{2}{5}$; $\frac{1}{3}$ is to the right of $\frac{1}{4}$

6. From the number-line picture above, we know that the mathematical sentence $\frac{2}{3} < \frac{3}{4}$ is a true sentence because $\frac{2}{3}$ is shown to the left of $\frac{3}{4}$. Can $\frac{2}{3}$ be renamed $\frac{8}{12}$? Yes $\frac{3}{4}$ can be renamed $\frac{9}{12}$. Is $\frac{8}{12} < \frac{9}{12}$ a true sentence? Yes

> If two fractions show the same denominator, the fraction showing the lesser numerator names the lesser number.

7. Is $\frac{17}{25}$ less than $\frac{23}{25}$? Yes Why? Both fractions show the same denominator, 25. $17 < 23$ Therefore, $\frac{17}{25} < \frac{23}{25}$

Teaching Pages 100 and 101

Pupil's Objectives

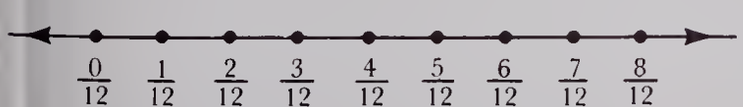
(a) To learn to determine which of two numbers pictured on a number line is the lesser; and (b) to learn which of two given fractions names the lesser rational number.

Background

Between any two points in the number line that represent rational numbers, there is an infinite number of points each of which represents a different rational number. Therefore, it is impossible to arrange rational numbers in an absolutely consecutive order as whole numbers can be arranged in consecutive order. For example, it is impossible to determine the rational number next greater than $\frac{1}{3}$. No matter what rational number is suggested as being the next greater, there is an infinite number of rational numbers greater than one third but less than the suggested rational number.

However, there is a way of determining which of two given rational numbers is the lesser. If two rational numbers are represented in a horizontal number-line picture, the point to the left always represents the lesser of the two rational numbers. This applies to any two rational numbers, regardless of whether they are positive, negative, or zero.

If two fractions show the same denominator, the fraction showing the lesser numerator names the lesser number. If two fractions show the same numerator, the fraction showing the lesser denominator names the greater number. If a unit length on a number line is separated into any number of segments of the same size, the points at the ends of each segment will represent rational numbers that can be named with fractions all showing the same denominator. The numerators will always be named in consecutive order.



For non-negative rational numbers, the numerators are in consecutive order increasing toward the right. From this it can be seen that if any two fractions show the same denominator, the fraction showing the lesser numerator names the lesser rational number.

If two fractions show different denominators, the numerator and denominator shown by either or both of the fractions may be multiplied by the identity element for multiplication in fraction form to produce fractions that name the same denominator. The fraction that names the lesser rational number can then be determined by comparing numerators.

On Teaching Page 96, the cross-products test was described for determining whether two fractions name the same rational number. $\frac{a}{b} = \frac{c}{d}$ if and only if $(a \times d) = (b \times c)$. There is also a cross-products test to determine which of two fractions names the lesser rational number. This test is stated as follows: "If $\frac{a}{b}$ and $\frac{c}{d}$ are fractions that name non-negative rational numbers, then $\frac{a}{b} > \frac{c}{d}$ if and only if $(a \times d) > (b \times c)$, and $\frac{a}{b} < \frac{c}{d}$ if and only if $(a \times d) < (b \times c)$." Also, $\frac{a}{d} = \frac{c}{d}$ if $a = c$; $\frac{a}{d} > \frac{c}{d}$ if $a > c$; and $\frac{a}{d} < \frac{c}{d}$ if $a < c$. It must be remembered that this is only valid for non-negative numbers.

Pre-Book Lesson

- Review use of the symbols ">" and "<," including exercises with whole numbers.
- Draw a diagram on the board such as the one under Individualizing Instruction for page 99. Try to have pupils discover where the greater numbers are represented on a number-line picture. Later, use this same diagram and, after writing a particular pair of fractions, have pupils insert the symbols $>$ or $<$ between them to make a true sentence.

Using the Text Page

- Ex. 1–3. It is important to emphasize that the rational numbers are ordered in a number-line picture. A mathematical sentence with a fraction on either side of an equals sign (=) is true only if the two fractions name the same rational number.
- Ex. 6. The generalization can be clearly demonstrated by the use of concrete objects which have been separated into halves, fourths, and so

on. Use a set of objects of the same size, and another set of objects each of which is a different size.

• Ex. 19–22. These exercises demonstrate how to determine which of two given fractions showing different denominators names the lesser rational number. The numerator and denominator shown by either or both of the fractions is multiplied by the identity element for multiplication in fraction form to rename with fractions naming the same denominator. The fraction naming the lesser numerator will then name the lesser rational number.

Individualizing Instruction

• *More capable pupils* may be guided to the discovery that, given any rational number, it is impossible to determine the next greater rational number. For example, it is impossible to determine a rational number next greater than $\frac{5}{8}$.

These pupils may also examine a number of true mathematical sentences with fractions that are statements of inequalities, such as $\frac{2}{3} < \frac{3}{4}$, and be led to discover the cross-products test for inequalities with fractions.

• *All pupils* may be led to discover a method for determining which of two fractions showing the same numerators but different denominators names the lesser rational number. The fraction showing the greater denominator names the lesser rational number. The fraction $\frac{2}{7}$ names a lesser rational number than does $\frac{2}{5}$. This can be verified by the method presented on the text page and by the use of concrete objects.

• *Slower learners* may be given a set of fractions such as $\{\frac{1}{8}, \frac{3}{4}, \frac{1}{2}, \frac{3}{8}, \frac{1}{4}, \frac{7}{8}\}$ and be asked to arrange the members of the set so that they name numbers in order of increasing value.

NOTES

8. Give a statement similar to the one following Ex. 6 which tells when one fraction names a greater number than another fraction. If two fractions show the same denominator, the fraction showing the greater numerator names the greater number.

9. For each of Ex. a–h, tell whether to use $<$ or $>$ or $=$ to make the sentence a true sentence.

a. $\frac{3}{9} \; \_?\_ \; \frac{8}{9}$ **b.** $\frac{14}{15} \; \_?\_ \; \frac{10}{15}$ **c.** $\frac{25}{25} \; \_?\_ \; \frac{24}{25}$ **d.** $\frac{3}{7} \; \_?\_ \; \frac{3}{7}$

e. $\frac{30}{40} \; \_?\_ \; \frac{31}{40}$ **f.** $\frac{5}{14} \; \_?\_ \; \frac{4}{14}$ **g.** $\frac{0}{8} \; \_?\_ \; \frac{1}{8}$ **h.** $\frac{29}{35} \; \_?\_ \; \frac{17}{35}$

10. In a number-line picture, would the point representing $\frac{7}{12}$ be shown to the left or to the right of the point representing $\frac{9}{12}$? $\frac{3}{12}$? $\frac{1}{2}$? left; right; right

[W]

Ex. 11–18. Copy and complete by writing $<$ or $>$ or $=$.

11. $\frac{5}{6} \; \_?\_ \; \frac{4}{6}$ **12.** $\frac{7}{11} \; \_?\_ \; \frac{9}{11}$ **13.** $\frac{2}{2} \; \_?\_ \; \frac{1}{2}$ **14.** $\frac{27}{28} \; \_?\_ \; \frac{28}{28}$

15. $\frac{30}{40} \; \_?\_ \; \frac{31}{40}$ **16.** $\frac{5}{14} \; \_?\_ \; \frac{4}{14}$ **17.** $\frac{0}{8} \; \_?\_ \; \frac{1}{8}$ **18.** $\frac{29}{35} \; \_?\_ \; \frac{17}{35}$

Ex. 19–22. Copy and complete.

19. $\frac{3}{5} = \frac{9}{?} \; \frac{}{15}$ and $\frac{2}{3} = \frac{10}{?} \; \frac{}{15}$, so $\frac{3}{5} \; \_?\_ \; \frac{2}{3}$.

20. $\frac{4}{5} = \frac{16}{?} \; \frac{}{20}$ and $\frac{3}{4} = \frac{15}{?} \; \frac{}{20}$, so $\frac{4}{5} \; \_?\_ \; \frac{3}{4}$.

21. $\frac{3}{4} = \frac{18}{?} \; \frac{}{24}$ and $\frac{6}{8} = \frac{18}{?} \; \frac{}{24}$, so $\frac{3}{4} \; \_?\_ \; \frac{6}{8}$.

22. $\frac{5}{16} = \frac{10}{?} \; \frac{}{32}$ and $\frac{1}{4} = \frac{8}{?} \; \frac{}{32}$, so $\frac{5}{16} \; \_?\_ \; \frac{1}{4}$.

Ex. 23–34. Write *T* or *F*.

23. $\frac{4}{5} < 1$ T **26.** $\frac{1}{3} < \frac{2}{5}$ T **29.** $\frac{7}{8} < \frac{5}{6}$ F **32.** $\frac{3}{7} > \frac{2}{5}$ T

24. $\frac{1}{2} < \frac{1}{3}$ F **27.** $\frac{3}{5} > \frac{9}{15}$ F **30.** $\frac{24}{5} > \frac{21}{4}$ F **33.** $\frac{1}{9} < \frac{2}{15}$ F

25. $\frac{14}{15} > \frac{29}{30}$ F **28.** $\frac{3}{4} > \frac{76}{100}$ F **31.** $\frac{5}{16} < \frac{1}{5}$ F **34.** $\frac{11}{12} > \frac{14}{15}$ F

35. Can a number be both *less than* and *greater than* another number? No

36. Can a number be both *less than* and *equal to* another number? No

37. Can a number be both *greater than* and *not equal to* another number? Yes

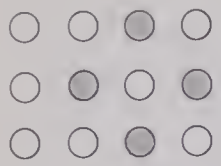
Ex. 38–41. Find the arithmetic mean for the measures.

38. 114, 256, 382, 464 304 **39.** 169, 254, 98, 65, 114 140

40. 1,963, 2,854, 1,561 2,126 **41.** 32, 38, 38, 42, 30 36

How Well Do You Remember?

[W]



1. Look at the diagram at the left and then write a fraction in simplest form which tells what part of the dots are colored. $\frac{1}{3}$

2. For the number $\frac{5}{7}$, write a fraction showing denominator

a. 14. $\frac{10}{14}$ b. 35. $\frac{25}{35}$ c. 70. $\frac{50}{70}$ d. 140. $\frac{100}{140}$ e. 700. $\frac{500}{700}$

3. Write fractions for Ex. a–c.

a. Two ninths $\frac{2}{9}$ b. Twelve fifths $\frac{12}{5}$ c. Seven tenths $\frac{7}{10}$

Ex. 4–6. Copy and complete.

4. $\frac{2}{3} = \frac{?4}{6} = \frac{18}{?27}$ 5. $\frac{23}{31} = \frac{92}{?124}$ 6. $\frac{3}{10} = \frac{?15}{50} = \frac{?30}{100} = \frac{?150}{500}$

Ex. 7–11. Write a fraction in simplest form for the number.

7. $\frac{32}{88}$ $\frac{4}{11}$ 8. $\frac{470}{900}$ $\frac{47}{90}$ 9. $\frac{115}{25}$ $\frac{23}{5}$ 10. $\frac{23+85}{200}$ $\frac{27}{50}$ 11. $\frac{7\times48}{6\times4}$ $\frac{14}{1}$

Ex. 12–15. Copy and complete by writing = or < or >.

12. $\frac{3}{7}$ $\_?\_$ $\frac{4}{7}$ 13. $\frac{62}{25}$ $\_?\_$ $\frac{47}{25}$ 14. $\frac{13}{15}$ $\_?\_$ $\frac{27}{30}$ 15. $\frac{5}{6}$ $\_?\_$ $\frac{7}{8}$

To Keep in Practice

[W]

Ex. 1–5. Copy and add.

1.	2.	3.	4.	5.
702	6,038	267	$438.75	$300.06
79	5,556	88	40.50	869.34
781	11,594	929	125.88	438.99
		1,284	$605.13	$1,608.39

Ex. 6–10. Copy and subtract.

6.	7.	8.	9.	10.
4,766	83,429	$45.89	$836.50	$1,000.00
2,988	1,463	1.99	22.88	498.99
1,778	81,966	$43.90	$813.62	$ 501.01

Ex. 11–15. Copy and multiply.

11.	12.	13.	14.	15.
38	302	1,309	$389.50	$480.75
29	77	403	67	25
1,102	23,254	527,527	$26,096.50	$12,018.75

Ex. 16–20. Divide. Write and test a checking sentence.

16. $14\overline{)847}$ 60, R7 17. $82\overline{)9,758}$ 119, R0 18. $29\overline{)9,396}$ 324, R0 19. $340\overline{)67,890}$ 199, R230 20. $15\overline{)\$348.75}$ $ 23.25, R0

Teaching Page 102

Pupil's Objectives

(a) To check understandings of important ideas studied thus far in Chapter 3; and (b) to have mixed computational practice with whole numbers.

Background

The exercises at the top of this page are useful as a review and analysis of the important ideas presented to this point in Chapter 3. The main ideas involved in these exercises are: (a) meaning of fraction and rational number; (b) renaming rational numbers; (c) writing fractions in simplest form; and (d) ordering rational numbers.

The exercises at the bottom of the page give the pupil mixed computational practice with whole numbers and with decimals which name dollars and cents. These practice exercises are necessary to maintain skills already developed, and will help diagnose sources of difficulties of individual pupils.

Pre-Book Lesson

- You may wish to review briefly each of the ideas listed in the Background section. Use whatever form of review you consider necessary before proceeding with the written work.
- If you are going to use these exercises as a test, be sure to tell the pupils so that they can prepare for it as they would for a regular testing situation. Explain that the exercises at the top of the page are a diagnostic test which will help to evaluate their understanding of the ideas presented thus far in the chapter, and that the exercises at the bottom of the page will help maintain their computational skills and help discover their sources of errors.

Using The Text Page

- It may not be necessary for *all pupils* to do all the exercises on this page, particularly the practice exercises. You may select only certain exercises for the *more capable pupils* or for the *slower learners* to do.
- Make certain that *all pupils* understand what they are to do in each set of written exercises, particularly the set at the bottom of the page. Have pupils check their computation by using inverse operations, or you may wish to have them express the exercises in the form of mathematical sentences. For example, Ex. 3 may be shown as $(267 + 88) + 929 = n$, Ex. 12 may be shown as $302 \times 77 = n$, and Ex. 18 may be shown as $29 \times n = 9{,}396$.

Individualizing Instruction

- The *more capable pupils* may work mentally the exercises at the top of the page. They may work Extra Activities or Supplementary Activities when they have completed their work. If the situation permits, you may wish to have these pupils assist *slower learners* in any remedial work necessary.
- Have *all pupils* restudy appropriate pages in the chapter if weaknesses are evident in the ideas being tested by the summary exercises. Some may need to refer back to pages where the topic was originally taught.

An evaluation of the results of each pupil's work will give you information for grouping pupils for presentation of new work as well as identifying concepts and skills which need reviewing or reteaching.

Teaching Page 103

Pupil's Objectives

(a) To learn the meaning of addition of two rational numbers; and (b) to learn that if $\frac{a}{c}$ and $\frac{b}{c}$ name rational numbers, then $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$.

Background

If $\frac{a}{b}$ and $\frac{c}{d}$ name rational numbers, their sum is defined as being equal to $\frac{(a \times d) + (b \times c)}{b \times d}$. In teaching addition of rational numbers, it is mathematically correct to begin with this definition. However, it is not a satisfactory approach for teaching this topic if one of the purposes of the instruction is to have the pupils gain insight into the computational techniques for addition of rational numbers.

An excellent way to teach addition of rational numbers is to do it in two stages: (1) addition with fractions showing the same denominator, and (2) addition with fractions showing different denominators. On this text page, addition is confined to addition with fractions showing the same denominator.

Two useful tools that may be used in teaching addition with fractions naming the same denominator are geometric figures (usually a circular region or a rectangular region) and the number-line picture.

When a geometric figure is used, parts of the figure may be shaded to represent the numbers being added. The denominators indicate the number of parts of the same size into which the geometric figure has been separated. The numerator indicates the number of these parts being represented by the fraction. The sum of the two rational numbers may then be represented by the union of the regions representing each number. Thus the addition of rational numbers with the same denominator is related to addition of whole numbers. $\frac{1}{5} + \frac{2}{5}$ is related to the addition $1 + 2$.

Pre-Book Lesson

Review use of the number-line picture in demonstrating addition of whole numbers. Using models of segments with varying lengths, have pupils manipulate them so they show additions in such mathematical sentences as $2 + 3 = 5$ and $1 + 2 = 3$. Ask if these same models can be used to show addition of rational numbers. Have a segment partitioned to show fifths and have pupils manipulate the pieces to show addition.*

Using the Text Page

- Ex. 1–5. Mathematically speaking, the sum $\frac{a}{c} + \frac{b}{c}$ is simply defined as being $\frac{a+b}{c}$. The subdivided regions and the number-line picture are only used to give insight into the computational techniques used to determine the sum of rational numbers. You may wish to use other models with these and other examples before making the generalization.
- Ex. 6. Point out that the 3 and the 2 can be thought of as 3 and 2 pieces of a whole which has been separated into sixths.
- Ex. 11–18. You may have pupils work some examples involving factoring. Then have them find fractions in simplest form for some rational numbers chosen at random.

Individualizing Instruction

- *More capable pupils* may be asked to explain the addition $\frac{3}{5} + \frac{2}{5}$ by use of two regions each of which has been divided into five equal parts.
- *All pupils* may be asked to find the sum of three fractions naming the same denominator. They may also be asked to determine which of two sums is greater such as $\frac{2}{9} + \frac{5}{9}$ or $\frac{3}{7} + \frac{4}{7}$.
- *Slower learners* may benefit from using models of segments or regions with Ex. 6–18 before they proceed to do the work. The generalization may confuse them so it may be necessary to avoid using it until they have mastered the ideas preceding its deduction.

* A classroom size model of a number line will be especially useful here. See item 6 on page xix.

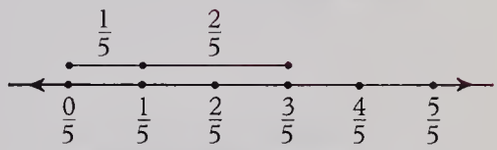
* Point out that unless directed otherwise, answers named with fractions should be in simplest form. If the answer is a whole number, it should be named with a standard numeral.

Addition of Rational Numbers

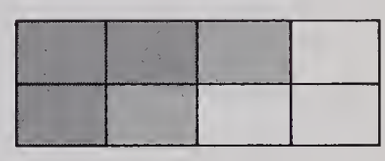
Meaning [O]

1. For the diagram at the right, $\frac{1}{5}$ tells what part is colored _?_ gray and $\frac{2}{5}$ tells what part is colored _?_ blue. What fraction tells what part of the diagram is colored? $\frac{3}{5}$ Does the diagram suggest that $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$? Yes

2. Explain how the number-line picture at the right suggests that $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$. The line segment above the number-line picture ends above $\frac{3}{5}$.



3. Explain how the diagram at the right suggests that $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$.



4. On the board, draw a number-line picture as in Ex. 2 which suggests that $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$.

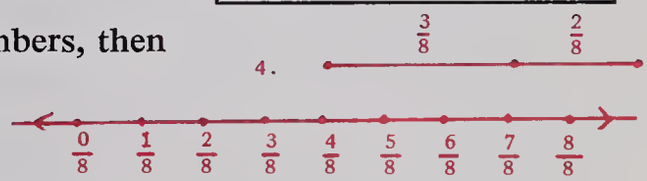
5. The work in the box shows another way to find the sum of $\frac{1}{5}$ and $\frac{2}{5}$. Explain each step shown. Find the sum of the numerators.

$$\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5} = \frac{3}{5}$$

If $\frac{a}{c}$ and $\frac{b}{c}$ are rational numbers, then

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}.$$

4.



6. Find fractions for the following as in the box.

a. $\frac{3}{6} + \frac{2}{6} = \frac{3+2}{6} = \frac{5}{6}$ **b.** $\frac{5}{7} + \frac{1}{7} = \frac{5+1}{7} = \frac{6}{7}$ **c.** $\frac{0}{8} + \frac{7}{8} = \frac{0+7}{8} = \frac{7}{8}$ **d.** $\frac{5}{9} + \frac{2}{9} = \frac{5+2}{9} = \frac{7}{9}$

[W]

Ex. 7–10. Draw a number-line picture which suggests that See below.

7. $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$.

8. $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$.

9. $\frac{3}{4} + \frac{1}{4} = 1$.

10. $\frac{7}{10} + \frac{2}{10} = \frac{9}{10}$.

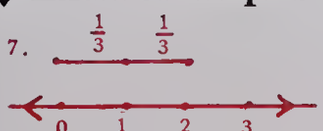
* Ex. 11–18. Add as in the box. Unless otherwise directed, when your answer is named by a fraction, always be sure the fraction is in simplest form. If your answer is a whole number, write the standard numeral for the whole number.

11. $\frac{3}{11} + \frac{4}{11}$ $\frac{7}{11}$ 12. $\frac{5}{20} + \frac{11}{20}$ $\frac{4}{5}$ 13. $\frac{1}{6} + \frac{2}{6}$ $\frac{1}{2}$ 14. $\frac{13}{25} + \frac{7}{25}$ $\frac{4}{5}$

15. $\frac{7}{12} + \frac{4}{12}$ $\frac{11}{12}$ 16. $\frac{23}{30} + \frac{6}{30}$ $\frac{29}{30}$ 17. $\frac{39}{50} + \frac{2}{50}$ $\frac{41}{50}$ 18. $\frac{1}{24} + \frac{5}{24}$ $\frac{1}{4}$

◆ **Extra Examples.** Sets 53–54.

7.



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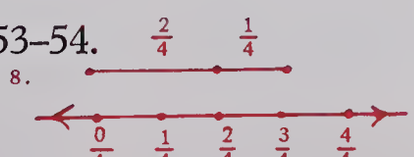

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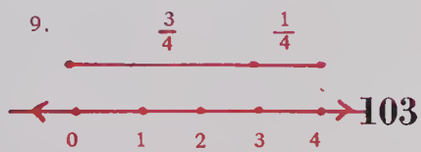
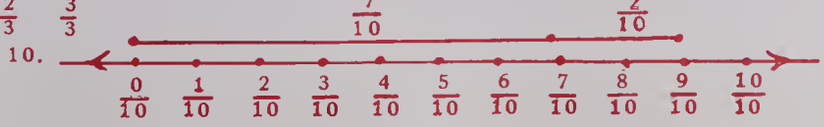


10.



*Note that by "least common multiple," we mean the least common counting-number multiple.

Least Common Denominator (L.C.D.)

Least common multiple [O]

A

$$n = \frac{1}{2} + \frac{2}{5}$$
$$= \frac{10}{20} + \frac{8}{20}$$
$$= \frac{10 + 8}{20}$$
$$n = \frac{18}{20}, \text{ or } \frac{9}{10}$$

B

$$n = \frac{1}{2} + \frac{2}{5}$$
$$= \frac{5}{10} + \frac{4}{10}$$
$$= \frac{5 + 4}{10}$$
$$n = \frac{9}{10}$$

1. Box A. To find the number represented by n in $n = \frac{1}{2} + \frac{2}{5}$, we can first rename $\frac{1}{2}$ and $\frac{2}{5}$ with the fractions $\frac{?}{20}$ (10) and $\frac{?}{20}$ (8). Explain each step shown. What is the answer expressed in simplest form? $\frac{9}{10}$

2. In box A, $\frac{1}{2}$ and $\frac{2}{5}$ were renamed with fractions both showing denominator $\_?\_$ (20). What common denominator is shown in box B? (10) Is 10 the *least common denominator* (*L.C.D.*)? (Yes) Explain the work in box B.

3. Box C shows a way to find the least common denominator of $\frac{3}{8}$ and $\frac{1}{6}$ by finding the least common counting-number multiple of the denominators.

C

Some multiples of 6:
6, 12, 18, (24), 30
Some multiples of 8:
8, 16, (24)
The L.C.M. of 6 and 8 is 24.

a. List some of the counting-number multiples of the lesser denominator, 6, starting with 6. Sample answer: 6, 12, 18, 24, 30

b. Start listing the counting-number multiples of 8, starting with 8. As you list each multiple, check to see if it is named in the list of multiples of 6. Sample answer: 8, 16, 24, 32, 40

* The least counting-number multiple of two numbers is called the *least common multiple* (*L.C.M.*). The least common denominator of $\frac{a}{b}$ and $\frac{c}{d}$ is the least common multiple of the denominators b and d.

4. $n = \frac{3}{8} + \frac{1}{6}$
$= \frac{9}{24} + \frac{4}{24}$
$= \frac{9 + 4}{24}$
$n = \frac{13}{24}$

** 4. Since the L.C.M. of 6 and 8 is 24, the L.C.D. of $\frac{3}{8}$ and $\frac{1}{6}$ is $\_?\_$ (24). At the board, solve $n = \frac{3}{8} + \frac{1}{6}$ as in box B.

[W]

Ex. 5–12. Solve by using the least common denominator as in box B. Use the method in Ex. 3 when finding the L.C.D.

5. $n = \frac{1}{2} + \frac{1}{3}$ $\frac{5}{6}$ 6. $n = \frac{2}{5} + \frac{1}{3}$ $\frac{11}{15}$ 7. $n = \frac{1}{4} + \frac{2}{3}$ $\frac{11}{12}$ 8. $n = \frac{2}{7} + \frac{1}{3}$ $\frac{13}{21}$
9. $n = \frac{2}{7} + \frac{1}{5}$ $\frac{17}{35}$ 10. $n = \frac{3}{4} + \frac{1}{5}$ $\frac{19}{20}$ 11. $n = \frac{1}{10} + \frac{3}{5}$ $\frac{7}{10}$ 12. $n = \frac{1}{3} + \frac{3}{10}$ $\frac{19}{30}$

♦ **Extra Examples.** Set 55.

** Note that we find the least common multiple of the denominators named by two fractions in order to find the least common denominator for the rational numbers named by the two fractions.

Teaching Page 104

Pupil's Objectives

(a) To learn the meaning of least common denominator; (b) to learn to find the least common multiple (L.C.M.) of two numbers; and (c) to learn to find the least common denominator for two fractions.

Background

The first stage in mastering the technique of adding two rational numbers is to learn to add with two fractions showing the same denominator. The second stage is to learn to add with two fractions showing different denominators.

To add with two fractions showing different denominators, first multiply the numerator and denominator expressed by one or both of the fractions by some factor so that the resulting fractions show the same denominator. For example, to find the sum of $\frac{2}{3}$ and $\frac{1}{5}$, the numerator and denominator of $\frac{2}{3}$ could be multiplied by 10 and the numerator and denominator of $\frac{1}{5}$ multiplied by 6.

$$\frac{2}{3} = \frac{2 \times 10}{3 \times 10} = \frac{20}{30} \qquad \frac{1}{5} = \frac{1 \times 6}{5 \times 6} = \frac{6}{30}$$

The resulting fractions show the same denominator, so their sum can be found by application of the principle

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}: \quad \frac{20}{30} + \frac{6}{30} = \frac{20+6}{30} = \frac{26}{30}.$$

However, the final sum is not shown with a fraction in simplest form. Both the numerator and denominator have a factor of 2. To find the fraction in simplest form, the numerator and denominator must be divided by 2: $\frac{26}{30} = \frac{26 \div 2}{30 \div 2} = \frac{13}{15}$.

The least common multiple of 3 and 5 is 15. If the numerator and denominator of $\frac{2}{3}$ and the numerator and denominator of $\frac{1}{5}$ had been multiplied by some factors so that the resulting fractions would show denominators that were the L.C.M., then the sum would have been shown by a fraction in simplest form. If the denominator shown by the resulting fractions is the least common multiple of the denominators of the fraction numbers being added, this denominator is called the *least common denominator*.

Pre-Book Lesson

- Have pupils review the concepts of the L.C.M. and factoring. Let pupils work some examples where they must factor and find the L.C.M.
- Review the principle used in solving exercises 19–34 on page 101 in showing two rational numbers with fractions that name the same denominator.

Using the Text Page

- Ex. 1–2. Make sure pupils understand the analogy between the work here and previous work with the L.C.M. and factoring. Point out that the method shown in box A and in box B both result in the correct answer. However, the method shown in box B results in a sum that is more often in simplest form.
- Ex. 3. The pupils may prefer to show, alternately, a multiple of 6 and then a multiple of 8 until a common multiple is discovered. Have pupils show counting-number multiples of other pairs of numbers to determine the L.C.D. for fractions showing these numbers for denominators.

Individualizing Instruction

- *More capable pupils* may describe faster methods of determining the L.C.D. Check their methods for mathematical correctness. They may also be encouraged to state such generalizations for $\frac{a}{b} + \frac{c}{d}$ as $\frac{(a \times d) + (b \times c)}{b \times d}$ and to illustrate this with several examples.
- *All pupils* may be given three fractions with different denominators and be asked to find the least common denominator of all three rational numbers and then proceed to find their sum.
- *Slower learners* may have some difficulty with use of the abbreviations "L.C.D." and "L.C.M." It may be best to use the terms "least common denominator" and "least common multiple" with these pupils.

By using models of segments of varying length, show these pupils the necessity of having segments separated into units of the same size if they are to be combined to form a single long segment. This is analagous to having like denominators in order to add rational numbers.

Teaching Page 105

Pupil's Objective

To learn the meaning of subtraction of rational numbers.

Background

The subtraction of two rational numbers named by fractions showing the same denominator is defined as $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$.

If the numbers to be subtracted have different denominators, the numerator and denominator of either or both of the numbers are multiplied by some factor so that the resulting numbers have like denominators. The subtraction can then be performed applying the above definition:

$$\frac{a}{b} - \frac{c}{d} = \frac{a \times d}{b \times d} - \frac{b \times c}{b \times d}$$
$$= \frac{(a \times d) - (b \times c)}{b \times d}$$

To give the pupils insight into the computational technique of subtracting rational numbers, a number-line picture is very useful. The pupils are familiar with the use of the number-line picture in defining the subtraction of whole numbers, and this same principle applies to the set of rational numbers.

Pre-Book Lesson

• Using the methods from page 103, review the use of the number-line picture in explaining the subtraction of whole numbers.

• Ask pupils to state mathematical sentences which show addition and then change them so they show subtraction. Have pupils label the addends and the sum.

Using the Text Page

• Ex. 1. In this exercise, the sum and one addend are given and the unknown addend is to be determined. Therefore, it is a subtraction problem; the relationship being unknown addend = sum − known addend. Pupils may need work with more examples before being able to make any generalizations.

• Ex. 3–11. When giving the pupils exercises involving subtraction, be certain that the sum is greater than or equal to the known addend. Otherwise, the answer would be a negative number.

Individualizing Instruction

• *More capable pupils* may be asked to represent with letters the common denominator for the fraction represented by $\frac{a}{b}$ and $\frac{c}{d}$. They could then develop the general formulas:

$$\frac{a}{b} + \frac{c}{d} = \frac{(a \times d) + (b \times c)}{b \times d}$$
$$\frac{a}{b} - \frac{c}{d} = \frac{(a \times d) - (b \times c)}{b \times d}$$

• *All pupils* may be given exercises such as $(\frac{7}{8} - \frac{2}{3}) - \frac{1}{24}$ which involve more than one subtraction.

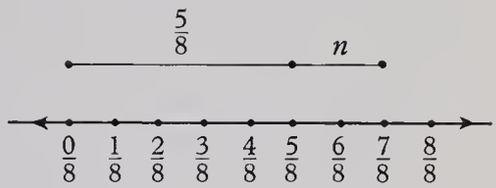
• Subtraction of rational numbers may be explained to the *slower learners* by use of pieces of cardboard or blocks of wood. If a piece of cardboard is cut into 8 parts of the same size and seven of these are considered, the subtraction $\frac{7}{8} - \frac{5}{8}$ may be explained by removing 5 of the 7 eighths and determining how many eighths are left, or by determining how many eighths must be added to 5 eighths to give 7 eighths. These pupils may need to use number-line pictures or objects when working Ex. 3–11.

* Emphasize that some of these exercises will require renaming of one or sometimes both rational numbers in order to have a common denominator.

Subtraction of Rational Numbers

[O]

1. Jane rides $\frac{7}{8}$ mile and Sue rides $\frac{5}{8}$ mile in the school bus. How much farther does Jane ride than Sue?



a. Explain how the mathematical sentence $\frac{5}{8} + n = \frac{7}{8}$ shows the relationship expressed in the problem. It shows the addends-sum relationship. Does the number-line picture at the right show this relationship? Yes

b. Does $n = \frac{7}{8} - \frac{5}{8}$ show the same relationship as $\frac{5}{8} + n = \frac{7}{8}$? Yes Explain the work in box A.

c. How much farther does Jane ride than Sue? $\frac{1}{4}$ mile

A

$$n = \frac{7}{8} - \frac{5}{8}$$
$$= \frac{7 - 5}{8}$$
$$= \frac{2}{8}, \text{ or } \frac{1}{4}$$

B

$$n = \frac{1}{2} - \frac{1}{3}$$
$$= \frac{3}{6} - \frac{2}{6}$$
$$= \frac{1}{6}$$

If $\frac{a}{c}$ and $\frac{b}{c}$ are rational numbers, then $\frac{a}{c} - \frac{b}{c} = \frac{a - b}{c}$.

* 2. Box B. To solve $n = \frac{1}{2} - \frac{1}{3}$, the numbers $\frac{1}{2}$ and $\frac{1}{3}$ were first renamed $\frac{?^3}{6}$ and $\frac{?^2}{6}$. What is the L.C.D. of $\frac{1}{2}$ and $\frac{1}{3}$? 6 Explain each step shown. The numbers are renamed with a common denominator. Then the numerators are subtracted.

[W]

Ex. 3–11. Solve. Find the L.C.D. as in Ex. 3, page 104.

3. $n = \frac{5}{7} - \frac{2}{7}$ $\frac{3}{7}$

4. $y = \frac{1}{4} - \frac{1}{6}$ $\frac{1}{12}$

5. $n = \frac{8}{7} - \frac{3}{14}$ $\frac{13}{14}$

6. $x = \frac{12}{15} - \frac{1}{15}$ $\frac{11}{15}$

7. $n = \frac{2}{3} - \frac{2}{5}$ $\frac{4}{15}$

8. $\frac{5}{4} - \frac{2}{3} = x$ $\frac{7}{12}$

9. $n = \frac{3}{4} - \frac{1}{8}$ $\frac{5}{8}$

10. $x = 1 - \frac{3}{7}$ $\frac{4}{7}$

11. $n = \frac{7}{8} - \frac{2}{5}$ $\frac{19}{40}$

◆ **Extra Examples.** Set 56.

* Pupils are familiar with these properties in relation to the set of whole numbers. No examine the properties to see if they also apply to the set of rational numbers.

Remembering the Addends-Sum Relationship

[O]

A

$$\frac{5}{7} = \frac{3}{7} + \frac{2}{7}$$
$$\frac{5}{7} - \frac{3}{7} = \frac{2}{7}$$
$$\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$$
$$\frac{5}{7} = \frac{2}{7} + \frac{3}{7}$$

B

$$n - \frac{3}{7} = \frac{1}{7}$$
$$n = \frac{1}{7} + \frac{3}{7}$$
$$= \frac{4}{7}$$

C

$$\frac{5}{9} - n = \frac{4}{9}$$
$$n = \frac{5}{9} - \frac{4}{9}$$
$$= \frac{1}{9}$$

D

$$n + \frac{3}{8} = \frac{3}{4}$$
$$n = \frac{3}{4} - \frac{3}{8}$$
$$= \frac{6}{8} - \frac{3}{8}$$
$$= \frac{3}{8}$$

1. Do the four sentences in box A illustrate the addends-sum relationship? Yes Which numerals name the sum? $\frac{5}{7}$ Which numerals name the addends? $\frac{2}{7}$ and $\frac{3}{7}$

2. Using $\frac{1}{3}$ and $\frac{1}{2}$ for the addends and n to represent the sum, write four sentences which illustrate the addends-sum relationship. $n = \frac{1}{3} + \frac{1}{2}$, $n - \frac{1}{3} = \frac{1}{2}$, $n - \frac{1}{2} = \frac{1}{3}$, $n = \frac{1}{2} + \frac{1}{3}$

3. In $n = \frac{3}{5} + \frac{2}{5}$, does n represent the sum or an addend? When the addends are known, we add to find the sum. Name the number represented by n. 1

4. Box B. In $n - \frac{3}{7} = \frac{1}{7}$, does n represent the sum or an addend? Explain the work shown. The addends are added to find the sum.

5. Box C. In $\frac{5}{9} - n = \frac{4}{9}$, does n represent the sum or an addend? When the sum and one of two addends are known, we subtract to find the other addend. Explain the work shown. The subtraction is done to find the other addend.

6. Box D. In $n + \frac{3}{8} = \frac{3}{4}$, does n represent the sum or an addend? Explain the work shown. The subtraction is done to find the other addend.

7. At the board, solve each of Ex. a–c. Express the answers in simplest form.

a. $\frac{7}{15} - n = \frac{2}{15}$ $\frac{1}{3}$ **b.** $n + \frac{1}{6} = \frac{7}{12}$ $\frac{5}{12}$ **c.** $\frac{2}{3} - n = \frac{1}{4}$ $\frac{5}{12}$

8. For each of Ex. 9–17, tell whether the letter represents the sum or an addend.

[W]

Ex. 9–17. Find the number represented by the letter. Remember to express the answer in simplest form.

9. $n - \frac{3}{11} = \frac{7}{11}$ $\frac{10}{11}$ sum;
10. $x - \frac{4}{7} = \frac{1}{7}$ $\frac{5}{7}$ sum;
11. $n - \frac{1}{8} = \frac{1}{2}$ $\frac{5}{8}$ sum;
12. $\frac{2}{3} - y = \frac{1}{3}$ $\frac{1}{3}$ addend;
13. $\frac{2}{5} - x = \frac{1}{10}$ $\frac{3}{10}$ addend;
14. $n + \frac{1}{3} = \frac{3}{4}$ $\frac{5}{12}$ addend;
15. $n + \frac{7}{12} = \frac{2}{3}$ addend; $\frac{1}{12}$
16. $\frac{5}{12} - x = \frac{1}{12}$ addend; $\frac{1}{3}$
17. $n - \frac{3}{5} = \frac{1}{3}$ sum; $\frac{14}{15}$

♦ **Extra Examples.** Set 57.

Teaching Page 106

Pupil's Objectives

(a) To review the addends-sum relationship; and (b) to practice using this relationship with rational numbers.

Background

In the addition of any two whole numbers, the relationship among the addends and the sum may be expressed in terms of addition or in terms of subtraction. If a, b, and c are whole numbers, and if $a = b + c$, then $a - b = c$ and $a - c = b$. These may also be written $c = a - b$ and $b = a - c$.

This same relationship exists in the set of rational numbers. If $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$ are fractions naming rational numbers, and if $\frac{a}{b} = \frac{c}{d} + \frac{e}{f}$, then $\frac{a}{b} - \frac{c}{d} = \frac{e}{f}$ and $\frac{a}{b} - \frac{e}{f} = \frac{c}{d}$. This may be expressed as

$$\text{sum} = \text{first addend} + \text{second addend}$$
$$\text{sum} - \text{first addend} = \text{second addend}$$
$$\text{sum} - \text{second addend} = \text{first addend}$$

In solving problems involving the addends-sum relationship, it must first be determined whether it is the sum that is unknown or whether it is an addend that is unknown. If the sum is unknown and the addends are given, the sum is found by adding the addends. Examples of this type of exercise are

$$n - \tfrac{7}{8} = \tfrac{1}{3} \quad \text{and} \quad n = \tfrac{1}{5} + \tfrac{3}{4}$$

If one of the addends is unknown, but the sum and the other addend are given, the unknown addend is found by subtracting the known addend from the sum. Examples of this type of exercise are

$$\tfrac{7}{8} - n = \tfrac{1}{3} \qquad n + \tfrac{3}{4} = \tfrac{7}{8} \qquad \tfrac{2}{5} + n = \tfrac{8}{9}$$

Pre-Book Lesson

Review the addends-sum relationship as it applies to whole numbers. Have pupils express the relationship with several examples using whole numbers and labeling the numerals for addends and the sum as shown in different positions in number sentences.

Using the Text Page

• Ex. 1. The first three mathematical sentences in box A express the addends-sum relationship that

$$\text{if, } \tfrac{a}{b} = \tfrac{c}{d} + \tfrac{e}{f}$$
$$\text{then, } \tfrac{a}{b} - \tfrac{c}{d} = \tfrac{e}{f} \text{ and } \tfrac{a}{b} - \tfrac{e}{f} = \tfrac{c}{d}.$$

The fourth mathematical sentence in box A is a restatement of the first mathematical sentence, the addends being reversed by applying the Commutative Property of Addition.

• Ex. 3–5. To find an unknown sum, the addends are added. To find an unknown addend, the known addend is subtracted from the sum.

• Ex. 7. Some pupils may need help with finding the simplest form. Have them factor pairs of numbers as review.

• Ex. 9, 10, 11, and 17. The sum is unknown so addition is to be performed. In Ex. 12–16, one addend is unknown so subtraction is to be performed.

Individualizing Instruction

• *More capable pupils* may be asked to create word problems containing an addends-sum relationship that could be expressed by one of the mathematical sentences in Ex. 9–17.

• *All pupils* may be given word problems and be asked to express the addends-sum relationship as a mathematical sentence. They may then change the problem so rational numbers are used in place of whole numbers.

• *Slower learners* may be given pieces of cardboard of the same size to illustrate and explain the addends-sum relationships expressed by the mathematical sentences in Ex. 9–17.

Pupil's Objectives

(a) To review the properties of addition; and (b) to learn to express these properties with rational numbers.

Background

There are four addition properties of rational numbers: (1) the set of rational numbers is closed under addition; (2) addition is commutative; (3) addition is associative; and (4) there is an additive identity element in the set of rational numbers. The addition of two rational numbers with the same denominator is defined as $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$. Since $\frac{a}{c}$ and $\frac{b}{c}$ are rationals, a and b are integers; $a + b$ is an integer; and c is a counting number. Therefore, $\frac{a+b}{c}$ is a rational number. Since any two rational numbers may be expressed with fractions showing the same denominator, the sum of any two rational numbers is always a rational number. We say the set of rational numbers is closed under the operation addition.

The Commutative Property of Addition in the set of rational numbers may be demonstrated by a comparison of the sum of $\frac{a}{b}$ and $\frac{c}{d}$ with the sum of $\frac{c}{d}$ and $\frac{a}{b}$.

$$\frac{a}{b} + \frac{c}{d} = \frac{(a \times d) + (b \times c)}{b \times d}; \quad \frac{c}{d} + \frac{a}{b} = \frac{(c \times b) + (d \times a)}{d \times b}$$
$$= \frac{(a \times d) + (b \times c)}{b \times d}$$

The Associative Property of Addition in the set of rational numbers may be demonstrated by comparing the sum $\left(\frac{a}{b} + \frac{c}{d}\right) + \frac{e}{f}$ with the sum $\frac{a}{b} + \left(\frac{c}{d} + \frac{e}{f}\right)$.

$$\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f} = \frac{(a \times d) + (b \times c)}{b \times d} + \frac{e}{f}$$
$$= \frac{(a \times d) \times f + (b \times c) \times f + (b \times d) \times e}{b \times d \times f}$$
$$= \frac{(a \times d \times f) + (b \times c \times f) + (b \times d \times e)}{b \times d \times f}$$
$$\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right) = \frac{a}{b} + \frac{(c \times f) + (d \times e)}{d \times f}$$
$$= \frac{(a \times d \times f) + (b \times c \times f) + (b \times d \times e)}{b \times d \times f}$$

The rational number that can be named by a fraction showing the numerator "0" is the number zero. If the fraction $\frac{0}{c}$ is used to represent the rational number zero, then $\frac{a}{b} + \frac{0}{c} = \frac{(a \times c) + (b \times 0)}{b \times c}$

$$= \frac{(a \times c) + 0}{b \times c}$$
$$= \frac{(a \times c)}{(b \times c)}$$
$$= \frac{a}{b}$$

The sum is the same as the given addend; therefore, $\frac{0}{c}$ is the identity element for addition in the set of rational numbers.

Pre-Book Lesson

Review the properties of addition in the set of whole numbers by working examples at the board. Some pupils may be helped by thinking of activities which can be performed in any order, and those which must be performed in a particular order.

Using the Text Page

- Ex. 1–4. Remind the pupils that in the statement of the Commutative Property of Addition the order of the addends is changed, while in the statement of the Associative Property of Addition, the order of the addends is not changed, but the grouping of the addends is changed.
- Ex. 9–17. Have the pupils try to determine the correct answer without using pencil and paper. Some pupils may need to review work using the symbols $<$ and $>$.

Individualizing Instruction

- *More capable pupils* may be asked if the sum of any two rational numbers is always a rational number and to give evidence to support their conclusion. They may also be asked if there is an identity element for addition in the set of rational numbers. They may use the generalizations and work examples to show their answers.
- *All pupils* may be asked to demonstrate the Commutative Property of Addition and the Associative Property of Addition by use of a number line picture.

* Pupils are familiar with these properties in relation to the set of whole numbers. Now examine the properties to see if they also apply to the set of rational numbers.

a. $5 + 97 = 97 + 5$	**d.** $(3 + 17) + 89 = 3 + (17 + 89)$
b. $(\frac{1}{2} + \frac{3}{4}) + \frac{5}{6} = \frac{1}{2} + (\frac{3}{4} + \frac{5}{6})$	**e.** $(\frac{1}{2} + \frac{2}{7}) + \frac{6}{7} = \frac{6}{7} + (\frac{1}{2} + \frac{2}{7})$
c. $\frac{7}{12} + \frac{4}{12} = \frac{4}{12} + \frac{7}{12}$	**f.** $\frac{7}{15} + \frac{3}{4} = \frac{3}{4} + \frac{7}{15}$

* Remembering Properties of Addition

[o]

1. At the board, find $\frac{3}{7} + \frac{2}{7}$ and then find $\frac{2}{7} + \frac{3}{7}$. $\frac{5}{7}$ Is it true that $\frac{3}{7} + \frac{2}{7} = \frac{2}{7} + \frac{3}{7}$? Yes This is an illustration of the _?_ Property of Addition. Commutative

2. Which of Ex. a–f in the box are illustrations of the Commutative Property of Addition? a, c, e, f

3. At the board, find $(\frac{3}{10} + \frac{4}{10}) + \frac{2}{10}$. $\frac{9}{10}$ Did you first perform the operation indicated within the parentheses? Yes Now find $\frac{3}{10} + (\frac{4}{10} + \frac{2}{10})$. $\frac{9}{10}$ Is $(\frac{3}{10} + \frac{4}{10}) + \frac{2}{10} = \frac{3}{10} + (\frac{4}{10} + \frac{2}{10})$ a true mathematical sentence? Yes This is an illustration of the _?_ Property of Addition. Associative

4. Which of Ex. a–f in the box are illustrations of the Associative Property of Addition? b, d

[w]

Ex. 5–8. Copy and complete by thinking about properties of addition.

5. $\frac{35}{71} + \_? = \frac{27}{50} + \frac{35}{71}$ $\frac{27}{50}$

6. $(\frac{1}{9} + \frac{17}{20}) + \frac{13}{15} = \frac{1}{9} + (\frac{17}{20} + \_?)$ $\frac{13}{15}$

7. $(\frac{5}{9} + \frac{20}{21}) + \frac{3}{4} = (\_? + \frac{5}{9}) + \frac{3}{4}$ $\frac{20}{21}$

8. $(\frac{1}{4} + \frac{2}{3}) + \frac{7}{8} = \frac{7}{8} + (\_? + \frac{2}{3})$ $\frac{1}{4}$

Ex. 9–17. Write *T* or *F*.

9. $\frac{2}{5} + \frac{1}{5} < \frac{3}{5}$ F

10. $\frac{7}{9} - \frac{4}{9} > \frac{2}{9}$ T

11. $\frac{13}{25} = \frac{2}{5} + \frac{3}{25}$ T

12. $\frac{5}{6} = \frac{30}{6} - \frac{25}{6}$ T

13. $\frac{1}{6} + \frac{5}{12} < \frac{3}{4}$ T

14. $\frac{7}{9} + (\frac{2}{5} + \frac{1}{5}) = 2$ F

15. $\frac{7}{11} = (\frac{10}{11} + \frac{4}{11}) - \frac{3}{11}$ F

16. $\frac{3}{7} < \frac{3}{14} + \frac{4}{14}$ T

17. $\frac{13}{21} > \frac{25}{42}$ T

Ex. 18–23. Solve.

18. $x + 1{,}786 = 2{,}184$ 398

19. $n = 26{,}486 \div 34$ 779

20. $3{,}342 - n = 794$ 2,548

21. $394 \times 67 = n$ 26,398

22. $y \div 126 = 49$ 6,174

23. $168 \div n = 14$ 12

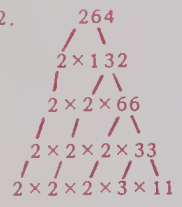
* Some pupils may need to review pages 21-23 before naming the numbers with base-five numerals.

To Keep in Practice

[W]

1. List within braces the factors of **a.** 26. **b.** 48.
a. {1, 2, 13, 26}; b. {1, 2, 3, 4, 6, 8, 12, 16, 24, 48}

2. Make a factor tree for 264.

2.


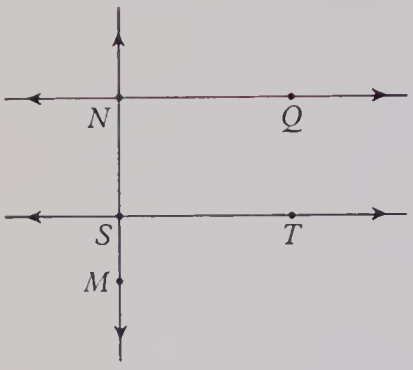
* Ex. 3–8. Write a base-five numeral for the number.

3. 9 14_{five} 4. 18 33_{five} 5. 25 100_{five} 6. 39 124_{five} 7. 125 1000_{five} 8. 130 1010_{five}

Ex. 9–13. Write a base-ten numeral for the number.

9. 21_{five} 11 10. 110_{five} 30 11. CCXIX 219 12. 62_{eight} 50 13. 66_{seven} 48

14. Find $A \cup B$ and $A \cap B$ if $A = \{5, 7, 8\}$ and $B = \{6, 8, 9\}$. $A \cup B = \{5, 6, 7, 8, 9\}$; $A \cap B = \{8\}$



Ex. 15–18. Refer to the figure at the left.

15. Name a pair of parallel segments. $\overline{NQ}$ and $\overline{ST}$

16. Name two pairs of perpendicular lines. $\overleftrightarrow{NQ}$ and $\overleftrightarrow{NS}$; $\overleftrightarrow{ST}$ and $\overleftrightarrow{NS}$

17. Find $\overline{NQ} \cap \overleftrightarrow{MN}$. {N} 18. Find $\overrightarrow{MN} \cap \overleftrightarrow{SN}$. $\overrightarrow{SN}$

19. Which members of $\{\frac{3}{4}, \frac{2}{3}, \frac{7}{8}\}$ are greater than $\frac{4}{5}$? $\frac{7}{8}$

Try These Exercises

Enrichment [W]

** If the greatest common factor of two numbers is 1, then the numbers are said to be *relatively prime*. For example, the numbers 4 and 15 are relatively prime because their greatest common factor is 1. However, the numbers 4 and 10 are not relatively prime because their greatest common factor is not 1.

1. Name the greatest common factor of 4 and 10. 2

Ex. 2–6. Write *Yes* if the numbers are relatively prime and write *No* if they are not.

2. 13, 24 Yes 3. 9, 51 No 4. 50, 63 Yes 5. 100, 103 Yes 6. 230, 605 No

7. If a fraction is in simplest form, is it true that the numerator and the denominator shown in the fraction are relatively prime? Yes

** If the numerator and denominator named by a fraction are relatively prime, then the fraction is in simplest form.

Teaching Page 108

Pupil's Objective

To evaluate concepts and skills presented in earlier chapters.

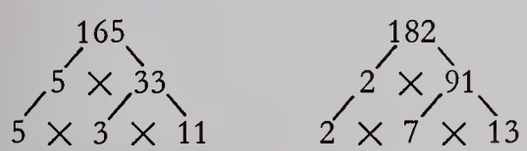
Background

The practice exercises on this page cover a variety of skills and ideas that have been presented in Chapters 1 and 2. They serve as a brief cumulative review of the following topics:

(a) factors of a number
(b) use of the factor tree
(c) changing from base ten to base five
(d) union and intersection of sets
(e) rounding numbers
(f) parallel lines and perpendicular lines
(g) intersection of sets of points
(h) ordering rational numbers

The enrichment exercises at the bottom of the page pertain to the idea of two numbers being *relatively prime*. Two numbers are said to be relatively prime if their greatest common factor is 1. This concept has application to the work in the chapter because when we name a number with a fraction in simplest form we are actually naming it with a fraction showing a numerator and denominator which are relatively prime. This idea can be used in defining a fraction in simplest form.

One method of determining whether two numbers are relatively prime is to compare their prime factorizations and look for common factors. To find if 165 and 182 are relatively prime:



The two numbers have no common factors other than 1 so they are relatively prime.

Pre-Book Lesson

• If a brief review of the topics listed in the Background section is held before the written work is assigned, encourage the pupils to ask questions about ideas which they do not completely understand.

• If you wish, you may use these exercises as a diagnostic test. Another possible use is an open-book exercise in which the pupils who are not sure how to do certain examples may use the index to find the topic and restudy the page on which the topic was introduced. This might work well with *more capable pupils*. *Slower learners* would probably need some help in using the index and in restudying the appropriate pages.

Using the Text Page

• Correct the work as soon as it is completed so that pupils with the same weaknesses may be grouped for remedial work. Assign Extra Examples or special selected examples to be worked with your assistance.

• *More capable pupils* who do not need remedial work may work the enrichment exercises while you are supervising the remedial work.

Individualizing Instruction

• The enrichment exercises at the bottom of the page are self-explanatory and the *more capable pupils* should be encouraged to try them on their own. If they finish their work early, they may be directed to work Supplementary Activities.

• In giving individual help to pupils, it may work as well to have some of the *more capable pupils* help the *slower learners* correct their mistakes and guide them in remedial work if the situation permits.

• If the class has little difficulty with the exercises, you may wish to discuss with *all pupils* the exercises at the bottom of the page. These examples and others similar to them may be worked at the board.

Pupil's Objectives

(a) To practice differentiating between addition and subtraction as operations to use in solving problems; and (b) to solve problems requiring more than one operation and involving fractions.

Background

Verbal problems provide an opportunity to employ basic arithmetic skills in a variety of situations and may be considered as opportunities for motivated drill. Although we are interested in whether the pupil obtains a correct or incorrect answer, we are *more* interested in whether he is learning how to analyze data and how to select the proper method of solution.

When evaluating the work done in solving problems, do not be as concerned with "Did he get the right answer?" as with "Did he select the right operation? Did he write a correct mathematical sentence?" The exercises at the bottom of the page will furnish sufficient practice at computation to evaluate this part of his mathematical ability.

Pre-Book Lesson

• Briefly review the four mathematical operations, asking for examples of problems requiring each operation. Help the pupils to state generalizations similar to:

a. add for a total when addends are unlike;

b. multiply for a total when the addends are alike;

c. multiply to find the total number of elements when the number of equivalent sets and the number of elements in each set are given;

d. subtract to find a missing addend when the sum and one addend are given;

e. subtract to find how much greater or less one quantity is than another;

f. divide to find a missing factor when the product and one factor are given.

• Before assigning the written work, explain the importance of selecting the correct operation and writing a correct mathematical sentence in solving the problems at the top of the page. Review the use of parentheses in solving two-step problems. Explain that the purpose of the exercises at the top of the page is to practice or evaluate the pupil's knowledge of *how* to solve problems, while the exercises at the bottom of the page pertain to computational skill.

Using the Text Page

During the oral work, have the pupils try to justify the operation they have selected in each problem, whether it is the correct operation or an incorrect one. When they select an incorrect operation, guide them through questioning to understanding why the operation they selected will not correctly solve the problem. Vocabulary and reading difficulties should be clarified at this point.

Individualizing Instruction

• *All pupils* may be asked to explain possible differences between problem situations for which addition is the correct operation and problem situations for which multiplication is the correct operation. Do the same for the operations addition and subtraction and for the operations multiplication and division.

• Give special attention to the *slower learners* in diagnosing and correcting difficulties in problem-solving. If necessary, do more oral work with them, having them determine what information is given, what question is asked, what operation is appropriate, and what mathematical sentence will result in the correct answer.

• Write on the board mathematical sentences such as $2 + \left(\frac{2}{3} - \frac{1}{4}\right) = n$ and $3 - \left(\frac{5}{6} - \frac{2}{5}\right) = n$, and ask *more capable pupils* to write problems for each mathematical sentence.

Solving Problems

[O]

For each of Ex. 1–5, tell what operation or operations to use to find the answer for the problem.

1. At a candy shop, Betty bought $\frac{3}{8}$ lb. of chocolate peppermints and Sue bought $\frac{1}{4}$ lb. They can buy $\_?\frac{5}{8}$ lb. of chocolate peppermints in all? Addition; $n=\frac{3}{8}+\frac{1}{4}$

2. With 25¢, John can buy either $\frac{5}{8}$ lb. of lemon drops or $\frac{1}{4}$ lb. of fudge. He can buy $\_?\frac{3}{8}$ lb. more of lemon drops than of fudge? Subtraction; $n=\frac{5}{8}-\frac{1}{4}$

3. Carl walks $\frac{2}{5}$ mi. to the school-bus stop. Then he rides in the bus for $\frac{1}{2}$ mi. That is how many miles in all? Addition; $n=\frac{2}{5}+\frac{1}{2}$; $\frac{9}{10}$

4. One day, Marge spent $\frac{1}{6}$ hr. on her music lesson in the morning and $\frac{1}{4}$ hr. in the afternoon. How much more time did she need to practice to meet her goal of $\frac{1}{2}$ hr. for that day?
Addition and subtraction; $n=\frac{1}{2}-\left(\frac{1}{6}+\frac{1}{4}\right)$; $\frac{1}{12}$ hr.

5. Alex bought two candy bars weighing $\frac{3}{8}$ lb. and $\frac{5}{16}$ lb. How much less than one pound of candy was that?
Addition and subtraction; $n=1-\left(\frac{3}{8}+\frac{5}{16}\right)$; $\frac{5}{16}$ lb.

[W]

Write and solve a mathematical sentence for Ex. 1–5.

♦ **Extra Problems.** Set 142.

To Keep in Practice

[W]

Ex. 1–6. Copy and multiply.

1.	2.	3.	4.	5.	6.
234	5,062	689	$14.75	$375.20	1,892
79	35	343	15	9	587
18,486	177,170	236,327	$221.25	$3,376.80	1,110,604

Ex. 7–11. Divide.

7. $37\overline{)789}$ 21, R12 8. $56\overline{)5{,}829}$ 104, R5 9. $9\overline{)\$109.89}$ $12.21, R0 10. $736\overline{)8{,}498}$ 11, R402 11. $94\overline{)76{,}098}$ 809, R52

Ex. 12–17. Find the number represented by the letter.

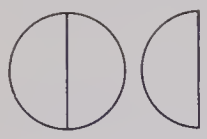
12. $n=\frac{5}{7}-\left(\frac{3}{14}+\frac{5}{14}\right)$ $\frac{1}{7}$ 13. $x-\frac{5}{21}=\frac{3}{7}$ $\frac{2}{3}$ 14. $n=\frac{1}{7}+\left(\frac{1}{3}+\frac{2}{21}\right)$ $\frac{4}{7}$

15. $n=\left(\frac{7}{9}+\frac{2}{5}\right)-\frac{3}{15}$ $\frac{44}{45}$ 16. $\frac{23}{25}-n=\frac{3}{5}$ $\frac{8}{25}$ 17. $y=\frac{4}{7}+\frac{1}{6}$ $\frac{31}{42}$

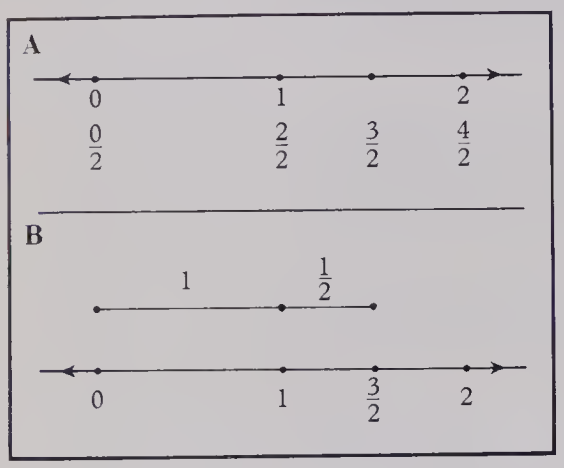
* Avoid saying "mixed number." Note that fractions and mixed forms are numerals

Rational Numbers Greater Than One

Resurvey; mixed forms [O]



1. How many halves are shown at the left? 3 To show three halves, we may write, $\frac{?}{2}$. 3



2. Box A. The number 0 has been renamed $\frac{?}{2}$; 0 1 has been renamed $\frac{?}{2}$; 2 and 2 has been renamed $\frac{?}{2}$. 4 Since the point labeled $\frac{3}{2}$ is shown to the right of 1 and to the _?_ (left) of 2, $\frac{3}{2}$ is _?_ (greater) than 1 and is _?_ (less) than 2.

3. Box B. Explain how the number-line picture shows that $\frac{3}{2}$ names the same rational number as $1 + \frac{1}{2}$. (The line segment showing $1 + \frac{1}{2}$ ends at $\frac{3}{2}$.) Another way to name this number is to write, $1\frac{1}{2}$. Such a numeral is called a *mixed form.* *

C

$$\frac{9}{4} = \frac{4}{4} + \frac{4}{4} + \frac{1}{4}$$
$$= 1 + 1 + \frac{1}{4}$$
$$= 2 + \frac{1}{4}$$
$$\frac{9}{4} = 2\frac{1}{4}$$

D

$$3\frac{2}{5} = 3 + \frac{2}{5}$$
$$= \frac{15}{5} + \frac{2}{5}$$
$$= \frac{17}{5}$$

4. Box C. Explain each step shown. Is it true that the mixed form $2\frac{1}{4}$ names the same number as the fraction $\frac{9}{4}$? Yes

5. At the board, find a mixed form as in box C for each of Ex. a–d. **a.** $\frac{5}{4}$ $1\frac{1}{4}$ **b.** $\frac{7}{5}$ $1\frac{2}{5}$ **c.** $\frac{12}{5}$ $2\frac{2}{5}$ **d.** $\frac{11}{3}$ $3\frac{2}{3}$

6. Box D. To find a fraction for the number named by the mixed form $3\frac{2}{5}$, we may first show _?_ 3 + $\frac{?}{5}$. 2 Then we may rename 3 with the fraction $\frac{?}{5}$. 15 Explain each step shown.

7. At the board, find a fraction as in box D for each of Ex. a–e.
a. $2\frac{1}{3}$ $\frac{7}{3}$ **b.** $3\frac{1}{2}$ $\frac{7}{2}$ **c.** $5\frac{1}{4}$ $\frac{21}{4}$ **d.** $4\frac{3}{5}$ $\frac{23}{5}$ **e.** $12\frac{13}{15}$ $\frac{193}{15}$

E

$\frac{9}{4}$ = _?_

$$4\overline{)9}$$
$$2$$
$$\underline{8}$$
$$R\ 1$$

$\frac{9}{4} = 2\frac{1}{4}$

8. Another way to find a mixed form for the number $\frac{9}{4}$ is to divide as shown in box E. Explain each step shown. To find a mixed form, the numerator was divided by the _?_ (denominator).

9. At the board, find mixed forms for Ex. a–e by using division. Divide mentally if you can.
a. $\frac{5}{2}$ $2\frac{1}{2}$ **b.** $\frac{7}{4}$ $1\frac{3}{4}$ **c.** $\frac{13}{3}$ $4\frac{1}{3}$ **d.** $\frac{15}{2}$ $7\frac{1}{2}$ **e.** $\frac{7}{6}$ $1\frac{1}{6}$

Teaching Pages 110 and 111

Pupil's Objective

To review naming rational numbers greater than one with fractions and mixed forms.

Background

The set of rational numbers is ordered. Given any two different rational numbers, one is less than the other. The set of rational numbers may be considered as being the union of two distinct subsets, one consisting of all rational numbers less than one, and the other consisting of all rational numbers equal to or greater than one.

Any rational number less than one is named by a fraction showing a numerator less than its denominator. Any rational number equal to or greater than one is named by a fraction showing a numerator equal to or greater than its denominator. The fractions for these numbers were formerly called *proper fractions* and *improper fractions*. These terms were somewhat misleading and have fallen into disuse.

Every rational number that is named by a fraction showing a numerator greater than or equal to its denominator is either a whole number or a number that can be expressed as the sum of a whole number and a rational number less than one. For example,

$$\begin{aligned} \tfrac{32}{9} &= \tfrac{27}{9} + \tfrac{5}{9} \\ &= 3 + \tfrac{5}{9} \end{aligned}$$

This sum may be abbreviated by omitting the sign of operation and may be written $3\frac{5}{9}$. In the past, names for rational numbers in this form have been called *mixed numbers*. However, there is nothing mixed about the *number* being represented. $3\frac{5}{9}$ names a unique rational number, represented by a single, unique position on the number line. The only thing that is mixed is the *numeral*, which consists of a whole number name and a fraction. It is, therefore, more nearly correct to refer to them as *mixed numerals* or *mixed forms*.

Although the plus sign (+) is omitted when naming a rational number shown in mixed form, the sign is inferred by the word "and" when reading the numeral, as "three *and* five ninths."

If the numerator named by a fraction is a multiple of the denominator, the fraction names a whole number.

$$\tfrac{11}{11} = 1 \qquad \tfrac{9}{3} = 3 \qquad \tfrac{36}{12} = 3 \qquad \tfrac{72}{4} = 18$$

To rename a number greater than one with a mixed form, the number may be expressed as the sum of two numbers each with the same denominator as the given number. The numerator shown by one of the fractions will be the greatest multiple of the denominator that does not exceed the numerator shown by the original fraction.

$$\tfrac{32}{9} = \tfrac{27}{9} + \tfrac{5}{9}$$

The fraction that names a whole number is then expressed as a whole number and the sign of operation is omitted.

$$\tfrac{32}{9} = 3\tfrac{5}{9}$$

Finding the greatest multiple of the denominator that does not exceed the numerator shown by the given fraction is actually division, with the remainder forming the numerator for the fraction part of the mixed form. This procedure may also be described as "Divide the numerator by the denominator to find the whole number for the mixed form, then express the remainder as a numerator for a fraction naming the denominator for the original fraction."

$$\tfrac{32}{9} = 9\overline{)\,32}\,\begin{matrix}3\\ \underline{27}\\ 5\end{matrix} = 3\tfrac{5}{9}$$

To change from a mixed form to a fraction form, the whole number named by the mixed form is renamed with a fraction showing the same denominator as the fraction in the mixed form, then the implied addition is performed:

$$\begin{aligned} 2\tfrac{7}{9} &= \tfrac{18}{9} + \tfrac{7}{9} \\ &= \tfrac{25}{9} \end{aligned}$$

Pre-Book Lesson

- The pupils may be given a problem such as the number of candy bars each child would receive

if there were three candy bars and two children. Some pupils will probably suggest $1\frac{1}{2}$ as an answer. Use cardboard to illustrate the example. The question as to whether or not $1\frac{1}{2}$ is a rational number can then be raised and discussed.

• Have some models to represent $3\frac{3}{8}$ pies. Let pupils use a piece of cardboard representing $\frac{1}{8}$ and find how many $\frac{1}{8}$'s there are in $3\frac{3}{8}$. Point out that each $\frac{8}{8}$ is equal to 1. Repeat the procedure with other examples. Try to have pupils work out a method for renaming from fractions to mixed forms.

Using the Text Pages

• Ex. 2–5. You may use cardboard models as well as models on the chalkboard to illustrate the numbers.

• Ex. 6. Notice that the frequently used short cut is not introduced here.

$$3\frac{2}{5} = \frac{(5 \times 3) + 2}{5} = \frac{17}{5}$$

The method shown in box D gives the pupil full understanding of the mathematics involved. The short cut often becomes nothing more than a poorly understood rote, mechanical procedure.

• Ex. 10. Have the pupils compare the procedure shown in box F with that shown in box C and point out the advantages of the method shown in box F especially as used with greater numbers. Explain that this procedure always results in an answer in simplest form. Some material from the Background which relates multiples of the denominator to division may be used at the teacher's discretion.

• Ex. 12. These exercises help point out the fact that the quotient of any two whole numbers (division by zero excluded) is a rational number whose numerator is the dividend and whose denominator is the divisor. $a \div b = \frac{a}{b}$ where a is a whole number and b is a counting number.

Individualizing Instruction

• *All pupils* may be given sets of rational numbers named with various forms, and be asked to arrange them in order of increasing value. For example: $\{2, 1\frac{3}{4}, 25 \div 18, \frac{20}{10}\}$

• *Slower learners* may need oral assistance and may need to use models when working Ex. 13–52. These pupils can very easily become confused when working with a factors-product relationship and fractions.

NOTES

10. Explain the work in box F. What was done to express the answer in simplest mixed form? ∨ When an answer is named by a mixed form, always have the fraction in simplest form unless otherwise directed.

The numerator was divided by the denominator.

F

$\frac{270}{12} = \text{-?-}$

$$\begin{array}{r} 22 \\ 12\overline{)270} \\ \underline{24} \\ 30 \\ \underline{24} \\ \text{R}\quad 6 \end{array}$$

$\frac{270}{12} = 22\frac{6}{12}$

$= 22\frac{1}{2}$

11. In box E, a mixed form was found for $\frac{9}{4}$ by dividing 9 by 4. In box F, a mixed form was found for $\frac{270}{12}$ by dividing -?- by -?-.

270 12

$2\frac{1}{4} = 9 \div \text{-?-}^{4}$ $22\frac{1}{2} = 270 \div \text{-?-}^{12}$

12. Express each of Ex. a–d with a mixed form.

a. $16 \div 3$ $5\frac{1}{3}$ **b.** $92 \div 5$ $18\frac{2}{5}$ **c.** $126 \div 12$ $10\frac{1}{2}$ **d.** $318 \div 43$ $7\frac{17}{43}$

[W]

Ex. 13–24. Find a mixed form as in box E or box F. Divide mentally when you can.

13. $\frac{7}{4}$ $1\frac{3}{4}$ **16.** $\frac{44}{15}$ $2\frac{14}{15}$ **19.** $\frac{505}{12}$ $42\frac{1}{12}$ **22.** $\frac{184}{9}$ $20\frac{4}{9}$

14. $\frac{14}{3}$ $4\frac{2}{3}$ **17.** $\frac{233}{16}$ $14\frac{9}{16}$ **20.** $\frac{400}{15}$ $26\frac{2}{3}$ **23.** $\frac{562}{18}$ $31\frac{2}{9}$

15. $\frac{23}{11}$ $2\frac{1}{11}$ **18.** $\frac{270}{25}$ $10\frac{4}{5}$ **21.** $\frac{526}{15}$ $35\frac{1}{15}$ **24.** $\frac{110}{24}$ $4\frac{7}{12}$

Ex. 25–34. Rename with a fraction.

25. $3\frac{2}{7}$ $\frac{23}{7}$ **26.** $9\frac{1}{10}$ $\frac{91}{10}$ **27.** $6\frac{2}{3}$ $\frac{20}{3}$ **28.** $5\frac{6}{7}$ $\frac{41}{7}$ **29.** $10\frac{3}{5}$ $\frac{53}{5}$

30. $4\frac{3}{16}$ $\frac{67}{16}$ **31.** $12\frac{5}{12}$ $\frac{149}{12}$ **32.** $3\frac{11}{15}$ $\frac{56}{15}$ **33.** $15\frac{21}{25}$ $\frac{396}{25}$ **34.** $9\frac{17}{24}$ $\frac{233}{24}$

Ex. 35–40. Find the number represented by n. Express the answer with a mixed form.

35. $n = 7 \div 4$ $1\frac{3}{4}$ **36.** $n = \frac{18}{4}$ $4\frac{1}{2}$ **37.** $262 \div 14 = n$ $18\frac{5}{7}$

38. $n = 43 \div 20$ $2\frac{3}{20}$ **39.** $n = \frac{167}{13}$ $12\frac{11}{13}$ **40.** $n = 451 \div 4$ $112\frac{3}{4}$

Ex. 41–46. Copy and complete.

41. $6 = 5\frac{?}{6}^{6}$

42. $7\frac{10}{12} = 7 + \frac{?}{12}^{10} = 7 + \frac{?}{6}^{5} = 7\frac{?}{6}^{5}$

43. $9 \div 5 = \frac{?}{5}^{9} = 1\frac{?}{5}^{4}$

44. $6\frac{5}{8} = 6\frac{?}{16}^{10} = 5\frac{?}{16}^{26}$

45. $5\frac{12}{8} = 5\frac{?}{?}^{3}_{2} = 6\frac{?}{?}^{1}_{2}$

46. $10\frac{5}{8} = 10\frac{?}{24}^{15} = 9\frac{?}{24}^{39}$

Ex. 47–52. Write *T* or *F*.

47. $3\frac{2}{7} < \frac{21}{7}$ F **48.** $\frac{32}{9} > 3$ T **49.** $\frac{1}{5} + \frac{7}{5} = 1\frac{3}{5}$ T

50. $\frac{100}{15} < 6$ F **51.** $9\frac{1}{10} = 9\frac{3}{30}$ T **52.** $\frac{23}{6} = 3 + \frac{5}{6}$ T

* Review finding the L.C.D.

Adding Numbers Greater Than One

Renaming with fractions [O]

A

$n = 1\frac{1}{2} + 2\frac{1}{4}$
$= \frac{3}{2} + \frac{9}{4}$ (1)
$= \frac{6}{4} + \frac{9}{4}$ (2)
$n = \frac{15}{4}$, or $3\frac{3}{4}$ (3)

B

$n - 2\frac{2}{5} = 3\frac{1}{3}$
$n = 3\frac{1}{3} + 2\frac{2}{5}$
$= \frac{10}{3} + \frac{12}{5}$
$= \frac{50}{15} + \frac{36}{15}$
$n = \frac{86}{15}$, or $5\frac{11}{15}$

1. Box A shows one way to find a mixed form for the number represented by n in $n = 1\frac{1}{2} + 2\frac{1}{4}$.

Step 1: Rename the addends with fractions.

$1\frac{1}{2} = \_?\_ + \frac{1}{2} = \frac{?^2}{2} + \frac{1}{2} = \frac{?^3}{2}$
$2\frac{1}{4} = \_?\_ + \frac{1}{4} = \frac{?^8}{4} + \frac{1}{4} = \frac{?^9}{4}$

Step 2: Express $\frac{3}{2}$ and $\frac{9}{4}$ with fractions showing the same denominator.

* The L.C.D. of $\frac{3}{2}$ and $\frac{9}{4}$ is $\_?\_$. $\frac{3}{2} = \frac{?^2 \times 3}{2 \times 2} = \frac{?^6}{4}$

Step 3: $\frac{6}{4}$ and $\frac{9}{4}$ show the same denominator so to find the sum, we $\_?^{\text{add}}\_$ the numerators.

To add two numbers named with mixed forms, we may first rename the numbers with fractions.

2. $n = 2\frac{2}{3} + 1\frac{4}{5}$
$= \frac{8}{3} + \frac{9}{5}$
$= \frac{40}{15} + \frac{27}{15}$
$n = \frac{67}{15}$, or $4\frac{7}{15}$

2. At the board, solve $n = 2\frac{2}{3} + 1\frac{4}{5}$ as in box A.

3. Box B. In the first sentence, does n represent the sum or an addend?

a. The L.C.D. of $\frac{10}{3}$ and $\frac{12}{5}$ is $\_?^{15}\_$.

b. Explain the work shown. Key ideas: The addends are renamed to show L.C.D., numerators added, and the sum renamed with a mixed form.

4. At the board, solve each of Ex. a–f as in the boxes.

a. $n = 1\frac{2}{3} + 2\frac{3}{4}$ $4\frac{5}{12}$ b. $n - 2\frac{3}{4} = 5\frac{1}{3}$ $8\frac{1}{12}$ c. $n = 4\frac{3}{4} + 7\frac{1}{2}$ $12\frac{1}{4}$
d. $x = 3\frac{2}{5} + 1\frac{1}{6}$ $4\frac{17}{30}$ e. $n - 1\frac{3}{8} = 2\frac{2}{3}$ $4\frac{1}{24}$ f. $y = 6 + 4\frac{3}{7}$ $10\frac{3}{7}$

[W]

Ex. 5–16. Solve. First rename the addends with fractions.

5. $n = 2\frac{3}{4} + 1\frac{1}{5}$ $3\frac{19}{20}$
6. $n = 2\frac{6}{7} + 3\frac{1}{7}$ 6
7. $y = 5\frac{3}{10} + 7\frac{1}{5}$ $12\frac{1}{2}$
8. $n = 5\frac{2}{3} + 2\frac{1}{6}$ $7\frac{5}{6}$
9. $x - 1\frac{1}{2} = 3\frac{1}{3}$ $4\frac{5}{6}$
10. $n = 6\frac{5}{12} + 7\frac{1}{6}$ $13\frac{7}{12}$
11. $n = 3\frac{5}{8} + 2\frac{1}{8}$ $5\frac{3}{4}$
12. $n - 7\frac{2}{7} = 8\frac{1}{3}$ $15\frac{13}{21}$
13. $2\frac{2}{5} = x - 8\frac{2}{15}$ $10\frac{8}{15}$
14. $n - 12\frac{14}{100} = 32\frac{93}{100}$ $45\frac{7}{100}$
15. $18\frac{51}{64} + 11\frac{3}{8} = x$ $30\frac{11}{64}$
16. $122\frac{7}{8} + 83\frac{5}{12} = n$ $206\frac{7}{24}$

Ex. 17–26. Rename with a mixed form.

17. $\frac{24}{10}$ $2\frac{2}{5}$ 18. $\frac{76}{8}$ $9\frac{1}{2}$ 19. $\frac{22}{16}$ $1\frac{3}{8}$ 20. $\frac{139}{13}$ $10\frac{9}{13}$ 21. $\frac{153}{12}$ $12\frac{3}{4}$
22. $\frac{183}{32}$ $5\frac{23}{32}$ 23. $\frac{148}{64}$ $2\frac{5}{16}$ 24. $\frac{399}{33}$ $12\frac{1}{11}$ 25. $\frac{324}{24}$ $13\frac{1}{2}$ 26. $\frac{1,025}{125}$ $8\frac{1}{5}$

Teaching Page 112

Pupil's Objective

To learn to add rational numbers greater than one by first changing from mixed forms to fractions.

Background

There are two basic methods of performing the addition of rational numbers when one or more of the addends is shown in mixed form. One method involves changing from mixed forms to fractions. The second method involves applying the commutative and associative properties of addition to add the whole-number parts for the mixed forms and the fraction-number parts for the mixed forms independently. The first of these two methods is the one introduced on this text page. The second method is introduced on page 113.

To add two rationals, one or more of which are expressed in mixed form, the numbers may first be renamed with fractions and then the addition performed by the method described on pages 103 and 104.

The procedure may be described by the following steps:

(1) Rename with fractions.

(2) Determine the least common denominator of the addends and express the addends with fractions showing denominators that are least common denominators.

(3) Perform the addition applying the principle that $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$.

(4) If the numerator of the resulting sum is greater than the denominator, express the sum with a mixed form.

(1) $1\frac{2}{3} + 4\frac{1}{2} = \frac{5}{3} + \frac{9}{2}$

(2) $= \frac{10}{6} + \frac{27}{6}$

(3) $= \frac{37}{6}$

(4) $= 6\frac{1}{6}$

Pre-Book Lesson

Review addition with fractions, referring back to pages 103 and 104 if necessary.

Using the Text Page

• Ex. 1. It is usually preferred that the final answer be expressed with a mixed form. This is particularly true when the pupil is asked to first estimate the answer, then calculate it and compare the calculated answer with the estimate. It is usually much easier to compare rational numbers greater than one when they are expressed with mixed forms than when they are expressed with fractions.

• Ex. 4–16. The pupil should recognize these as expressions of the addends-sum relationship. It must first be determined whether the variable represents the sum or the missing addend.

Individualizing Instruction

• *More capable pupils* may be lead to the realization that this is a very cumbersome method of adding rational numbers expressed with mixed forms when the whole-number part for the mixed forms is great or when the denominator shown by the fraction part of the mixed form is great. Ex. 14–16 illustrate this quite clearly. Pupils may even have suggestions for a more efficient method of performing the addition.

• *All pupils* may be asked to make up word problems involving an addends-sum relationship that may be expressed by one of the mathematical sentences in Ex. 4–16.

• *Slower learners* may need to work orally with the teacher on such examples as Ex. 14–26. They may benefit by using cardboard models after they have renamed with fractions.

Teaching Page 113

Pupil's Objective

To learn to add rational numbers greater than one using the vertical form for addition.

Background

The development of the vertical form for addition of rational numbers involves application of the commutative and associative properties of addition.

By definition, the numeral $6\frac{3}{7}$ means $6 + \frac{3}{7}$, and the numeral $9\frac{2}{7}$ means $9 + \frac{2}{7}$. Therefore,

$$6\tfrac{3}{7} + 9\tfrac{2}{7} = (6 + \tfrac{3}{7}) + (9 + \tfrac{2}{7})$$

Addition is commutative and associative, so we can write

$$\begin{aligned} 6\tfrac{3}{7} + 9\tfrac{2}{7} &= (6 + 9) + (\tfrac{3}{7} + \tfrac{2}{7}) \\ &= 15 + \tfrac{5}{7} \\ &= 15\tfrac{5}{7} \end{aligned}$$

This may be written in vertical form:

$$\begin{aligned} 6\tfrac{3}{7} &= 6 + \tfrac{3}{7} \\ \underline{9\tfrac{2}{7}} &= \underline{9 + \tfrac{2}{7}} \\ & \quad 15 + \tfrac{5}{7} = 15\tfrac{5}{7} \end{aligned}$$

or more simply as:

$$\begin{array}{r} 6\frac{3}{7} \\ \underline{9\frac{2}{7}} \\ 15\frac{5}{7} \end{array}$$

A mixed form is not in simplest mixed form if the fraction shows a numerator equal to or greater than the denominator. For example, in

$$\begin{array}{r} 6\frac{5}{7} \\ \underline{11\frac{4}{7}} \\ 17\frac{9}{7} \end{array}$$

the answer is not in simplest mixed form. The fraction must be written in mixed form, and then the implied addition performed.

$$17\tfrac{9}{7} = 17 + 1\tfrac{2}{7} = 18\tfrac{2}{7}$$

Pre-Book Lesson

• Have the pupils perform several additions, some using the horizontal form and some using the vertical form. Have the pupils express their preference and the reasons for this preference.

• Have pupils practice adding some rational numbers and show the answers in simplest form. Have them change the examples by using the commutative and associative properties. Ask if they might add numbers shown in mixed form using the vertical form.

Using the Text Page

• Ex. 1. To demonstrate the validity of the vertical form and to show the advantage of the vertical form over the horizontal form, this addition may be explained by use of the following:

$$\begin{aligned} 2\tfrac{3}{5} + 4\tfrac{1}{5} &= (2 + \tfrac{2}{5}) + (4 + \tfrac{1}{5}) \\ &= (2 + 4) + (\tfrac{2}{5} + \tfrac{1}{5}) \\ &= 6 + \tfrac{3}{5} \\ &= 6\tfrac{3}{5} \end{aligned}$$

• Ex. 3. The work may be shown by use of the horizontal form:

$$\begin{aligned} 3\tfrac{3}{5} + 6\tfrac{4}{5} &= (3 + \tfrac{3}{5}) + (6 + \tfrac{4}{5}) \\ &= (3 + 6) + (\tfrac{3}{5} + \tfrac{4}{5}) \\ &= 9 + \tfrac{7}{5} \\ &= 9 + 1\tfrac{2}{5} \\ &= 10\tfrac{2}{5} \end{aligned}$$

• Ex. 15. Have the pupils attempt to solve this using the method of renaming the addends with fractions. Then have them attempt to solve it using the method in box C. This should impress them as to the advantage of the method shown in box C.

Individualizing Instruction

• *More capable pupils* may be asked to perform the addition shown in box A by using the horizontal form and justifying each step in the procedure, indicating where the commutative and associative properties of addition are applied.

• The teacher may wish to work Ex. 10–18 orally with the *slower learners* using cardboard models to show fraction numbers and how the numbers can be renamed for easier adding.

* Emphasize that since the denominators were not the same, we had to rename to show a common denominator before performing the operation.

Vertical Form for Addition

[o]

1. Box A shows, in vertical form, a way to add with mixed forms without renaming the addends. $\frac{3}{5} + \frac{1}{5} = \_?\frac{4}{5}$, $2 + 4 = \_?6$, so $2\frac{3}{5} + 4\frac{1}{5} = \_?6\frac{4}{5}$

A

Add

$2\frac{3}{5}$

$4\frac{1}{5}$

$6\frac{4}{5}$

2. Add and give the missing numerals.

a. $3\frac{5}{12}$ / $4\frac{2}{12}$ / $\_?\frac{7}{12}$ **b.** $5\frac{3}{8}$ / $7\frac{2}{8}$ / $12\frac{?}{8}$ **c.** $12\frac{2}{7}$ / $3\frac{3}{7}$ / $\_?\frac{15}{7}$ **d.** 5 / $2\frac{1}{3}$ / $\_?\frac{?}{3}$ **e.** $5\frac{5}{12}$ / 3 / $\_?\frac{?}{12}$

3. For the addition shown in box B, the sum is not expressed in simplest mixed form. On the board, show the work and explain each step for expressing $9\frac{7}{5}$ in simplest mixed form. See below.

B

Add

$3\frac{3}{5}$

$6\frac{4}{5}$

$9\frac{7}{5}$

4. On the board, express each of the following in simplest mixed form.

a. $12\frac{4}{3}$ $13\frac{1}{3}$ **b.** $7\frac{20}{11}$ $8\frac{9}{11}$ **c.** $14\frac{7}{2}$ $17\frac{1}{2}$ **d.** $20\frac{8}{3}$ $22\frac{2}{3}$ **e.** $15\frac{14}{3}$ $19\frac{2}{3}$

The addends are renamed to show a common denominator. Then the addition is done.

* 5. Explain the work in box C. ∧ Why was $1\frac{1}{2}$ renamed $1\frac{2}{4}$? ∧ Is $3\frac{3}{4}$ in simplest mixed form? Yes

4 is the common denominator.

C

$n = 1\frac{1}{2} + 2\frac{1}{4}$

Add

$1\frac{1}{2} = 1\frac{2}{4}$

$2\frac{1}{4} = 2\frac{1}{4}$

$3\frac{3}{4}$

$n = 3\frac{3}{4}$

[w]

Ex. 6–9. Copy and add.

6. $3\frac{1}{5}$ / $7\frac{3}{5}$ / $10\frac{4}{5}$ 7. $5\frac{6}{13}$ / $3\frac{1}{13}$ / $8\frac{7}{13}$ 8. $2\frac{3}{10}$ / $7\frac{7}{10}$ / 10 9. $7\frac{2}{5}$ / $8\frac{1}{2}$ / $15\frac{9}{10}$

Ex. 10–12. Solve. First rename the addends with fractions.

10. $x = 3\frac{3}{4} + 5\frac{1}{6}$ $8\frac{11}{12}$ 11. $n = 7\frac{3}{8} + 4\frac{1}{5}$ $11\frac{23}{40}$ 12. $y - 2\frac{9}{20} = 7\frac{6}{50}$ $9\frac{57}{100}$

Ex. 13–21. Solve. Use whichever method seems best for you. Remember to express each fraction in simplest form.

13. $n = 3\frac{1}{2} + 5\frac{3}{4}$ $9\frac{1}{4}$ 16. $n = 5\frac{4}{5} + 2\frac{1}{2}$ $8\frac{3}{10}$ 19. $x = 30\frac{2}{3} + 14\frac{11}{12}$ $45\frac{7}{12}$

14. $n - 3 = 5\frac{5}{6}$ $8\frac{5}{6}$ 17. $y = 7\frac{4}{5} + 3\frac{7}{10}$ $11\frac{1}{2}$ 20. $n - 4\frac{4}{5} = 13\frac{2}{3}$ $18\frac{7}{15}$

15. $y = 4\frac{5}{8} + 2\frac{2}{3}$ $7\frac{7}{24}$ 18. $12\frac{1}{2} + 3\frac{7}{10} = n$ ∨ $16\frac{1}{5}$ 21. $n - 9\frac{2}{3} = 4\frac{5}{6}$ $14\frac{1}{2}$

◆ **Extra Examples.** Sets 58–60.

3. $9\frac{7}{5} = 9 + \frac{7}{5}$

$= 9 + 1\frac{2}{5}$

$= 10\frac{2}{5}$

* Point out that there are two renaming steps; first to show a common denominator and second to show more $\frac{1}{12}$'s so that the subtraction could be done.

Subtracting with Mixed Forms

Renaming for subtraction [O]

A

$$n = 3\tfrac{1}{12} - 1\tfrac{1}{3}$$
$$= \tfrac{37}{12} - \tfrac{4}{3}$$
$$= \tfrac{37}{12} - \tfrac{16}{12}$$
$$= \tfrac{37 - 16}{12}$$
$$= \tfrac{21}{12}$$
$$n = 1\tfrac{9}{12}, \text{ or } 1\tfrac{3}{4}$$

1. Box A shows one way to solve $n = 3\frac{1}{12} - 1\frac{1}{3}$. Have both the sum and the known addend been renamed with fractions? Yes Explain each step shown.

If two numbers are named with mixed forms, then to subtract one number from the other we may first rename the numbers with fractions.

2. At the board, solve $n = 4\frac{2}{3} - 3\frac{1}{2}$ as in box A. $1\frac{1}{6}$

B

Subtract

$5\frac{4}{7}$

$2\frac{1}{7}$

$3\frac{3}{7}$

C

Subtract

$6\frac{1}{2} = 6\frac{3}{6}$

$2\frac{1}{3} = 2\frac{2}{6}$

$4\frac{1}{6}$

3. Box B shows a vertical form for finding $5\frac{4}{7} - 2\frac{1}{7}$. $\frac{4}{7} - \frac{1}{7} =$ _?$\frac{3}{7}$, $5 - 2 =$ _?3, so $5\frac{4}{7} - 2\frac{1}{7} =$ _?$3\frac{3}{7}$

4. Explain the work in box C. After renaming to show a common denominator, the subtraction is done.

5. At the board, solve $n = 7\frac{2}{5} - 3\frac{1}{3}$ by working as in box C. $4\frac{1}{5}$

D

$$n = 3\tfrac{1}{12} - 1\tfrac{1}{3}$$

Subtract

$$3\tfrac{1}{12} = 3\tfrac{1}{12} = 2\tfrac{13}{12}$$
$$1\tfrac{1}{3} = 1\tfrac{4}{12} = 1\tfrac{4}{12}$$
$$1\tfrac{9}{12}, \text{ or } 1\tfrac{3}{4}$$

* 6. In box D, Ex. 1 was solved by subtracting in vertical form.

a. Can $\frac{4}{12}$ be subtracted from $\frac{1}{12}$? No

b. $3\frac{1}{12}$ must be renamed to show more $\frac{1}{12}$'s.

$3\frac{1}{12} = (2 + 1) + \frac{1}{12} = 2 + (\frac{?}{12}^{12} + \frac{1}{12}) = 2\frac{?}{12}^{13}$

c. Why was this renaming necessary? To show more $\frac{1}{12}$'s.

d. Explain each step shown in box D.

Key idea: Rename $3\frac{1}{12}$ to show more $\frac{1}{12}$'s so that the subtraction can be done.

[W]

Ex. 7–18. Copy and subtract. Remember to express each fraction in simplest form.

7.	8.	9.	10.	11.	12.
$8\frac{3}{5}$	$6\frac{4}{9}$	$6\frac{3}{5}$	7	$8\frac{2}{3}$	$5\frac{6}{7}$
$2\frac{1}{5}$	$2\frac{1}{9}$	3	$5\frac{1}{2}$	$4\frac{3}{4}$	$1\frac{6}{7}$
$6\frac{2}{5}$	$4\frac{1}{3}$	$3\frac{3}{5}$	$1\frac{1}{2}$	$3\frac{11}{12}$	4
13.	**14.**	**15.**	**16.**	**17.**	**18.**
$2\frac{3}{5}$	$5\frac{6}{11}$	$3\frac{5}{8}$	$3\frac{7}{20}$	$7\frac{8}{15}$	$40\frac{5}{8}$
$\frac{2}{5}$	$1\frac{1}{22}$	$\frac{3}{4}$	$\frac{9}{20}$	$2\frac{11}{15}$	$31\frac{1}{4}$
$2\frac{1}{5}$	$4\frac{1}{2}$	$2\frac{7}{8}$	$2\frac{9}{10}$	$4\frac{4}{5}$	$9\frac{3}{8}$

▲ **Reteaching.** Set 10. ◆ **Extra Examples.** Sets 61–63.

Teaching Page 114

Pupil's Objective

To learn to subtract one rational number from another when one or both are named with mixed forms.

Background

If two rational numbers are named with mixed forms, the numbers may be renamed with fractions showing like denominators before subtracting. The subtraction is then performed by the method shown on pupil's page 105:

$$\begin{aligned} 3\tfrac{4}{5} &= \tfrac{19}{5} = \tfrac{57}{15} \\ \underline{1\tfrac{1}{3} = \tfrac{4}{3} = \tfrac{20}{15}} \\ \tfrac{37}{15} \text{ or } 2\tfrac{7}{15} \end{aligned}$$

A second method of subtracting one rational number from another when each is named with a mixed form involves subtracting the whole numbers named in the mixed forms and then subtracting the fraction-number parts. The mathematical justification of this method would seem to be quite simple. However, it is quite involved as becomes apparent when the subtraction is expressed using the horizontal form:

$$\begin{aligned} 18\tfrac{3}{5} - 6\tfrac{1}{5} &= (18 + \tfrac{3}{5}) - (6 + \tfrac{1}{5}) \\ &= (18 - 6) + (\tfrac{3}{5} - \tfrac{1}{5}) \\ &= 12 + \tfrac{2}{5}, \text{ or } 12\tfrac{2}{5} \end{aligned}$$

The teacher is advised against attempting to justify this method of subtraction, except intuitively in terms with which the pupil is already familiar. It can be shown that this method is analogous to subtracting one whole number from another.

$$\begin{aligned} 54 - 31 &= (5 \text{ tens} + 4 \text{ ones}) - (3 \text{ tens} + 1 \text{ one}) \\ &= (5 \text{ tens} - 3 \text{ tens}) + (4 \text{ ones} - 1 \text{ one}) \\ &= (2 \text{ tens}) + (3 \text{ ones}) \\ &= 23 \end{aligned}$$

When written in vertical form this becomes

$$\begin{aligned} 54 &= 5 \text{ tens} + 4 \text{ ones} \\ \underline{31} &= \underline{3 \text{ tens} + 1 \text{ one}} \\ 2 \text{ tens} + 3 \text{ ones, or } 23 \end{aligned}$$

This is analogous to the subtraction $18\tfrac{3}{5} - 6\tfrac{1}{5}$:

$$\begin{aligned} 18\tfrac{3}{5} &= 18 + \tfrac{3}{5} \\ \underline{6\tfrac{1}{5}} &= \underline{6 + \tfrac{1}{5}} \\ 12 + \tfrac{2}{5}, \text{ or } 12\tfrac{2}{5} \end{aligned}$$

When the fraction in the mixed form for the sum is less than the fraction in the mixed form for the known addend, a renaming of the sum is necessary. The subtraction $18\tfrac{1}{5} - 6\tfrac{3}{5}$ can be shown to be analogous to the subtraction $36 - 18$:

$$\begin{aligned} 36 &= \overset{2}{\cancel{3}} \text{ tens} + \overset{16}{\cancel{6}} \text{ ones} \\ \underline{18} &= \underline{1 \text{ ten} + 8 \text{ ones}} \\ 1 \text{ tens} + 8 \text{ ones} &= 18 \end{aligned}$$

$$\begin{aligned} 18\tfrac{1}{5} &= \overset{17}{\cancel{18}} + \overset{6}{\cancel{\tfrac{1}{5}}} \\ \underline{6\tfrac{3}{5}} &= \underline{6 + \tfrac{3}{5}} \\ 11 + \tfrac{3}{5}, \text{ or } 11\tfrac{3}{5} \end{aligned}$$

Pre-Book Lesson

- Review the subtraction of whole numbers as shown in the Background section above.
- Have pupils work subtraction examples involving fractions showing like and unlike denominators. Have pupils change these examples by using the commutative and associative properties.

Using the Text Page

- Ex. 1. This is based on the principle $\frac{a}{c} - \frac{b}{c} = \frac{a - b}{c}$ shown on page 103. Some pupils will forget to rename the numbers so they have the same denominators. Make sure pupils do not eliminate this step in any of their work.
- Ex. 3. This can be shown to be analogous to the first subtraction for $54 - 31$ as described in the Background section.
- Ex. 4. The fractions must show common denominators in order for the operation subtraction to be performed.

Individualizing Instruction

- *All pupils* may create word problems which may be solved by any of Ex. 7–18.
- *Slower learners* may be given simple word problems involving subtraction of rational numbers using mixed forms. They may work Ex. 7–21 orally where renaming of the sum is necessary.

Teaching Page 115

Pupil's Objective

To learn to add three rational numbers named with mixed forms, using either horizontal form or vertical form.

Background

Because addition is associative, the sum of three rational numbers will be the same regardless of whether the sum of the first two numbers is added to the third, or the first number is added to the sum of the second and third numbers.

$$(\tfrac{5}{6} + 1\tfrac{3}{4}) + 2\tfrac{1}{2} = \tfrac{5}{6} + (1\tfrac{3}{4} + 2\tfrac{1}{2})$$

The addition above may be performed using the vertical form in which the whole numbers and the fraction numbers are added independently. The following justification of this method involves use of the commutative and associative properties of addition:

$$\begin{aligned} \tfrac{5}{6} + 1\tfrac{3}{4} + 2\tfrac{1}{2} &= \tfrac{5}{6} + 1 + \tfrac{3}{4} + 2 + \tfrac{1}{2} \\ &= (1 + 2) + (\tfrac{5}{6} + \tfrac{3}{4} + \tfrac{1}{2}) \\ &= 3 + (\tfrac{10}{12} + \tfrac{9}{12} + \tfrac{6}{12}) \\ &= 3 + \tfrac{25}{12} \\ &= 3 + 2 + \tfrac{1}{12} \\ &= 5\tfrac{1}{12} \end{aligned}$$

This may be written in vertical form as:

$$\begin{aligned} \tfrac{5}{6} &= \tfrac{10}{12} \\ 1\tfrac{3}{4} &= 1\tfrac{9}{12} \\ \underline{2\tfrac{1}{2}} &= \underline{2\tfrac{6}{12}} \\ 3\tfrac{25}{12} &= 3 + 2\tfrac{1}{12} \\ &= 5\tfrac{1}{12} \end{aligned}$$

Still another method may be used where the mixed form is first renamed with fractions, the common denominator found, and the additions performed. However, this method is more lengthy.

Pre-Book Lesson

• Review the use of the associative property in the addends-sum relationship. Review addition of rational numbers.

• Discuss and illustrate material from the Background together with discussion of the oral part of the page.

Using the Text Page

• Ex. 1–2. In box A is the horizontal form of the work in which the sum of the first two numbers is added to the third number. In box B is the horizontal form of the work in which the first number is added to the sum of the second and third numbers. In box C is the vertical form of the work in which the whole numbers and the numbers named by the fractions are added independently.

Pupils may be asked to show other forms for doing the same work. The mixed forms may first be renamed with fractions and then shown with common denominators.

• Ex. 3. A fourth method of showing the work involves the renaming of the addends with fractions. Of these four methods of showing the work in performing addition of rational numbers, the method shown in box C is probably the most convenient and the most commonly used.

Individualizing Instruction

• *More capable pupils* may be given a "magic square" that is partially completed and be asked to complete it so that the sum of the numbers shown in each of the rows, in each of the columns, and along each diagonal is $3\frac{3}{4}$.

	$\frac{1}{4}$	$1\frac{1}{2}$
$\frac{3}{4}$		
	$2\frac{1}{4}$	$\frac{1}{2}$

• *Slower learners* may work Ex. 4–11 with the teacher, using the mixed forms or first renaming the mixed forms with fractions. Make sure these pupils understand that both the commutative and associative properties are used when performing addition with three or more addends. The teacher may find use of fraction parts and a flannel board helpful here.*

* See 1 and 2, page xix. Both these devices will be most useful in all lessons involving fractions and rational numbers.

A

$$\begin{aligned} n &= \tfrac{3}{4} + 2\tfrac{1}{2} + 1\tfrac{1}{8} \\ &= (\tfrac{3}{4} + 2\tfrac{1}{2}) + 1\tfrac{1}{8} \\ &= (\tfrac{3}{4} + 2\tfrac{2}{4}) + 1\tfrac{1}{8} \\ &= 2\tfrac{5}{4} + 1\tfrac{1}{8} \\ &= 3\tfrac{1}{4} + 1\tfrac{1}{8} \\ &= 3\tfrac{2}{8} + 1\tfrac{1}{8} \\ n &= 4\tfrac{3}{8} \end{aligned}$$

B

$$\begin{aligned} n &= \tfrac{3}{4} + 2\tfrac{1}{2} + 1\tfrac{1}{8} \\ &= \tfrac{3}{4} + (2\tfrac{4}{8} + 1\tfrac{1}{8}) \\ &= \tfrac{3}{4} + 3\tfrac{5}{8} \\ &= \tfrac{6}{8} + 3\tfrac{5}{8} \\ &= 3\tfrac{11}{8} \\ n &= 4\tfrac{3}{8} \end{aligned}$$

Adding Three Rational Numbers

C Add

$$\begin{aligned} \tfrac{3}{4} &= \tfrac{6}{8} \\ 2\tfrac{1}{2} &= 2\tfrac{4}{8} \\ 1\tfrac{1}{8} &= 1\tfrac{1}{8} \\ 3\tfrac{11}{8}, \text{ or } &4\tfrac{3}{8} \end{aligned}$$

[O]

1. For a candy sale, Jean made $\frac{3}{4}$ lb. of fudge on Monday, $2\frac{1}{2}$ lb. on Tuesday, and $1\frac{1}{8}$ lb. on Wednesday. How many pounds of fudge did she make during that three-day period?

a. Explain how the sentence $n = \frac{3}{4} + 2\frac{1}{2} + 1\frac{1}{8}$ shows the relationship expressed in the problem. It expresses the addends-sum relationship.

b. To solve, we may work as in box A or as in box B. Explain the work in these two boxes. Two addends are added and the third addend is added to their sum. What property of addition permits us to work either as in box A or as in box B to obtain the answer? Associative

c. Explain each step in box C. The addends are shown with a common denominator. Then their sum is found. Is this an easier way to find the answer? Yes How many pounds of fudge did Jean make? $4\frac{3}{8}$

2. At the board, solve $n = 1\frac{2}{3} + \frac{5}{6} + 3\frac{7}{12}$ first as in box A, then as in box B, and then as in box C. $n = 6\frac{1}{12}$

3. Could we also solve the sentence for Ex. 2 by first renaming all the addends with fractions? Yes Do this on the board. Sample answer:

$$\begin{aligned} n &= 1\tfrac{2}{3} + \tfrac{5}{6} + 3\tfrac{7}{12} \\ &= (\tfrac{5}{3} + \tfrac{5}{6}) + \tfrac{43}{12} \\ &= (\tfrac{10}{6} + \tfrac{5}{6}) + \tfrac{43}{12} \\ &= \tfrac{15}{6} + \tfrac{43}{12} \\ &= \tfrac{30}{12} + \tfrac{43}{12} \\ &= \tfrac{73}{12} \end{aligned}$$

[W] $n = 6\frac{1}{12}$

Ex. 4–11. Copy and add as in box C.

4.	5.	6.	7.	8.	9.	10.	11.
$1\frac{5}{8}$	7	$7\frac{1}{5}$	$6\frac{2}{3}$	$10\frac{1}{12}$	$7\frac{1}{15}$	$11\frac{2}{5}$	$7\frac{1}{4}$
$\frac{3}{4}$	$2\frac{1}{2}$	$2\frac{2}{3}$	$7\frac{1}{6}$	$2\frac{1}{6}$	8	$48\frac{1}{10}$	$3\frac{2}{3}$
$2\frac{3}{8}$ $4\frac{3}{4}$	$3\frac{5}{6}$ $13\frac{1}{3}$	8	$3\frac{5}{6}$ $17\frac{2}{3}$	$3\frac{2}{3}$ $15\frac{11}{12}$	$4\frac{2}{3}$ $19\frac{11}{15}$	$2\frac{3}{5}$ $62\frac{1}{10}$	$12\frac{1}{9}$ $23\frac{1}{36}$

▲ **Reteaching.** Set 11. $17\frac{13}{15}$ ◆ **Extra Examples.** Sets 64–65.

* It is a good practice to sometimes have pupils record the numbers they used to make estimates so that you can check their method of estimating.

Estimating the Answer

[O]

1. Jerry is $62\frac{3}{4}$ in. tall and John is $58\frac{1}{8}$ in. tall. Jerry is how many inches taller than John?

a. To estimate the answer, we may round each number to the nearest whole number. Rounded to the nearest whole number, $62\frac{3}{4}$ is $\_?^{63}\_$ and $58\frac{1}{8}$ is $\_?^{58}\_$. $63 - 58 = 5$, so 5 in. is an estimate of the correct answer.

b. At the board, find the correct answer for the problem. $4\frac{5}{8}$ in.

* For each of Ex. 2–7, give an estimate of the correct answer by rounding each number to the nearest whole number. Do not work the problems yet.

2. Ray dug $6\frac{3}{4}$ bu. of potatoes in the morning and $8\frac{3}{4}$ bu. in the afternoon. That was how many bushels of potatoes? $7 + 9 = 16$; 16 $n = 6\frac{3}{4} + 8\frac{3}{4}$; $15\frac{1}{2}$

3. One year, Linda's weight increased from $98\frac{3}{4}$ lb. to $104\frac{1}{4}$ lb. That was an increase of how many pounds? $104 - 99 = 5$; 5 $n = 104\frac{1}{4} - 98\frac{3}{4}$; $5\frac{1}{2}$

4. At the cheese counter, Mrs. Arlen bought a package of Danish cheese which weighed $8\frac{1}{6}$ oz. and a package of Swiss cheese which weighed $2\frac{2}{3}$ oz. How many ounces more did the package of Danish cheese weigh than the package of Swiss cheese? $8 - 3 = 5$; 5 oz.; $n = 8\frac{1}{6} - 2\frac{2}{3}$; $5\frac{1}{2}$ oz.

5. At the meat counter, Mrs. Arlen bought three roasts weighing $7\frac{5}{8}$ lb., $6\frac{7}{8}$ lb., and $5\frac{1}{4}$ lb. What was the total weight of the three roasts? $8 + 7 + 5 = 20$; 20 lb.; $n = 7\frac{5}{8} + 6\frac{7}{8} + 5\frac{1}{4}$; $19\frac{3}{4}$ lb.

6. One house lot is $85\frac{3}{4}$ ft. wide and the lot next to it is $98\frac{1}{8}$ ft. wide. What is the total width of the two lots? $86 + 98 = 184$; 184 ft.; $n = 85\frac{3}{4} + 98\frac{1}{8}$; $183\frac{7}{8}$ ft.

7. Lucy rode her bicycle $3\frac{7}{8}$ mi. and pushed it $\frac{5}{6}$ mi. That was how far in all? $4 + 1 = 5$; 5 mi.; $n = 3\frac{7}{8} + \frac{5}{6}$; $4\frac{17}{24}$ mi.

[W]

Now write and solve a mathematical sentence to find the correct answer for each of Ex. 2–7.

♦ **Extra Problems.** Set 143.

Teaching Page 116

Pupil's Objective

To estimate the answer to a problem whose solution requires the addition of rational numbers.

Background

In estimating the sum of two or more rational numbers, each of the addends may be rounded to the nearest whole number. In estimating the sum $6\frac{2}{3} + 5\frac{3}{4} + 4\frac{1}{8}$, the addends may be rounded to the nearest whole number so that the addition becomes $7 + 6 + 4$. The estimated answer would therefore be about 17. The calculated answer is:

$$\begin{aligned} 6\tfrac{2}{3} &= 6\tfrac{16}{24} \\ 5\tfrac{3}{4} &= 5\tfrac{18}{24} \\ \underline{4\tfrac{1}{8}} &= \underline{4\tfrac{3}{24}} \\ 15\tfrac{37}{24} &= 15 + 1\tfrac{13}{24}, \text{ or } 16\tfrac{13}{24} \end{aligned}$$

Our calculated answer is reasonably approximate to the estimated answer. In rounding these three numbers, two of them were increased and one of them was decreased, so it is reasonable that the estimate would be slightly greater than the calculated answer. Rounded numbers may be used in problems involving any of the four operations.

Pre-Book Lesson

• Review the idea of rounding numbers and using them in problem-solving. Have pupils recall the advantages of estimating the answer when solving problems.

• Review the steps to follow in reasoning through the solution to a problem. Have pupils explain how to differentiate between pertinent and non-pertinent data in a problem.

• Write some simple problems on the board and have pupils explain how they would proceed toward a solution. Have them tell whether the data can be expressed with the addends-sum relationship or the factors-product relationship and have them write a mathematical sentence for each problem.

Using the Text Page

• Ex. 1. The pupils must be able to recognize whether a fraction names a number greater than one half or less than one half in order to know how to round a number to the nearest whole number. If the numerator is greater than one half the denominator, then the fraction names a rational number greater than one half. If the numerator is less than half the denominator, then the fraction names a rational number less than one half. If the numerator is one half the denominator, then round to the greater number.

Be sure pupils have mastered the technique of rounding numbers to the nearest whole number when the addends are named with fractions and mixed forms and have mastered estimating sums before they calculate the answers to these problems. They may practice rounding whole numbers first. When writing mathematical sentences for the problems, make sure pupils use either addition or subtraction, whichever the problem indicates.

• Ex. 2–7. Discuss all problems. Make sure pupils can select the pertinent data and determine whether the sum or an addend is missing. Have pupils tell how they would set up and solve some of the mathematical sentences.

Individualizing Instruction

• *More capable pupils* may find the maximum difference between an estimated sum of three rational numbers named with mixed forms and the calculated sum under each of the following circumstances: (1) all three addends are rounded to the next greater whole number; (2) all three addends are rounded to the next lesser whole number. Under circumstance 1 above, the estimated answer would be greater but less than 3 greater; under circumstance 2, the estimated answer would be less than the calculated answer but not more than 3 less.

• *All pupils* may be asked to estimate the sum in each of Ex. 4–11 on page 115.

These pupils may also discuss rounding $2\frac{1}{2}$ to the nearest whole number. It is mathematically correct to round it to either 2 or 3. However, it is by common convention that it is rounded to the next greater whole number, 3.

Teaching Page 117

Pupil's Objectives

(a) To practice adding and subtracting with mixed forms; (b) to practice solving mathematical sentences requiring adding and subtracting with mixed forms; and (d) to practice ordering rational numbers named with mixed forms.

Background

In the exercises on this page, the pupil is required to apply to mixed forms many of the concepts learned in earlier work. This includes writing mathematical sentences involving the addends-sum relationship, solving these mathematical sentences, applying the Commutative Property of Addition, applying the Associative Property of Addition, and ordering rational numbers. These exercises not only serve as a measure of how well a pupil can compute with mixed forms, but also how well he has mastered the ideas named above.

You may wish to refer to Background sections for pages on which the topics listed above were first presented.

Pre-Book Lesson

• You may wish to have the pupils do one or two of each type of exercise found on this page as a review before assigning the exercises as written work. Or you may wish to select or write other exercises related to the material on the page for review.

• Any instructions for use of the page should be given before the book is opened.

• You may wish to have *more capable pupils* work enrichment exercises or Supplementary Activities when they have completed their work. These pupils may need to work only a few selected examples before proceeding with other work.

Using the Text Page

• The exercises on this page may be used in a variety of ways. Ex. 1–18 may be used as a set of practice exercises, as a test of computational skill, or the odd-numbered exercises may be used as practice exercises and the even-numbered exercises used as a computational test. You may wish to make Ex. 23–25 oral instead of written exercises.

• Make certain that *all pupils* understand what they are to do in each of the exercises, particularly in Ex. 19–31.

• Ex. 19–22. Have *all pupils* show their work so that you can determine whether or not they understand how to solve a mathematical sentence involving an addends-sum relationship.

• The number of exercises on this page may be excessive for the *slower learner*. You may wish to select only certain exercises for them to do.

Individualizing Instruction

• Ask the *more capable pupils* to explain the use of the parentheses in Ex. 20, 21, and 29. Would removing the parentheses have any effect on the answer?

• After the exercises have been completed, and the papers returned, have *all pupils* correct their mistakes and restudy appropriate pages in the text or do Extra Examples at the end of the book to remedy any weaknesses revealed by the exercises.

• When explaining to the *slower learners* the errors they made in Ex. 19–25, it may be helpful to have them use frames $\square$, $\square$, $\triangle$, or letters a, b, c to illustrate the addends-sum relationship, Commutative Property of Addition, and Associative Property of Addition before using mixed forms.

* It is not wrong if pupils were to rename both addends to show a common denominator 20, but point out that their work would be easier if they used the least common denominator 10.

To Keep in Practice

A. and S. with mixed forms [W]

Ex. 1–12. Copy and add.

1.	* 2.	3.	4.	5.	6.
$6\frac{7}{12}$	$15\frac{2}{5}$	$6\frac{3}{4}$	$323\frac{5}{21}$	$1{,}026\frac{1}{2}$	$18\frac{5}{6}$
$7\frac{3}{12}$	$7\frac{1}{10}$	$4\frac{5}{8}$	$46\frac{20}{21}$	$538\frac{2}{5}$	$13\frac{2}{3}$
$13\frac{5}{6}$	$22\frac{1}{2}$	$11\frac{3}{8}$	$370\frac{4}{21}$	$1{,}564\frac{9}{10}$	$32\frac{1}{2}$

7.	8.	9.	10.	11.	12.
$17\frac{2}{9}$	$20\frac{3}{4}$	$93\frac{1}{2}$	$37\frac{7}{8}$	$\frac{21}{25}$	$4\frac{1}{3}$
$\frac{5}{9}$	$19\frac{1}{4}$	$6\frac{2}{5}$	$\frac{5}{8}$	$\frac{17}{50}$	$8\frac{3}{4}$
$75\frac{1}{9}$	$5\frac{3}{4}$	$127\frac{7}{10}$	$\frac{7}{8}$	$16\frac{3}{5}$	$13\frac{1}{2}$
$92\frac{8}{9}$	$45\frac{3}{4}$	$227\frac{3}{5}$	$39\frac{3}{8}$	$17\frac{39}{50}$	$26\frac{7}{12}$

Ex. 13–18. Copy and subtract.

13.	14.	15.	16.	17.	18.
$367\frac{17}{30}$	$100\frac{3}{11}$	$32\frac{3}{4}$	$1{,}603\frac{4}{50}$	$87\frac{1}{7}$	$20\frac{2}{5}$
$89\frac{13}{30}$	$78\frac{5}{11}$	$15\frac{1}{2}$	$873\frac{23}{25}$	$23\frac{1}{3}$	$11\frac{3}{4}$
$278\frac{2}{15}$	$21\frac{9}{11}$	$17\frac{1}{4}$	$729\frac{4}{25}$	$63\frac{17}{21}$	$8\frac{13}{20}$

Ex. 19–22. Find the number represented by the letter.

19. $n = 23\frac{1}{5} + 12\frac{3}{8} + 6\frac{2}{5}$ $41\frac{39}{40}$

20. $y = 307\frac{10}{11} - (3\frac{1}{3} + 27\frac{5}{6})$ $276\frac{49}{66}$

21. $x - (23 + 16\frac{7}{8}) = 55\frac{3}{8}$ $95\frac{1}{4}$

22. $n + 49\frac{1}{2} = 73\frac{2}{5}$ $23\frac{9}{10}$

23. Using $3\frac{2}{5}$ and $5\frac{1}{5}$ for the addends and n to represent the sum, write four mathematical sentences which illustrate the addends-sum relationship. $n = 3\frac{2}{5} + 5\frac{1}{5}$; $n - 3\frac{2}{5} = 5\frac{1}{5}$; $n - 5\frac{1}{5} = 3\frac{2}{5}$; $n = 5\frac{1}{5} + 3\frac{2}{5}$

24. Using mixed forms, write a mathematical sentence which illustrates the Commutative Property of Addition. Sample answer: $2\frac{1}{2} + 7\frac{3}{4} = 7\frac{3}{4} + 2\frac{1}{2}$

25. Using mixed forms, write a mathematical sentence which illustrates the Associative Property of Addition. Sample answer: $(1\frac{2}{3} + 3\frac{1}{2}) + 6\frac{3}{4} = 1\frac{2}{3} + (3\frac{1}{2} + 6\frac{3}{4})$

Ex. 26–31. Copy and complete by writing $=$ or $<$ or $>$.

26. $3\frac{7}{8} - 1\frac{1}{2}$ _?_ $2\frac{1}{3}$ ($>$)

27. $5 + 17\frac{3}{4}$ _?_ $22\frac{5}{6}$ ($<$)

28. $37\frac{2}{5} - 26\frac{1}{10}$ _?_ 11 ($>$)

29. 15 _?_ $(3\frac{1}{4} + 12\frac{2}{3}) - \frac{1}{2}$ ($\leq$)

30. $345\frac{7}{9} + 16\frac{1}{3} + 5\frac{5}{6}$ _?_ $367\frac{17}{18}$ ($=$)

31. $69\frac{4}{25} + 18\frac{38}{100}$ _?_ $120\frac{2}{3} - 33\frac{1}{6}$ ($>$)

32. List within braces the whole numbers which make

a. $n < 607 - 598$ a true mathematical sentence. {0, 1, 2, 3, 4, 5, 6, 7, 8}

b. $n < 6\frac{7}{8} - 4\frac{3}{4}$ a true mathematical sentence. {0, 1, 2}

c. $3\frac{2}{3} + 2\frac{3}{5} > n$ a true mathematical sentence. {0, 1, 2, 3, 4, 5, 6}

Solving Problems

[o]

Every time Tim placed his cat on the scales, it jumped off before he could read how much the cat weighed. Then Tim thought of an easy way to find the cat's weight. Holding the cat, Tim stood on the scales and found that their combined weight was $92\frac{1}{2}$ lb. Tim then found that, without the cat, he weighed 85 lb. He then subtracted 85 from $92\frac{1}{2}$.

1. Why did Tim subtract 85 from $92\frac{1}{2}$? To find the number of pounds difference between their weights.

2. Explain the subtraction shown in the box. Tim's cat weighed $7\frac{1}{2}$ lb. Key idea: 92 is renamed to show more ones.

Subtract

$$\begin{array}{r} {}^{8}\,\cancel{9}\,{}^{(12)}\,2\frac{1}{2} \\ -\;85 \\ \hline 7\frac{1}{2} \end{array}$$

3. If a pail itself weighs $\frac{3}{4}$ lb., explain how to find how many pounds of water the pail would hold if full. Sample answer: Weigh a full pail. Subtract $\frac{3}{4}$ from the number of pounds the full pail weighs.

4. A scientist weighed a container of acid and found that the combined weight of the container and the acid was $5\frac{5}{8}$ oz. He knew that the container alone weighed $2\frac{5}{8}$ oz. Give a mathematical sentence which may be used to find the weight of the acid. Then solve the sentence and give the weight of the acid.

$n = 5\frac{5}{8} - 2\frac{5}{8}$; 3 oz.

Teaching Pages 118 and 119

Pupil's Objectives

(a) To practice solving problems involving an addends-sum relationship which requires two operations; and (b) to work with measures expressed in mixed form.

Background

Many of the problems on these pages involve the addends-sum relationship in which the sum and one addend are given and the missing addend is to be determined. This means that the solution requires subtraction. However, in this case answering a hidden question is a prerequisite to being able to answer the question stated in the problem. In such problems, the given addend is itself a sum, and this sum must be determined first before the required subtraction can be performed. If n represents the answer to the question asked in the problem, the relationship in the problem may be expressed as:

$$n = a - (b + c)$$

The sum $(a + b)$ must be found before the subtraction is performed. These are sometimes called "two-step problems." The importance of the parentheses in this type of problem must be stressed. The pupils must realize that $a - (b + c)$ is not the same as $a - b + c$.

$$12 - (4 + 3) \neq 12 - 4 + 3$$
$$12 - 7 \neq 8 + 3$$
$$5 \neq 11$$

By common convention, $a - b + c$ is always to be interpreted as $(a - b) + c$. If there are no parentheses, the operations of addition and subtraction are to be performed in the order in which they occur from left to right.

One reason for confusion in this matter is that $(a + b) - c$ *is* equivalent to $a + (b - c)$ and to $a + b - c$. Using this convention, if the parentheses are inadvertently omitted, the answer will not be affected. However, omitting the parentheses in $a - (b + c)$ *will* affect the answer.

The proper use of parentheses can be summarized as:

$$\begin{aligned}
(1)\ a + (b + c) &= (a + b) + c \\
&= a + b + c \\
(2)\ a + (b - c) &= (a + b) - c \\
&= a + b - c \\
(3)\ a - (b + c) &\neq (a - b) + c \\
&\neq a - b + c \\
(4)\ a - (b - c) &\neq (a - b) - c \\
&\neq a - b - c
\end{aligned}$$

It is not intended that each of the principles above be discused with the pupils, but the teacher should be aware that they are involved. It is sufficient just to have the pupils *always* show the parentheses, performing the operation indicated within the parentheses first.

Pre-Book Lesson

- Review how to differentiate between a problem requiring addition, a problem requiring subtraction, and a problem requiring both addition and subtraction. Find applications based on situations with which the pupils are familiar. Make up some simple problems involving a school or other social situation and have pupils differentiate between the operations involved in solving the problems.
- State problems that contain no numbers, similar to the following:

  "You know how much an empty glass weighs and how much it weighs when full of water. How can you tell how much the water in the glass weighs?"

  Make up other such problems involving a hidden question and have pupils pick out the hidden question.
- Quickly review the addends-sum relationship in terms of mathematical sentences, including how to determine when the n in the sentence stands for an addend or a sum, and how to determine what operation is required.

Using the Text Pages

- In the oral Ex. 1–5, each exercise contains the addends-sum relationship in which the sum and one of the addends are given and the missing addend is to be determined. This requires sub-

traction. However, in Ex. 5, the known addend is itself a sum which must be determined before the required subtraction can be performed. Before assigning the written work, give the pupils more practice, if necessary, at analyzing problems of this type and writing mathematical sentences to express the relationship in the problem. Emphasize the proper use of parentheses. Remind the pupils that in any number sentence, the operation shown within the parentheses is to be performed first.

• Praise pupils for analyzing the problems correctly and writing correct mathematical sentences, even though they may make computational errors.

• Children with reading and vocabulary difficulties may require special attention. You may wish to have pupils read all the problems aloud so that any difficulties of this nature may be rectified.

• If the *slower learners* are assigned fewer exercises than the other pupils, Ex. 6, 7, 11, and 13 will serve as a representative sampling of the exercises.

• Remember that there are often several ways to express a relationship in a problem. The following are four ways for expressing the relationship in Ex. 8:

(1) $n = 25 - (5\frac{1}{2} + 6\frac{3}{4} + 7\frac{1}{3})$
(2) $25 = n + (5\frac{1}{2} + 6\frac{3}{4} + 7\frac{1}{3})$
(3) $25 - (5\frac{1}{2} + 6\frac{3}{4} + 7\frac{1}{3}) = n$
(4) $(5\frac{1}{2} + 6\frac{3}{4} + 7\frac{1}{3}) + n = 25$

Individualizing Instruction

• Have *more capable pupils* write the correct operation symbol in place of □ to make each a true sentence:

(a) $48 = 36 \square (19 - 7)$
(b) $(4 \square 3) = (6 \square 6)$
(c) $24 = 30 \square (3 \square 2)$
(d) $0 = 18 \square (9 \square 2)$
(e) $12 = 4 \square (2 \square 1)$

They may write generalizations similar to those in the Background for the different ways of expressing the addends-sum relationship with three addends.

• *Slower learners* may need additional work in writing mathematical sentences for problems requiring two operations. They may need to write a separate sentence for each operation and then combine them into a single sentence. You may give them more problems and have them write a correct mathematical sentence without solving it. They may also benefit from having you work the written section with them.

• *All pupils* may find a challenge in estimating the answers to Ex. 6–11 before solving the mathematical sentence.

These pupils may make up problems which require two operations for their solution and which involve an addends-sum relationship.

NOTES

5. An uncut cheese weighed $12\frac{1}{2}$ lb. One customer bought $1\frac{1}{4}$ lb. and a second customer bought $2\frac{3}{8}$ lb. How much cheese was left?

a. To find the answer, we can use the mathematical sentence $n = \_? - (1\frac{1}{4} + \_?)$. $12\frac{1}{2}$ $2\frac{3}{8}$

b. At the board, find the answer for the problem. $8\frac{7}{8}$ lb.

[W]

Ex. 6–11. Write and solve a mathematical sentence to find the answer for the problem.

6. The weight of a container full of sand was $16\frac{3}{8}$ lb. The container alone weighed $1\frac{3}{4}$ lb. How many pounds of sand were in the container? $n = 16\frac{3}{8} - 1\frac{3}{4}$; $14\frac{5}{8}$

7. During a 3-week period, Mr. Wyman bought $40\frac{1}{10}$ gal. of gasoline. He bought $12\frac{9}{10}$ gal. the first week and $17\frac{3}{10}$ gal. the second week. How many gallons did he buy the third week? $n = 40\frac{1}{10} - \left(12\frac{9}{10} + 17\frac{3}{10}\right)$; $9\frac{9}{10}$

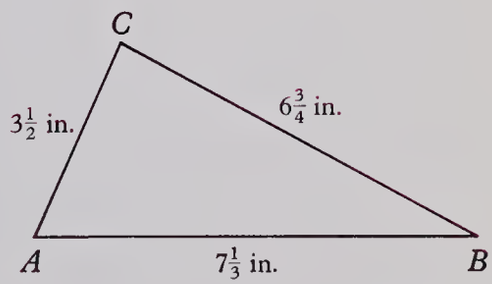
8. A bolt of cloth contained 25 yd. of material. One customer bought $5\frac{1}{2}$ yd., a second customer bought $6\frac{3}{4}$ yd., and a third customer bought $7\frac{1}{3}$ yd. How many yards of material were left? $n = 25 - \left(5\frac{1}{2} + 6\frac{3}{4} + 7\frac{1}{3}\right)$; $5\frac{5}{12}$

9. One Saturday, Mary and Irene worked making decorations for the class party. Mary worked $3\frac{1}{2}$ hr. in the morning and $3\frac{3}{4}$ hours in the afternoon. In all, Irene worked $6\frac{1}{3}$ hours. Mary worked how many hours more than Irene? $n = \left(3\frac{1}{2} + 3\frac{3}{4}\right) - 6\frac{1}{3}$; $\frac{11}{12}$ hr.

10. From a spool containing 20 ft. of ribbon, Mary cut a 4-ft. length, a $5\frac{1}{2}$-ft. length, and a $3\frac{5}{12}$-ft. length. How much ribbon was left on the spool? $n = 20 - \left(4 + 5\frac{1}{2} + 3\frac{5}{12}\right)$; $7\frac{1}{12}$ ft.

11. The sum of $4\frac{3}{5}$ and $2\frac{7}{10}$ is how much greater than the sum of $3\frac{3}{4}$ and $1\frac{1}{5}$? $n = \left(4\frac{3}{5} + 2\frac{7}{10}\right) - \left(3\frac{3}{4} + 1\frac{1}{5}\right)$; $2\frac{7}{20}$

Ex. 12–13. Refer to the drawing at the right.



12. Side AB is how many inches longer than side AC? $3\frac{5}{6}$ in.

13. Side AC is how many inches shorter than side BC? $3\frac{1}{4}$ in.

* Encourage pupils to write "0.15" instead of ".15."

Name of Place	Hundred's	Ten's	One's		Tenth's	Hundredth's	Thousandth's
a.			0	.	3		hundredth's
b.			4	.	2	3	5
c.		1	3	.	2	2	one's
d.	5	1	8	.	0	5	3 thousandth's
Value of Place	100	10	1		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$

Decimals

Resurvey [O]

1. Numerals such as those in Ex. a–d in the chart above are called *decimals*. The dot in a decimal is called the *decimal point*. Give the name and the value of each place shown in the chart. Name the first three places to the right of the decimal point. hundred, 100; ten, 10; one, 1; tenth, $\frac{1}{10}$; hundredth, $\frac{1}{100}$; thousandth, $\frac{1}{1,000}$ tenth's, hundredth's, thousandth's Name the next two places. ten-thousondth's, hundred-thousondth's

2. In 0.3, the digit 3 is in _?_ place. Give the name of the place that the 3 is in for each of the decimals in Ex. b–d above. tenth's See above.

3. In 9,728.3452, the 9 is in _?_ place and the 5 is in _?_ place. thousond's thousandth's

4. The decimal 0.3 may be read, *three tenths*. 4.235 may be read, *four and two hundred thirty-five thousandths*. Note the plan of reading "and" for the decimal point. Read the decimals in Ex. c–d above in this way. See below.

* 5. On the board, write a decimal for each of Ex. a–d.

a. Fifteen hundredths 0.15 b. Twenty-seven thousandths 0.027

c. Six and five tenths 6.5 d. Thirty and forty-one hundredths 30.41

6. Read the following decimals. See below.

a. 0.376 b. 10.62 c. 192.084 d. 0.368

4. c. Thirteen and twenty-two hundredths
d. Five hundred eighteen and fifty-three thousandths
6. a. Three hundred seventy-six thousondths
b. Ten and sixty-two hundredths
c. One hundred ninety-two and eighty-four thousondths
d. Three hundred sixty-eight thousondths

Teaching Pages 120 and 121

Pupil's Objectives

(a) To review reading decimals; (b) to review renaming from fractions to decimals; and (c) to review renaming from decimals to fractions.

Background

In our Hindu-Arabic system of numeration, each place in a numeral has a value ten times as great as that of the place to its right, and a value $\frac{1}{10}$ as great as that of the place to its left. When the system of numeration is extended to include places to the right of one's place, the place to the right of one's place has a value of $\frac{1}{10}$, and the name of the place is *tenth's place*. The value of the next place to the right is $\frac{1}{100}$, and the name of the place is *hundredth's place*. This pattern of place values continues indefinitely to the right. The numeral 51.237 means $(5 \times 10) + (1 \times 1) + (2 \times \frac{1}{10}) + (3 \times \frac{1}{100}) + (7 \times \frac{1}{1,000})$. The place values may be expressed in various ways such as:

$$\cdots 100, 10, 1, \frac{1}{10}, \frac{1}{100}, \frac{1}{1,000}, \frac{1}{10,000}, \cdots$$

$$\cdots 10^2, 10^1, 10^0, \frac{1}{10^1}, \frac{1}{10^2}, \frac{1}{10^3}, \frac{1}{10^4}, \cdots$$

$$\cdots 10^2, 10^1, 10^0, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, \cdots$$

The last two forms are called *exponent forms*.

Use of the decimal point is necessary to identify one's place, which is always the place just to the left of the decimal point. The decimal point also separates the numeral into two parts, the part naming a whole number and the part naming a number less than 1. If a decimal shows only a rational number between 0 and 1, such as .9237, it should be written as 0.9237 to more clearly indicate that the whole number is 0.

Sometimes the term *decimal fraction* is used instead of the term *decimal*. However, some mathematicians feel that since 0.9237 is a decimal and since $\frac{9,237}{10,000}$ is a fraction, *decimal fraction* is a contradiction of terms.

There are two common methods for reading a decimal. One method is to read each digit from left to right, without reference to place value named. The numeral 92.37 may be read "nine, two, point, three, seven." What is probably the most widely used method of reading decimals is as follows:

(1) Read the part of the numeral to the left of the decimal point as you would read a numeral for a whole number.

(2) Read the decimal point as "and."

(3) Read the part of the numeral to the right of the decimal point as a whole number followed by the name of the place of the digit at the extreme right.

Using this method, 41.679 would be read "forty-one and six hundred seventy-nine thousandths."

The correct use of the word *and* is very important in reading decimals. It should not be used after reading the hundred's digit such as reading 360 as "three hundred and sixty." 360 should be read "three hundred sixty." Notice the importance of the word *and* in the following:
0.417 is read "four hundred seventeen thousandths"
400.017 is read "four hundred *and* seventeen thousandths"

To change from a decimal to a fraction, replace the part of the numeral to the right of the decimal point with a fraction showing a denominator which is the place value of the final digit on the right in the decimal. For 0.28 we may write $\frac{28}{100}$. The justification for this is:

$$\begin{aligned} 0.28 &= (2 \times \tfrac{1}{10}) + (8 \times \tfrac{1}{100}) \\ &= \tfrac{2}{10} + \tfrac{8}{100} \\ &= \tfrac{20}{100} + \tfrac{8}{100} \\ &= \tfrac{28}{100} \end{aligned}$$

When the denominator is a power of 10, the factors of the denominator other than itself and 1 are 2 and 5. If the numerator is not divisible by 2 or 5, the fraction is in simplest form. Since the numerator of $\frac{28}{100}$ is divisible by 2, the fraction is not in simplest form $(\frac{28}{100} = \frac{28 \div 4}{100 \div 4} = \frac{7}{25})$.

Any number less than one shown with a denominator which is a power of 10, may easily be renamed with a decimal because the language used to read the fraction is the same as the language

used to read the decimal that names the number. 0.37 and $\frac{37}{100}$ are each read "thirty-seven hundredths."

To name with a decimal a number shown with a denominator which is not a power of 10, we must first multiply the numerator and denominator by a factor such that the resulting fraction will name a denominator that is a power of 10. For example:

$$\frac{117}{250} = \frac{117 \times 4}{250 \times 4} = \frac{468}{1{,}000} = 0.468$$

Pre-Book Lesson

• Review the pattern of place values in our Hindu-Arabic system of numeration. List several numerals on the board and have pupils give the place value for each digit. Reestablish the idea of powers of ten as a means of showing place value.

• Review finding fractions which are in simplest form for various numbers and finding fractions which name denominators that are powers of ten.

• Suggest to pupils that they record in their notebooks a chart such as in the Background to show place values and various decimals and fractions. New terms and methods of reading decimals should also be included.

• The method of changing from a decimal to a fraction should be well established in the oral part of the lesson. Material from the Background may be used freely.

Using the Text Pages

• Ex. 1. The distinction between number and numeral should be established for decimals. Notice that the naming of the places in a decimal is symmetrical with respect to one's place and is not symmetrical with respect to the decimal point.

• Ex. 4. Make sure pupils understand that the decimal is named by the place of the digit farthest to the right. The correct reading of the word "and" should be stressed. Also make sure pupils include the zero in writing such decimals as 0.3, 0.004.

• Ex. 10–11. If the denominator shown by a fraction is a power of 10, the decimal may be written directly. If not, pupils must multiply the numerator and denominator shown by the fraction by a factor so that the resulting fraction will show a denominator which is a power of 10.

• Ex. 16–23. The completely factored form for any power of 10 contains only 2's and 5's. For this reason, any numerator which does not have a factor of 2 or 5 is relatively prime with respect to a denominator for a decimal. The numerator for the fraction in Ex. 16 and 20 does not contain 2 or 5 as a factor. Therefore, the numerator and denominator are relatively prime and the fraction is in simplest form. The numerator shown by the fraction in each of the other exercises in this group contains a factor of 2 or a factor of 5, so each has a factor in common with the denominator and the fraction is not in simplest form.

• Ex. 24–35. Notice that in each of these exercises the denominator shown by the given fraction is a factor of a power of 10. There is a whole number such that its product with the denominator is a power of 10. In giving pupils any exercises of this type to supplement these examples, be certain the denominator of the given fraction number is a factor of a power of 10.

Individualizing Instruction

• *More capable pupils* may be asked to describe the difficulty encountered in attempting to rename numbers such as $\frac{2}{3}$ and $\frac{5}{7}$ with decimals. These pupils may be asked to explain why a fraction is in simplest form if the numerator shown does not contain a factor of 2 or 5 when the denominator is a power of ten. The importance of prime factors can be brought out here.

• *All pupils* may be asked to list reasons why decimals were invented. Have them try to imagine what our money system might be like if there were no decimals. Have pupils compare our money system with our decimal system.

• This topic of decimals may be introduced to the *slower learners* through the use of our money system with which they are familiar. These pupils may also benefit from naming the numbers in Ex. 5 and 6 with fractions. Help pupils read the decimals using the correct terminology.

7. Box A. The digit 3 is in tenth's _?_ place so it indicates the number $3 \times$ _?_, or $\frac{?}{10}$. In 0.02, the digit 2 is in hundredth's _?_ place so it indicates the number $2 \times$ _?_, or $\frac{?}{100}$.

A

$0.3 \rightarrow 3 \times \frac{1}{10} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10}$

$0.02 \rightarrow 2 \times \frac{1}{100} = \frac{1}{100} + \frac{1}{100} = \frac{2}{100}$

Key idea: The number is renamed by a fraction in simplest form.

8. Box B. Explain the work shown. ^ Does 0.32 name the same number as $\frac{32}{100}$? Yes ^ as $\frac{8}{25}$? Yes

B

$0.32 = \frac{3}{10} + \frac{2}{100} = \frac{30}{100} + \frac{2}{100} = \frac{32}{100}$, or $\frac{8}{25}$

9. On the board, write fractions in simplest form for the following numbers.

a. 0.7 $\frac{7}{10}$ **b.** 0.29 $\frac{29}{100}$ **c.** 0.003 $\frac{3}{1,000}$ **d.** 0.038 $\frac{19}{500}$

Key idea: The number is renamed with a decimal.

10. Box C. Explain the work shown. ^ To rename $\frac{2}{5}$ with a decimal, can we first rename with a fraction showing a denominator which is a power of 10? Yes

C

$\frac{2}{5} = \frac{2 \times 2}{2 \times 5} = \frac{4}{10} = 0.4$

11. On the board, show the work for renaming the following with decimals as in box C.

a. $\frac{3}{20} = \frac{5 \times 3}{5 \times 20} = \frac{15}{100} = 0.15$ **b.** $\frac{19}{25} = \frac{4 \times 19}{4 \times 25} = \frac{76}{100} = 0.76$ **c.** $\frac{3}{4} = \frac{25 \times 3}{25 \times 4} = \frac{75}{100} = 0.75$ **d.** $\frac{4}{5} = \frac{2 \times 4}{2 \times 5} = \frac{8}{10} = 0.8$ **e.** $\frac{1}{2} = \frac{5 \times 1}{5 \times 2} = \frac{5}{10} = 0.5$ [W]

Ex. 12–15. Write a decimal for the number.

12. Five and thirty-three hundredths 5.33

13. Two and six thousandths 2.006

14. Seventeen and one hundred sixty-four thousandths 17.164

15. Six hundred ninety-one and eighty-four hundredths 691.84

Ex. 16–23. Rename with a fraction in simplest form.

16. 0.27 $\frac{27}{100}$ **17.** 0.005 $\frac{1}{200}$ **18.** 0.0008 $\frac{1}{1,250}$ **19.** 0.4005 $\frac{801}{2,000}$

20. 0.047 $\frac{47}{1,000}$ **21.** 0.65 $\frac{13}{20}$ **22.** 0.055 $\frac{11}{200}$ **23.** 0.875 $\frac{7}{8}$

Ex. 24–35. Rename with a decimal.

24. $\frac{3}{10}$ 0.3 **25.** $\frac{777}{1,000}$ 0.777 **26.** $\frac{9}{100}$ 0.09 **27.** $\frac{3}{5}$ 0.6 **28.** $\frac{3}{50}$ 0.06 **29.** $\frac{113}{200}$ 0.565

30. $\frac{245}{250}$ 0.98 **31.** $\frac{192}{500}$ 0.384 **32.** $\frac{7}{20}$ 0.35 **33.** $\frac{7}{10}$ 0.7 **34.** $\frac{19}{50}$ 0.38 **35.** $\frac{21}{25}$ 0.84

▲ **Reteaching.** Set 12. ♦ **Extra Examples.** Set 66.

* Have the fraction in each mixed form show a denominator which is a power of 10.

Renaming Numbers Greater Than One

Key idea: The number is renamed with a fraction. Resurvey [O]

A	$2.3 = 2 + 0.3$ $= 2 + \frac{3}{10}$ $= 2\frac{3}{10}$, or $\frac{23}{10}$
B	$2.3 = 2\frac{3}{10}$
C	$2.3 = \frac{20}{10} + \frac{3}{10}$ $= \frac{23}{10}$
D	$\frac{41}{10} = 4\frac{1}{10} = 4.1$
E	$\frac{7}{5} = \frac{14}{10}$ $= 1\frac{4}{10}$ $= 1.4$

1. Explain the work in box A. Is it true that 2.3, $2\frac{3}{10}$, and $\frac{23}{10}$ all name the same number? Yes

2. At the board, find both a mixed form and a fraction for 5.7 as in box A. $5.7 = 5 + 0.7 = 5 + \frac{7}{10}$ $= 5\frac{7}{10}$, or $\frac{57}{10}$

3. Since 2.3 may be read, *two and three tenths*, we can avoid working as in box A and directly write a mixed form as in box B. Read each of the following in this way and then write a mixed form for each number. * See lower left.

a. 1.9 **b.** 3.5 **c.** 15.7 **d.** 28.4 **e.** 315.2

4. Since 2 is equal to 20 tenths, we may also read 2.3 as *twenty-three tenths*. We may think, *20 tenths plus 3 tenths equals* $\frac{23}{10}$ as in box C. Read each of the following as a number of tenths or as a number of hundredths and then write fractions. See lower left.

a. 7.7 **b.** 39.5 **c.** 4.13 **d.** 22.4 **e.** 627.39

5. To rename $\frac{41}{10}$ with a decimal, we may first rename with the mixed form $4\frac{?}{10}$ and then with 4.1 as in box D. Explain the work in box E to find a decimal for $\frac{7}{5}$. $\frac{7}{5}$ is renamed with $\frac{14}{10}$, with a mixed form, and then with a decimal.

6. At the board, find a decimal for each of the following:

a. $\frac{11}{10}$ 1.1 **b.** $\frac{29}{10}$ 2.9 **c.** $\frac{13}{2}$ 6.5 **d.** $\frac{271}{100}$ 2.71 **e.** $\frac{113}{50}$ 2.26 **f.** $5\frac{23}{1,000}$ 5.023

[W]

Ex. 7–12. Write a mixed form and a fraction for the number.

7. 7.9 $7\frac{9}{10}, \frac{79}{10}$ 8. 23.7 $23\frac{7}{10}, \frac{237}{10}$ 9. 8.71 $8\frac{71}{100}, \frac{871}{100}$ 10. 4.03 $4\frac{3}{100}, \frac{403}{100}$ 11. 51.007 $51\frac{7}{1,000}, \frac{51,007}{1,000}$ 12. 6,015 $6\frac{15}{1,000}, \frac{6,015}{1,000}$ (or $6\frac{3}{200}, \frac{1,203}{200}$)

Ex. 13–18. Write a decimal for the number.

13. $\frac{11}{2}$ 5.5 14. $\frac{239}{10}$ 23.9 15. $\frac{33}{10}$ 3.3 16. $24\frac{7}{1,000}$ 24.007 ^ 17. $16\frac{2}{5}$ 16.4 ^ 18. $3\frac{1}{50}$ 3.2

Ex. 19–24. Write *T* or *F*.

19. $3\frac{1}{2} = 3.5$ T 20. $2\frac{3}{10} < 2\frac{1}{5}$ F 21. $5.65 > 5\frac{61}{100}$ T

22. $\frac{5}{2} < 2.6$ T 23. $7\frac{1}{10} = \frac{7}{10}$ F 24. $3.265 < 3.27$ T

◆ **Extra Examples.** Set 67.

3. a. One and nine tenths, $1\frac{9}{10}$

b. Three and five tenths, $3\frac{5}{10}$

c. Fifteen and seven tenths, $15\frac{7}{10}$

d. Twenty-eight and four tenths, $28\frac{4}{10}$

e. Three hundred fifteen and two tenths, $315\frac{2}{10}$

4. a. Seventy-seven tenths, $\frac{77}{10}$

b. Three hundred ninety-five tenths, $\frac{395}{10}$

c. Four hundred thirteen hundredths, $\frac{413}{100}$

d. Two hundred twenty-four tenths, $\frac{224}{10}$

e. Sixty-two thousand seven hundred thirty-nine hundredths, $\frac{62,739}{100}$

Teaching Page 122

Pupil's Objective

To review naming a rational number greater than one with a decimal, a fraction, or a mixed form.

Background

Any rational number greater than one that cannot be named with a whole number may be named with a decimal, with a fraction, or with a mixed form. The rational number three and seven tenths may be named 3.7, $3\frac{7}{10}$, or $\frac{37}{10}$.

Renaming from a decimal to a mixed form is performed by simply reading the decimal and writing directly the mixed form that would be read in the same way. The decimal 5.9 is read "five and nine tenths," which is the same way the mixed form $5\frac{9}{10}$ is read.

To change from a decimal to a fraction, the numerator will be the same as the number named by the decimal if the decimal point is removed. The denominator will be the place value of the last digit at the extreme right of the decimal. For example, 539.21 may be named with the fraction $\frac{53,921}{100}$. This may be explained in the following manner:

$$\begin{aligned} 539.21 &= \tfrac{539}{1} + \tfrac{21}{100} \\ &= \tfrac{53,900}{100} + \tfrac{21}{100} \\ &= \tfrac{53,921}{100} \end{aligned}$$

To change from a fraction to a decimal, rename from the fraction to a mixed form by the method shown on page 111. The fraction part of the mixed form is then replaced with a decimal by the method shown on page 121. In some cases, before naming with a decimal is possible, the denominator may have to be changed to a power of 10.

$$\begin{aligned} \tfrac{37}{10} &= 3\tfrac{7}{10} \\ &= 3.7 \end{aligned}$$

Pre-Book Lesson

• Review changing from fraction to mixed form as explained on page 111, and changing from mixed form to a decimal as explained on page 121.

• Give pupils practice finding denominators which are powers of 10. They may also practice showing the place value of a denominator which is a power of 10 by telling how many places to the right of the decimal point the last digit would lie.

Using the Text Page

• Ex. 1–3. Pupils may need to work more examples of this type before they are able to write the mixed form directly from the decimal.

• Ex. 4. After the pupils have completed this exercise, you may want to lead them to the discovery that the numerator of the resulting fraction is the same number as named by the given decimal with the decimal point removed. The denominator names the place value of the last digit in the decimal.

• Ex. 5–6, 13–18. Exercises involving the renaming from fractions to decimals are limited here to fractions showing a denominator which is a factor of a power of 10. This means the denominator is a multiple of 2 or a multiple of 5. Many pupils will need repeated work with these numbers unless they learn to first name the denominator as a power of 10.

Individualizing Instruction

• *More capable pupils* may be asked to express as a mixed form base-ten numeral the base-four numeral 3.13_{four}.

$$\begin{aligned} 3.13_{four} &= (3 \times 1) + (1 \times \tfrac{1}{4}) + (3 \times \tfrac{1}{16}) \\ &= 3 + \tfrac{1}{4} + \tfrac{3}{16} \\ &= 3 + \tfrac{4}{16} + \tfrac{3}{16} \\ &= 3\,\tfrac{7}{16} \end{aligned}$$

They may first make a chart showing how places to the right of one's place are symmetrical with those to the left of one's place just as in our base-ten system.

• *Slower learners* may need to work with the teacher on Ex. 7–24 in order to establish the principles whereby rational numbers greater than one may be renamed with fractions, decimals, and mixed forms.

Pupil's Objectives

(a) To learn to round rational numbers named with mixed forms to the nearest whole number; and (b) to round rational numbers named with decimals to any given place.

Background

To round a number named in mixed form to the nearest whole number, it is only necessary to examine the fraction part of the mixed form. If the fraction names a number greater than one half, then the number named by the mixed form is rounded to the next greater whole number. If the fraction names a number less than one half, then the number named by the mixed form is rounded to the next lesser whole number. If the fraction names the number one half, it is mathematically correct to round the number named by the mixed form to either the next greater or the next lesser whole number. However, it is by common convention that a number named by such a mixed form is always rounded to the next greater whole number.

To round a number named by a decimal to any specific place, the digit to the right of this place is examined. If this digit is 5, 6, 7, 8, or 9, the digit in the place to which the decimal is being rounded is changed to name a number greater by 1, and all the digits to the right are dropped. For example, to round 3.4509 to the nearest tenth, the digit in hundredth's place is examined. This digit is 5. Therefore, the digit in tenth's place is changed to show an increase of 1 and the digits to the right are dropped. 3.4509 rounded to the nearest tenth is, therefore, 3.5.

If the digit to the right of the place to which the decimal is being rounded is 0, 1, 2, 3, or 4, then the digits to the right of the place to which the decimal is being rounded are simply dropped. The decimal 3.44999 rounded to the nearest tenth is 3.4.

Pre-Book Lesson

• Review rounding of whole numbers and provide examples which may be worked at the board.

• A problem which might present itself with some pupils is the recognition of rational numbers less than, equal to, or greater than $\frac{1}{2}$. These pupils may need practice finding many names for a rational number before proceeding with additional work. A number-line picture with decimals, fractions, and mixed forms showing the same number would help pupils "see" which numerals are closer to a given numeral and consequently which numbers are more approximate to a given number.

Using the Text Page

• Ex. 5. Point out that numbers named with mixed forms may be rounded to the nearest whole number without difficulty, but that a number named by a decimal may be rounded to any desired place within the decimal. Compare the difficulty of rounding $6\frac{23}{500}$ to the nearest tenth with the ease of rounding 1.246 to the nearest tenth. Have pupils first rename from fractions to decimals, then let them round to the required place. A meterstick marked off in millimeters would be a good device to show rounding to hundredths or tenths.

• Ex. 11–15. Ten cents is one tenth of a dollar. These decimals represent a number of dollars. Therefore, they represent numbers which are to be rounded to show the nearest tenth of a dollar. Flannel board with models of dimes and dollars may prove helpful here for some pupils.

Individualizing Instruction

• *All pupils* may be given numbers named with mixed forms such as $5\frac{23}{50}$ and be asked to round to the nearest tenth by first renaming from the mixed form to a decimal.

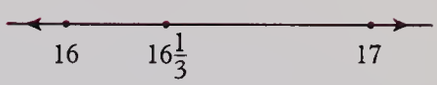
• *More capable pupils* may be asked to name different amounts of money with fractions not decimals. They may then make up their own money system, explain equivalences in our money system, and have other pupils name different amounts of money in our system with decimals, mixed forms, and fractions.

* Write "12.44" rather than "12.440." However, 6.304 rounded to the nearest hundredth is 6.30.

Rounding Numbers

[O]

1. The three numbers 16, $16\frac{1}{3}$, and 17 are shown in the number-line picture at the right. Is $16\frac{1}{3}$ nearer 16 than 17? Yes Then $16\frac{1}{3}$ rounded to the nearest whole number is _? 16.



2. On the board, draw a number-line picture like the one for Ex. 1 and locate $16\frac{3}{4}$. Is $16\frac{3}{4}$ nearer 16 than 17? No Is it nearer 17 than 16? Yes Then $16\frac{3}{4}$ rounded to the nearest whole number is _? 17.

3. In a number-line picture, would $16\frac{1}{2}$ be located halfway between 16 and 17? Yes In such a case, we round to the greater number. We say that $16\frac{1}{2}$ rounded to the nearest whole number is _? 17.

4. On the board, show each of Ex. a–e rounded to the nearest whole number. **a.** $5\frac{4}{7}$ 6 **b.** $39\frac{2}{9}$ 39 **c.** $17\frac{1}{2}$ 18 **d.** $205\frac{21}{40}$ 206 **e.** $\frac{15}{28}$ 1

5. To round a number named with a decimal, we round as with whole numbers named with standard numerals. For example, to round 12.438 to the nearest hundredth, we examine the digit in thousandth's place. Since that digit is 8, we round 12.438 * to _? 12.44. If the digit in thousandth's place had been 0, 1, 2, 3, or 4, the number would have been rounded to _? 12.43.

6. In 8.35, the digit in hundredth's place is _? 5, so 8.35 rounded to the nearest tenth is _? 8.4

[W]

** Ex. 7–10. Round the number to the nearest hundredth; to the nearest tenth; to the nearest whole number.

7. 35.478	8. 4.903	9. 34.6798	10. 900.0608
35.48; 35.5; 35	4.90; 4.9; 5	34.68; 34.7; 35	900.06; 900.1; 900

Ex. 11–15. Round to the nearest 10¢; to the nearest dollar.

11. $43.57	12. $0.89	13. $400.93	14. $5.55	15. $6.08
$43.60; $44	$0.90; $1	$400.90; $401	$5.60; $6	$6.10; $6

Ex. 16–20. Round to the nearest whole number.

16. $3\frac{7}{8}$ 4 17. $\frac{12}{19}$ 1 18. $37\frac{45}{90}$ 38 19. $87\frac{1}{2}$ 88 20. $534\frac{5}{11}$ 534

◆ **Extra Examples.** Sets 68–69.

** Do not round a number more than once; return to the original number each time when rounding.

Thinking about Units of Measurement

Resurvey; standard and non-standard units [O]

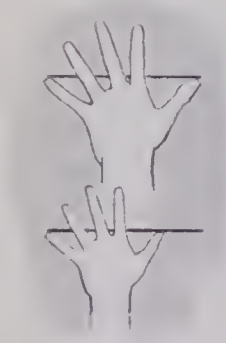
Mr. Dann paced the width of his garden and said it was 10 paces wide. Mrs. Dann said the width was 14 paces and Jane said it was 15 paces.

1. Mr. Dann, Mrs. Dann, and Jane each gave a *measurement* of the width of the garden. For Mr. Dann's measurement, the *measure* was 10 and the *unit of measurement* was Mr. Dann's pace. Give the measure and unit of measurement for each of Ex. a–f.

a. 31 paces 31; pace **b.** 27 miles 27; mile **c.** 5 feet 5; foot

d. 7 hand-spans 7; hand-span **e.** 3 pencil-lengths 3; pencil-length **f.** 17 inches 17; inch

A measurement may be expressed by a numeral followed by the name of a unit of measurement. The number of units is called the measure.



2. Bill said, "I have a piece of string as long as my arm" and Joe said, "I have a piece of rope as long as my arm." Do you think that the string and the rope are both the same length? Can't tell - probably "no." Give a better way to measure such things. Sample answers: Use a ruler or yardstick

3. Why is "hand-span" not a particularly useful unit of measurement? Everyone does not have the same size hand.

4. If everyone had the same size hand, would a hand-span be a useful unit of measurement? Yes

Teaching Pages 124 and 125

Pupil's Objectives

(a) To learn the precise meaning of the terms measure, measurement, and unit of measurement; and (b) to review the meaning and use of standard and non-standard units of measurement.

Background

An object usually has several different properties such as color, hardness, length, weight, temperature, luster, or height. Some of these properties are measurable. That is, they may be described in terms of a unit of measurement. For example, the length of a garden may be described in such units of measurement as paces, feet, yards, rods, or meters.

Measurement is a term sometimes used to describe a property of an object. This consists of the *measure* expressing the number of standard units of measurement, followed by the name of the unit of measurement. If a garden is described as being 50 feet long, the measure is *50*, the unit of measurement is *feet*, and the measurement is *50 feet*.

There is a difference of opinion among mathematicians and scientists as to how the measurement 50 feet is to be interpreted. Most mathematicians feel that the unit of measurement is a part of the physical description of the object not a part of the mathematical description. To such mathematicians, the term *50 feet* means "the length of the garden in feet is 50."

Scientists tend to have a different interpretation of the term 50 feet. Scientists interpret a measurement as being a product of two factors, the measure and the unit of measurement. To them, *50 feet* means (50) $\times$ (1 foot). They then use units of measurement in their computations as though they are factors. Below is the work showing that a car traveling at 37 miles per hour will travel 111 miles in 3 hours. Notice how the numerator and denominator were divided by "hour" to give an expression in simplest form.

$$\begin{aligned}\text{Distance} &= \text{rate} \times \text{time} \\ &= 37 \times \frac{\text{mile}}{\text{hour}} \times 3 \times \text{hour} \\ &= \frac{37 \times 3 \times \text{mile} \times \text{hour}}{\text{hour}} \\ &= 111 \times \text{mile} \\ &= 111 \text{ miles}\end{aligned}$$

The scientist will often treat units of measurement as factors in creating new units of measurement. For example,

$$12 \text{ kilowatts} \times 7 \text{ hours} = 84 \text{ kilowatt-hours}$$

These are examples of dimensional analysis which scientists and engineers find to be of great practical value. However, it is important that pupils first master the mathematical concept of measurement, and this is the concept of measurement presented in this text.

Every measurement must contain a unit of measurement, either standard or non-standard. A non-standard unit of measurement is one that is descriptive of the measurement but one that is not precisely defined for general use. Examples of non-standard units of measurement are the pace, arm-length, hand-span, pinch, knee-high, and sky-high. "Pace" means the distance covered by one step, and length in paces means the number of steps in a particular amount of length. A standard unit of measurement is precisely defined, set by law, and in this country controlled by the National Bureau of Standards. The listing and definition of our standard units of measurement occupies hundreds of pages.

The definitions of many of our basic units of measurement are very complicated. For example, an *inch* is defined as $\frac{1}{36}$ of a yard. But what is a yard? A yard is defined as $\frac{3,600}{3,937}$ of a meter. But what is a meter? A meter is the distance between the central mark of a group of three marks at each end of a certain platinum-iridium bar kept at the International Bureau of Weights and Measures near Paris. It is supported on two rollers at least one centimeter in diameter placed symmetrically 572 millimeters apart, the bar being at a temperature of 0° C. and subject to standard atmospheric pressure. A meter is therefore the distance between two lines on a certain bar measured under certain

standard conditions. Copies of this bar are in Washington, D. C.

For more precise work, the meter may also be defined as being 1,650,763.73 times the wavelength of the orange-red light of excited krypton 86 gas.

From this it can be seen that having pupils learn precise definitions of our basic units of measurement is impractical. Below is a listing of some of the most widely used standard units of measurement:

Length	*Weight*	*Volume*
inch	ounce	cubic inch
foot (12 in.)	pound (16 oz.)	cubic foot
yard (3 ft.)	ton (2,000 lb.)	cubic yard
rod (16½ ft.)	gram	pint
miles (5,280 ft.)	kilogram (1,000 g.)	quart (2 pts.)
millimeter		gallon (4 qts.)
centimeter (10 mm.)		cubic centimeter
meter (100 cm.)		liter
kilometer (1,000 m.)		

Pre-Book Lesson

• Select an object and list as many of its properties as possible, then determine which of these properties are easily measured and which are very difficult to measure. If an object is blue, it is very difficult to measure exactly how blue it is.

• Place two marks at either end of the floor. Have a few pupils find the distance between the two marks in paces, and record these distances on the board. Let pupils explain the difference among the measurements and let them determine if they would use this method if they wished to measure a distance carefully. Have pupils think of other ways in which they would estimate the same distance.

Using the Text Pages

• The purpose of these exercises is to review the concept of measurement in terms of standard and non-standard units of measurement. It is not intended that the pupil at this time master or review the standard units of measurement or the relationship among standard units of measurement.

• Ex. 1. Make sure pupils understand that the measure is a number while the unit of measurement is not. This has important application later when pupils are asked to use operations involving measurements.

• Ex. 2–5. The disadvantage of non-standard units of measurement is that they do not have precise meaning. To a person with a large hand, a "hand-span" probably is quite different from what it is to a person with a small hand. If everyone had the same size hand then "hand-span" would be a meaningful term.

• Ex. 15. Pupils should refer to a dictionary or other such source to find meanings for the terms.

Individualizing Instruction

• *More capable pupils* may be asked to report to the class on some of the newest standard units of measurement such as "mach number" to describe the speed of an airplane, or "g's" of force acting on an astronaut due to acceleration. They may be asked also to find the largest standard unit of measurement for distance (the light-year) and the smallest standard unit of measurement for distance (the Angstrom).

• *All pupils* may be given a stick or piece of cardboard of arbitrary length which is to be defined as being one *golub* in length. They then may be asked to estimate the dimensions of various objects in the room in terms of the golub.

These pupils may refer back to Ex. 6 and read outside sources having information concerning the historical background of the various systems of measurement, their origins, and the process of their universal adoption.

• *Slower learners* may need help with Ex. 11, 14, and 15 since they may not recall some units of measurement. You may wish to have them use a dictionary or other source material to find the necessary information. These pupils may also benefit by listing some standard units of measurement, their use, and a description in their math notes or a notebook. This will ease the memory load and enable them to become familiar with the necessary material through use rather than by rote.

5. Units of measurement such as pace, arm-length, and hand-span are called *non-standard* units of measurement. Why is non-standard a good name for such units? Key idea: They do not always mean the same thing. Name some other non-standard units of measurement. Answers will vary.

6. Units such as inch, foot, yard, mile, quart, pound, and ton mean the same to everyone. Such units are called *standard* units of measurement. For the United States, the "size" of a standard unit is set by law and is controlled by the National Bureau of Standards. For each of the following, tell whether a standard unit or a non-standard unit is used.

a. He is as strong as an ox. non-standard
b. He drove his car 5 miles. standard
c. The corn is knee-high. non-standard
d. She drank a half-pint of milk. standard
e. The kite flew sky-high. non-standard
f. She used a pinch of salt. non-standard

[W]

Ex. 7–10. Copy and complete by writing the name of a standard unit of measurement to make a reasonable sentence.

7. The top of her desk is about 30 inches from the floor.
8. Jim is in the sixth grade; he is about 5 feet tall.
9. The motorboat race was held on a lake about 4 miles long.
10. Bill's father is about 2 yards tall.

11. Name ten different standard units of measurement; five for measuring length, three for measuring liquids, and two for measuring weight. Sample answers: inch, foot, yard, mile, rod; cup, pint, quart; ounce, pound

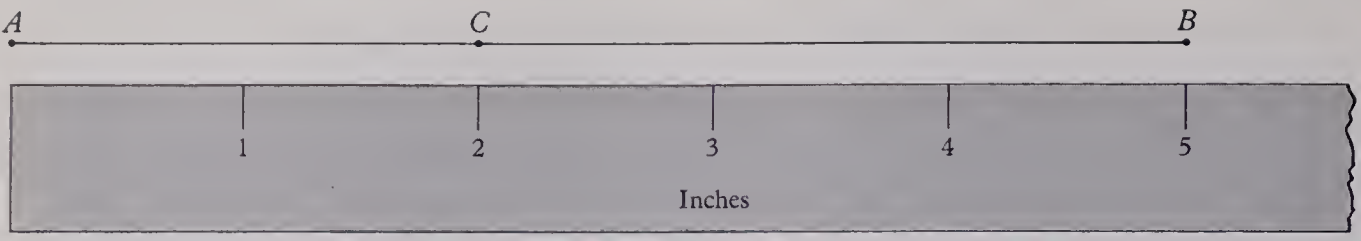
12. Write five sentences using standard units. Do not use any of the sentences on this page. Answers will vary.

13. Write five sentences using non-standard units. Do not use any of the sentences on this page. Answers will vary.

14. Name four standard units of measurement which are used to measure time. Sample answer: second, minute, hour, day

15. Tell something that is measured by the following units. Answers will vary.

a. degree Sample answers: temperature, angles
b. cord wood
c. cubic centimeter liquid volume
d. calorie heat, energy
e. acre land
f. fathom water depth
g. liter liquid volume
h. peck dry goods



* "m($\overline{AB}$)" means "the measure of $\overline{AB}$" so it would be redundant to write "the m($\overline{AB}$)."

Measures of Models of Segments

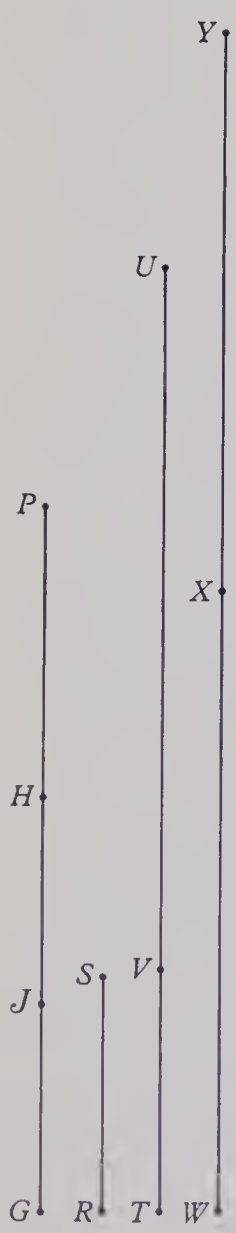
Resurvey [O]

1. A model of line segment AB is shown above. The measurement of the model of $\overline{AB}$ is _?5 in. The measure is 5 and the unit of measurement is inch _?_. Instead of saying, *the measure of the model of* $\overline{AB}$, we just say, *the measure of* $\overline{AB}$.

2. A unit of measurement used to find length is called a *unit of length*. What unit of length was used in measuring $\overline{AB}$? inch Name some other units of length. Sample answers: foot, yard, mile

3. As shown above, point C partitions $\overline{AB}$ into two segments, AC and _?CB What is the measure of $\overline{AC}$? 2 of $\overline{CB}$? 3 Is the measure of $\overline{AB}$ equal to the sum of the measures of $\overline{AC}$ and $\overline{CB}$? Yes

The measure of a segment is equal to the sum of the measures of the segments into which the given segment is partitioned.



* 4. Instead of writing, *the measure of* $\overline{AB}$, we may write, $m(\overline{AB})$. For the segment shown above, is it true that $m(\overline{AB}) = m(\overline{AC}) + m(\overline{CB})$? Yes

5. Refer to the segments shown and complete the following:

a. $m(\overline{GH}) + m(\overline{HP}) = m(\overline{\text{GP}}\text{?}\_)$

b. $m(\overline{TV}) + m(\overline{\text{VU}}\text{?}\_) = m(\overline{TU})$

c. $m(\overline{HP}) + m(\overline{JH}) + m(\overline{GJ}) = m(\overline{\text{GP}}\text{?}\_)$

[W]

6. Express $m(\overline{WY})$ as the sum of two measures. $m(\overline{WX}) + m(\overline{XY})$

7. Using a unit of 1 inch, copy and complete the following:

a. $m(\overline{GP})$ = _?3 in inches

b. $m(\overline{TU})$ = _?4 in inches

c. $m(\overline{RS})$ = _?1 in inches

d. $m(\overline{WY})$ = _?5 in inches

Teaching Page 126

Pupil's Objectives

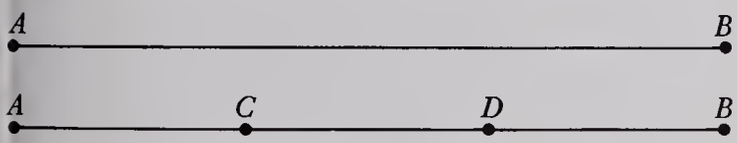
(a) To learn the meaning of the measure of a model of a segment; and (b) to learn to determine the measure of a model of a segment.

Background

We cannot determine the measurement of a geometric figure such as a segment because such figures are only concepts or ideas. However, we can draw a model to represent a segment and then determine the measurement of the model. The model may be a pencil mark on paper, chalk on the chalkboard, or the edge of an object.

The measure of a segment is equal to the sum of the measures of the segments into which the given segment is partitioned. If the given segment is partitioned into a number of segments, the measurement of each being one standard unit of measurement such as 1 inch, then the measure of the entire segment can easily be determined.

To determine the measure of $\overline{AB}$, the segment is partitioned into segments each of which has a measurement of one standard unit of measurement.



In this example, the measurement of each segment is one inch. The measure of model $\overline{AB}$ is equal to the sum of the measures of the segments into which it has been partitioned. The measure of $\overline{AB}$ may be written $m(\overline{AB})$. Therefore,

$$\begin{aligned} m(\overline{AB}) &= m(\overline{AC}) + m(\overline{CD}) + m(\overline{DB}) \\ &= 1 + 1 + 1 \\ &= 3 \end{aligned}$$

The unit of measurement is the inch so $m(\overline{AB}) = 3$ in inches. The device used to partition this model of a segment was a ruler. The ruler has already been partitioned into sections, the edge of each representing a segment whose measure is 1 in inches. $m(\overline{AB})$ means "the measure of the model representing segment AB." However, it is usually read, "the measure of segment AB."

Pre-Book Lesson

• Review the idea of a line segment. It has no width or height but has length. It is only a concept but we can draw models to represent it.

• Picture several partitioned segments on the board. Using the idea of the union of sets of points, have pupils find the union of the smaller segments into which the large segment has been partitioned.

Using the Text Page

• Ex. 1. It may be pointed out that the drawing is not a segment but only a model representing a segment, although we commonly refer to it as "segment AB" rather than "the model representing segment AB."

• Ex. 4. $m(\overline{AB})$ means the measure of $\overline{AB}$, not the measurement of $\overline{AB}$. The measurement of $\overline{AB}$ is 5 inches. The measure of $\overline{AB}$ is 5. One of the reasons for this distinction is that since only measures are numbers, only measures, not measurements may be operated on.

Place several models of segments and their corresponding measurements on the board. Have pupils list their measurements and then their measures using the form $m(\overline{AB}) = n$.

Individualizing Instruction

• *More capable pupils* may be led in a discussion concerning the actual length of an inch. An inch is $\frac{1}{36}$ of a yard; a yard is $\frac{3,600}{3,937}$ of a meter; a meter is the distance between two lines on a bar in Paris. However, no physical object can be measured exactly because of the nature of matter.

• *All pupils* may be asked to explain how we use a ruler to determine the measure of a model of a segment. They may be asked to suggest a convenient standard unit of measurement to use in determining the measure of segments of various lengths such as the width of the desk, the length of the room, the length of the building, the distance between two cities, and the distance between two galaxies.

Teaching Page 127

Pupil's Objectives

(a) To check understanding of important ideas studied in Chapter 3; and (b) to review concepts pertaining to angles.

Background

The exercises on this page may be helpful in evaluating how well pupils have learned the important ideas presented in the chapter. The exercises also help to maintain and reinforce the ideas and skills which have been developed.

The exercises pertain to changing from a mixed form to a fraction; changing from a fraction to a mixed form; changing from fractions and mixed forms to decimals; writing decimals; solving mathematical sentences requiring adding or subtracting with mixed forms; and finding fractions in simplest form.

At the bottom of the page is a review of some of the ideas pertaining to angles, including meaning of right angle, acute angle, and obtuse angle; definition of angle; and meaning of vertex of an angle.

Pre-Book Lesson

• You may wish to use some exercises for oral work or you may wish to have other review before assigning the written work. Select pupils to tell how to do the exercises in each of the groups.

• Remind pupils that they are nearing the end of the chapter and the end-of-chapter tests are not too far away. Explain that these exercises afford an excellent opportunity to review and reinforce concepts and skills presented previously in the chapter.

Using the Text Page

• Below is a list which may be used for reteaching and review for errors in specific exercises.

For errors in	*Review pages*
Ex. 1–6	110, 111
7–12	110, 111
13–18	120, 121
19–20	120, 121
21–29	120, 121
30–34	97

• Ex. 1–6. Pupils may draw diagrams to illustrate each of the examples at the bottom of the page.

• Pupils who finish the work early may be assigned Extra Activities or Supplementary Activities.

Individualizing Instruction

• After the exercises have been corrected, group *all pupils* according to areas of weakness and provide individual help by special selected examples, Reteaching exercises, and Extra Examples.

• *More capable pupils* may make up exercises similar to those at the bottom of the page and help *slower learners* work them, thus strengthening concepts in these areas.

* Point out that each number is to be named with a fraction in simplest form even though the numerator may be greater than the denominator.

How Well Do You Remember?

[W]

Ex. 1–6. Rename with a fraction.

1. $16\frac{1}{2}$ $\frac{33}{2}$ 2. $8\frac{3}{4}$ $\frac{35}{4}$ 3. $17\frac{1}{3}$ $\frac{52}{3}$ 4. $41\frac{5}{12}$ $\frac{497}{12}$ 5. $13 + \frac{8}{9}$ $\frac{125}{9}$ 6. $4\frac{7}{12}$ $\frac{55}{12}$

Ex. 7–12. Rename with a mixed form.

7. $\frac{23}{5}$ $4\frac{3}{5}$ 8. $\frac{80}{17}$ $4\frac{12}{17}$ 9. $\frac{189}{25}$ $7\frac{14}{25}$ 10. $17 + \frac{4}{9}$ $17\frac{4}{9}$ 11. 25.27 $25\frac{27}{100}$ 12. $\frac{44}{12}$ $3\frac{2}{3}$

Ex. 13–18. Rename with a decimal.

13. $\frac{7}{10}$ 0.7 14. $\frac{81}{100}$ 0.81 15. $\frac{21}{50}$ 0.42 16. $\frac{7}{2}$ 3.5 17. $37\frac{39}{1{,}000}$ 37.039 18. $18\frac{4}{25}$ 18.16

Ex. 19–20. Write a decimal for the number.

19. Twenty-three and sixty-two hundredths 23.62

20. One hundred and five hundred four thousandths 100.504

Ex. 21–29. Find the number represented by the letter.

21. $n = 23\frac{1}{2} + 13\frac{2}{5}$ $36\frac{9}{10}$ 24. $x = 5 + 49\frac{2}{3} + 19\frac{5}{6}$ $74\frac{1}{2}$ 27. $18\frac{3}{5} - n = 12\frac{1}{3}$ $6\frac{4}{15}$

22. $n = 45\frac{3}{4} - 31\frac{1}{3}$ $14\frac{5}{12}$ 25. $x = 35\frac{1}{7} - (7 + 8\frac{2}{3})$ $19\frac{10}{21}$ 28. $y = 199 \div 12$ $16\frac{7}{12}$

23. $n - 4\frac{7}{10} = 15\frac{3}{5}$ $20\frac{3}{10}$ 26. $209\frac{5}{6} - n = 134\frac{1}{12}$ $75\frac{3}{4}$ 29. $632 \div 16 = n$ $39\frac{1}{2}$

* Ex. 30–34. Find a fraction in simplest form for the number.

30. $\frac{80}{88}$ $\frac{10}{11}$ 31. $\frac{100}{125}$ $\frac{4}{5}$ 32. $\frac{36}{20}$ $\frac{9}{5}$ 33. $\frac{56}{161}$ $\frac{8}{23}$ 34. $45\frac{6}{8}$ $\frac{183}{4}$

Remembering about Angles

Resurvey [W]

Copy and complete.

1. The sides of an angle are geometric figures called _?_. rays

2. If the sides of an angle form a square corner, the angle is called a right _?_ angle.

3. An angle ABC is the union of rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

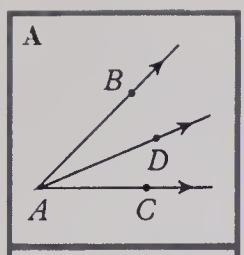
4. The sides of an angle intersect in a point called the vertex _?_.

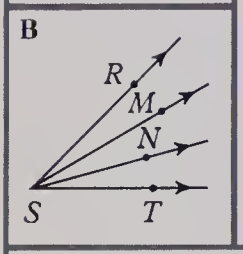
5. An angle smaller than a right angle is called an acute _?_ angle.

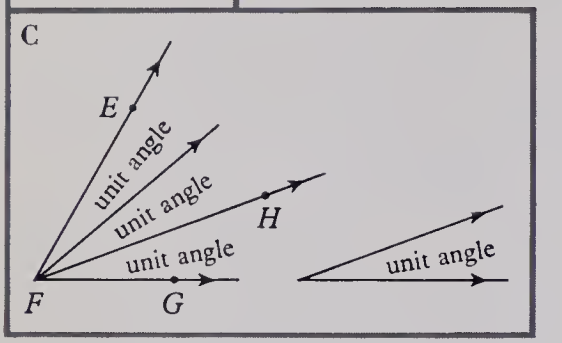
6. An angle larger than a right angle is called an obtuse _?_ angle.

Measures of Models of Angles

Using non-standard unit angles [O]







1. Box A. Is the end point of ray AD also the vertex of angle BAC? Yes Do the other points in $\overrightarrow{AD}$ lie in the interior for $\angle BAC$? Yes $\overrightarrow{AD}$ *partitions* $\angle BAC$ and its interior. Two angles, $\angle BAD$ and $\angle DAC$, are formed. No points in the interior of $\angle BAD$ are in the interior of $\angle DAC$. We say, $\overrightarrow{AD}$ *partitions* $\angle BAC$ *into two angles.*

2. Box B. Rays SM and SN partition $\angle RST$ into three angles. Name these angles. $\angle RSM, \angle MSN, \angle NST$

3. On the board, show an angle partitioned into three angles; five angles.

4. To measure an angle, we use a *unit angle.* In box C, $\angle EFG$ is shown partitioned into how many unit angles? 3 Then the measure of $\angle EFG$ is _?_ 3 in unit angles.

5. Would the measure of $\angle EFG$ be greater if we used a smaller unit angle? Yes The unit angle shown in box C is a non-standard unit angle. Explain why we need a standard unit angle.

Key idea: The size of angles can be expressed in terms of an agreed-upon unit.

6. Box C. In unit angles, what is the measure of $\angle EFH$? 2 of $\angle HFG$? 1 Is the sum of the measures of angles EFH and HFG equal to the measure of angle EFG? Yes We write,

$$m(\angle EFH) + m(\angle HFG) = m(\angle EFG).$$

[W]

7. Box B. The measure of $\angle RST$ may be expressed as $m(\angle RSM) + m(\angle MST)$. Express $m(\angle RST)$ in another way. $m(\angle RSN) + m(\angle NST)$

Ex. 8–11. Copy and complete. Refer to box B.

8. $m(\angle MST) = m(\angle MSN) + m(\angle \underset{NST}{\_?\_})$
9. $m(\angle MSN) = m(\angle RSN) - m(\angle \underset{MSN}{\_?\_})$
10. $m(\angle NST) = m(\angle MST) - m(\angle \_?\_)$
11. $m(\angle RSM) = m(\angle \underset{RSN}{\_?\_}) - m(\angle NSM)$

Teaching Page 128

Pupil's Objective

To learn to determine the measure of a model representing an angle using non-standard units of measurement.

Background

An angle is a set of points formed by the union of two rays with a common end point, called the vertex. $\angle ABC$, Fig. 1, is the union of $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

A third ray may be used with $\angle ABC$ to form two angles such that the interiors for the two angles have no points in common. Actually, three angles are formed, $\angle ABC$, $\angle ABD$, and $\angle DBC$, but only two, $\angle ABD$ and $\angle DBC$, have non-intersecting interiors.

If we partition an angle and its interior to form a number of unit angles, either standard or non-standard unit angles, then we can determine the measure of the given angle in terms of this unit angle. For example, if $\angle ABC$ and $\overrightarrow{BD}$ form two angles, $\angle ABD$ and $\angle DBC$, each of which is a unit angle, then the measure of $\angle ABC$, written $m(\angle ABC)$, is 2 in unit angles.

The measure of an angle is equal to the sum of the measures of the angles formed when the given angle and its interior is partitioned. To find the measure of an angle, we partition it and its interior so that a number of angles are formed each of which is a unit angle having a measure of 1. In the example above, $m(\angle ABC) = m(\angle ABD) + m(\angle DBC)$. The addends-sum relationship may also be expressed as $m(\angle ABD) = m(\angle ABC) - m(\angle DBC)$, and as $m(\angle DBC) = m(\angle ABC) - m(\angle ABD)$.

Pre-Book Lesson

• Review definition of angle and definition of interior for an angle. Have pupils draw models to represent angles and name points in the regions interior and exterior to the angles. Let them practice partitioning the models of the angles and their interiors and naming points in the intersection or union of various angles.

• Have pupils draw models to represent segments and review finding measures of several segments and several partitioned segments using non-standard units of measurement.

Using the Text Page

• Ex. 1. When a given angle is partitioned, we are only interested in the resulting angles that have non-intersecting interiors. Although the drawing in box A shows three angles, $\angle DAC$, $\angle BAD$, and $\angle BAC$, only two of these, $\angle DAC$ and $\angle DAB$ have non-intersecting interiors.

• Ex. 4–5. The purpose of these exercises is to show how a unit angle is used in determining the measure of an angle, and to show the need for a standard unit angle. The measure of $\angle EFG$ is 3, and the unit of measurement is the unit angle shown in box C. However, this is a non-standard unit angle. What is needed is a convenient unit angle, precisely defined and agreed upon, and used by everyone for naming the measure of an angle.

• Ex. 6. Point out that it is the measures that are being added and not the measurements.

• Ex. 7–11. On the board, show several angles and their interiors which have been partitioned. Using a cardboard model of a unit angle and its interior, have pupils find the measure of the models in unit angles.

Individualizing Instruction

• *More capable pupils* may draw models of partitioned angles and their interiors and have other pupils explain how many unit angles they measure.

• *All pupils* may be asked to cut from paper a semicircular region and then partition it into angles and their interiors each of which represents a unit angle as shown in box C. The pupils then may be given several angles and asked to determine their measure using this unit angle as a unit of measurement.

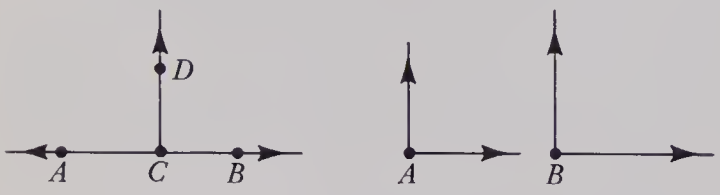
Pupil's Objectives

(a) To learn the meaning of the standard unit angle; (b) to review the meaning of acute angle and obtuse angle; and (c) to learn the meaning of acute and obtuse angle in terms of a measure in degrees.

Background

The standard unit for measuring an angle is a standard unit angle called *one degree*, which is written $1°$. Such an angle is formed by partitioning a whole plane region about a point so that there are 360 congruent angles (angles of the same size). The measure of the standard unit angle in degrees is 1.

Since an entire plane may be partitioned so that there are 360 standard unit angles, or degrees, a half plane may be partitioned into 180 standard unit angles or degrees. In the drawing below, $\overleftrightarrow{AB}$ separates the plane into two half planes, each of which may be partitioned into 180 congruent angles, the measure of each being 1 in degrees. If $\overrightarrow{CD}$ is drawn to partition one half plane so that there are two congruent angles, the measure of each of these angles formed will be 90 in degrees. An angle whose measure in degrees is 90 is called a *right angle*.



An angle whose measure in degrees is less than 90 is called an *acute angle*, and an angle whose measure in degrees is greater than 90 but less than 180 is called an *obtuse angle*. It should be noted that the defining of a standard unit angle with its interior as being $\frac{1}{360}$ part of a plane subdivided about a point is purely arbitrary. It was defined this way centuries ago and has been in use ever since.

Pre-Book Lesson

• Ask for suggestions for several desirable properties of a standard unit of measurement for angles. It should be possible to partition a square corner into an integral number of standard unit angles.

• Ask pupils to make up some standard unit for measuring angles and explain how they would use it in measuring models represented on the board.

Using the Text Page

• Ex. 2. Point out that the measure of an angle is not dependent in any way on the portion of the angle shown in a drawing. $\angle A$ and $\angle B$ represented in column A have the same measure even though the measures of the length of the models of the rays forming $\angle A$ are not the same as the measures of the models of the rays forming $\angle B$. A "square corner" is formed by the sides of an angle whose measure in degrees is 90. Therefore, the square corner of a piece of paper can be used for determining whether a given angle is a right angle.

• Ex. 5–15. Since $\overrightarrow{EA}$ is perpendicular to $\overleftrightarrow{FD}$, $\angle AED$ is a right angle and $\angle FEA$ is a right angle. Since each of these right angles is partitioned into 3 angles of the same size, they must each have a measure in degrees of 30.

Individualizing Instruction

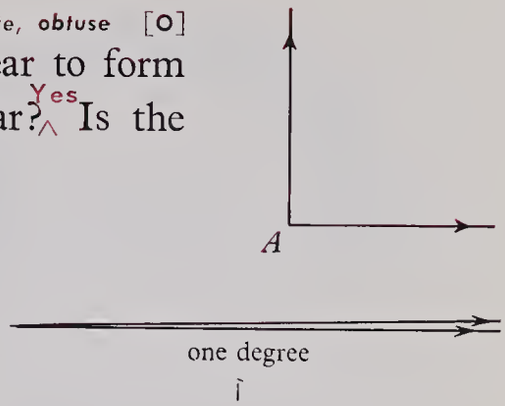
• You may wish to work Ex. 5–14 orally with *slower learners*. Draw representations of angles and make a cardboard model of a ten-degree angle by using a protractor. Use this model to measure the representations drawn.

• *More capable pupils* may be given the more technical definition of a standard unit angle as being one of the angles formed by partitioning a plane about a point so as to form 360 congruent angles. A directional compass can be used to show how subdividing a plane to form standard unit angles has important practical applications in indicating directions. Ask for possible reasons why the number 360 was probably selected as the number of parts into which the plane is subdivided in defining a standard unit angle. Ask for examples of standard units of measurement that are multiples of 12 smaller units (dozen, gross, minute, hour, day, year, foot, yard, and mile to name just a few).

* Pupils may use the square corner of a sheet of paper as a model of a right angle if they need help in deciding which angles are acute and which are obtuse.

A Standard Unit Angle

Resurvey; acute, obtuse [O]



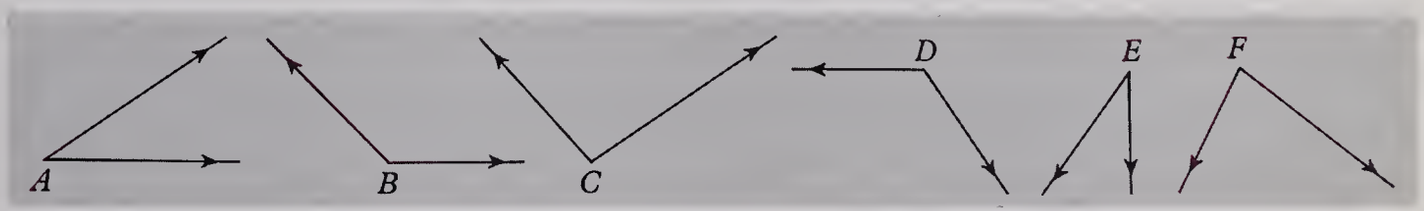
1. Do the sides of $\angle A$ shown at the right appear to form a square corner? Yes Then are the sides perpendicular? Yes Is the angle a *right angle*? Yes

2. Think of $\angle A$ as partitioned into 90 smaller angles, each of the same size. Then each small angle is a standard unit angle called *one degree*, written, $1°$. What is the measure in degrees of $\angle A$? 90 We write, $m(\angle A) = 90$ *in degrees*. We say that a *measurement* of $\angle A$ is $90°$.

3. If a right angle is partitioned into two angles of the same size, then what is the measure in degrees of each of the two angles? 45

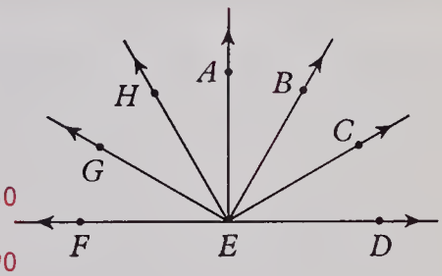
4. An angle whose measure in degrees is less than 90 is called an *acute angle*. An angle whose measure in degrees is greater than 90 and less than 180 is called an *obtuse angle*.

* Which of the angles shown in the box appear to be acute angles? A, E, F obtuse angles? B, C, D



[W]

At the right, $\overrightarrow{EA}$ is shown perpendicular to $\overleftrightarrow{FD}$. Also, $\angle FEA$ and $\angle AED$ are each partitioned into three angles of the same size. In degrees, what is



5. $m(\angle FEA)$? 90 6. $m(\angle AED)$? 90 7. $m(\angle GEA)$? 60

8. $m(\angle HEA)$? 30 9. $m(\angle GEB)$? 90 10. $m(\angle HEC)$? 90

What is the measure in degrees of

11. $\angle AEC$? 60 12. $\angle AEG$? 60 13. $\angle CEG$? 120 14. $\angle BEH$? 60

15. Name two angles shown each of whose measure in degrees is 150. Name two acute angles; two obtuse angles.
Sample answers: ∠FEC, ∠GED; ∠FEG, ∠GEH; ∠FEB, ∠HED

* An estimate of the measure of an angle will avoid the mistake of pupils reading the wrong scale.

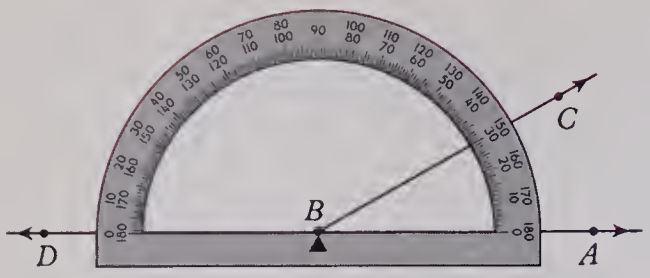


Fig. 1

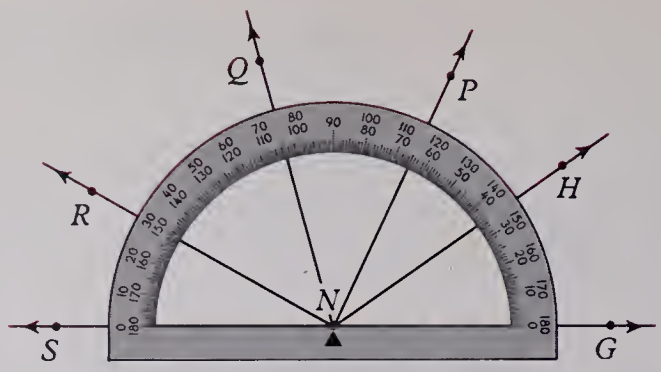


Fig. 2

An Instrument for Measuring Models of Angles

[O]

The instrument pictured above is called a *protractor*. It is used to measure and to draw models of angles. When we say, *measure an angle*, we mean, *measure a model of an angle*.

1. A protractor has scales with which we can find the measure in degrees of a given angle. In Fig. 1, to measure $\angle ABC$ the protractor was placed with its *base line* along the picture of ray BA such that the small center mark $\triangle$ points to the picture of the vertex of the angle.

* **a.** Give the readings on the two scales where the picture of ray BC crosses the curved edges of the protractor. 150, 30

b. Is $\angle ABC$ an acute angle or an obtuse angle? acute Then is its measure in degrees less than or greater than 90? less Which reading should you use for a reasonable answer? 30 $m(\angle ABC) = \_?$ 30

2. $\angle CBD$ is also shown in Fig. 1.

a. Which scale shows "O" where the picture of ray BD crosses the scale? The one on the outside edge. What is the reading on this scale where the picture of ray BC crosses? 150 $m(\angle CBD) = \_?$ 150

b. Is angle CBD an obtuse angle? Yes What is the sum of the measures in degrees of $\angle ABC$ and $\angle CBD$? 180

[W]

Use Fig. 2 to find the measure in degrees of

3. $\angle GNH$. 35 4. $\angle HNS$. 145 5. $\angle QNS$. 75 6. $\angle GNR$. 150

● **Extra Activity.** Set 164.

Teaching Page 130

Pupil's Objective

To learn the use of the protractor in determining the measure of a model representing an angle.

Background

The pupils are familiar with the use of a ruler in determining the measure of the length of a segment. The ruler is partitioned into a number of segments, each of which is a standard unit of measurement such as an inch or a millimeter. A protractor is something like a ruler. It is an instrument that serves as a model of a half plane that has been partitioned into 180 standard unit angles, the measure of each being 1 in degrees.

By placing the protractor properly on the model representing an angle, the partitions for the standard unit angles may be read from the scale on the protractor, and the measure in degrees determined directly. Care must be taken to place the protractor properly on the model representing the angle. The small center mark must point to the vertex of the given angle and the base line of the protractor must coincide with one ray forming the angle so that the other ray may be seen through the opening in the protractor.

Care must also be taken to be sure that the correct scale on the protractor is read. The protractor usually has two scales, one reading from 0 to 180 from right to left and one reading 0 to 180 from left to right. The measure of the angle is found by reading the numeral where the second ray of the angle crosses the scale which indicates 0 for the first ray of the angle. As a check to insure that the correct scale is being read, it is best to first determine by inspection whether the given angle is greater than 90° or less than 90°.

Pre-Book Lesson

• Ask for an explanation as to how a ruler is used to determine the measure of a segment, and what the numbers and markings on the ruler mean. Ask for suggestions as to how to make an instrument that could be used for determining the measure of an angle.

• A flat piece of paper may be folded like a fan to represent a device to measure angles. It can be unfolded as needed to measure angles of varying sizes. Pupils can see how advantageous a protractor is after using the fan.

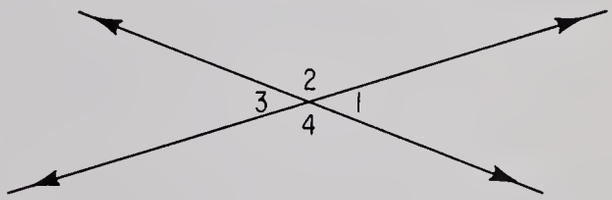
Using the Text Page

• Ex. 1–2. Ask for possible reasons why a protractor has two scales. Ask for ways in which a person can check that he has read the correct scale.

• Ex. 3–6. You may obtain a large, demonstration size protractor and use it at the board while pupils follow each step with their own protractors from the back of the book.

Individualizing Instruction

• *More capable pupils* may be given pairs of intersecting lines, asked to find the measure of each of the four angles formed, and look for a relationship among the measures of the four angles. They



should discover that when two lines intersect, the vertical angles are equal in measure. In the diagram above, $\angle 1$ and $\angle 3$ form a pair of vertical angles and $\angle 2$ and $\angle 4$ form a pair of vertical angles.

• *All pupils* may be given models representing a group of angles and be asked to determine the measure of each angle using the protractor. Check to see that each pupil is placing his protractor correctly on the given angle, and that the correct scale is being read.

All pupils may be asked to draw models representing angles of given sizes by using the protractor.

• *Slower learners* may need to work Ex. 3–6 with the teacher's assistance. They will gain more confidence with the use of the protractor when someone is at hand to help them.

Pupil's Objectives

(a) To learn the meaning of an angle of a triangle; and (b) to learn that the sum of the measures of the angles of any triangle is 180 in degrees.

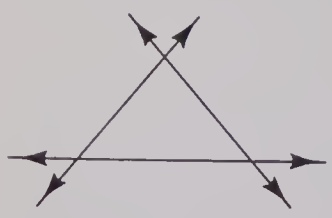
Background

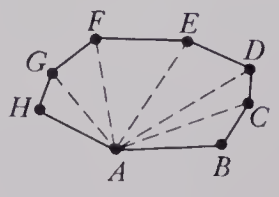
In geometry, an angle is the union of two rays which intersect in a common end point. Two segments that intersect in an end point may resemble an angle, but do not form an angle. We say that the segments *determine* an angle whose vertex is the common end point and whose sides contain the given segments. Therefore, any two adjacent sides of a polygon determine an angle, and this is called an angle of the polygon. In fact, polygons, except the quadrilateral, are named for their number of angles, the suffix *-gon* meaning *angle*. The polygon we call the triangle is obviously named for its number of angles. *Pentagon* means *five angles*, *hexagon* means *six angles*, and so forth.

The sum of the measures of the angles of a polygon is always equal to $(n - 2) \times 180$ in degrees, where n represents the number of sides, or the number of angles, of the polygon. The sum of the measure of the angles of a triangle is $(3 - 2) \times 180$, or 180 in degrees. The proof of this theorem is not appropriate here, but the principle can be demonstrated by cutting out the corners of a triangular region and fitting them together so that they form a straight angle whose measure is 180 in degrees.

Pre-Book Lesson

• Have pupils draw a model representing three lines intersecting in such a manner as to form a triangle as shown below.





Have them measure the angles of the triangle with their protractor and determine the sum of their measures. Record the results on the chalkboard and compare the sums obtained.

• Review the definition of ray, segment, line, angle, and triangle. Ask about the intersection or union of pairs of these figures to determine which of them form angles and which form triangles. Make a distinction between unions and intersections which *seem* to form angles, and those which actually do form angles.

Using the Text Page

• Ex. 2. The method of naming the angles of a triangle is the same as that of naming any angle. In $\triangle ABC$, the angle determined by $\overline{AB}$ and $\overline{AC}$ may be named $\angle A$, $\angle BAC$, or $\angle CAB$.

• Ex. 4. A triangle is named by the names of the three points forming the vertices. The order in which the vertices are named is irrelevant, but they are usually written in alphabetical order where practical. Make sure pupils understand the use of the symbol $\triangle$.

• Ex. 5. Have the pupils try to draw the model representing a triangle so that when its corners are cut out and fitted together, the sum of the measures of the angles does *not* appear to be 180 in degrees. When they find it is impossible to do this, they will be more convinced of the truthfullness of the statement that the sum of the measure of the angles of a triangle is always 180 in degrees.

Individualizing Instruction

• Ask the *more capable pupils* to determine the relationship between the sum of the measure of the angles of the octagon *ABCDEFGH* pictured in column A and the sum of the measure of the angles of the six triangles. Ask them to determine the sum of the measures of the angles of the octagon.

• For Ex. 5, ask *all pupils* if this is a proof that the sum of the measure of the angles of a triangle is 180 in degrees. It is not a proof; it is only supporting evidence.

• *Slower learners* may need to work with the teacher in solving Ex. 6. They may try to draw representations of triangles so that the sum of the angle measures is either less than or greater than 180 in degrees.

* If pupils measure the angles of a triangle, the sum of the measures may not be 180 in degrees because of the approximate nature of each measurement.

Measures of Angles of a Triangle

[O]

1. At the right, Fig. 1 shows two line segments with a common end point and Fig. 2 shows two rays with a common end point. Which is a picture of an angle? Fig. 2 How is an angle defined in geometry? The union of two rays, not in the same line, with a common end point.

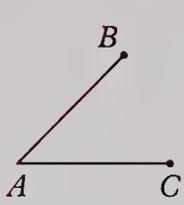


Fig. 1

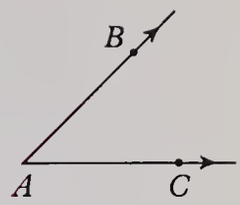
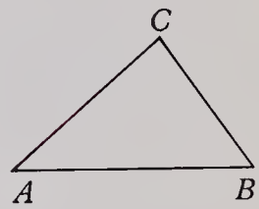


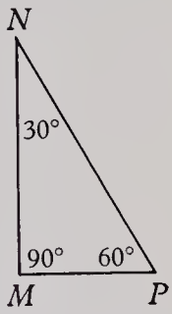
Fig. 2

2. When two segments have a common end point as in Fig. 1, they *determine* an angle whose vertex is that common end point and whose sides contain the given segments. For triangle ABC name the angle determined by $\overline{AB}$ and $\overline{AC}$; $\overline{AB}$ and $\overline{BC}$; $\overline{AC}$ and $\overline{BC}$. These angles are called *the angles of triangle ABC.* ∠A or ∠CAB ∠B or ∠CBA ∠C or ∠ACB



3. How many angles are there in a triangle? 3 *Tri* means *three*, so *triangle* means three *angle*.

4. *Triangle MNP* may be written, $\triangle MNP$. Name the angles of $\triangle MNP$ and give their measures in degrees. What is the sum of the measures? 180 ∠NMP, 90; ∠MNP, 30; ∠MPN, 60



5. On your paper, draw a large picture of a triangular region and color the corners as in Fig. 3. Then cut out the picture and tear off the corners. Then fit the corners together and paste on another sheet of paper as shown in Fig. 4. How does this suggest that the sum of the measures in degrees of the angles of a triangle is 180? The outside edges fall in a line. Measured with a protractor, it would measure 180 in degrees.

Fig. 3

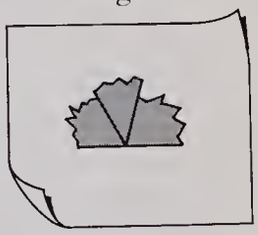
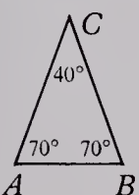


Fig. 4

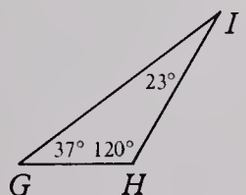
* **In degrees, the sum of the measures of the angles of a triangle is 180.**

[W]

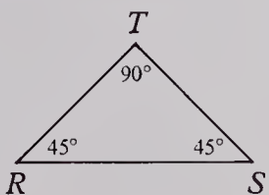
6. For each triangle shown below, give the measure of each angle and then find the sum of the measures of the angles of the triangle. Use the form $m(\angle CAB) = 70$ in degrees.



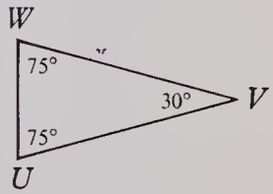
m (∠CAB) = 70 in degrees
m (∠ACB) = 40 in degrees
m (∠CBA) = 70 in degrees
180



m (∠IGH) = 37 in degrees
m (∠GHI) = 120 in degrees
m (∠HIG) = 23 in degrees
180



m (∠TRS) = 45 in degrees
m (∠RST) = 45 in degrees
m (∠STR) = 90 in degrees
180

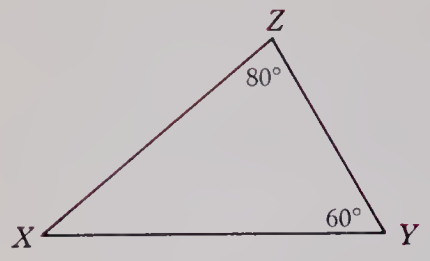


m (∠UWV) = 75 in degrees
m (∠WVU) = 30 in degrees
m (∠VUW) = 75 in degrees
180

* Point out that parentheses must be used in the mathematical sentence to indicate that the sum of the measures of two angles is to be subtracted. $180 - (26+63) \neq 180-26+63$

Finding Measures of Angles of a Triangle

[o]



$$\begin{aligned} n &= 180 - (26 + 63) \\ &= 180 - 89 \\ &= 91 \end{aligned}$$

1. We know that, for a triangle, the sum of the measures in degrees of its angles is 180. In degrees, $m(\angle Y) + m(\angle Z) =$ _?_. 140 Then to find $m(\angle X)$, we _?_ 140 from 180. What is $m(\angle X)$ in degrees? 40 ^

subtract

* **2.** If 26° and 63° are measurements of two angles of a triangle, what is the measure in degrees of the third angle? 91

a. In the mathematical sentence in the box, n represents the measure in degrees of the third angle. Explain how the sentence shows the relationship expressed in the problem. Key idea: It expresses the addends-sum relationship.

b. Explain the work in the box. Key idea: add then subtract.

c. The measure of the third angle is _?_. 91

3. At the board, find the measure in degrees of the third angle of a triangle if measurements of two of its angles are

a. 37° and 65°. $n = 180 - (37 + 65)$ $= 180 - 102$ $= 78$

b. 127° and 23°. $n = 180 - (127 + 23)$ $= 180 - 150$ $= 30$

4. One angle of a triangle measures 70 in degrees. The other two angles are both the same size. At the board, find the measure in degrees of each of these two angles.

a. Give a mathematical sentence for the problem. $n = (180 - 70) \div 2$ ^

b. Each of the other two angles measures _?_ 55 in degrees.

5. Could the three angles of a triangle measure 52, 61, and 73 in degrees? No ^ Explain your reasoning. The sum of the measures of the angles of a triangle should be 180 in degrees.

[w]

Ex. 6–13. Measurements of two angles of a triangle are given. Find the measure in degrees of the third angle. Write a mathematical sentence and show your work.

6. 28°, 55° 97 **7.** 77°, 30° 73 **8.** 61°, 61° 58 **9.** 34°, 140° 6

10. 100°, 19° 61 **11.** 16°, 5° 159 **12.** 90°, 30° 60 **13.** 90°, 45° 45

● **Extra Activity.** Set 165.

Teaching Page 132

Pupil's Objective

To learn to determine the measure of one angle of a triangle when the measures of each of the other two angles is given.

Background

If $\angle A$, $\angle B$ and $\angle C$ are the three angles of any triangle, the measure of $\angle A$ and the measure of $\angle B$ being given and the measure of $\angle C$ to be determined, the relationship among these three angles may be expressed as:

(1) $[m(\angle A) + m(\angle B)] + m(\angle C) = 180$

(2) $m(\angle C) = 180 - [m(\angle A) + m(\angle B)]$

Mathematical sentence (2) is used to find the measure of one angle when the measure of each of the other two angles is given.

The addends-sum relationship expressed in mathematical sentence (1) may also be written:

(3) $[m(\angle A) + m(\angle B)] = 180 - m(\angle C)$

If the measure of $\angle A$ is equal to the measure of $\angle B$, mathematical sentence (3) may be expressed as:

(4) $2 \times m(\angle A) = 180 - m(\angle C)$

This is a factors-product relationship, so it may also be expressed as:

(5) $m(\angle A) = [180 - m(\angle C)] \div 2$

Sentence (5) is used to find the measure of each of two angles of a triangle that are equal in measure when the measure of the third angle is given.

Pre-Book Lesson

- Give pupils practice finding the unknown addend when the sum is given and one of 2 addends is given, and when two of 3 addends are given.
- Have pupils change mathematical sentences showing the addends-sum relationship with two like addends to sentences showing the factors-product relationship.
- Remind pupils that even in work with angles, they are adding measures, not measurements.

Using the Text Page

- Ex. 1. Emphasize the addends-sum relationship here, using additional exercises if necessary.
- Ex. 2. Emphasize the importance of the parentheses in writing the relationship expressed in the problem. Writing $n = 180 - 26 + 63$ is incorrect because $(180 - 26) + 63 \neq 180 - (26 + 63)$. $n = 180 - (26 + 63)$ is the correct form and makes it easier for the pupil to recognize this as an addends-sum relationship and perform the operations in the proper order.
- Ex. 4. The relationship in the problem may be expressed as a factors-product relationship:

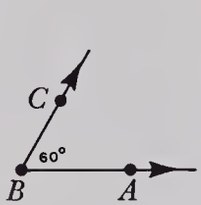
$$2 \times n = (180 - 70)$$
$$2 \times n = 110$$

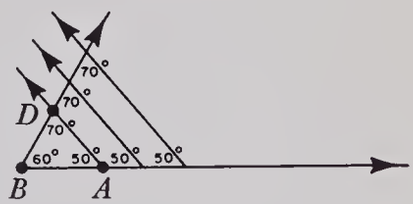
The product and one factor are given and the other factor is to be found. This indicates division.

$$n = 110 \div 2$$

Individualizing Instruction

- Have *more capable pupils* draw a model representing $\angle ABC$, the measure of which is 60 in degrees. Then have them draw a model representing $\angle BAD$, the measure of which is 50 in degrees, to form a triangle from the angle.





No matter where on $\overrightarrow{BA}$ the 50° angle is drawn, the measure of the third angle will always be 70 in degrees. This supports the theorem that when the measure of each of two angles of a triangle are given, the measure of the third angle is determined, regardless of the lengths of the sides of the triangle.

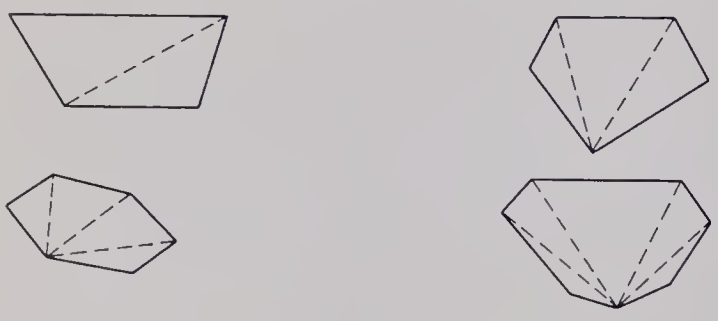
- You may need to work Ex. 6–13 orally with *slower learners* in order for them to be able to work effectively. Models representing triangles with measures marked at each angle may be good devices for visualizing the work which needs to be done.

Pupil's Objectives

(a) To learn that the sum of the measures of the angles of a quadrilateral is 360 in degrees; and (b) to learn to find the measure of one angle of a quadrilateral when the measure of each of the other three angles is given.

Background

The sum of the measure of the angles of any polygon of n number of sides is $(n - 2) \times 180$ in degrees. Therefore, the sum of the measure of the angles of a quadrilateral is $(4 - 2) \times 180$, or 360 in degrees. This relationship between the number of sides of a polygon and the sum of the measure of the angles of the polygon can be illustrated by partitioning the polygon and its interior into a number of non-intersecting triangular regions.



The number of triangular regions formed is always two less than the number of sides of the polygon. The sum of the measure of the angles of each triangle is 180 in degrees so the sum of the measures of the angles of all the triangles formed is $(n - 2) \times 180$.

The relationship among the four angles of the quadrilateral may be expressed as: $m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D) = 360$.

If the measure of $\angle A$, $\angle B$, and $\angle C$ are given, and the measure of $\angle D$ is to be found, $m(\angle D) = 360 - [m(\angle A) + m(\angle B) + m(\angle C)]$.

Pre-Book Lesson

• Have pupils practice solving mathematical sentences expressing the addends-sum relationship in which 3 of the 4 addends and the sum are given and the fourth addend is to be found.

• Review finding the sum of the measures of the three angles of a triangle, and finding the measure of one angle of a triangle when the other two are given.

• Review the idea of partitioning various polygonal regions into triangular regions.

Using the Text Page

• Ex. 1–2. Using material from the Background, work from the idea of measures of angles of a triangle to partitioning polygonal regions into triangular regions and finding measures of angles of the triangles formed.

• Ex. 3. The work for solving this problem may be written as

$$\begin{aligned} 25 + 68 + 150 + n &= 360 \\ 243 + n &= 360 \\ n &= 360 - 243 \\ n &= 117 \end{aligned}$$

• In Ex. 9–12, have pupils verify that the sum of the measure of angles of each quadrilateral is 360 in degrees by showing quadrilaterals and their interiors partitioned into two triangular regions.

Individualizing Instruction

• *More capable pupils* may be asked to partition models representing pentagons, hexagons, septagons, and octagons and their interiors into triangular regions in such a manner that the sum of measures of the angles of the triangles formed is equal to the sum of the measure of the angles of the polygon. Have them determine the relationship between the number of sides of the polygon and the number of triangles formed by partitioning the polygon and its interior. They can then be led to discover the relationship that the sum of the measure of the angles of a polygon is $(n - 2) \times 180$ in degrees, where n is the number of sides of the polygon.

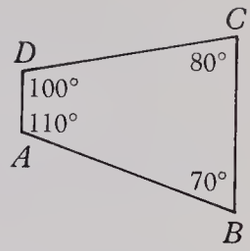
• *Slower learners* may work at the board with large models representing quadrangular, pentagonal, and hexagonal regions using a chalkboard protractor to find the measure of each angle. They may then partition the models into triangular regions and compare the sum of the measures of the angles of the triangles with the sum of the measures of the angles of each of the other figures.

* Be sure pupils understand that they will need to use parentheses in the mathematical sentence.

Measures of Angles of a Quadrilateral

[O]

1. Name the angles of quadrilateral *ABCD* and give their measures in degrees. What is the sum of the four measures? 360



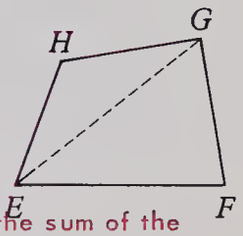
∠DAB, 110; ∠ABC, 70; ∠BCD, 80; ∠CDA, 100

2. The quadrilateral shown at the right is a special quadrilateral called a rectangle. Is each angle of a rectangle a right angle? Yes The measure in degrees of a right angle is 90, so the sum of the measures in degrees of the angles of a rectangle is 360.

In degrees, the sum of the measures of the angles of a quadrilateral is 360.

3. If the measures in degrees of three angles of a quadrilateral are known, tell how to find the measure in degrees of the fourth angle. Subtract the sum of their measures from 360. At the board, find the measure in degrees of the fourth angle of a quadrilateral if three of its angles measure 25, 68, and 150 in degrees. 117 Give a mathematical sentence for the problem.* $n = 360 - (25 + 68 + 150)$

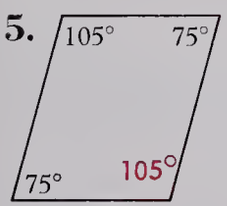
4. Quadrangular region *EFGH* is shown partitioned into two triangular regions. Name the two triangles suggested. △HEG, △GEF What is the sum of the measures in degrees of the angles of each of the triangles? 180 How does this suggest that the sum of the measures in degrees of the angles of a quadrilateral is 360? For the two triangles, the sum of the measures in degrees of their angles is 360.



[W]

Ex. 5–8. Write the missing measurement for the quadrilateral.

5.



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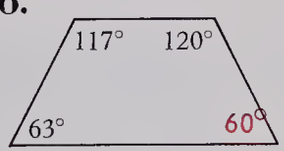

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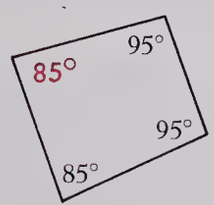

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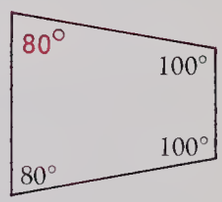
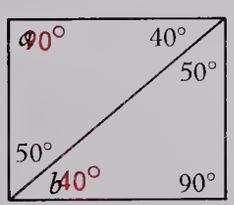


Ex. 9–12. Write measurements for angles *a* and *b*.

9.



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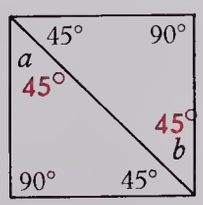

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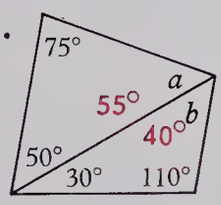

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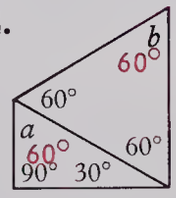
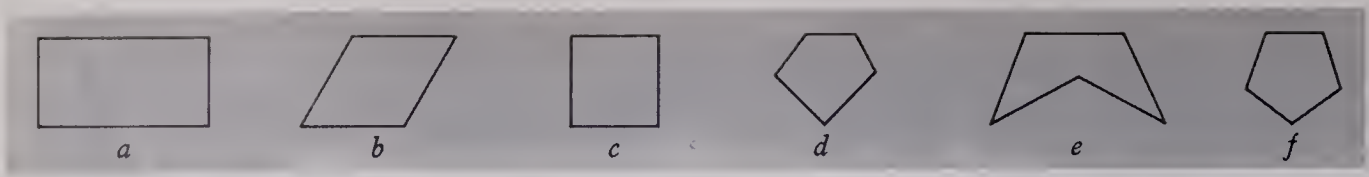




Regular Polygons

Resurvey [O]

1. The geometric figures shown in the box are called polygons. If the sides of a polygon are all the same length, then the polygon is equilateral. Which polygons shown in the box appear to be equilateral? Which do not? a, d

2. What would be a good name for a polygon whose angles all have the same measure? equiangular Which polygons shown in the box appear to be equiangular? a, c, f Which do not? b, d, e

3. Which polygons shown in the box appear to be both equilateral and equiangular? c, f Such polygons are called *regular polygons*.

A regular polygon is a polygon which is both equilateral and equiangular.

4. Are the polygons shown at the left hexagons? Yes Give a definition of a hexagon. A polygon with six sides. Which hexagon shown appears to be a regular hexagon? a

5. Find some models of regular polygons in your classroom. Answers will vary.

6. All equilateral triangles are also equiangular. Are all equilateral quadrilaterals also equiangular? No Explain. Key idea: A rhombus is equilateral, but not equiangular. (Note figure b at top of page.)

[W]

7. Copy and complete: The measure in degrees of each angle of a regular quadrilateral is 90. (Hint: In degrees, what is the sum of the measures of the angles?)

Ex. 8–11. Write *T* or *F*.

8. A square is a regular quadrilateral. T

9. A rectangle is an equiangular quadrilateral. T

10. All rhombuses are regular polygons. F

11. A regular hexagon is a parallelogram. F

Teaching Page 134

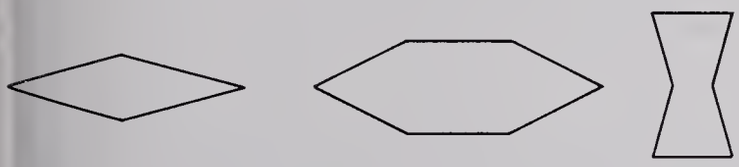
Pupil's Objective

To learn that a regular polygon is one that is both equilateral and equiangular.

Background

A polygon is any simple closed curve consisting entirely of segments. If the segments are all of the same length, the polygon is equilateral. If the angles of the polygon all have the same measure, the polygon is equiangular. If a polygon is both equilateral and equiangular, it is called a regular polygon.

Every square is both equilateral and equiangular. Therefore, every square is a regular polygon. Every equilateral triangle is also equiangular and is therefore a regular polygon. Polygons other than triangles may be equilateral and yet not be regular polygons because they are not equiangular. Below are some models representing equilateral polygons that are not equiangular and so are not regular polygons.



Pre-Book Lesson

• Review the definition of a *polygon* and the naming of polygons. Place models of different polygons on the board and have pupils identify and describe them. Make sure they know the characteristics representative of each type of polygon.

• Review the meaning of the terms equilateral and equiangular. Have pupils draw models of some equilateral polygons and models of some equiangular polygons.

• Review finding the sum of the measures of the angles of a quadrilateral and finding the measure of one angle when measures of the other three are given.

Using the Text Page

• Ex. 4. Although both hexagons are equilateral, hexagon *b* is not equiangular and is, therefore, not a regular polygon.

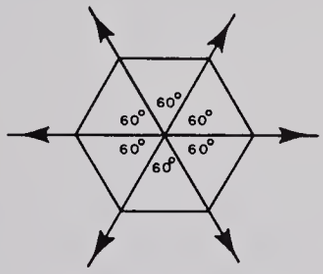
• Ex. 5. When asking pupils for models showing regular polygons, be sure to differentiate between a polygon and a polygonal region.

• Ex. 6. Have pupils draw on the chalkboard models showing equilateral quadrilaterals that are not equiangular.

• Ex. 10. A rhombus is an equilateral quadrilateral but it is not equiangular. Draw on the chalkboard a model showing a rhombus and have pupils point out two angles that are not equal in measure.

Individualizing Instruction

• Have the *more capable pupils* partition a plane about a given point into six angles whose measure is 60 in degrees. On each ray have them mark off a specific distance from the given point and connect these points with segments.



Have them determine if the resulting figure represents a regular polygon.

Repeat the procedure, having them partition a plane about a point into 4, 5, 8, 9, 10, or 12 equal parts, marking off on each ray a specific distance from the given point and then connecting the points to form a polygon. Have them determine whether the resulting polygon is always a regular polygon.

• For Ex. 7–11, *slower learners* may need to work with the teacher using cardboard or paper models and rulers and protractors to find if the lengths of sides and measures of angles of polygons are the same.

Pupil's Objectives

(a) To review the meaning of congruent figures as those that have the same size and shape; (b) to find that circular regions of the same size are congruent; and (c) to find that segments having the same length are congruent.

Background

Two geometric figures are congruent if models representing their corresponding parts are equal in measure. If two figures are congruent, the models showing these figures should match perfectly when one is placed on top of the other.

Congruent figures have the same size and the same shape. There are certain geometric figures that always have the same shape. All segments have the same shape, all circles have the same shape, and all squares have the same shape. Figures that have the same shape will be congruent if they have the same size. This means two segments having the same length are congruent, two circles having the same circumference are congruent, and two squares that have the same length sides are congruent. The symbol for congruence is $\cong$. $\overline{AB} \cong \overline{CD}$ is read "segment AB is congruent to segment CD."

One part of a geometric figure may be congruent to another part of the same figure. Each side of an equilateral polygon is congruent to each of the other sides of that polygon. Each side of a square or a rhombus is congruent to each of the other sides.

The question sometimes arises as to whether two geometric figures that are mirror images of each other are congruent. That is, are Fig. 1 and Fig. 2 below models of congruent figures?

Fig. 1

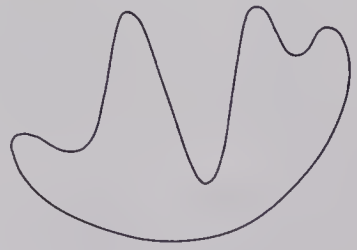


Fig. 2

If Fig. 2 is rotated and moved only within its plane, it cannot be made to match with Fig. 1. However, if a model for Fig. 2 is cut out and turned over, it can then be placed on Fig. 1 and will match it perfectly. Such figures are congruent.

In attempting to determine whether two geometric figures are congruent, it is permissible to "turn over" or rotate one of the figures 180° about an axis in the plane to make the models of the figures match. Figures that are of the same size and same shape are congruent even though they may be mirror images of each other.

Pre-Book Lesson

- On the blackboard draw a model of two segments each having the same length. Ask for ways in which the two segments are alike and ways in which they differ. The segments have the same length but are different in position. Point out that we do not say two segments are equal because in mathematics equality means identity, and these segments are not the same segment.
- Repeat the same exercise using two squares having sides of the same length, and two circles having the same circumference.

Using the Text Page

- Ex. 1–3. To demonstrate that the figures are congruent, a tracing of one of the models may be placed over the other model. Make other models having the same size, but different shapes. Show they do not match by placing one over the other.
- Ex. 4. Pieces of string can be used to show lengths of circumferences of circles.
- Ex. 5. Pupils may be shown how to use a pair of compasses to determine if two segments have the same length and are, therefore, congruent.

Ex. 11. Explain that two regions bounded by congruent figures are congruent regions.

Individualizing Instruction

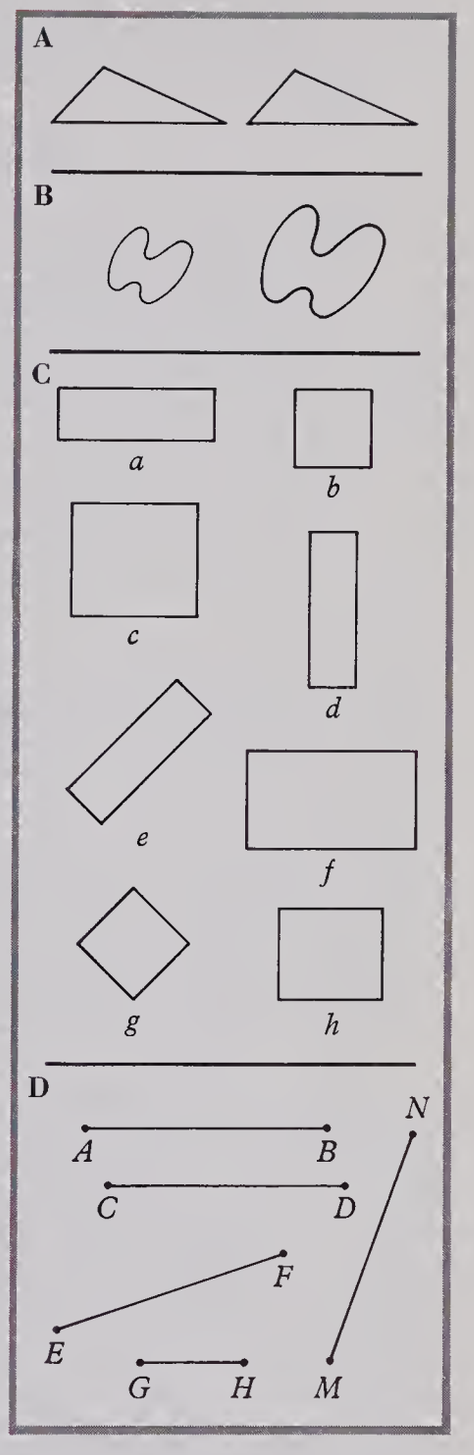
More capable pupils may be given two figures such as in the Background section and asked to determine if they are congruent. They can then be given other pairs of figures that are mirror images of each other and asked to determine if they are congruent.

* Geometric figures which are the same shape, but not necessarily the same size, are said to be "similar."

Congruent Figures

Resurvey [O]

1. In geometry, figures which have the same shape and size are said to be *congruent*. Do the triangles shown in box A appear to have the same shape and size? Yes Then do the triangles appear to be congruent? Yes



* 2. The figures shown in box B have the same shape. Do they have the same size? No Then are they congruent? No

3. Box C. Which of figures *c*–*h* appear to be congruent to figure *a*? d, e to figure *b*? g

4. Do all circles have the same shape? Yes the same size? No See if you can find some models of circular regions in your classroom that are congruent. Answers will vary.

5. Line segments are congruent if they have the same size. To find the size of a segment, we measure its length. Which segments shown in box D appear to be congruent? $\overline{AB}$, $\overline{CD}$, $\overline{EF}$, $\overline{MN}$

[W]

Ex. 6–10. Write *T* or *F*.

6. All squares have the same shape. T

7. All squares are congruent. F

8. The sides of a square are congruent. T

9. The sides of an equilateral triangle are not congruent. F

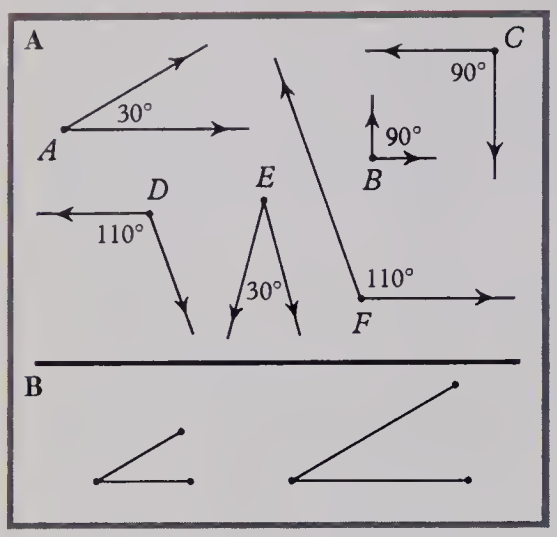
10. All right triangles are congruent. F

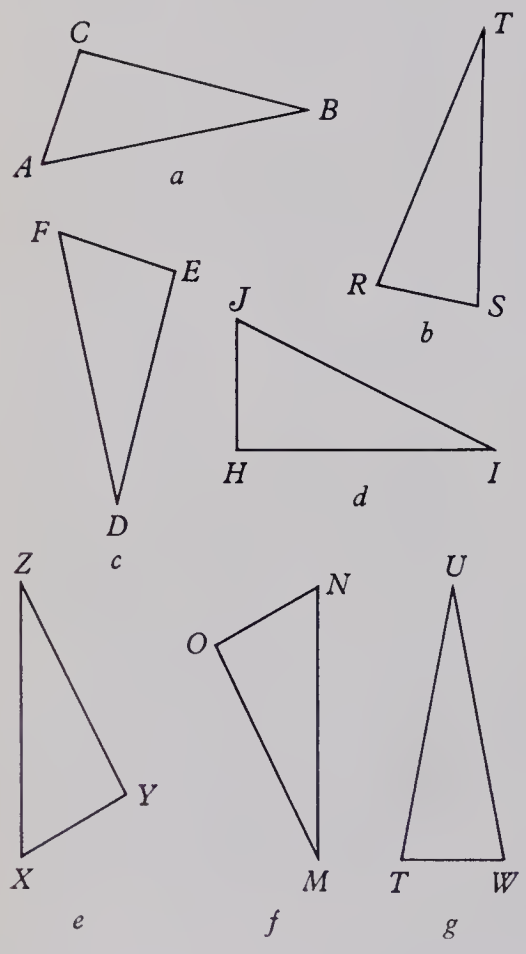
11. Name some models of congruent regions. For example, the top and the bottom of a cereal box are usually models of congruent rectangular regions. Answers will vary.

● **Extra Activity.** Set 166.

* One way of showing that two angles are congruent is to show that the model for one angle fits exactly on the model for the other angle. When we say that two angles are congruent because they have the same measure, we are assuming that the measurements are exact.

Congruent Angles and Triangles





[O]

* 1. Angles are congruent if they have the same size. Do angles have the same size if their measures in degrees are the same? Yes Then if two angles have the same measure in degrees, they are congruent. Do the *models* of the rays which form two angles have to be the same length for the angles to be congruent? No

2. Box A. Name three pairs of congruent angles. ∠A and ∠E, ∠B and ∠C, ∠D and ∠F

3. Are all right angles congruent? Yes Why? They all have the same measure in degrees.

4. Box B. Each of the two figures shown is the union of two line segments. Are the figures congruent? No Does it appear that the angles determined by the figures are congruent? Yes

5. Make a tracing of Fig. *a* on thin paper and compare the tracing to each of Fig. *b*–*g*. Which figures are congruent to $\triangle ABC$? c, e, f

[W]

Ex. 6–8. Write *T* or *F*.

6. The angles of a square are congruent. T

7. All acute angles are congruent. F

8. All equilateral triangles are congruent. F

Ex. 9–10. Refer to the figures shown at the left. Make tracings if they will help you.

9. Is $\triangle RST$ congruent to $\triangle DEF$? No to $\triangle TWU$? Yes

10. Is $\angle RST$ congruent to $\angle ABC$? No to $\angle TUW$? No

● **Extra Activity.** Set 167.

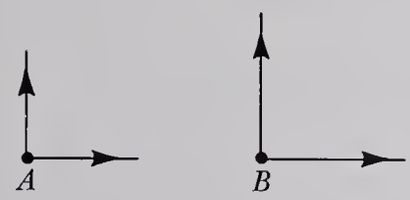
Teaching Page 136

Pupil's Objective

To learn the meaning of congruent angles, and the meaning of congruent triangles.

Background

If the measures of each of two angles is the same, they are said to have the same size and shape and so are congruent. However, the models representing the two angles do not have to have the same size. For example, $\angle A$ and $\angle B$ below are congruent even though the models of the rays forming the angles are not the same length.



It is understood that all rays are infinite in length so the actual rays forming the angles are equal in length, regardless of the lengths of their models.

There are three theorems that pertain to congruent triangles:

(1) If the lengths of the sides of one triangle are equal respectively to the lengths of the three sides of another triangle, the two triangles are congruent.

(2) If two triangles have the measure of two sides and the included angle in one triangle equal respectively to the measure of two sides and the included angle of the other, they are congruent.

(3) If two triangles have the measure of two angles and the included side equal respectively to the measure of two angles and the included side of the other, then the two triangles are congruent.

Pre-Book Lesson

• Ask how the definition of congruent figures given on page 135 might be applied to defining congruent angles.

• Review the characteristics of angles having the same size. Ask pupils about the importance of showing rays having the same length when picturing angles.

Using the Text Page

• Ex. 1. Point out that all rays are infinite in length regardless of the length of the model.

• Ex. 4. The pair of segments on the left in box B is not congruent to the pair of segments to the right. However, the angle determined by the pair of segments to the left is congruent to the angle determined by the pair of segments to the right.

Individualizing Instruction

• The *more capable pupils* may be asked to do each of the following:

(1) try to make models showing two triangles, the lengths of the sides of one triangle being the same as the lengths of the sides of the other triangle, such that the two triangles are not congruent;

(2) try to make models of two triangles, the measure of two sides and the included angle of one triangle being equal respectively to the measure of the two sides and the included angle of the other triangle, such that the two triangles are not congruent;

(3) try to make models of two triangles, the measure of two angles and the included side of one triangle being equal respectively to the measure of two angles and the included side of the other triangle, such that the two triangles are not congruent. Have them summarize their conclusions reached as a result of their efforts to make these models.

• *All pupils* may write an explanation for and draw models for Ex. 6–10. Their explanations should reveal whether or not they have a grasp of the true meaning of congruence.

Teaching Pages 137, 138 and 139

Pupil's Objective

To take the regular end-of-chapter tests including (a) Test of Information and Meaning 3; (b) Diagnostic Test 3; (c) Problem Test 3; and (d) Computation Test 3.

Background

It is a good idea to establish a regular procedure for administering, scoring, discussing, and recording end-of-chapter tests. These tests measure the ideas, skills, and concepts which have been developed or extended in Chapter 3.

The results of the tests should be carefully recorded and analyzed. They serve as a measurement of each pupil's ability and as a means of diagnosing and remedying weaknesses through the Extra Practice Sets and individual instruction.

Pre-Book Lesson

• Chapters 1 and 2 contain a great deal of resurvey ideas and skills presented in earlier grades. A sizable portion of Chapter 3 is devoted to extension of these ideas and skills. The new material should be reviewed before the end-of-chapter tests are administered.

• One convenient method of conducting a cumulative review of the chapter is to select exercises from each of the sets of summary exercises on pages 102 and 127, and from the sets of practice exercises on pages 102, 108, 109, and 117.

Using the Text Pages

• The tests may be given over a period of two or more days in any order. No more than two tests should be given in any one day.

• Before each test, pupils should scan the test, read the directions and understand exactly what to do. Any instructions for use of paper or recording answers should be given at this time.

• An item analysis of these tests will provide you with information as to which items are missed most frequently. If a sizable group of pupils shows difficulty with any item or items, provide individual help and reteaching.

Individualizing Instruction

• After completing the tests *all pupils* should correct their mistakes, be grouped according to areas of common weaknesses and work appropriate Extra Examples and special selected exercises to strengthen weak areas. After reteaching work has been completed, some pupils may wish to work Supplementary Activities on Teacher's Pages 138–139.

• *More capable pupils* may help *slower learners* with remedial work if the situation permits. They should work Supplementary Activities after they have completed test work.

The table of per cents shown below will enable you to determine quickly the per cent for each pupil's score.

Table of Per Cents for Chapter 3 Scores

Problem Test		Computation Test					
Score	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*
1	11	1	4	10	40	18	72
2	22	2	8	11	44	19	76
3	33	3	12	12	48	20	80
4	44	4	16	13	52	21	84
5	56	5	20	14	56	22	88
6	67	6	24	15	60	23	92
7	78	7	28	16	64	24	96
8	89	8	32	17	68	25	100
9	100	9	36				

* Since to estimate, $7\frac{2}{3}$ and $10\frac{4}{5}$ are rounded "up," a better estimate is obtained by rounding $6\frac{1}{2}$ "down" to 6.

Do You Understand?

Test of Information and Meaning 3

Ex. 1–3. Estimate the sum by rounding each addend to the nearest whole number.

1. $12\frac{5}{9} + 3\frac{2}{3}$ 17 * **2.** $7\frac{2}{3} + 6\frac{1}{2} + 10\frac{4}{5}$ 25 or 26 **3.** $35\frac{7}{20} + \frac{8}{9} + 21\frac{1}{2}$ 58

Ex. 4–7. Copy and complete by writing $=$ or $<$ or $>$.

4. $\frac{3}{5}$ _?_ $\frac{7}{5}$ **5.** $\frac{12}{13}$ _?_ $\frac{7+5}{13}$ **6.** $\frac{2}{3}$ _?_ $\frac{4}{5}$ **7.** $\frac{4}{7}$ _?_ $\frac{5}{6}$

** Ex. 8–12. Rename with a fraction in simplest form.

8. $\frac{22}{30}$ $\frac{11}{15}$ **9.** $\frac{57}{99}$ $\frac{19}{33}$ **10.** 23.5 $\frac{47}{2}$ **11.** $8\frac{2}{9}$ $\frac{74}{9}$ **12.** $\frac{5}{8} + \frac{7}{8}$ $\frac{3}{2}$

Ex. 13–17. Rename with a mixed form.

13. $\frac{17}{5}$ $3\frac{2}{5}$ **14.** 3.25 $3\frac{1}{4}$ **15.** 201.37 $201\frac{37}{100}$ **16.** $\frac{356}{24}$ $14\frac{5}{6}$ **17.** $19 \div 3$ $6\frac{1}{3}$

Ex. 18–22. Rename with a decimal.

18. $\frac{37}{100}$ 0.37 **19.** $\frac{3}{50}$ 0.06 **20.** $\frac{119}{100}$ 1.19 **21.** $3\frac{7}{10}$ 3.7 **22.** $16 + \frac{23}{1{,}000}$ 16.023

Ex. 23–27. Round to the nearest hundredth.

23. 2.889 2.89 **24.** 0.036 0.04 **25.** 45.222 45.22 **26.** 7.635 7.64 **27.** 89.872 89.87

28. The least common denominator of $\frac{3}{14}$ and $\frac{17}{21}$ is _?_ 42.

Ex. 29–31. Tell whether n represents the sum or an addend and then solve.

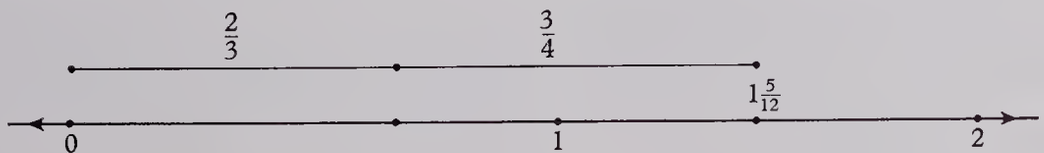
29. $n - 23\frac{3}{4} = 4\frac{3}{4}$ sum, $27\frac{3}{4}$ **30.** $17\frac{8}{9} - n = 12\frac{1}{3}$ addend, $5\frac{5}{9}$ **31.** $2\frac{7}{8} + 30\frac{4}{5} = n$ sum, $33\frac{27}{40}$

32. Which of figures *b*–*e* appear to be congruent to figure *a*? e

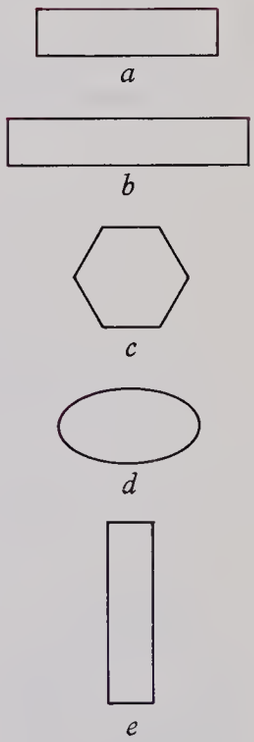
33. Which of figures *a*–*e* appear to be regular polygons? c

34. In degrees, the sum of the measures of the angles of a triangle is _?_ 180.

35. Write four mathematical sentences which show the relationship expressed in the following number-line picture.



$1\frac{5}{12} = \frac{2}{3} + \frac{3}{4}$; $1\frac{5}{12} - \frac{2}{3} = \frac{3}{4}$; $1\frac{5}{12} - \frac{3}{4} = \frac{2}{3}$; $1\frac{5}{12} = \frac{3}{4} + \frac{2}{3}$



** The numerator shown may be greater than the denominator for some fractions.

* Depending on how the question is worded, some answers will require labels. Notice that the answer for Ex. 1 is a number and that the answer for Ex. 3 is a number and a unit. You may require pupils to label all answers, only make sure that pupils understand the type of answers that you expect before giving the test.

Do You Make Mistakes?

Diagnostic Test 3

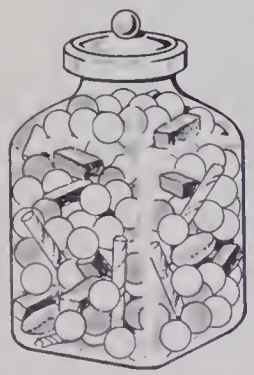
Copy and work the examples in rows 1-3.

	a	b	c	d	Study Pages	Practice: Use Sets
1. Add	$36\frac{7}{8}$ $47\frac{5}{8}$ $84\frac{1}{2}$	$20\frac{2}{3}$ $91\frac{5}{7}$ $112\frac{8}{21}$	$\frac{7}{9}$ $28\frac{2}{3}$ $29\frac{4}{9}$	$376\frac{7}{8}$ $83\frac{9}{12}$ $460\frac{5}{8}$	103–104, 112–113	58–59
2. Subtract	$40\frac{7}{8}$ $17\frac{3}{8}$ $23\frac{1}{2}$	$298\frac{3}{4}$ $88\frac{5}{6}$ $209\frac{11}{12}$	$10\frac{1}{2}$ $\frac{8}{9}$ $9\frac{11}{18}$	$1{,}609\frac{4}{5}$ $907\frac{2}{3}$ $702\frac{2}{15}$	105, 114	61–62
3. Add	$30\frac{1}{2}$ $6\frac{3}{4}$ 17 $54\frac{1}{4}$	$43\frac{2}{3}$ $78\frac{5}{7}$ $202\frac{1}{7}$ $324\frac{11}{21}$	$\frac{5}{6}$ $38\frac{2}{3}$ $3\frac{7}{8}$ $43\frac{3}{8}$	$372\frac{5}{8}$ $37\frac{1}{4}$ $526\frac{1}{2}$ $936\frac{3}{8}$	115	64–65

Can You Solve Problems?

Problem Test 3

Let n represent the number which will answer the question in the problem. Write a mathematical sentence for the problem and then solve to find the answer for the problem.



* 1. Ed watched television $1\frac{1}{4}$ hr. on Friday and $2\frac{4}{5}$ hr. on Saturday. How many hours was that in all? $n = 1\frac{1}{4} + 2\frac{4}{5}$; $4\frac{1}{20}$

2. A year ago, Paula weighed $92\frac{3}{4}$ lb. She now weighs $101\frac{1}{2}$ lb. How many pounds has she gained? $n = 101\frac{1}{2} - 92\frac{3}{4}$; $8\frac{3}{4}$

3. A jar filled with hard candies weighs $2\frac{3}{4}$ lb. It contains $1\frac{15}{16}$ lb. of candy. How much does the jar weigh? $n = 2\frac{3}{4} - 1\frac{15}{16}$; $\frac{13}{16}$ lb.

4. Jim said he needed $4\frac{1}{2}$ yd. of cord to rig a model sailboat. His mother gave him $6\frac{1}{3}$ yd. That was how much more than he said he needed? $n = 6\frac{1}{3} - 4\frac{1}{2}$; $1\frac{5}{6}$ yd.

Supplementary Activities

• Ask the *more capable pupils* to write = or < or > in the blank to make true mathematical sentences. All numerators are whole numbers and all denominators are counting numbers.

If $\frac{a}{b} = \frac{c}{d}$, and $d > b$, then $c \_\_ a$.
If $\frac{a}{b} = \frac{c}{d}$, and $b = d$, then $a \_\_ c$.

• Ask *all pupils* what whole number a represents if $\frac{2}{3} = \frac{2+a}{3+a}$. What counting numbers can a and b represent in each of the following?

$\frac{a}{b} = \frac{a+1}{b+1}$ $(a = 1, b = 1)$
$\frac{a}{b} = \frac{a+4}{b+3}$ $(a = 4, b = 3)$
$\frac{a}{b} = \frac{b}{a}$ $(a = b)$

	$\frac{1}{3}$	
$1\frac{2}{3}$		
$2\frac{1}{3}$		1

• Complete the magic square so that the sum of the numbers in each row, each column, and along each diagonal is 9.

• Picture two angles whose intersection is (a) one point; (b) two points; (c) three points; (d) four points; (e) zero points; (f) a ray.

• How many pictures of angles can you find in Fig. 1? (10) Name them.

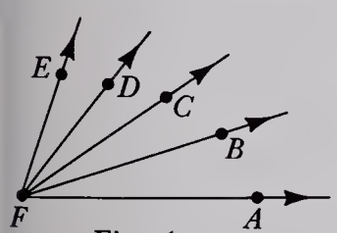


Fig. 1

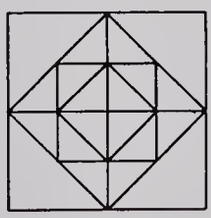


Fig. 2

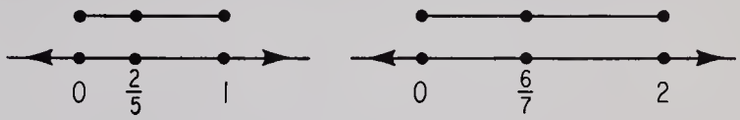
• How many squares are pictured in Fig. 2? (12) How many pictures of triangles do you see? (36)

• What digit does each of the letters in the problem stand for?

(Proceed as follows: if $A - P = A$, then P is 0 in line 5; if $W - 5 = 0$ in line 5 then W is 5; substitute 5 for all W's; if $7 - F = W$ in line 4 and $W = 5$, and $M - X = F$ in line 4, F must be less than 5, substitute 1 for F and proceed from there.)

(1)	F HM	Answer:
(2)	2F5)M 7 F 9 W	$A = 6$
(3)	X F W	$F = 1$
(4)	F W A 9	$H = 7$
(5)	F 5 P 5	$M = 3$
(6)	A 4 W	$P = 0$
	A 4 W	$W = 5$

• For each of the number-line pictures below, write a mathematical sentence using the symbol for addition, then write two mathematical sentences using the symbol for subtraction.



• In each of the following, name four different rational numbers that when substituted for n would make the sentence true.

$\frac{3}{4} < n < 1$
$\frac{3}{4} < n < \frac{4}{5}$
$1\frac{2}{3} < n < 1\frac{5}{6}$

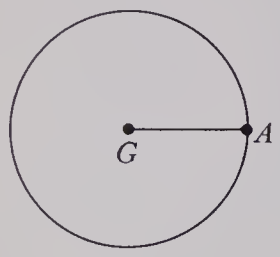
• Show that subtraction is not commutative and is not associative in the set of rational numbers by determining whole numbers for a, b, and c and counting numbers for d, e, and f which when substituted for the letters would make each of the following false:

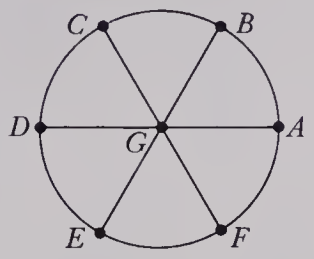
$\frac{a}{d} - \frac{b}{e} = \frac{b}{e} - \frac{a}{d}$
$\left(\frac{a}{d} - \frac{b}{e}\right) - \frac{c}{f} = \frac{a}{d} - \left(\frac{b}{e} - \frac{c}{f}\right)$

• In Ex. 11–18 on page 101, what is the relationship between $(a \times d)$ and $(b \times c)$ when $\frac{a}{b} > \frac{c}{d}$? when $\frac{a}{b} < \frac{c}{d}$? when $\frac{a}{b} = \frac{c}{d}$? Lead the pupils to the realization that:

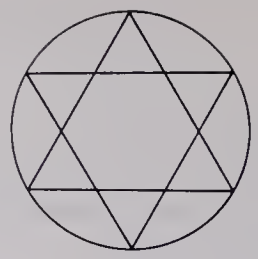
if $\frac{a}{b} > \frac{c}{d}$ then $(a \times d) > (b \times c)$
if $\frac{a}{b} < \frac{c}{d}$ then $(a \times d) < (b \times c)$
if $\frac{a}{b} = \frac{c}{d}$ then $(a \times d) = (b \times c)$

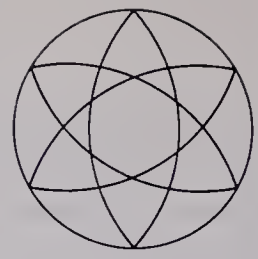
• Draw a picture to represent a circle and show a radius AG. Using the center for the circle as an end point, show six adjacent 60° angles so they intersect the circle at points A, B, C, D, E, and F.





Join pairs of points in various ways to make different geometric designs.





• Draw a picture to represent a circle and show a radius AG. Using the center for the circle as an end point, show eight 45° angles so they intersect the circle at points A, B, C, D, E, F, G, H. Join the points shown and shade in various regions to make interesting geometric designs. Pupils may draw several such designs.

NOTES

5. Mr. Oaks cooked the dinner when his club met at his house. He made his favorite beef stew. To the cooked beef, he added $1\frac{3}{8}$ cups of sliced onions, $7\frac{1}{2}$ cups of diced potatoes, $1\frac{1}{4}$ cups of diced carrots, and $1\frac{2}{3}$ cups of diced turnips. That was a total of how many cups of vegetables? $n = 1\frac{3}{8} + 7\frac{1}{2} + 1\frac{1}{4} + 1\frac{2}{3}$; $11\frac{19}{24}$

6. When Mr. Oaks started to make a dressing for the salad, he found there was only $1\frac{1}{8}$ cups of olive oil in the house. The recipe called for $2\frac{1}{2}$ cups of oil, so he used peanut oil to make up the difference. How many cups of peanut oil did he use? $n = 2\frac{1}{2} - 1\frac{1}{8}$; $1\frac{3}{8}$

7. $48\frac{7}{8}$ is how much greater than the sum of $26\frac{1}{2}$ and $5\frac{3}{5}$? $n = 48\frac{1}{8} - (26\frac{1}{2} + 5\frac{3}{5})$; $16\frac{31}{40}$

8. Find the arithmetic mean for $12\frac{1}{2}$, $11\frac{3}{4}$, $11\frac{1}{4}$, $13\frac{1}{8}$, and $12\frac{3}{8}$. See below.

9. What is the measure in degrees of the third angle of a triangle if measurements of the other two angles are 38° and 65°? $n = 180 - (38 + 65)$; 77

How Well Can You Compute?

Computation Test 3

Ex. 1–7. Subtract the number from $1\frac{1}{2}$.

1. $\frac{3}{4}$ $\frac{3}{4}$ 2. $\frac{4}{5}$ $\frac{7}{10}$ 3. $\frac{7}{8}$ $\frac{5}{8}$ 4. $\frac{1}{10}$ $1\frac{2}{5}$ 5. $1\frac{1}{3}$ $\frac{1}{6}$ 6. $1\frac{1}{5}$ $\frac{3}{10}$ 7. $\frac{4}{25}$ $1\frac{17}{50}$

Ex. 8–13. Add.

8. 37,869 + 4,899 = 42,768

9. $356\frac{7}{8} + 38\frac{1}{3} = 395\frac{5}{24}$

10. \$65.89 + 38.09 = \$103.98

11. $6\frac{17}{24} + 13\frac{1}{4} = 19\frac{23}{24}$

12. $38\frac{2}{3} + 79\frac{6}{11} = 118\frac{7}{33}$

13. $6\frac{7}{8} + 7\frac{5}{6} = 14\frac{17}{24}$

Ex. 14–16. Multiply.

14. $47 \times 18 = 846$

15. $398 \times 736 = 292,928$

16. $\$67.75 \times 14 = \$948.50$

Ex. 17–19. Divide. Express the answer with a mixed form.

17. $46\overline{)89,967} = 1,955\frac{37}{46}$

18. $37\overline{)45,001} = 1,216\frac{9}{37}$

19. $19\overline{)8,198} = 431\frac{9}{19}$

Ex. 20–25. Find the number represented by the letter.

20. $n = 4\frac{3}{4} - 3\frac{1}{3}$ $1\frac{5}{12}$

21. $45\frac{1}{3} - (34\frac{1}{2} + \frac{6}{7}) = x$ $9\frac{41}{42}$

22. $n - 389\frac{3}{5} = 47\frac{2}{3}$ $437\frac{4}{15}$

23. $n = 45 + (4\frac{3}{5} + \frac{1}{2} + \frac{3}{5})$ $50\frac{7}{10}$

24. $x = 63 - (36 + 7\frac{1}{2} + 10\frac{2}{3})$ $8\frac{5}{6}$

25. $n + 34\frac{6}{7} = 67\frac{11}{14}$ $32\frac{13}{14}$

8. $n = (12\frac{1}{2} + 11\frac{3}{4} + 11\frac{1}{4} + 13\frac{1}{8} + 12\frac{3}{8}) \div 5$; $12\frac{1}{5}$

4

Measurements for the Same Amount

Resurvey [O]

1. One day in August, the Jackson Dairy sold 40 gal. of ice cream. Is 40 gal. a *measurement* of the amount of ice cream sold? Yes ^ Is 160 qt. also a measurement of the amount of ice cream sold? Yes ^ Explain. Since 1 gal. = 4 qt., 40 gal. = 160 qt.

2. To show that both 40 gal. and 160 qt. are measurements for the same amount, we write, *40 gal. = 160 qt.* Is it true that both 1 ft. and 12 in. are measurements for the same length? Yes ^ Then 1 ft. = _?_ 12 in. Complete the following:

a. 3 ft. = _?_ 36 in. **b.** 1 yd. = _?_ 3 ft. = _?_ 36 in.

3. During a week in August, the Jackson Dairy sold 944 qt. of ice cream. That was how many gallons?

a. To find the number of gallons, we divide 944 by _?_ 4.

b. Does $n = 944 \div 4$ show the relationship expressed in the problem? Yes

c. On the board, show the work for finding $944 \div 4$.

d. 944 qt. = _?_ 236 gal.

```
236
4)944
 8
 14
 12
 24
 24
R  0
```

Overview—Chapter 4

• Much of the work in Chapter 4 pertains to measurement, adding and subtracting with decimals, and areas of regions. The work with measurement includes English and metric units of measurement; equivalent measurements; decimal concepts with measurements; using a centimeter scale; unit of precision and greatest possible error; use of a micrometer; measuring segments; finding perimeters and areas of regions; renaming measurements of and comparing sizes of area; and congruent regions.

• **Problem-Solving.** Pupils are given experience solving problems involving the addends-sum relationship with decimals; differentiation between addition and subtraction; social situations with geometric ideas, measurement of perimeter and areas, and use of the micrometer; use of decimals in a hospital; measurement of land and money involved in home remodeling.

• **Maintenance.** Summary and practice exercises are presented throughout the chapter and on pages 147, 158, 159, 161, 168, and 172. Topics covered include computation involving measures in both the English and the metric systems, expressed with decimals or fractions; equivalent measurements; arithmetic mean; precision and greatest possible error; sets and geometry; the addends-sum relationship; inequalities; the four operations with fractions and decimals; simplest form; rounding numbers; and exponent form.

• **Enrichment.** Topics presented on the enrichment pages include problems involving work and computing foot-pounds; meaning of and naming of adjacent angles in geometric figures; geometric figures as presenting optical illusions; and use of a planimeter to find areas of regions when enclosed by a simple closed curve. Supplementary Activities found at the end of the chapter may also be used for enrichment.

• **Tests.** The regular end-of-chapter tests evaluate such concepts as inequalities; measurement involving metric and non-metric units; greatest possible error and unit of precision; the addends-sum relationship as expressed with decimals; use of parentheses in solving mathematical sentences; problem-solving in social situations involving measurement with metric and non-metric units; and finding perimeter and area of rectangular regions. Selected practice pages may also be used for diagnostic and testing purposes.

Teaching Pages 140 and 141

Pupil's Objectives

(a) To review the concept of equivalent measurements; and (b) to review expressing a measurement in larger or smaller units.

Background

Since "equal" is a term usually restricted to use with numbers, some mathematicians prefer the word "equivalent" instead of the word "equal" when defining measurements. Their feeling is that *1 foot* means a segment whose measure is 1 in feet, while *12 inches* means 12 segments, the measure of each being 1 in inches. The 12 one-inch segments have the same length as the 1 one-foot segment, so the 1 one-foot segment may be used in place of the 12 one-inch segments in expressing measurements. They are not one and the same segment. When we write "1 ft. = 12 in.," we state measurements for the same length. The *measures* are not equal ($1 \neq 12$), and the *units of measurement* are not the same (foot is not the same as inch).

Discuss the preceding information with *more capable pupils*. It is not intended that *all pupils* be required to use the term *equivalent* in place of the more commonly used term *equal*.

When a pupil is given a measurement and is asked to find an equivalent measurement using either larger or smaller units, he is often faced with the problem of whether to multiply or divide. If a measurement is given and the equivalent measurement is to be expressed in a smaller unit, the measure in that smaller unit may be found as shown in (1); the measure for a larger

unit may be found as shown in (2).

(1) measure in smaller unit = measure in larger unit × number of smaller units equivalent to one larger unit

(2) measure in larger unit = measure in smaller unit ÷ number of smaller units equivalent to one larger unit

Pre-Book Lesson

• Ask questions pertaining to the table of units of measurement on page 371 to be sure the pupils understand these tables and can use them in doing the exercises on these pages.

• Discuss the uses of measurement and the importance of a familiarity with equivalent measurements. Ask for examples from the pupil's experience of equivalent measurements.

Using the Text Pages

• Ex. 1–2. While we use "=" with measurements of length as a short cut notation, we really mean "is equivalent to," or "is the same length as." For example, we write "3 ft. = 1 yd.," but we mean "3 ft. is the same length as 1 yd." This should be pointed out to *all pupils*. The more definitive distinction presented in the Background should be pointed out to the *more capable pupils*.

List a few measurements on the board and have pupils find equivalent measurements. Use rulers and yardsticks, pint, quart, and gallon containers to demonstrate equivalence.

• Ex. 3. The relationship may first be expressed in words, as in (1) and (2) in the Background section. Ask why units of measurement are never shown in a mathematical sentence. Measures, not units of measurement, are numbers.

• Ex. 4. The explanation of why $\frac{5}{60}$ hr. is equivalent to 5 min. should not involve multiplication with fractions because this topic has not yet been resurveyed. It should be explained in terms of the relationship $\frac{1}{60}$ hr. = 1 min., so $\frac{5}{60}$ hr. = 5 min.

• Ex. 6–11. Make sure pupils know when to multiply and when to divide. Leave the expressions from the Background on the board.

• Ex. 16. This is a two-step problem involving first multiplication, then division.

• Ex. 18–23. Refer to Ex. 4 and the work in the box to see the relationship between mixed forms, division with a remainder, and expressing the remainder in fraction form.

• Ex. 30–35. The measure to be shown in the last blank in each of these exercises is named with a mixed form. Write several similar exercises on the board. If the situation permits, have *more capable pupils* relate these exercises to Ex. 4, the work in the box, and Ex. 18–23. Have pupils explain their thinking to the entire class.

Individualizing Instruction

• Have *more capable pupils* explain the general relationship among measurements described in the Background, and answer such questions as:

(a) If a measurement of 6 *snouts* is equivalent to a measurement of 27 *blats*, what is the relationship between *snouts* and *blats*?

(b) If 1 *doot* = 7 *koots*, a measurement of 32 *doots* is equivalent to a measurement of how many *koots*? A measurement of 9 *doots* is equivalent to a measurement of how many *koots*?

• Have *all pupils* turn to the tables on page 371 and name an object whose measurement is usually given in each unit of measurement listed.

• Emphasize for the *slower learners* that when a measurement is expressed in terms of a larger unit, the measure will always be a lesser number than that of the given measurement.

Use gallons and quarts, a yard and three rulers, dozen and half-dozen cartons, to demonstrate equivalence. A measurement involving both gallons and quarts may be expressed as a measurement in gallons only. The measure is expressed with a mixed form and can easily be illustrated by the use of the containers.

*A "furlong" is a unit of length used in describing distances for horse racing.

4. To express 185 min. as a number of hours, we divide _?_ (185) by _?_ (60). Explain the work in the box. Instead of writing $3\frac{5}{60}$ hr., we may write, *3 hr.* _?_ (5) *min.* Explain. $\frac{5}{60}$ hr. means 5 min.

185 min. = _?_ hr.

$$60\overline{)185}\quad 3,\ 180,\ R\ 5$$

185 min. = $3\frac{5}{60}$ hr.
= 3 hr. 5 min.

5. On the board, express 127 in. in terms of feet and inches. Explain your work.

127 in. = $10\frac{7}{12}$ ft. = 10 ft. 7 in.

[W]

Ex. 6–11. Copy and complete. Use the tables on page 371 if you need help.

6. 3 mi. = _?_ (15,840) ft. **7.** 7 yd. = _?_ (252) in. **8.** 9 T. = _?_ (18,000) lb.

9. 3 bu. = _?_ (12) pk. **10.** 3 hr. = _?_ (10,800) sec. **11.** 5 gal. = _?_ (40) pt.

Ex. 12–15. Write the names and abbreviations of

12. 5 units of length. inch, in.; foot, ft.; yard, yd.; mile, mi.; rod, rd.

13. 5 units of time. second, sec.; minute, min.; hour, hr.; day, da.; year, yr.

14. 3 units of weight. ounce, oz.; pound, lb.; ton, T.

15. 3 units of liquid measure. pint, pt.; quart, qt.; gallon, gal.

16. Bob studies 45 min. each evening. How many *hours* does he study in 4 evenings? 3

*__17.__ An eighth of a mile is sometimes called a "furlong." Express one furlong in terms of feet. 660 feet

Ex. 18–23. Solve. Express the answer with a mixed form.

18. $n = 235 \div 60$ ($3\frac{11}{12}$) **19.** $n = 6{,}050 \div 2{,}000$ ($3\frac{1}{40}$) **20.** $x = 27 \div 4$ ($6\frac{3}{4}$)

21. $n = 54 \div 16$ ($3\frac{3}{8}$) **22.** $x = 1{,}500 \div 360$ ($4\frac{1}{6}$) **23.** $n = 720 \div 320$ ($2\frac{1}{4}$)

Ex. 24–35. Copy and complete.

24. 24 in. = _?_ (2) ft.

25. 48 mo. = _?_ (4) yr.

26. 1,095 da. = _?_ (3) yr.

27. 300 pk. = _?_ (75) bu.

28. 72 qt. = _?_ (18) gal.

29. 86 pt. = _?_ (43) qt.

30. 80 ft. = _?_ (26) yd. _?_ (2) ft. = _?_ ($26\frac{2}{3}$) yd.

31. 24 da. = _?_ (3) wk. _?_ (3) da. = _?_ ($3\frac{3}{7}$) wk.

32. 151 qt. = _?_ (37) gal. _?_ (3) qt. = _?_ ($37\frac{3}{4}$) gal.

33. 35 oz. = _?_ (2) lb. _?_ (3) oz. = _?_ ($2\frac{3}{16}$) lb.

34. 340 rd. = _?_ (1) mi. _?_ (20) rd. = _?_ ($2\frac{1}{16}$) mi.

35. 7,680 lb. = _?_ (3) T. _?_ (1,680) lb. = _?_ ($3\frac{21}{25}$) T.

♦ **Extra Examples.** Sets 70–73.

* Point out that the units of the metric system are based on powers of ten.

Units for Measuring Length

Metric units of length [O]

1. Measurements such as 2 in., 5 ft., and 4 yd. are measurements of length. A measurement of length is called a *linear measurement*. In a linear measurement, the unit of measurement is a unit of length.

2. Which of Ex. a–e are linear measurements? b, d Which are not? a, c, e

a. 3 gal. **b.** 52 ft. **c.** 18 T. **d.** 23 rd. **e.** 2 qt.

3. *Inch*, *foot*, *yard*, *rod*, and *mile* are units of length in the *English system of measurement*. *Centimeter* and *meter* are units of length in what system? Metric system

1 meter (m.)	=	10 decimeters (dm.)
1 decimeter	=	10 centimeters (cm.)
1 meter	=	100 centimeters

4. In many countries, the metric system of measurement is commonly used. In the United States, it has been an official system since 1893. Some standard units of length in the metric system are centimeter (cm.), decimeter (dm.), and meter (m.). Using the information in the box, give fractions to complete Ex. a–c.

* **a.** 1 dm. = $\frac{1}{10}$ m. **b.** 1 cm. = $\frac{1}{10}$ dm. **c.** 1 cm. = $\frac{1}{100}$ m.

5. 1 m. = 100 cm., so 5 m. = 500 cm. On the board, copy and complete Ex. a–f.

a. 8 m. = 800 cm. **b.** 17 dm. = 170 cm. **c.** 31 m. = 310 dm.

d. 9 dm. = 90 cm. **e.** 12 m. = 120 dm. **f.** 64 m. = 6,400 cm.

6. Is it true that 1 cm. = 0.01 m.? Yes Then 3 cm. = 0.03 m.

[W]

Ex. 7–18. Copy and complete.

7. 14 m. = 1,400 cm.
8. 5 cm. = 0.05 m.
9. 8 m. = 800 cm.
10. 640 cm. = 6.4 m.
11. 10 m. = 100 dm.
12. 1 dm. = 0.1 m.
13. 640 cm. = 64 dm.
14. 22 m. = 220 dm.
15. 23 dm. = 230 cm.
16. 3 dm. = 0.3 m.
17. 1,300 cm. = 13 m.
18. 3 cm. = 0.3 dm.

Teaching Page 142

Pupil's Objectives

(a) To learn the standard units of length in the metric system; and (b) to express measurements in the metric system with larger and smaller units.

Background

The metric system of measurement originated in France after the French Revolution. In this system, there is a basic unit of measurement of distance (the meter), of capacity (the liter), and of weight and mass (the gram). A smaller unit is usually one tenth as large as the next larger unit.

The meter is defined as 1,650,763.73 wave lengths of the orange-red light from the isotope krypton 86. This definition was adopted by the International Bureau of Weights and Measures in 1960.

The standard unit of weight and mass, the gram, weighs slightly more than 0.0022 pounds, and is used to weigh drugs and medicine.

The standard unit of capacity is the liter which is defined as the volume occupied by one kilogram of pure water at 4°C. One milliliter (one thousandth of a liter) or one cubic centimeter of pure water has a weight of one gram. 1 liter = 1,000.027 cubic centimeters, but the approximation 1 liter = 1,000 cubic centimeters is sufficient for practical purposes. A can 3 inches square and 6.8 inches high holds about a liter of water.

The metric system is ideal for computations in our base-ten system of numeration, in which place values are powers of 10. A distance of 9 meters 3 decimeters 7 centimeters may be expressed as 9.37 meters, as 93.7 decimeters, or as 937 centimeters. To express any given measurement in a smaller or larger unit, the measure is multiplied or divided by a power of 10.

Pre-Book Lesson

• Review multiplication and division by 10 and 100.

• Give pupils practice at changing various fractions showing denominators other than powers of ten to those showing denominators which are powers of ten.

Using the Text Page

• Ex. 3–4. Because pupils may not use the metric system very often in their daily living experiences, they may not realize the importance of mastering the concept of these units of measurement. Point out that the metric system is used extensively in pharmacy, foreign trade, Civil Service, and Engineering. Most measurements made in science classes in later years are in the metric system (refer to Background).

• Ex. 5–18. Mention that when expressing a measurement in the metric system in smaller units, the given measure is multiplied by 10; 100; 1,000. Have pupils use the short cut method of eliminating or annexing zeros when dividing or multiplying by powers of ten.

Compare the difficulty of expressing 172 inches as a measurement in feet with the ease of expressing 172 centimeters as a measurement in meters. You may refer to the Background of pages 140 and 141 to review briefly expressing a measurement in larger or smaller units.

Individualizing Instruction

• Ask *all pupils* for examples of use of the metric system with which they are familiar. Some examples are width of movie film (8 mm., 16 mm.); displacement of the engines of foreign cars (liters or cubic centimeters); the size of the bore of cannon and rifles (mm.).

Discuss the possible difficulties involved in changing to common use of the metric system in a country like the United States which predominantly uses the English system.

• For the *slower learners* converting a measurement from meters to centimeters, the measure in meters may be first expressed in terms of decimeters, and then this expressed in terms of centimeters. In this way, each step involves only multiplication by 10.

Teaching Page 143

Pupil's Objectives

(a) To compare the size of the centimeter and the inch; and (b) to gain experience using a centimeter scale.

Background

One of the purposes of the exercises on this page is to give the pupil a concept of the relative size of one centimeter as compared to one inch. It is important for a person already familiar with the units of measurement in one system to have some concept as to the size of a measurement expressed in a unit of the other system.

One inch is approximately equivalent to $2\frac{1}{2}$ centimeters. A distance of 10 inches will be $2\frac{1}{2}$ times as great as a distance of 10 centimeters. A distance of 10 centimeters is less than one half as great as a distance of 10 inches.

If a pupil is given a model representing a segment and is told it is x inches in length, he should have some concept as to where he should place a mark if he is told to mark off a segment x centimeters from one end. He should estimate the location of the center of the segment and then mark off a segment that is slightly less than half the length of the given segment.

If a pupil is given a model representing a segment and is told it is y centimeters in length, he should be able to draw a model of a segment which is about y inches in length by marking off a segment $2\frac{1}{2}$ times as great as the given segment. Exercises such as these give the pupil some concept as to the relative sizes of the inch and the centimeter.

Pre-Book Lesson

Ask, "What kind of ruler do we need to measure with metric units?" What is needed, of course, is a ruler that has been marked off into segments of the same size, the measure of each being 1 in a standard metric unit.

Using the Text Page

• Ex. 1–2. Have the pupils also express the relationship between the inch and the centimeter in more general terms such as, "An inch is about $2\frac{1}{2}$ times as great as a centimeter" and as "A centimeter is less than half an inch." Have pupils give examples of items about one inch long or about one centimeter long.

• Ex. 5–6. If pupils do not have centimeter scales, they can make their own by using the picture on the text page as a guide. Holding the edge of a piece of paper up to the picture, they can mark off segments one centimeter in length. If possible, after cutting out their scales, they can mount them on cardboard.

The measure of a segment is hardly ever both a whole number in inches and also a whole number in centimeters. For this reason, the measurements should be expressed to the nearest inch or to the nearest centimeter.

Some pupils may decide to express the measurement of each segment in inches and then multiply the measure by 2.54 to obtain the measure in centimeters. Discourage this practice as this defeats the purpose of the exercises which is to give the pupil experience in using a metric scale for determining the measurement of a segment.

Individualizing Instruction

Have *all pupils* use their centimeter scales to determine the measure of various things in the classroom in centimeters. Then have other pupils estimate their length in inches without using a ruler.

Before completing Ex. 6, have these pupils put their centimeter scale next to the scale on a ruler and compare the measurements. Give them measurements in centimeters and ask for each corresponding measurement to the nearest inch, and vice versa.

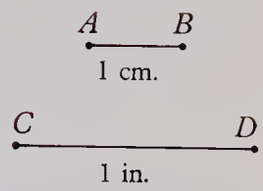
* The relationship between the inch and the centimeter is stated so that pupils can acquire some understanding of relative lengths. Do not require pupils to convert measurements in English units to metric units or metric units to English units.

Using a Centimeter Scale

[O]

1. Give the lengths of segments AB and CD shown at the right. Which is the longer unit, centimeter or inch? inch

1 cm. 1 in.

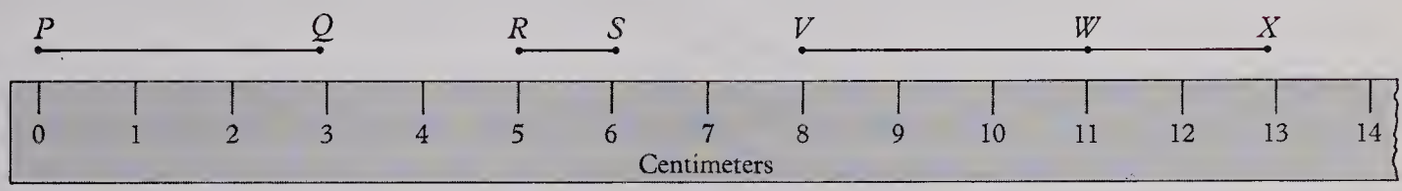


2. Does it seem that the length of a 1-in. segment is more than twice the length of a 1-cm. segment? Yes A segment which measures 1 in inches measures *approximately* 2.54 in centimeters.

* **1 inch is about 2.54 centimeters.**

3. A centimeter scale is shown below. Give the following measures in centimeters.

a. $m(\overline{PQ})$ 3 **b.** $m(\overline{RS})$ 1 **c.** $m(\overline{VW})$ 3 **d.** $m(\overline{WX})$ 2 **e.** $m(\overline{VX})$ 5



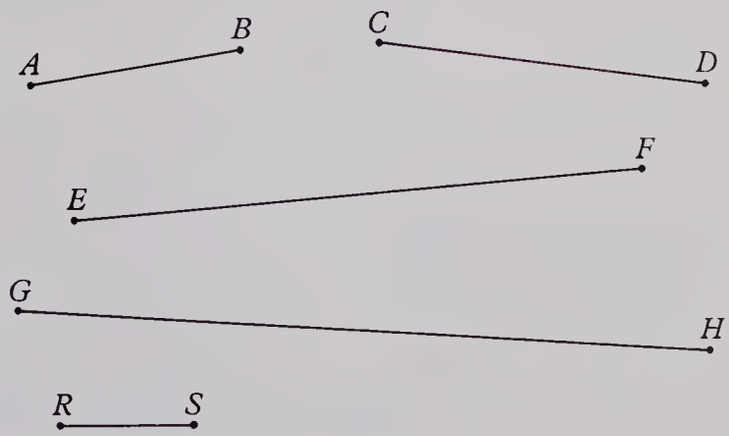
4. With an inch scale, measure $\overline{MN}$ to the nearest inch. 4 in.

5. If you do not have a centimeter scale, make a scale like the one shown above. Measure $\overline{MN}$ to the nearest centimeter. 10 cm.

[W]

6. Copy and complete the table for the segments shown at the right. The first one is done for you.

Segment	Measurement to the nearest centimeter	Measurement to the nearest inch
AB	2 cm.	1 in.
CD	_?3_ cm.	_?1_ in.
EF	_?6_ cm.	_?2_ in.
GH	_?7_ cm.	_?3_ in.
RS	_?1_ cm.	_?1_ in.



* This comparison is made only for the purpose of pupils acquiring some understanding of relative lengths.

Some Other Metric Units of Length

Kilometer and millimeter [O]

1. In the picture above, John is shown measuring the width of a doorway with a *meterstick*. How many centimeters are shown on a *meterstick*? 100

A meterstick is about 39.37 in.

* 2. Compare a meterstick with a yardstick. Which is longer, the meterstick or the yardstick? Meterstick

3. In the English system, to give the distance between Boston and Seattle, would it be better to use yards or miles? miles In the metric system, *kilometers* would be used. *Kilo* means thousand, so kilometer (km.) means 1,000 meters. Complete Ex. a–c.

a. 2 km. = 2,000 m. **b.** $\frac{1}{2}$ km. = 500 m. **c.** 200 m. = $\frac{1}{5}$ km.

4. What do you think is meant by *deci* in decimeter (dm.)? tenth *centi* in centimeter (cm.)? hundredth

5. A *millimeter* (mm.) is a shorter metric unit than a centimeter. In the word millimeter, *milli* means thousandth, so 1 mm. = 0.001 m.

[W]

Ex. 6–14. Copy and complete.

6. 1 m. = 1,000 mm.
7. 3 km. = 3,000 m.
8. 3 m. = 30 dm.
9. 1 cm. = 10 mm.
10. 500 mm. = 50 cm.
11. 5,000 m. = $\frac{5}{2.5}$ km.
12. 5 cm. = 50 mm.
13. 2,000 mm. = $\frac{2}{2.5}$ m.
14. 250 cm. = 2.5 m.

◆ **Extra Examples.** Set 74.

Teaching Page 144

Pupil's Objectives

(a) To compare the meter and the yard; (b) to learn the meaning of kilometer and millimeter; and (c) to compare measurements of length within the metric system.

Background

A meter is equivalent to 39.37 inches. The prefix *kilo* means *thousand*, so *kilometer* means *one thousand meters*. Since one meter is slightly longer than three feet, one kilometer is a little longer than 3,000 feet, or approximately three-fifths of a mile. Five kilometers is approximately equivalent to 3 miles. To obtain an equivalent measurement in miles, a measure in kilometers may be rounded to the nearest multiple of 5, then divided by 5 and multiplied by 3.

The prefixes *deci* (tenth), *centi* (hundredth), and *milli* (thousandth) found in *decimeter*, *centimeter*, and *millimeter* mean one tenth, one hundredth, and one thousandth of a meter respectively.

Pre-Book Lesson

• One mile is equivalent to approximately 1,600 meters. Have pupils express in meters the distance between two cities that are 700 miles apart. The answer 1,120,000 meters suggests that a unit of measurement larger than the meter would be useful for measuring long distances.

• Conduct a general discussion of the use of metric units in school sports such as track and swimming; and Olympic events such as track, auto racing, skiing, and swimming.

Using the Text Page

• Ex. 1–2. If possible, give the pupils a meterstick and a yardstick to examine and compare. Review the prefixes *deci*, *centi*, and *milli*.

• Ex. 3. Ask for a rough comparison of the kilometer and the mile. You may wish to discuss the information from the Background.

• Ex. 5. Ask for the probable meaning of a *mill*. It means one thousandth of a dollar. There is no coin called a mill, but it is a unit of money sometimes useful in computing tax rates or for values of various trading stamps.

Individualizing Instruction

• *All pupils* may be asked to name the metric unit most convenient for measuring such things as:

(a) the distance from New York to Boston
(b) pupil's height
(c) length of a pencil
(d) diameter of a piece of chalk
(e) thickness of a book
(f) length of the classroom
(g) distance from Earth to an orbiting satellite.

These pupils may discuss the possible advantages of the metric system over the English system of measurement. Compare this discussion with others held previously on the same topic. The following items could be discussed:

(1) The uniform system of names
(2) Prefixes added to basic unit of measurement to name other units
(3) Relative ease of renaming in a system based on powers of ten
(4) Naming very small or very large measurements with a system based on the powers of ten.

• Ask the *more capable pupils* for examples of the use of *kilo* other than kilometer. Some examples are *kilowatt*, a unit of measurement of electricity, and *kilocycles*, a unit of measurement of radio frequencies.

You may have these pupils look up in a general reference work or sports regulation manual the specifications for measuring a baseball diamond, a tennis court, and a basketball court, and express these specifications in the metric system.

• The *slower learners* may be given a meterstick that has been marked off into centimeters and be asked to determine the measure of various objects to the nearest centimeter or decimeter.

• Have *all pupils* prepare and conduct a debate on changing to common use of the metric system.

Pupil's Objectives

(a) To add measures by expressing all measurements in terms of the smallest unit given; and (b) to add the measures in larger units and the measures in smaller units independently.

Background

A measurement consists of a measure and a unit of measurement. Because measures are numbers, they can be operated on. If the sum of any two measures is to be found, the units of measurement must be the same. Otherwise, the answer is meaningless. If we have a measurement of 5 gallons and a measurement of 3 yards, we obtain a sum of 8 when we add measures. But 8 what? The sum has no meaning.

Very often a measurement is given in two units such as 9 lb. 14 oz. (pounds and ounces) for the weight of a bag of potatoes. There are two methods of finding the sum of measures for measurements expressed this way. The first method involves expressing all the measurements in terms of the smallest unit given, adding the measures, and then expressing the sum in terms of the largest unit.

The second method involves finding the sum of the measures of the measurements expressed in larger units and finding the sum of the measures of the measurements expressed in smaller units independently. For the English system of measurement, this is by far the simpler and more preferred method.

Pre-Book Lesson

- Give pupils practice changing from decimals and fractions to mixed forms.
- Let the pupils practice expressing measurements in the largest possible unit of measurement, such as expressing 25 ounces as 1 lb. 9 oz.

Using the Text Page

- Ex. 2. Ask why the method shown in box B is preferable to the method shown in box A, especially if the units of measurement are expressed in the English system. Ask for a detailed explanation of how pupils express the total number of ounces in terms of pounds and ounces.
- Ex. 4. Adding in this way resembles the procedure for adding rational numbers and expressing the results in simplest form.
- Ex. 11 and 13. To point out that the method shown in box B does not have such a great advantage over the method shown in box A in the metric system as in the English system, have the pupils do these two exercises by the method shown in box A.

Individualizing Instruction

- Ask the *more capable pupils* to explain why the sum of two measures is meaningless unless the measures are in the same unit of measurements.
- *All pupils* may be given exercises involving three units of measurement and be asked to find the sum of the measures. For example,

hr.	min.	sec.
5	42	17
4	18	29
2	22	51

Examples with tons, pounds, and ounces; or yards, feet, and inches could also be used.

- Separate cubical counting blocks into two sets, one set containing 16 members and the other containing 8 members. Let each block represent 1 ounce of cheese and have *slower learners* find the total weight of cheese in ounces (24 oz.); in pounds and ounces (1 lb. 8 oz.); and in pounds ($1\frac{1}{2}$ lb.). The last measure is expressed with a mixed form. This type of exercise will illustrate methods of adding measures and reinforce the presentation of the pupil's page.

Adding Measures

[O]

1. Mrs. Wilson bought two cans of tomatoes. Read the measurements shown on the cans. One way to find the total weight is to express the measurements in terms of ounces as in box A.

a. 1 lb. = _?16oz., so 1 lb. 8 oz. = _?24oz. and 1 lb. 4 oz. = _?20oz.

b. 44 ÷ 16 = $2\frac{?}{16}$, $\frac{12}{}$ so 44 oz. = 2 lb. _?12oz.

2. Box B shows a better way to find the total weight of the tomatoes. Adding pounds and ounces is somewhat like adding tens and ones _?_. Explain why we do not have to rename the 12 shown in ounce's column? 12 ounces are less than 1 pound.

3. Mrs. Wilson bought 2 lb. 6 oz. of ham and 1 lb. 12 oz. of steak. How much meat in all did Mrs. Wilson buy (box C)? 18 oz. = 1 lb. _?2 oz., so 3 lb. 18 oz. = _?4 lb. _?2 oz.

4. Work the following additions at the board.

a.

lb.	oz.
6	3
3	14
10 lb.	1 oz.

b.

yd.	ft.
5	2
7	2
13 yd.	1 ft.

c.

m.	cm.
12	91
15	40
28 m.	31 cm.

[W]

5. Sue is 12 yr. 7 mo. old. Kay is 2 yr. 8 mo. older than Sue. How old is Kay? 15 yr. 3 mo.

A Add

24
20
44

44 oz. = 2 lb. 12 oz.

B Add

lb.	oz.
1	8
1	4
2	12

2 lb. 12 oz.

C Add

lb.	oz.
2	6
1	12
3	18

4 lb. _?2 oz.

Ex. 6–13. Copy and add as in box C.

6.

yd.	ft.
30	1
42	1
72 yd.	2 ft.

7.

hr.	min.
7	41
23	13
30 hr.	54 min.

8.

gal.	qt.
4	2
5	1
9 gal.	3 qt.

9.

gal.	qt.
8	3
7	2
16 gal.	1 qt.

10.

bu.	pk.
3	2
4	3
8 bu.	1 pk.

11.

cm.	mm.
52	7
24	9
77 cm.	6 mm.

12.

gal.	qt.
6	3
25	3
32 gal.	2 qt.

13.

m.	dm.
8	9
20	3
29 m.	2 dm.

* When renaming 5 da. 2 hr. 12 min., remind pupils that the renaming is based on the fact that 1 da. = 24 hr. and 1 hr. = 60 min.

Subtracting Measures

[o]

A

Subtract

lb.	oz.
1	8
1	4
0	4

4 oz.

B

Subtract

lb.	oz.
2	(20)
~~3~~	~~4~~
1	8
1	12

1 lb. 12 oz.

1. 1 lb. 4 oz. of tomatoes weighs how much less than 1 lb. 8 oz.? 4 oz. Explain the work in box A.

2. At the board, find the answer for Ex. 1 by first expressing each measurement in terms of ounces. $\frac{24}{20}$ 4 oz.

3. A book that weighs 3 lb. 4 oz. is how much heavier than a book that weighs 1 lb. 8 oz. (box B)?

a. Can 8 be subtracted from 4? No

b. Is it true that 3 lb. 4 oz. = 2 lb. 20 oz.? Yes Explain. Key idea: 1 pound can be renamed as 16 oz.

c. Explain the work in box B. What is the answer for the problem? 1 lb. 12 oz.

4. At the board, find how much more time 2 hr. 10 min. is than 50 min. Work as in box B. 1 hr. 20 min.

5. In which of Ex. 6–13, will you have to rename one of the measurements before you subtract? 7, 9, 10, 11, 12, 13

[w]

Ex. 6–13. Subtract.

6.

yd.	ft.
7	2
4	1
3 yd.	1 ft.

7.

yd.	ft.
13	1
7	2
5 yd.	2 ft.

8.

hr.	min.
12	32
9	27
3 hr.	5 min.

9.

m.	cm.
27	20
15	47
11 m.	73 cm.

10.

gal.	qt.
525	2
64	3
460 gal.	4 qt.

11.

T.	lb.
5	300
	700
4 T.	1,600 lb.

12.

min.	sec.
10	0
3	48
6 min.	12 sec.

* 13.

da.	hr.	min.
5	2	12
3	5	30
1 da.	20 hr.	42 min.

14. John bought 2 lb. 8 oz. of bacon and 3 lb. 10 oz. of hamburger. He also bought 4 lb. 6 oz. of bananas. The total weight of the bacon and hamburger was how much more than the weight of the bananas? 1 lb. 12 oz.

15. The film shown at the left is 16 mm. wide. Its width is how much less than 2 cm.? 4 mm.

♦ **Extra Examples.** Set 75.

Teaching Page 146

Pupil's Objectives

(a) To learn to subtract measures by first expressing all measurements in terms of the smallest units given; and (b) to subtract the measures in larger units and smaller units independently.

Background

When subtracting the measure for a compound measurement such as 3 lb. 4 oz. from another such as 5 lb. 2 oz., it is usually simpler to subtract the measures in larger units and then subtract the measures in smaller units independently rather than first to express all measurements in terms of the smallest unit given. However, when subtracting measures in this manner, it may be necessary to rename the measure for the sum so that the subtraction with the smaller units may be performed. The subtraction indicated at the left below cannot be performed until the 5 lb. 2 oz. is renamed as 4 lb. 18 oz.

Subtract

lb.	oz.
5	2
3	4

lb.	oz.
4	18
3	4
1	14

1 lb. 14 oz.

Care should be taken to avoid the use of the traditional term "borrow" in renaming the 5 lb. 2 oz. as 4 lb. 18 oz. "Borrow" is not a mathematical term.

Pre-Book Lesson

Review subtractions such as $8\frac{1}{2} - 5\frac{3}{4}$ or $92 - 76$ in which renaming of the sum is necessary. Review expressing measurements in larger or smaller units using the method and mathematical sentences presented on Teaching pages 140–141.

Using the Text Page

- Ex. 1–2. Ask pupils which method they prefer, the method shown in box A or the method suggested in Ex. 2. Ask them the reason for their preference.
- Ex. 3b. Do not permit the use of the term "borrowing." Have pupils use the term "rename." You may have pupils work a few extra examples involving renaming of sums and known addends.
- Ex. 6–13. When performing the *addition* of measures for compound measurements, it is often necessary to rename the sum so that it is expressed in the desired form, such as renaming 3 lb. 18 oz. as 4 lb. 2 oz. When performing the subtraction of measures for compound measurements, it is not necessary to rename to express the answer in the desired form. Point this out and have pupils give the reason for this.

Individualizing Instruction

- *All pupils* may be given exercises involving subtraction of measures in which the units of measurements are not the same, such as:

(1) How much longer is 5 yd. 2 in. than 2 ft. 11 in.?

(2) What weight must be added to 14 oz. to give a total weight of 3 lb.?

- *More capable pupils* may be asked to compare the renaming necessary in subtractions such as $15\frac{1}{4} - 7\frac{3}{4}$ and $51 - 47$ with the renaming necessary in Ex. 9–13.

Teaching Page 147

Pupil's Objectives

(a) To satisfactorily complete the review and maintenance exercises; and (b) to recognize and restudy areas where weaknesses are revealed.

Background

The purpose of these exercises is to make certain that pupils have mastered the concepts presented up to this point in the chapter before proceeding with extension of these topics on the pages that follow. The review affords an opportunity to reinforce the skills in computations involving addition and subtraction with mixed forms.

For errors on Ex.	*Review*
(top of page 147)	
1	page 142
2–4	page 142
5	page 144
6, 7	pages 79, 145
(bottom of page 147)	
1–5	pages 113, 115
6–15	pages 112, 114
16, 17	page 145

Pre-Book Lesson

If the exercises are used as a test, you may wish to have a brief review prior to assigning the work. Review examples may be selected at random from previous pages. Reteaching prior to the test would be valuable in an instance where pupils have difficulties in specific areas. The test scores would then be an indication of the reteaching effectiveness.

Using the Text Page

Instructions for working the exercises, use of paper, and correcting the results should be given at this time. You may wish to have pupils work the top of the page first, correct their results, and then work the bottom of the page. In this way, many errors due to carelessness can be avoided.

Individualizing Instruction

- After the tests have been scored, they should be returned to *all pupils* so that they may correct any errors. Pupils with the same errors may be grouped for additional help. Extra exercises should be given as remedial work.
- *More capable pupils* may work Supplementary Activities or Extra Activities Sets if they do not need additional work with the material at hand. If the situation permits, you may wish to have these pupils help *slower learners* with remedial work which you may assign after specific weaknesses have been revealed.

NOTES

How Well Do You Remember?

[W]

1. Write the names of four units of length in Sample answers:
   a. the English system. inch, foot, yard, mile b. the metric system. meter, centimeter, decimeter, kilometer

Ex. 2–5. Copy and complete.

2. 5 m. = 500 cm. 3. 750 cm. = 7.5 m.
4. 3,000 km. = 3,000,000 m. 5. 12 cm. = 120 mm.

6. At a mean average of 80 kilometers per hour, how many hours will it take Mr. Whiting to drive 500 kilometers? Express the answer in terms of hours and minutes. 6 hr. 15 min.

7. Jerry recorded the lengths of 5 metal rods as follows: 13 cm., 12 cm., 17 cm., 14 cm., 14 cm. Find the arithmetic mean for these data. 14 cm.

8. If the mean average for the weights of 20 metal rods is 3 oz., find the total weight in terms of pounds and ounces. 3 lb. 12 oz.

To Keep in Practice

A., S. [W]

Ex. 1–5. Copy and add.

1. $45\frac{3}{7}$ + $63\frac{5}{7}$ = $109\frac{1}{7}$
2. $103\frac{1}{2}$ + $77\frac{3}{4}$ = $181\frac{1}{4}$
3. $60\frac{2}{3}$ + $5\frac{3}{5}$ = $66\frac{4}{15}$
4. $287\frac{5}{8}$ + $57\frac{4}{5}$ = $345\frac{17}{40}$
5. $1{,}603\frac{2}{3}$ + $567\frac{5}{6}$ = $2{,}171\frac{1}{2}$

Ex. 6–15. Find the number represented by the letter.

6. $n = 23\frac{1}{2} - 13\frac{2}{3}$ $9\frac{5}{16}$
7. $45\frac{7}{8} - 34\frac{3}{4} = x$ $11\frac{1}{8}$
8. $n - 10\frac{1}{6} = 23\frac{5}{7}$ $33\frac{37}{42}$
9. $47 - n = 23\frac{3}{4}$ $23\frac{1}{4}$
10. $x = 10 + 34\frac{1}{2} + \frac{7}{8}$ $45\frac{3}{8}$
11. $n = 3\frac{1}{4} - (\frac{7}{8} + \frac{3}{4})$ $1\frac{5}{8}$
12. $x = (25 + 567) - (23\frac{1}{2} + 308\frac{1}{3})$ $260\frac{1}{6}$
13. $x - 67\frac{3}{8} = 35\frac{4}{5} + 62\frac{1}{2}$ $165\frac{27}{40}$
14. $n = 23\frac{1}{5} + 1\frac{3}{5} + 28\frac{2}{3} + 100\frac{1}{3}$ $153\frac{4}{5}$
15. $n = 3{,}497 - 2{,}778\frac{3}{10}$ $718\frac{7}{10}$

Ex. 16–18. Copy and add.

16.

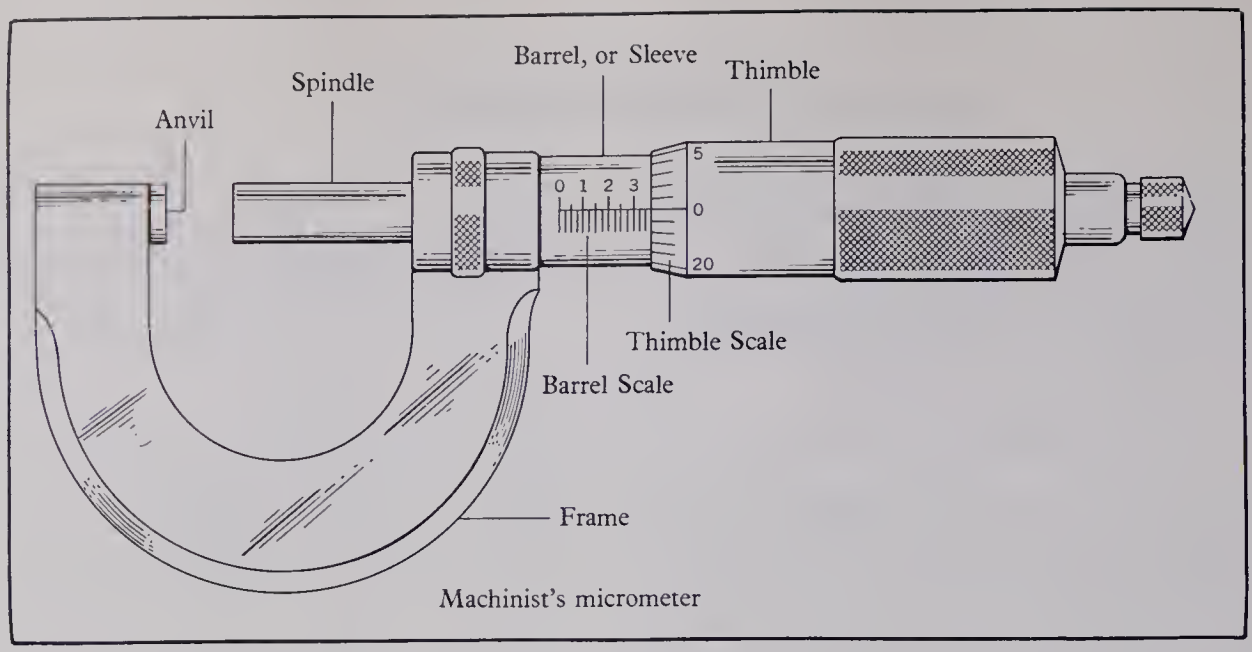
ft.	in.
6	8
12	5
19 ft.	1 in.

17.

cm.	mm.
27	6
18	7
46 cm.	3 mm.

18.

mi.	ft.
3	400
2	5,080
6 mi.	200 ft.



Machinist's micrometer

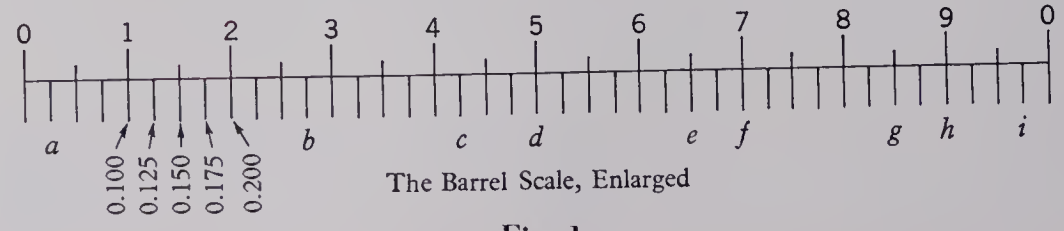
What Is a Micrometer?

[o]

A *micrometer* is an instrument for measuring very small things. The micrometer shown above is the kind used by machinists to measure to thousandths of an inch.

1. Name the parts of the micrometer shown above. When the thimble is turned, the spindle moves. The object to be measured is placed between the spindle and the anvil. Then the thimble is turned until both the anvil and the spindle touch the object. Then a measurement is obtained by reading the barrel scale and the thimble scale.

2. Fig. 1. Each labeled unit on the barrel scale represents 0.100 in. (we write 0.100 to show thousandths of an inch). Each unlabeled unit represents 0.025 in. Then the reading for Ex. a is 0.025 in. Give the readings for Ex. b–i.



The Barrel Scale, Enlarged

Fig. 1

b. 0.275; c. 0.425; d. 0.500; e. 0.650; f. 0.700; g. 0.850; h. 0.900; i. 0.975

Teaching Pages 148 and 149

Pupil's Objective

To become familiar with a measuring instrument called the *micrometer*.

Background

The precision possible in measuring very small objects with a ruler is very limited. If each unit on a ruler which represents one inch is partitioned into 64 congruent parts so that each new division represents $\frac{1}{64}$ inch, the markings on the ruler are so close together it is difficult to distinguish one mark from the next. It is difficult to determine the number of sixty-fourths of an inch any one mark represents. Using a ruler to measure to thousandths of an inch is obviously impractical.

To meet the need for an instrument that could measure to thousandths of an inch, the micrometer was developed. The micrometer is based on the principle of the screw. As the thimble is turned, the spindle moves forward or back, closing or lengthening the gap between it and the anvil. The object to be measured is placed between the anvil and the spindle, and the thimble is turned until both the anvil and the spindle touch it. As the thimble is turned, it moves uniformly along the barrel scale. The screw device is designed so that one complete turn of the thimble moves the spindle only 0.025 inch. The thimble moves the same distance along the barrel scale.

On the barrel scale each labeled unit represents one tenth of an inch, but because the micrometer shows thousandths of an inch, this is written 0.100 instead of 0.1. This is not the same as "one hundred-thousandth" which is written "0.00001." Each labeled unit on the barrel scale is partitioned into four unlabeled congruent parts, each of which represent 0.025 in.

The thimble scale is partitioned into 25 units. Since, in one complete rotation of the thimble scale the spindle moves 0.025 in., each unit on the thimble scale represents 0.001 in. When the thimble scale reads "0," the end of the thimble is at one of the marks on the barrel scale, and only the barrel scale needs to be read. Use of the thimble scale for readings other than "0" is not introduced until page 171. Work on this page is limited to examples where the thimble scale reads 0."

Pre-Book Lesson

- Discuss the limitations of a ruler in measuring small objects. Ask "Why is a ruler impractical for measuring to thousandths of an inch?" Try to obtain a ruler which measures to $\frac{1}{64}$ in. and show it to the class. Point out the difficulty of using such a ruler and the problems involved in designing and the impracticality in using a ruler which would measure to thousandths of an inch.
- Discuss the difficulty of measuring diameters of round or cylindrical objects using a ruler.
- Ask pupils about the tasks involved in a machinist's job. Discuss various industries in which machinists are employed. Emphasize that the nature of the work requires great precision in measurement.
- Try to obtain a machinist's micrometer and a demonstration-size micrometer to use with the class. If they are not available, a rough model of a micrometer can be made by using cylinders of cardboard, and pupils can identify parts on the model using a diagram on the chalkboard.

Using the Text Pages

- Ex. 1. Be sure pupils understand that the spindle moves ahead and back as the thimble is turned. The thimble moves in the same direction and exactly the same distance along the barrel scale. Pupils might not immediately see that the thimble and spindle are engaged in both a rotary motion and a back-and-forth motion at the same time. You can use the analogy of a wood screw. As it is turned clockwise, it also goes into the wood.

Make sure pupils can identify the parts of the micrometer and are able to read the scales correctly before proceeding with other exercises.

- Ex. 2. Stress that each of the numerals on the barrel scale represents one tenth of an inch, expressed as 0.100 in. Ask pupils the meaning of the zeros in hundredth's and thousandth's places in 0.100. Be sure they do not confuse this with 0.00001 in.

Explain to pupils that the use of the markings other than "0" on the thimble scale will be introduced later. Each mark on the thimble scale indicates a measurement of 0.001 inch. There are 25 marks in all on this scale.

Individualizing Instruction

• Draw on the board an enlarged model of a micrometer. Have *all pupils* come to the board, one at a time, and indicate where on the barrel scale the end of the thimble would be when objects with the following diameters are measured:

(a) 0.075 in. (b) 0.425 in. (c) 0.900 in.

• *Slower learners* may be grouped so that they are able to work some examples using a real micrometer or a cardboard model. They may then complete the remaining exercises using only the diagrams on pupil's page 149.

• *More capable pupils* may use a machinist's handbook to find how and in what instances a micrometer is used. They may write problems and examples, using information from the handbook.

NOTES

3. When the thimble shows "0," then the end of the thimble is at one of the marks on the barrel scale. In such a case, only the barrel scale need be read. The reading for Fig. 2 is 0.075 in. Give the readings for each of Fig. 3–4.

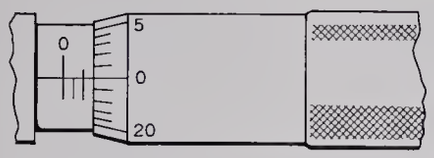


Fig. 2

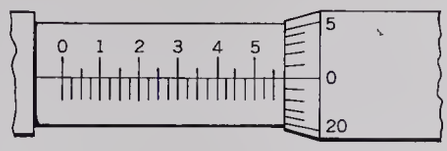


Fig. 3 0.575 in.

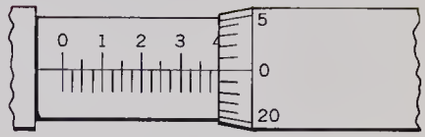
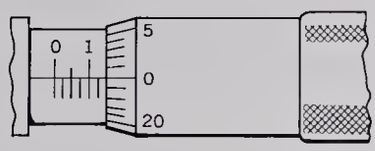


Fig. 4 0.400 in.

4. Give the reading for the micrometer shown in the box on the opposite page. 0.375 in.

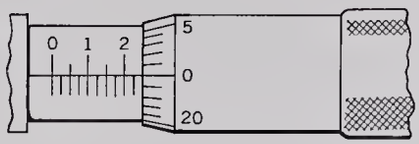
Give the reading for each of Ex. 5–10. Remember, [W] the decimal should show 3 decimal places. Also, write "in." for each reading to show the unit of measurement.

5.



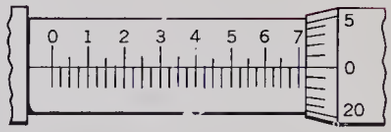
0.150 in.

6.



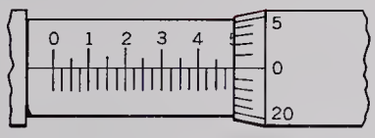
0.250 in.

7.



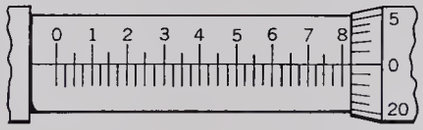
0.725 in.

8.



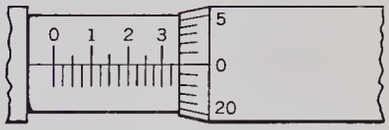
0.500 in.

9.



0.825 in.

10.



0.350 in.

11. Copy and complete. The first one is done for you.

Micrometer Reading	Barrel Scale		Thimble Scale
	Labeled Units	Unlabeled Units	
0.450 in.	4	2	0
0.375 in.	_?_3	_?_3	0
0.025 in.	_?_0	_?_1	0
0.675 in. _?_	6	3	0
0.925 in. _?_	9	1	0

* The perimeter is expressed with a measure and a unit. The computations done to find the perimeter are performed on measures.

Perimeters of Polygons

[o]

1. A simple closed curve formed by line segments is called a polygon.

2. Box A. Is each of the figures a polygon? Yes Give a special name for each polygon shown. Sample answers: triangle; rectangle; parallelagram; pentagon

3. Box A. The lengths of the sides of triangle ABC are given. What is the sum of the measures? 14

> * **The total distance along the sides of a polygon is called the *perimeter* of the polygon. The measure of the perimeter of a polygon is equal to the sum of the measures of its sides.**

4. The perimeter of triangle ABC is 14 inches, or 1 ft. 2 in. Find the perimeters of the other polygons shown in box A. 28 in., 20 in., 23 in.

5. If the lengths of the sides of a triangle are 2 ft. 3 in., 4 ft. 7 in., and 5 ft. 10 in., what is its perimeter? 12 ft. 8 in. Explain the work in box B. The sum of the measures of the sides is found.

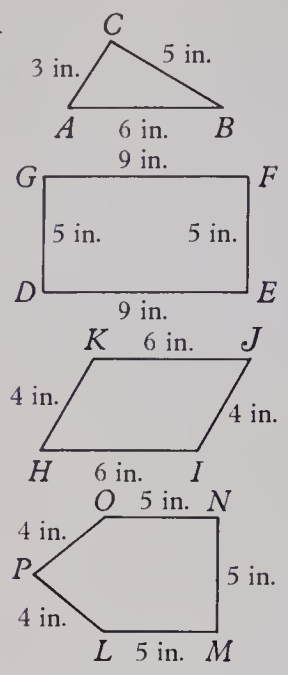
6. Box C. Explain the work shown for finding the perimeter of a quadrilateral whose sides measure 4, $4\frac{2}{3}$, $5\frac{1}{2}$, and $6\frac{1}{3}$ in centimeters. The perimeter is $20\frac{1}{2}$ cm.
The sum of the measures of the sides is found.

7. For any triangle, the sum of the measures of the three sides is the measure of the perimeter of the triangle. If P represents the measure of the perimeter and a, b, and c represent the measures of the sides of the triangle, then $P = a + b + c$.

8. On the board, write a mathematical sentence for finding the measure of the perimeter of a quadrilateral if a, b, c, and d represent the measures of the sides of the quadrilateral. P = a + b + c + d

9. If we know the perimeter and the lengths of two sides of a triangle, tell how we can find the length of the third side. Subtract the sum of the measures of the two sides from the measure of the perimeter.

A



B

Add

ft.	in.
2	3
4	7
5	10
11	20

12 ft. 8 in.

C

Add

$4 = 4$

$4\frac{2}{3} = 4\frac{4}{6}$

$5\frac{1}{2} = 5\frac{3}{6}$

$6\frac{1}{3} = 6\frac{2}{6}$

$19\frac{9}{6}$

or $20\frac{1}{2}$

Teaching Pages 150 and 151

Pupil's Objectives

(a) To learn the meaning of the perimeter of a polygon; (b) to learn that the measure of the perimeter of a polygon is equal to the sum of the measures of its sides; and (c) to learn to find the measure of one side of a polygon when the measure of the perimeter and the measures of each of the other sides are given.

Background

Since the perimeter of a polygon is the total distance along its sides, perimeter is a measurement. If the perimeter of a polygon is 16 feet, the measure is 16 in feet, and is equal to the sum of the measures of the sides. Such a sum is meaningful only if the lengths of the sides are expressed in the same unit of measurement.

The relationship among the measures of the perimeter of a polygon such as a quadrilateral and the measures of its sides may be expressed by $a + b + c + d = P$, where P represents the measure of the perimeter and where a, b, c, and d represent the measures of the sides. If the polygon is a triangle, the relationship may be expressed as $a + b + c = P$.

If the measure of the perimeter and measures of all the sides but one side are given, the measure of the one side may be found by expressing the above relationships as $c = P - (a + b)$ for the triangle and as $d = P - (a + b + c)$ for the quadrilateral.

In these equations, each of these letters represents a measure. This is a simpler notation than the use of $m(\ )$ to represent the measure of a side of a polygon.

Pre-Book Lesson

Give pupils practice in subtracting a sum of two measures from a third measure. Have pupils also practice finding an unknown addend in the addends-sum relationship involving three addends.

Using the Text Pages

• Ex. 1. Ask for the meaning of a simple closed curve. Ask for the difference between a simple closed curve and a closed curve that is not simple. Have pupils draw models on the chalkboard of closed curves that are not simple. Review the meaning of an equilateral geometric figure.

• Ex. 2. Have pupils give as many names to each of the polygons as possible. Polygon *DEFG* is a quadrilateral and also a rectangle. Polygon *HIJK* is a quadrilateral and also a parallelogram. Ask for definitions of the following: triangle, quadrilateral, parallelogram, rectangle, pentagon.

• Ex. 3. Point out that the perimeter of a polygon is a measurement and consists of a measure and a unit of measurement. Point out that the sum of the measures of the sides is only meaningful if the measurements have the same units. The measurements of all of the sides of each polygon in box A have the same unit of measurement.

• Ex. 7. Emphasize that P, a, b, c, and d represent measures, and that this is simpler notation than the $m(\ )$ notation.

• Ex. 9. Have pupils explain the development of the equation $c = P - (a + b)$ from the equation $P = a + b + c$ in Ex. 7. This is another way to express the addends-sum relationship.

• Ex. 12. The relationship $P = a + b + c + d$ may also be expressed as $d = P - (a + b + c)$. Point out that this is the relationship of the measure of the perimeter, the measure of the known sides, and the measure to be determined. Have pupils explain how the second equation may be developed from the first equation. Point out that this problem involves a known sum, a known addend, and an unknown addend, so the operation subtraction is indicated.

Individualizing Instruction

• Have *more capable pupils* determine by experimentation the effect on the measure of the perimeter of a polygon when the measure of each side of the polygon is doubled, and when the measure of each side of the polygon is tripled.

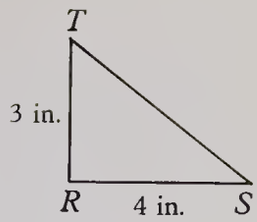
• Ask *all pupils* to find the length of each of two sides of an isosceles triangle that are the same length when the perimeter and the length of the third side are given.

These pupils may find the length of a rectangle if the perimeter and the width are given; and may determine the perimeter of a polygon for which the measurements of the sides are given. For example, have them express, in feet, the perimeter of a triangle for which the lengths of sides are: 1 yd., 2 ft., and 18 inches. Be sure to indicate in what units the perimeter is to be expressed.

• *Slower learners* may be given a ruler and some cardboard models of rectangular regions and asked to determine the perimeter of the polygons forming the boundary of the regions. They may be given models of polygons and asked to determine the perimeter of the polygons, rounding the measure of each side to the nearest whole number before performing the additions.

NOTES

10. The perimeter of triangle RST shown at the right is 12 in. Find the length of side ST.



a. Does the mathematical sentence $n = 12 - (3 + 4)$ show the relationship expressed in the problem? Yes Does n represent a *measure* or a *measurement*? measure Explain how to solve the mathematical sentence. Add 3 and 4. Subtract their sum from 12.

b. The length of side ST is ?5 in.

11. At the board, find the length of the third side of a triangle if the perimeter is 10 cm. and two sides each measure $3\frac{1}{5}$ in centimeters. Write a mathematical sentence as in Ex. 10. $n = 10 - (3\frac{1}{5} + 3\frac{1}{5})$; $3\frac{3}{5}$ cm.

12. The lengths of three sides of a quadrilateral are 2 ft. 5 in., 3 ft. 1 in., and 3 ft. 3 in., and the perimeter is 10 ft. 3 in. What is the length of the fourth side?

a. Explain the work in box D. The sum of the measures of the three sides is found and subtracted from the measure of the perimeter.

b. The length of the fourth side is ?1 ft. ?6 in.

D

Add

ft.	in.
2	5
3	1
3	3
8	9

Subtract

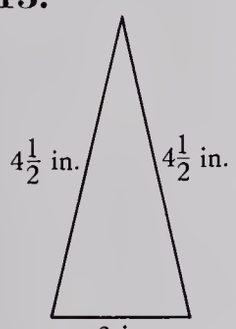
ft.	in.
9	⑮
~~10~~	~~3~~
8	9
1	6

1 ft. 6 in.

[W]

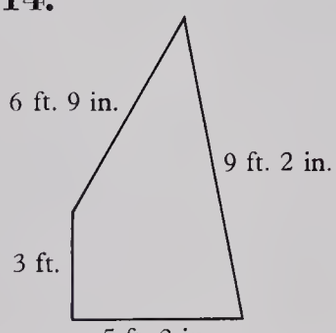
Ex. 13–15. Find the perimeter of the polygon shown.

13.



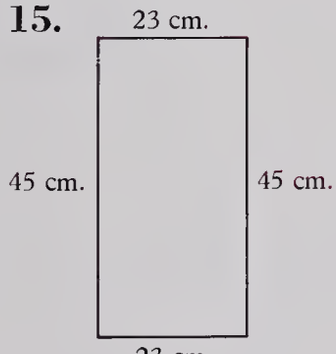
11 in.

14.



24 ft. 1 in.

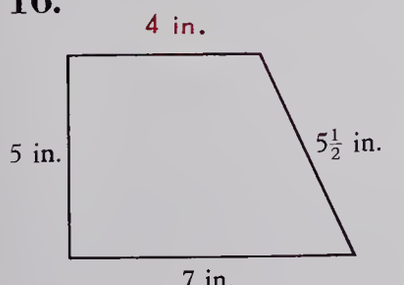
15.



1 m. 36 cm.

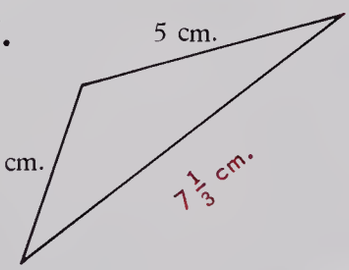
Ex. 16–18. Find the missing measurement.

16.



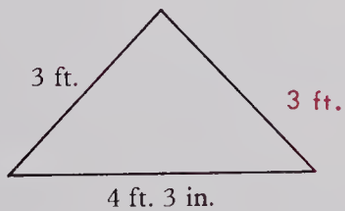
Perimeter: $21\frac{1}{2}$ in.

17.



Perimeter: $15\frac{5}{6}$ cm.

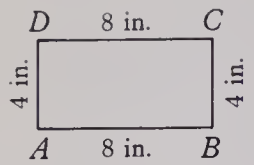
18.



Perimeter: 10 ft. 3 in.

* Note that the terms "length" and "width" are not used with all parallelograms, only rectangles.

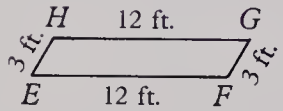
Finding Perimeters



1. To find the perimeter of rectangle $ABCD$, could we use $n = 4 + 8 + 4 + 8$? Yes $n = (2 \times 4) + (2 \times 8)$? Yes $n = 2 \times (4 + 8)$? Yes Does n represent a *measure* or a *measurement*? measure What is the perimeter of the rectangle? 24 in.

2. If P represents the measure of the perimeter and a and b represent the measures of the width and the length of a rectangle, then $P =$ _?2_ $\times (a + b)$.

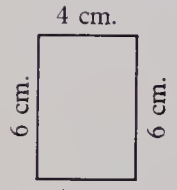
* 3. Could we use $P = 2 \times (a + b)$ for finding the perimeter of a parallelogram if a represents the measure of one of the two shorter sides and b represents the measure of one of the two longer sides? Yes



4. For parallelogram $EFGH$ shown at the left, $P = 2 \times (3 +$ _?12_ $) = 2 \times$ _?15_ $=$ _?30_. The perimeter is _?30_ ft., or _?10_ yd.

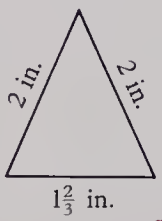
5. At the board, find the perimeters of the polygons shown below. $P = 2 \times (a + b)$ may be used for which two figures? a, c

a.



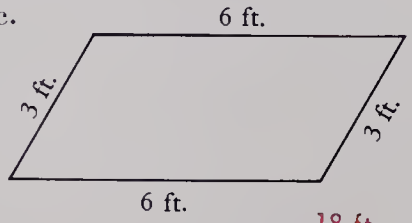
20 cm.

b.



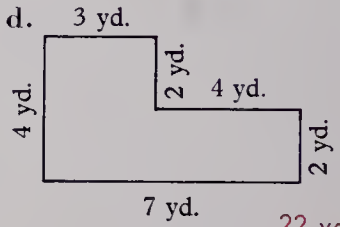
$5\frac{2}{3}$ in.

c.



18 ft.

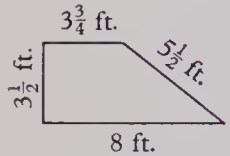
d.



22 yd.

[W]

6. Mr. Tuttle's vegetable garden is rectangular. It is 13 yd. wide and 25 yd. long. How many yards of fencing material does he need to completely fence in the garden? 76



7. The shape of Mrs. Tuttle's rock garden is shown at the left. Find the perimeter of the rock garden. $20\frac{3}{4}$ ft.

8. Find the perimeter of an isosceles triangle if the length of each of the two equal sides is 14 cm. and the length of the third side is $7\frac{1}{3}$ cm. $35\frac{1}{3}$ cm.

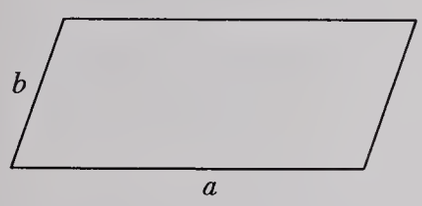
Teaching Page 152

Pupil's Objective

To learn to find the measure of the perimeter of a parallelogram.

Background

The measure of the perimeter of any quadrilateral is equal to the sum of the measures of its four sides. If the quadrilateral is a parallelogram, the opposite sides are equal in length, and the sum of the measures of any two adjacent sides is equal to one half the measure of the perimeter.



If a is the measure of one side and b is the measure of an adjacent side, then $a + b$ will be equal to half the measure of the perimeter. If P represents the measure of the perimeter, then $(a + b) = (P \div 2)$, or $P = 2 \times (a + b)$. This relationship may be developed as follows:

(1) $P = a + a + b + b$ because opposite sides of a parallelogram have the same length.

(2) $P = (2 \times a) + (2 \times b)$ because addition of like addends may be expressed as a product of one addend and the number of addends.

(3) $P = 2 \times (a + b)$ because multiplication is distributive over addition.

If a parallelogram is a rectangle, the measurement of the shorter side is called the width and the measurement of the longer side is called the length.

Pre-Book Lesson

• Ask for the definitions of parallelogram and rectangle. Ask for the meaning of the length and width of a rectangle. Try to arrive at the fact that the opposite sides of a parallelogram have the same length.

• Have pupils try to think of instances where they, their families, or friends had to work with perimeters in some situation. They may also list instances where they might encounter work with perimeters in the future.

Using the Text Page

• Ex. 1. Have the pupils justify going from the first mathematical sentence to the second, and the second mathematical sentence to the third. Remind pupils that the opposite sides of a parallelogram have the same length.*

• Ex. 3. The longer side of a parallelogram is not the *length*, and the shorter side is not the *width*. The terms *length* and *width* are used only in reference to the lengths of the sides of rectangles.

• Ex. 6. Some pupils may need to draw a diagram to represent the rectangular vegetable garden. The use of fencing material involves finding the "distance around," or the perimeter.

Individualizing Instruction

• Ask the *more capable pupils* to develop an equation for expressing the relationship between the measure of a perimeter of an isosceles triangle and the measure of its sides, such as $P = a + a + b$, or $P = (2 \times a) + b$ where a is the measure of one of the equal sides and b is the measure of the third side.

• Ask *all pupils* to determine by experimenting how the perimeter of a parallelogram changes when (a) the length of the longer side is doubled and the length of the shorter side is halved; (b) the length of both the longer side and the shorter sides are doubled; and (c) the length of the shorter sides is doubled and the length of the longer sides is unchanged.

• Have the *slower learners* find the measure of the perimeter of parallelograms first by finding the sum of the measure of the sides.

* See item 13, page xix. Cubical counting blocks can be used to make a model of a rectangle. Pupils can then count the number of blocks in the length and width, each block representing one unit of measurement. By manipulating the blocks they can more easily visualize the relationships in the mathematical sentences.

Teaching Page 153

Pupil's Objectives

(a) To review the definition of regular polygons; and (b) to learn that the measure of the perimeter of an equilateral polygon is equal to the product of the measure of one side and the number of sides.

Background

The measure of the perimeter of any polygon is equal to the sum of the measures of its sides. For a polygon of eight sides where each letter represents the measure of length of one side,

$$P = a + b + c + d + e + f + g + h$$

If the polygon is equilateral, each of the sides has the same measure, which we can represent by s.

$$P = s + s + s + s + s + s + s + s$$

Since the sum of like addends may be expressed as the product of one addend and the number of addends, the above sentence may be expressed as

$$P = 8 \times s$$

For any equilateral polygon, $P = n \times s$ where n represents the number of sides. This relationship may be used to find the perimeter of any equilateral polygon, including any regular polygon, when the number of sides and the measure of one of the sides are given. The factors-product relationship may also be expressed by $s = P \div n$ which may be used to find the measure of one side of an equilateral polygon when the measure of the perimeter and the number of sides are given. $n = P \div s$ may be used to find the number of sides of an equilateral polygon when the measure of the perimeter and the measure of one side are given.

Pre-Book Lesson

Review the concept that the sum of like addends may be expressed as the product of one addend and the number of addends. Also that one unknown addend of a series may be found given the sum and all but the unknown addend.

Using the Text Page

- Ex. 1. Have the pupils verify that the polygons shown on the text page are equilateral by marking off the length of one side on the edge of a piece of paper and then comparing the length of this segment to the lengths of each side of the polygon.
- Ex. 2. Emphasize that a regular polygon must be *both* equilateral and equiangular.
- Ex. 3. Make sure pupils understand the meaning of the terms *converse* and *reverse*.
- Ex. 4. Point out that the relationship $P = n \times s$ applies to any equilateral polygon, and not to just regular polygons.
- Ex. 6. Have the pupils express the factors-product relationship $P = n \times s$ in the form $n = P \div s$ which may be used to find the number of sides of an equilateral polygon when the measure of the perimeter and the measure of one side are given. Ask pupils to describe what would happen if the pentagon did not have all sides the same size and all angles the same size.

Individualizing Instruction

- Have *all pupils* predict the effect on the measure of the perimeter of an equilateral polygon if the length of each side were doubled. Let these pupils confirm their prediction by working several examples.

Give these pupils the measure of the side of an equilateral polygon in one unit such as inches and have them express the perimeter in another unit such as feet.

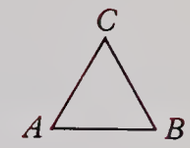
- Have the *slower learners* find the perimeter of an equilateral polygon first by adding the measures of all the sides and then by using $P = n \times s$ to show the advantage of using this relationship.

* An equilateral polygon has sides of the same length, but does not necessarily have angles of equal measure. For example, polygon IJKL shown at the right is an equilateral polygon.

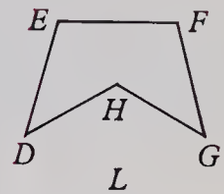
Perimeters of Equilateral Polygons

Resurvey; regular polygons [O]

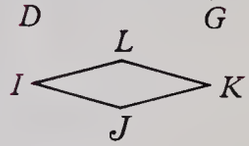
1. When the length of each side of a polygon is the same, then the polygon is called an equilateral polygon. Do all the polygons shown at the right appear to be equilateral polygons? Yes



2. Which of the polygons shown at the right appear to be regular polygons? Give a definition of a regular polygon. Equilateral polygon ABC, polygon MNOPQ and equiangular

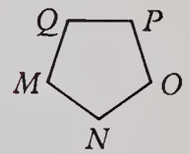


* **3.** Are all regular polygons equilateral polygons? Yes Is the converse (the reverse of the sentence) true? That is, are all equilateral polygons regular polygons? No



4. If the measure of each side of an equilateral polygon is 4, what is the measure of its perimeter if it has

a. 3 sides? 12 **b.** 5 sides? 20 **c.** 10 sides? 40 **d.** n sides? $4 \times n$



If P represents the measure of the perimeter and s represents the measure of each side of an n-sided equilateral polygon, then

$$P = n \times s.$$

5. In the mathematical sentence $P = n \times s$, which letters represent the factors? Which letter represents the product? P

n, s

6. Find the length of each side of a regular pentagon if its perimeter is 70 in.

$P = n \times s$
$70 = 5 \times s$
$s = 70 \div 5$
$= 14$

a. $P =$ 70 and $n =$ 5.

b. Explain each step in the box. When the product and one factor are known, we divide to find the unknown factor.

c. The length of each side of the pentagon is 14 in.

[W]

Ex. 7–8. Use $P = n \times s$ to find the perimeter.

7. A regular hexagon with each side 23 centimeters long 138 cm.

8. A 12-sided equilateral polygon with each side 27 feet long 324 ft.

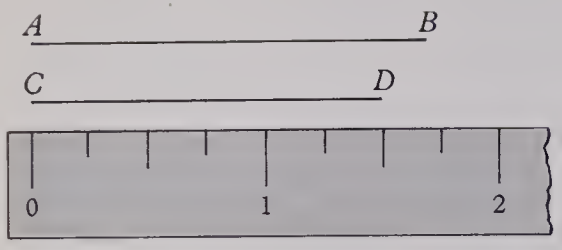
Ex. 9–10. Find the length of each side of the polygon as in the box.

9. A regular octagon with perimeter 104 inches 13 in.

10. A 20-sided equilateral polygon with perimeter 350 inches $17\frac{1}{2}$ in.

Measuring to the Nearest Unit

Unit of precision [O]



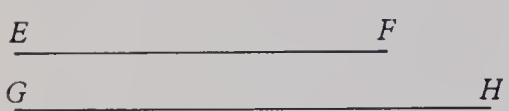
1. When we measure to find the length of a segment, do we expect to find the actual length of the segment? No Explain. Key idea: All measurements are approximate.

2. The measurement of segment AB (shown at the left) to the nearest inch is 2 in.; to the nearest $\frac{1}{2}$ in. is $1\frac{1}{2}$ in.; to the nearest $\frac{1}{4}$ inch is $1\frac{3}{4}$ in.

3. To the nearest inch, does the length of $\overline{CD}$ appear to be nearer 1 in. or 2 in.? It appears to be halfway between them. Since the length of $\overline{CD}$ appears to be halfway between 1 in. and 2 in., its length *to the nearest inch* could be given as 1 in. or as 2 in. In such cases, we agree to give the greater measurement.

4. If, to the nearest *inch*, the measurement of a segment is 7 in., then could its length have appeared to be $6\frac{1}{4}$ in.? No $6\frac{1}{2}$ in.? Yes $6\frac{7}{8}$ in.? Yes $7\frac{3}{8}$ in.? Yes $7\frac{1}{2}$ in.? No We say, *its length is either* $6\frac{1}{2}$ *in. or is between* $6\frac{1}{2}$ *in. and* $7\frac{1}{2}$ *in.*

5. If, to the nearest $\frac{1}{4}$ *inch*, the measurement of a segment is $5\frac{3}{4}$ in., then its length is either $5\frac{5}{8}$ in. or is between $5\frac{5}{8}$ in. and $5\frac{7}{8}$ in.



6. The measurements of $\overline{EF}$ and $\overline{GH}$ to the nearest $\frac{1}{8}$ *inch* are $1\frac{5}{8}$ in. and 2 in. Since both measurements are to the nearest $\frac{1}{8}$ inch, we say that the *unit of precision* for these measurements is $\frac{1}{8}$ inch. To show that the unit of precision for 2 in. is $\frac{1}{8}$ inch, we may write, $2\frac{0}{8}$ *in.* What is the unit of precision for each of the following measurements?

a. $5\frac{0}{2}$ in. $\frac{1}{2}$ inch **b.** $17\frac{0}{4}$ in. $\frac{1}{4}$ inch **c.** $2\frac{0}{4}$ cm. $\frac{1}{4}$ centimeter **d.** $5\frac{0}{10}$ in. $\frac{1}{10}$ inch **e.** $4\frac{0}{2}$ mi. $\frac{1}{2}$ mile

7. A measurement of $5\frac{0}{2}$ cm. means that the unit of precision is $\frac{1}{2}$ centimeter and that the length is $4\frac{3}{4}$ cm. or is between $4\frac{3}{4}$ cm. and $5\frac{1}{4}$ cm.

8. If $2\frac{1}{2}$ in. and 3 in. are measurements to the nearest $\frac{1}{4}$ inch, we may write, $2\frac{2}{4}$ *in.* and $3\frac{0}{4}$ *in.* to show the unit of precision.

Teaching Pages 154 and 155

Pupil's Objectives

(a) To learn that no measurement is exact; (b) to learn that measurements are made to the nearest unit of measurement being used; and (c) to learn the meaning of unit of precision.

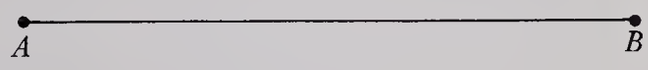
Background

To determine the measure of length of any segment, we need an instrument such as a ruler on which each standard unit of length has been marked off into a number of smaller units each the same size. For instance, one inch may be marked off into parts the same size such as sixteenths of an inch, so the length of a segment can be determined to the nearest $\frac{1}{16}$ inch. It cannot be determined to the nearest $\frac{1}{32}$ inch because such a unit is not marked off on the measuring device. The unit of precision of such a measurement is defined as being $\frac{1}{16}$ inch.

All measurements should be expressed in such a way as to indicate the unit of precision. If the unit of precision is $\frac{1}{16}$ inch, and if this has not been indicated in some other way, a measurement should be written as $3\frac{8}{16}$ inches rather than as $3\frac{4}{8}$ inches, $3\frac{2}{4}$ inches, or $3\frac{1}{2}$ inches. The measurement $3\frac{8}{16}$ inches clearly indicates that the unit of precision is $\frac{1}{16}$ inch and that the measurement to the nearest $\frac{1}{16}$ inch is $3\frac{8}{16}$ inches. The measurement $3\frac{4}{8}$ inches would indicate that the measurement was made to the nearest $\frac{1}{8}$ inch; the measurement $3\frac{2}{4}$ inches would indicate the measurement was made to the nearest $\frac{1}{4}$ inch, and the measurement $3\frac{1}{2}$ inches would indicate the measurement was made to the nearest $\frac{1}{2}$ inch.

When expressing measurements with fractions and mixed forms, the fractions should not be in simplest form. Changing a fraction in a measurement to a fraction in simplest form may not indicate the unit of precision. If the measurement of a segment is 2 inches to the nearest $\frac{1}{16}$ inch, this should be written as $2\frac{0}{16}$ inches to indicate the actual unit of precision used in making the measurement. Writing the measurement as 2 inches would indicate the unit of precision as being 1 inch, which is incorrect.

The length of a model of a segment can be determined only as precisely as the unit of precision of the scale being used. The length of $\overline{AB}$ could be expressed as any one of the measurements below depending on the unit of precision of the scale used.



Unit of Precision	1″	$\frac{1}{2}$″	$\frac{1}{4}$″	$\frac{1}{8}$″	$\frac{1}{16}$″
Measure of $\overline{AB}$ in inches	3	$2\frac{1}{2}$	$2\frac{2}{4}$	$2\frac{5}{8}$	$2\frac{9}{16}$

The measure of $\overline{AB}$ is approximately $2\frac{9}{16}$ in inches. This may be written "$m(\overline{AB}) \approx 2\frac{9}{16}$ in inches." The symbol "$\approx$" means "is approximately equal to." Since every measurement is approximate, it would be more mathematically correct to use the symbol "$\approx$" than to use the "$=$" symbol in expressing measures of segments. However, the use of the "$=$" symbol is so widespread that we shall simply note that when we write "$m(\overline{AB}) = 2\frac{9}{16}$ inches" we really mean "$m(\overline{AB}) \approx 2\frac{9}{16}$ in inches."

Units of precision may also be indicated by decimals. The measurement 976.25 miles indicates a unit of precision of 0.01 or $\frac{1}{100}$ mile. The measurement 970.0 miles indicates a unit of precision of 0.1 or $\frac{1}{10}$ mile. The measurement 970 miles does not indicate any unit of precision because we don't know whether the 0 indicates a rounded number, an actual reading of zero miles, or a measurement made by a device indicating only tens of miles. If the number of significant zeros is indicated, the unit of precision can be determined. For example, a measurement of 2,000 miles to the nearest mile clearly indicates a unit of precision of 1 mile.

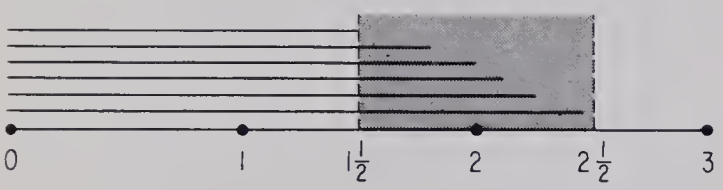
Pre-Book Lesson

Ask the pupils to express their age to the nearest year, month, and day to point out that the answer to the question "How old are you?" will depend on the precision of the measurement desired.

Using the Text Pages

• Ex. 1. It is impossible to find the actual measurement of any object because of the characteristics of matter and the inexact nature of measurement. Even though it may not be practical, it is theoretically possible to partition a very small unit of measurement into a still smaller unit. Matter is unstable and in a constant state of flux due to such physical influences as heat, pressure, loss of and movement of electrons. It is constantly expanding and contracting. The latter two considerations may be discussed with *more capable pupils.*

• Ex. 2. A measurement rounded to the nearest inch may refer to the length of any of the segments pictured below. To refer to a specific segment, a smaller unit of measurement must be used.



• Ex. 4. In describing measurements, sometimes the shorter terminology ". . . its length is $6\frac{1}{2}$ in. or is between $6\frac{1}{2}$ in. and $7\frac{1}{2}$ in." is used because the given measure is taken as a number which has been rounded to the next greater number. For instance in Ex. 5 and 7, the text reads "the length is . . . — or is between . . . — and . . . —." Actually, the length in Ex. 5 could be *either* $5\frac{3}{4}$ in. *or* between $5\frac{3}{4}$ in. and $6\frac{1}{4}$ in.

• Ex. 8. Point out that in expressing measurements so that the unit of precision is clearly indicated, fractions are not in simplest form. The denominator indicates the unit of precision used in making the measurement.

• Ex. 11–13. Most measurements in science are made with metric units and are expressed in decimal form. The unit of precision is the place value of the digit at the extreme right of the decimal. The measurement 39.36 inches indicates a unit of precision of 0.01 inch. The unit of precision is easy to determine in Ex. 11–13. Sometimes one of the zeros is underlined, such as 3,000 miles to indicate a unit of precision of 100 miles.

Individualizing Instruction

• Ask the *more capable pupils* in which system of measurement, the English or the metric, are measurements more likely to be expressed with fractions and mixed forms, and in which system are measurements more likely to be expressed with decimals. Ask them for the advantages and disadvantages of indicating units of precision when measuring in the metric system.

• *All pupils* may be given models of line segments and be asked to determine the measurement of each with a unit of precision of 1 inch, $\frac{1}{2}$ inch, $\frac{1}{4}$ inch, $\frac{1}{8}$ inch, and $\frac{1}{16}$ inch.

• Let *slower learners* determine which of the following people needs to measure weight with the smallest unit of precision:

(a) a man weighing the amount of coal being loaded on a truck

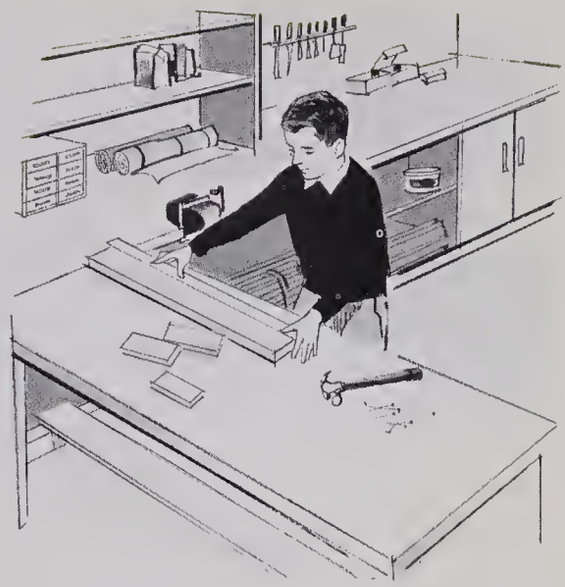
(b) a grocer weighing a bunch of carrots

(c) a jeweler weighing a diamond

Have them describe a situation where different people would measure the same object with different units of precision, such as a painter measuring a floor to determine how much paint it will take and a carpet-maker measuring to make a wall-to-wall carpet.

9. John measured three things using a unit of precision of $\frac{1}{8}$ inch and recorded his measurements as in Ex. a–c. On the board, rename the measurements to show the unit of precision.

a. 7 in. $7\frac{0}{8}$ in. **b.** $7\frac{1}{2}$ in. $7\frac{4}{8}$ in. **c.** $6\frac{3}{4}$ in. $6\frac{6}{8}$ in.



10. John said that a measurement of his workbench was 2 yd. Did he clearly indicate the unit of precision for the measurement? No What would have been the unit of precision if he had said

a. 2 yd., to the nearest yard? 1 yard

b. 2 yd., to the nearest foot? 1 foot

c. 2 yd. 0 in.? 1 inch

11. A measurement of 2.0 yd. indicates the unit of precision is $\frac{?}{10}$ yard $\left(\frac{1}{10}\right)$ while 5.30 m. indicates the unit of precision is $\frac{?}{100}$ meter, or 1 _?_ centimeter.

12. When the unit of precision is not clearly indicated, we will consider it to be the smallest *unit of measurement* shown. The unit of precision for 2 yd. is _?_ (1 yard); for 13 ft. is _?_ (1 foot); for 2.7 cm. is _?_ ($\frac{1}{10}$ centimeter).

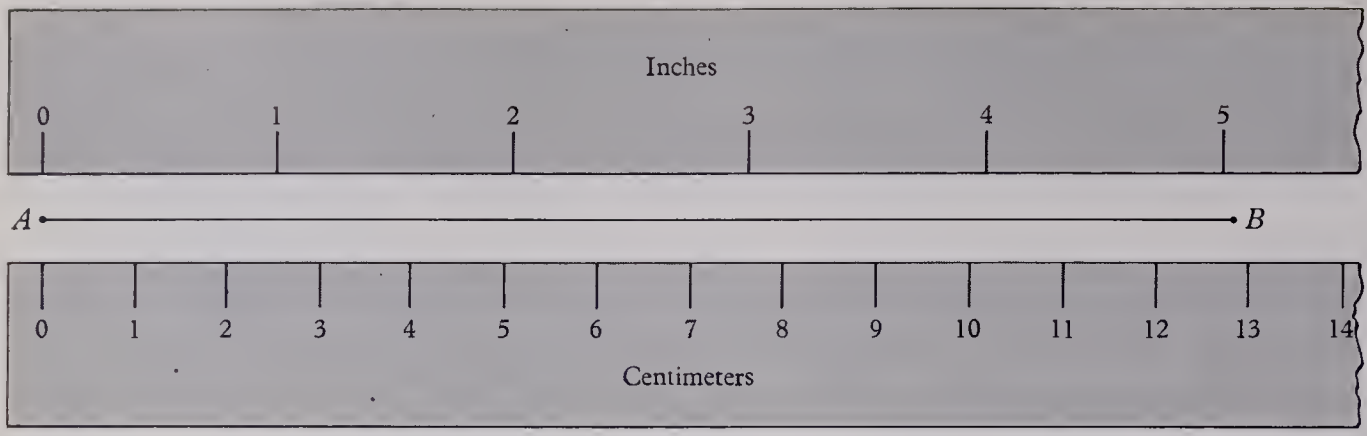
13. What do you think the unit of precision is for 15 ft.? 1 foot for 239 mi.? 1 mile for 3,482 mi.? 1 mile for $16\frac{0}{8}$ ft.? $\frac{1}{8}$ foot for 182.0 mi.? 0.1 mile [W]

Ex. 14–25. Name the unit of precision for the measurement.

14. $5\frac{0}{4}$ in. $\frac{1}{4}$ inch
15. $8\frac{2}{5}$ cm. $\frac{1}{5}$ centimeter
16. 9 yd. 1 yard
17. 5.00 ft. 0.01 foot
18. 3 yd. 2 ft. 1 foot
19. 4 yd. $2\frac{1}{2}$ ft. $\frac{1}{2}$ foot
20. 1,723 mi. 1 mile
21. 10.2 yd. 0.1 yard
22. 6 yd. 1 ft. 2 in. 1 inch
23. 8 yd. 2 ft. $3\frac{1}{2}$ in. $\frac{1}{2}$ inch
24. 20 m. 3 cm. 1 centimeter
25. 10.0 in. 0.1 inch

Ex. 26–35. Round to the nearest whole number.

26. $8\frac{2}{3}$ 9
27. 102.4 102
28. $16\frac{3}{5}$ 17
29. 99.09 99
30. 1,532.5 1,533
31. $7\frac{15}{30}$ 8
32. 10.51 11
33. 9.49 9
34. $8\frac{2}{5}$ 8
35. $49\frac{5}{11}$ 49



Meaning of Precision

Greatest possible error [0]

1. Segment AB is shown with an inch scale and a centimeter scale. To the nearest inch, the measurement of $\overline{AB}$ is _?5_ in. To the nearest centimeter, the measurement of $\overline{AB}$ is _?13_ cm.

2. What is the unit of precision for 5 in.? 1 inch for 13 cm.? 1 centimeter Since 1 centimeter is less than 1 inch, 13 cm. is a more precise measurement than 5 in. Which of these two measurements appears to be nearer the length of $\overline{AB}$? either When measuring a segment, is a more precise measurement *necessarily* nearer the length of the segment? No

3. To understand what precision means, suppose that we were told that 5 in. and 13 cm. were two measurements of $\overline{CD}$.

a. A measurement of 5 in. means that the length of $\overline{CD}$ is either $4\frac{1}{2}$ in. or is between $4\frac{1}{2}$ in. and $5\frac{1}{2}$ in. Then, the *greatest possible error* between 5 in. and the length of $\overline{CD}$ is _?$\frac{1}{2}$_ in.

b. A measurement of 13 cm. means that the length of $\overline{CD}$ is $12\frac{1}{2}$ cm. or is between _?$12\frac{1}{2}$_ cm. and _?$13\frac{1}{2}$_ cm. Then, the greatest possible error between 13 cm. and the length of $\overline{CD}$ is _?$\frac{1}{2}$_ cm.

c. Which is less, $\frac{1}{2}$ centimeter or $\frac{1}{2}$ inch? $\frac{1}{2}$ centimeter Is it true that the greatest possible error is less when 1 centimeter is the unit of precision than when 1 inch is the unit of precision? Yes

The greatest possible error is less when a smaller unit is used. For this reason, when a smaller unit is used, the measurement is more precise.

Teaching Pages 156 and 157

Pupil's Objectives

(a) To learn that a smaller unit of measurement means a more precise measurement; and (b) to learn that the greatest possible error in a measurement is one half the unit of precision.

Background

The precision of a measurement is an indication of the size of the unit of measurement used. A measurement made with a ruler whose edge has been marked off into sixteenths of an inch can be made to the nearest $\frac{1}{16}$ inch. It is incorrect to estimate such a measurement to the nearest $\frac{1}{32}$ inch. Such a measurement is more precise than a measurement made with a ruler whose edge has been marked off into half inches. With such a ruler, measurements can only be made to the nearest $\frac{1}{2}$ inch. The smaller the unit of measurement used, the more precise the measurement will be.

A measurement of $5\frac{1}{2}$ inches means that the unit of precision is $\frac{1}{2}$ inch and that the length is actually $5\frac{1}{4}$ inches or somewhere between $5\frac{1}{4}$ inches and $5\frac{3}{4}$ inches. The measurement $5\frac{1}{2}$ inches may be as much as $\frac{1}{4}$ inch greater than the true length of the segment or as much as $\frac{1}{4}$ inch less than the true length. This measurement could be written as "$5\frac{1}{2}$ inches $\pm$ $\frac{1}{4}$ inch." Since the greatest possible error is always one half the unit of precision, the notation showing $\pm$ $\frac{1}{4}$ inch is not necessary.

Greatest possible error must not be confused with relative error. The relative error of a measurement is an indication of how serious the greatest possible error is. It is usually expressed with a decimal or a per cent. Relative error can be found as follows:

$$\text{Relative error} = \frac{\text{measure of greatest possible error}}{\text{given measure}}$$

The relative error in the measurement $2\frac{1}{2}$ inches would be $\frac{1}{4} \div 2\frac{1}{2}$ or 10%. The relative error of the measurement $25\frac{0}{2}$ would be $\frac{1}{4} \div 25\frac{0}{2}$ or 1%. Notice that while the greatest possible error is the same for both measurements ($\frac{1}{2}$ inch), the relative error is ten times more serious in the first measurement than in the second measurement.

It is important to realize that a measurement in fraction form or mixed form should not be expressed with a decimal since this may be misleading in finding the greatest possible error. A measurement of $2\frac{1}{2}$ inches indicates a unit of precision of $\frac{1}{2}$ inch and a greatest possible error of $\frac{1}{4}$ inch. It implies that the true length of the segment is between $2\frac{1}{4}$ and $2\frac{3}{4}$ inches. A measurement of 2.5 inches indicates a unit of precision of $\frac{1}{10}$ inch and a greatest possible error of $\frac{1}{20}$ inch. It implies that the true length of the segment is between 2.45 inches and 2.55 inches. The statement $2\frac{1}{2}$ inches is the same as 2.5 inches is not entirely true unless the unit of precision is specified through some other means.

Pre-Book Lesson

- Give the pupils measurements of segments and have them determine the minimum and maximum lengths. Use a variety of units of precision.
- List on the board measurements whose measure is a unit fraction, such as $\frac{1}{2}$ inch, $\frac{1}{4}$ foot, and $\frac{1}{6}$ mile. Ask the pupils to find the measurement of a segment half as long.

Using the Text Pages

- Ex. 2–3. Precision is not a measurement of actual error; it is a measurement of possible error. The more precise the measurement, the smaller is the possible error. We cannot determine the exact error of a measurement because the length of a segment cannot be determined exactly.
- Ex. 4. Point out that when a measurement is made to the nearest unit of precision, the measurement may be half a unit of precision *greater* than the actual length of the segment or it may be half a unit of precision *less* than the actual length of the segment.
- Ex. 5. Be sure the pupils understand how to find a fraction number half as great as a given fraction number. The topic of multiplication with fractions has not been resurveyed, but the pupils should realize that a rational number half as great as a given rational number will have the same numerator as the given rational number but will

have a denominator twice as great. Half of $\frac{1}{8}$ is $\frac{1}{16}$; half of $\frac{1}{2}$ is $\frac{1}{4}$; half of $\frac{1}{3}$ is $\frac{1}{6}$. Be sure no one is making an error such as "half of $\frac{1}{2}$ is $\frac{2}{4}$." The use of models of rectangular or circular regions partitioned into sections will allow pupils to visualize one half of a given number of sections as representing a unit fraction.

• Ex. 6. If the greatest possible error is $\frac{1}{4}$ inch, the measurement may be $\frac{1}{4}$ inch greater than the true length or $\frac{1}{4}$ inch less than the true length. Therefore, the unit of precision is $\frac{1}{2}$ inch. The unit of precision is always double the greatest possible error.

Individualizing Instruction

• Have *all pupils* fill in a chart such as the following to point out that measurements expressed with fractions and measurements expressed with decimals equivalent to these fractions may have quite different units of precision and greatest possible errors. (Refer to the Background.)

Measurement	Unit of Precision	Greatest Possible Error
$2\frac{1}{2}$ inches	_____	_____
2.5 inches	_____	_____
10.75 feet	_____	_____
$10\frac{3}{4}$ feet	_____	_____
$5\frac{0}{5}$ miles	_____	_____
5.0 miles	_____	_____

Ask *all pupils* why "$5\frac{1}{2}$ inches" and "$5\frac{1}{2}$ inches $\pm$ $\frac{1}{4}$ inch" have the same meaning.

• If the *slower learners* have difficulty with Ex. 12, have them first state the unit of precision, then state the greatest possible error, and then indicate the possible range of lengths which could be expressed with that measurement. The unit of precision is $\frac{1}{4}$ inch. The greatest possible error is $\frac{1}{8}$ inch. Therefore the measurements of the segments must be in the range "5 inches $\pm$ $\frac{1}{8}$ inch" (equal to or greater than $4\frac{7}{8}$ inches but less than $5\frac{1}{8}$ inches). Give *slower learners* assistance in picturing models of such segments and assist them with the other written work by having them use rulers to better visualize the measurements and units of precision.

NOTES

4. If 1 inch is the unit of precision, then the greatest possible error is $\frac{1}{2}$ in. If 1 centimeter is the unit of precision, then the greatest possible error is _?_ $\frac{1}{2}$ cm. Give similar statements for the following units of precision.

a. 1 foot $\frac{1}{2}$ ft. **b.** 1 yard $\frac{1}{2}$ yd. **c.** $\frac{1}{2}$ mile $\frac{1}{4}$ mi. **d.** 1 meter $\frac{1}{2}$ m. **e.** $\frac{1}{4}$ millimeter $\frac{1}{8}$ mm.

For a measurement, the greatest possible error is one half the unit of precision.

5. For each of Ex. a–d, first give the unit of precision and then give the greatest possible error for the measurement.

a. $5\frac{1}{8}$ in. $\frac{1}{8}$ inch; $\frac{1}{16}$ in. **b.** 3 ft. $2\frac{1}{2}$ in. $\frac{1}{2}$ inch; $\frac{1}{4}$ in. **c.** 5 m. 37 cm. 1 centimeter; $\frac{1}{2}$ cm. **d.** $8\frac{1}{4}$ mm. $\frac{1}{4}$ millimeter; $\frac{1}{8}$ mm.

6. If, for a measurement, the greatest possible error is $\frac{1}{4}$ in., what unit of precision was used? Explain your reasoning. $\frac{1}{2}$ inch Key idea: $\frac{1}{4}$ inch is $\frac{1}{2}$ of $\frac{1}{2}$ inch. [W]

Ex. 7–10. Name the greatest possible error for each.

7. $2\frac{3}{4}$ ft. $\frac{1}{8}$ ft. **8.** $6\frac{0}{8}$ in. $\frac{1}{16}$ in. **9.** 7 m. 2 cm. $\frac{1}{2}$ cm. **10.** 4 yd. $3\frac{1}{2}$ in. $\frac{1}{4}$ in.

11. Copy and complete the following chart.

Unit of precision	1 inch	1 foot	1 yard	1 rod	1 mile	1 meter
Greatest possible error	$\frac{1}{2}$ in.	$\frac{1}{2}$ ft.	$\frac{1}{2}$ yd. _?_	$\frac{1}{2}$ rd. _?_	$\frac{1}{2}$ mi. _?_	$\frac{1}{2}$ m. _?_

12. Draw models of three segments of different lengths such that $5\frac{0}{4}$ in. would be a measurement of each segment.
Key idea: The measure in inches of each segment must be $4\frac{7}{8}$ or between $4\frac{7}{8}$ and $5\frac{1}{8}$.

13. Rename each of Ex. a–d to show that the greatest possible error is $\frac{1}{8}$ in. (*Hint:* What unit of precision must be shown?) $\frac{1}{4}$ in.

a. 5 in. $5\frac{0}{4}$ in. **b.** $3\frac{1}{2}$ in. $3\frac{2}{4}$ in. **c.** 3 ft. 3 ft. $\frac{0}{4}$ in. **d.** 4 ft. 2 in. 4 ft. $2\frac{0}{4}$ in.

14. If a measurement is given to the nearest meter, then is 50 cm. the greatest possible error? Yes

Ex. 15–19. For the given measurement, make a statement about the length of the segment measured as in Ex. 3a.

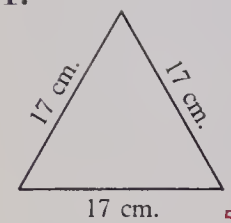
15. $5\frac{1}{2}$ in. **16.** 3 yd. **17.** $8\frac{1}{4}$ ft. **18.** 6 m. **19.** $4\frac{0}{4}$ mi.

Key idea: 15. either $5\frac{1}{4}$ in. or between $5\frac{1}{4}$ in. and $5\frac{3}{4}$ in.
16. either $2\frac{1}{2}$ yd. or between $2\frac{1}{2}$ yd. and $3\frac{1}{2}$ yd.
17. either $8\frac{1}{8}$ ft. or between $8\frac{1}{8}$ ft. and $8\frac{3}{8}$ ft.
18. either $5\frac{1}{2}$ m. or between $5\frac{1}{2}$ m. and $6\frac{1}{2}$ m.
19. either $3\frac{7}{8}$ mi. or is between $3\frac{7}{8}$ mi. and $4\frac{1}{8}$ mi.

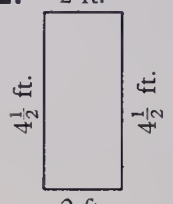
How Well Do You Remember?

[W]

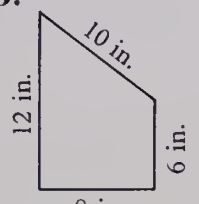
Ex. 1–5. Find the perimeter of the polygon shown. Use $P = n \times s$ or $P = 2 \times (a + b)$ whenever possible.

1. 

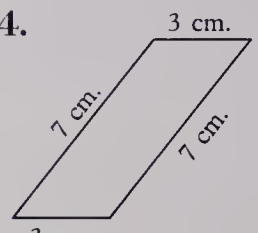
51 cm.

2. 

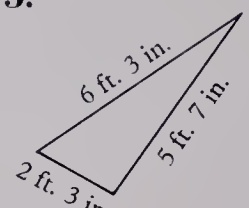
13 ft.

3. 

36 in.

4. 

20 cm.

5. 

14 ft. 1 in.

6. If the length of each side of a regular hexagon is 9 in., find its perimeter in terms of feet and inches. 4 ft. 6 in.

7. Name the unit of precision for a measurement if the greatest possible error for the measurement is $\frac{1}{16}$ in. $\frac{1}{8}$ inch

Ex. 8–11. Name the unit of precision for the measurement.

8. $3\frac{3}{4}$ cm. $\frac{1}{4}$ centimeter 9. $31\frac{1}{8}$ in. $\frac{1}{8}$ inch 10. $5\frac{0}{10}$ in. $\frac{1}{10}$ inch 11. 2 yd. $3\frac{1}{2}$ in. $\frac{1}{2}$ inch

12. Copy and complete: A measurement of $3\frac{0}{2}$ cm. means that the length is _?_ $2\frac{3}{4}$ cm. or is between _?_ $2\frac{3}{4}$ cm. and _?_ $3\frac{1}{4}$ cm.

Try These Exercises

Enrichment [W]

* The work done in lifting a certain object a certain distance is often measured in terms of *foot-pounds*. The number, W, of foot-pounds of work done in lifting an object weighing k pounds a distance of d feet can be found by using $W = k \times d$. For example, the work done in lifting a 3-lb. object a distance of 2 feet is 6 foot-pounds $(6 = 3 \times 2)$.

Find the amount of work done when a 5-lb. object is lifted

1. 1 foot. 5 foot-pounds 2. 3 feet. 15 foot-pounds 3. 16 feet. 80 foot-pounds 4. 100 feet. 500 foot-pounds

5. Jerry did 17 foot-pounds of work when he lifted an object 2 feet. How much did the object weigh? $8\frac{1}{2}$ pounds

6. How much does a rock weigh if the work done in lifting it 4 feet is the same as when lifting a 7-lb. rock 7 feet? $12\frac{1}{4}$ lb.

* The word "work" is used as in physics to mean the measure of the force times the measure of the distance moved, not "effort."

Teaching Page 158

Pupil's Objectives

(a) To satisfactorily complete the review and maintenance exercises; (b) to restudy material not completely understood; and (c) to study as an enrichment topic the relationship among weight, distance, and work.

Background

The purpose of these exercises is to maintain concepts presented in pages 150–157. They may also be used as a test.

The enrichment topic pertains to the measurement of an amount of work done in lifting an object. Work may be defined as "the product of the measures of the force exerted on a body, and the distance that the body moves in the direction of the force." In order to apply this definition to the mathematical sentence on the pupil's page, you need to know that the upward force that one exerts in lifting an object is the same as its weight. In the English system of measurement, the standard unit of measurement of work is the foot-pound. A force of one pound acting through a distance of one foot does one foot-pound of work. When an object is lifted, the measure of the work done, expressed in foot-pounds, is equal to the product of the measure of the weight in pounds and the measure of the distance in feet through which it is moved. This may be expressed by the relationship $W = k \times d$ (see text page).

The relationship may also be expressed as $k = W \div d$ to find the measure of weight in pounds of an object being lifted when the measure of work in foot-pounds and the measure of distance in feet are given. It may also be expressed as $d = W \div k$ to find the measure of distance.

Pre-Book Lesson

Instructions for using the text page should be given at this time. You may wish to use the exercises at the top of the page for review or as a test. Pupils may then work Extra Examples Sets from the back of the book if you feel additional work is necessary. Assign the enrichment topic to *more capable pupils*.

Using the Text Page

• Ex. 1–6 (bottom). You may wish to have *more capable pupils* attempt the enrichment exercises without previous discussion.

For errors on Ex.	*Review*
1–5	Pages 150, 152, 153
6	Page 153
7	Pages 156, 157
8–11	Page 154
12	Page 156

• Ex. 1–6. You may wish to have *more capable pupils* attempt the enrichment exercises without previous discussion.

Individualizing Instruction

• After Ex. 1–12 have been completed and returned to the class, *all pupils* should correct their errors and review the pages indicated under Using the Text Page. The teacher may work with those pupils needing additional help in understanding their mistakes and may give extra exercises to those needing remedial work.

• *More capable pupils*, who are doing the enrichment exercises, may be given the definition of work from the Background. Ask them the relationship between *force* and *weight* when lifting an object. From looking at both the written definition and the mathematical sentence on the pupil's page, they should realize that force is the same as weight when lifting an object.

These pupils may be asked for other possible units of measurement for work. For example, if weight was given in ounces and distance was given in inches, what would be the unit of measurement for the work done? (the inch-ounce) Other examples could be used such as the centimeter-gram, meter-kilogram, foot-ton, and so on.

Pupil's Objectives

(a) To review concepts and maintain skills taught in Chapters 1–3; and (b) to find where areas of weakness exist so that appropriate material may be restudied as necessary.

Background

A periodic review of the concepts and skills presented earlier in the text is necessary to maintain proficiency in the skills and mastery of the concepts. Some of the concepts presented earlier in the text may now be restudied in the light of new knowledge gained more recently. The teacher may wish to review previous Background material to reintegrate it with material more recently presented.

Pre-Book Lesson

If examples from the previous page have not been used as a test, you may wish to use this pupil's page as a quiz. In that case, select several examples for oral discussion before opening the text and assigning the work.

Using the Text Page

Ex. 1–31. Even if you decide to use the material as maintenance only, you may wish to choose random examples from the pupil's page for oral work with the class. Otherwise, pupils should be allowed to do the work on their own. Later, when the corrected work has been returned, pupils making errors in answering these questions may review the topic of the exercise on the corresponding pages listed below:

For errors on Ex.	*Review*
1	page 67
2–3	page 8
4	page 66
5	page 44
6–7	page 129
8	page 47
9	pages 28, 56
10	pages 45, 88
11	page 135
12	pages 30, 32
13–17	pages 61, 62
18–25	pages 71–75
27–31	pages 19, 69

Individualizing Instruction

- If several pupils exhibit weakness in any one skill or in understanding a particular concept, group them for reteaching. A short oral or written test may be given to ensure the effectiveness of the reteaching.
- *More capable pupils* may wish to work on Extra Activities which they have not had time to complete previously.
- *Slower learners* may need individual help when reviewing the topics on the pages listed above.

NOTES

*You may want to review that "4^2" means 4×4, not 4×2.

To Keep in Practice

[W]

1. List within braces the members of $\{3, 5, 7, 9, 11, 13\}$ which are primes. $\{3, 5, 7, 11, 13\}$

2. Find the union of $\{1, 3, 7\}$ and $\{2, 6, 7, 8\}$. $\{1, 2, 3, 6, 7, 8\}$

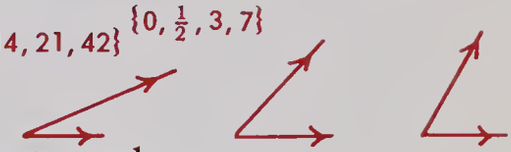
3. Find the intersection of $\{0, \frac{1}{2}, 3, 7\}$ and
a. the set of counting numbers. $\{3, 7\}$ **b.** the set of rational numbers. $\{0, \frac{1}{2}, 3, 7\}$

4. List within braces the factors of 42. $\{1, 2, 3, 6, 7, 14, 21, 42\}$

On your paper, show Sample answers:

5. two parallel segments.

6. three acute angles.

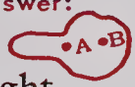


7. two perpendicular lines.

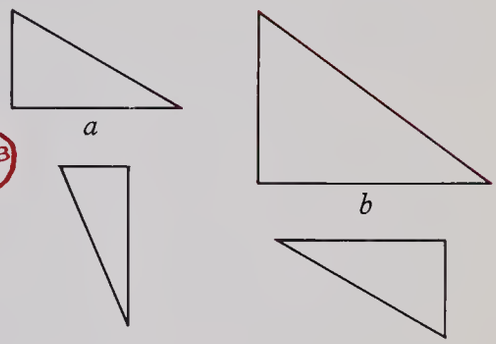
8. a scalene triangle.

9. Name the identity element for
a. addition. 0 **b.** multiplication. 1

10. Show a simple closed curve and then show points A and B in the interior for the curve. Sample answer:



11. Which of the four figures shown at the right appear to be congruent? a and d



12. Draw a number-line picture to show the relationship expressed by $3 + 8 = n$.

Ex. 13–17. Multiply.

13.	14.	15.	16.	17.
34	806	1,678	\$45.77	\$601.89
18	43	438	61	8
612	34,658	734,964	\$2,791.97	\$4,815.12

Ex. 18–25. Divide and check.

18.	19.	20.	21.
$9\overline{)3,678}$	$43\overline{)5,677}$	$109\overline{)22,305}$	$17\overline{)\$39.27}$
408, R6	132, R1	204, R69	\$ 2.31, R0

22.	23.	24.	25.
$48\overline{)2,432}$	$19\overline{)6,949}$	$552\overline{)61,384}$	$28\overline{)74,906}$
50, R32	365, R14	111, R112	2,675, R6

Ex. 26–31. Find the number represented by the letter.

26. $n \div 13 = 4{,}627$ 60,151
*27. $x = 4^2 + 5^3 + 2^4$ 157
28. $y = 3^2 \times 5 \times 7^2$ 2,205
29. $n = (8 \times 8 \times 8) + 34$ 546
30. $n = 5{,}518 \div (124 \div 2)$ 89
31. $x = (3 \times 5^2 \times 11) \div (3 \times 5)$ 55

829.46 means

8 hundreds + 2 tens + 9 ones + 4 tenths + 6 hundredths,

or 82 tens + 9 ones + 4 tenths + 6 hundredths,

or 829 ones + 4 tenths + 6 hundredths,

or 8,294 tenths + 6 hundredths,

or 82,946 hundredths,

or 829 ones + 46 hundredths.

Decimals

Remembering decimal concepts [O]

1. Read from the box above some of the meanings of 829.46 by completing the statement shown.

2. Can 829.46 be renamed with the fraction $\frac{82,946}{100}$? Yes Explain. 829.46 means 82,946 hundredths.

3. Can 829.46 be renamed with the mixed form $829\frac{46}{100}$? Yes with $829\frac{23}{50}$? Yes Explain. 829.46 means 829 ones + 46 hundredths. In simplest form, $\frac{46}{100}$ is $\frac{23}{50}$.

4. Rename 7.8 with a fraction showing denominator 10. $\frac{78}{10}$

5. On the board, write decimals for the following numbers.

a. $\frac{2}{10}$ 0.2 b. $3\frac{7}{10}$ 3.7 c. $7\frac{8}{100}$ 7.08 d. $9\frac{27}{100}$ 9.27 e. $27\frac{3}{100}$ 27.03

6. Give the meaning of 52.307 in as many ways as you can.

7. In 6.0079, the 7 is in thousandth's place and 9 is in ten-thousandth's place. The digit 7 in its place indicates $\frac{7}{1,000}$ and the digit 9 in its place indicates $\frac{9}{10,000}$.

8. What is the name of the 5th place to the right of the decimal point? ten-millionth's the 6th place? hundred-thousandth's the 7th place? millionth's

[W]

Ex. 9–14. Write a decimal for the number.

9. $7\frac{1}{100}$ 7.01 10. $\frac{23}{1,000}$ 0.023 11. $4\frac{51}{10,000}$ 4.0051 12. $15\frac{1}{2}$ 15.5 13. $\frac{256}{10}$ 25.6 14. $\frac{37}{100,000}$ 0.00037

15. Name in order from left to right the first five places to the right of the decimal point. tenth's, hundredth's, thousandth's, ten-thousandth's, hundred-thousandth's

Ex. 16–18. Which decimal names the greater number,

16. 0.83 or 0.73? 17. 0.52 or 0.5? 18. 0.06 or 0.17?

Teaching Page 160

Pupil's Objective

To review several concepts involving the meaning of decimals.

Background

The purpose of these exercises is to review some concepts involving decimals in order to prepare for the extension of these concepts to include addition and subtraction of rational numbers named with decimals.

A decimal is a common notation for expressing a rational number. The principle of place value for whole numbers which has been studied previously can be extended to include numbers represented to the right of the decimal point. Writing a decimal in expanded form helps to illustrate its various meanings. It is expressed as a sum, with each addend representing tenths, hundredths, thousandths, and so forth.

Pre-Book Lesson

Concepts of decimals may be introduced through a discussion of the various meanings of a whole number within the framework of place value. The whole number 347 may be interpreted as having any one of the following meanings:

(a) 3 hundreds + 4 tens + 7 ones
(b) 3 hundreds + 47 ones
(c) 347 ones
(d) 34 tens + 7 ones
(e) $(3 \times 100) + (4 \times 10) + (7 \times 1)$
(f) $(3 \times 100) + (47 \times 1)$
(g) (347×1)
(h) $(34 \times 10) + (7 \times 1)$

This concept may be extended to decimals so that the decimal 3.47 may be interpreted as having any one of the following meanings:

(a) 3 ones + 4 tenths + 7 hundredths
(b) 3 ones + 47 hundredths
(c) 34 tenths + 7 hundredths
(d) 347 hundredths

These meanings may also be expressed by the following:

(a) $3 + \frac{4}{10} + \frac{7}{100}$
(b) $3 + \frac{47}{100}$
(c) $\frac{34}{10} + \frac{7}{100}$
(d) $\frac{347}{100}$

Using the Text Page

• Ex. 2–3. In renaming 829.46 in Ex. 2 with a fraction, we use 82,946 hundredths from the box at the top of the page. In renaming 829.46 in Ex. 3 with a mixed form, use 829 ones + 4 _ ? _ + 6 _ ? _ from the box.

• Ex. 6. Have pupils use expanded form, exponent form, and fraction form for the answer.

• Ex. 9–14. Point out the advantage of a denominator which is a multiple of ten when renaming from a fraction to a decimal. The place value for a digit in a decimal should be stressed.

• Ex. 16–18. In determining which of two given fractions names the greater number, the numbers are first renamed to show like denominators. The fraction showing the greater numerator names the greater number. This same concept applies to decimals. It is easier to compare two fractions if they both show the same denominator. Two decimals may be more easily compared if they both have the same number of digits to the right of the decimal point. Zeros may be annexed as needed. For example, it is easier to compare 2.50 and 0.50 then 2.50 and 0.5.

Individualizing Instruction

• Ask the *more capable pupils* to compare the annexing of zeros on the right of a decimal point to the multiplication of the numerator and denominator shown by a fraction by some power of 10.

• *Slower learners* will need assistance to visualize the renaming of a number with different forms. They may benefit by using more illustrations showing the importance of place value.

Teaching Page 161

Pupil's Objectives

(a) To review renaming numbers; and (b) to review rounding numbers.

Background

A number is an idea or concept and may be named by any one of several different numerals. The number of the set $\{\triangle, \square, \square, \#, *\}$ is *five* and may be named by any of 5, V, ||||, 10_{five}, $3 + 2$, $\frac{5}{1}$, 5^1, $\frac{25}{5}$, 5.0, $5\frac{0}{2}$, and so on.

The pupils have studied naming numbers with base-five numerals, Roman numerals, base-ten numerals, exponent form, fractions, mixed forms, and decimals. They have also studied showing a number as a sum, product, unknown addend, or unknown factor of other numbers. The exercises on this page will give the pupils an opportunity to refresh their memories about various ways of naming a number.

Pre-Book Lesson

Review the difference between number and numeral. Have the pupils list on the chalkboard many ways of naming with various numerals a number such as sixteen and a number such as one half.

Using the Text Page

One example from each of the exercises may be used for oral work at the board prior to assigning the remaining exercises as written work. The teacher can determine from this whether there are pupils who need help with any of these concepts.

Individualizing Instruction

• *All pupils* who made errors consistently in certain exercises should restudy the appropriate pages listed below and should be given additional practice exercises.

For errors on Ex.	*Review*
1–5	page 13
6–10	page 23
11–15	page 19
16–20	page 97
21–25	page 110
26–30	page 122
31–34	pages 120, 121

• *More capable pupils* may work on Extra Activities Sets or Supplementary Activities after they have completed the written work.

NOTES

*You may want to review naming numbers with base-five numerals on pages 21-23.

Practice in Renaming Numbers

[W]

Ex. 1–5. Write a Roman numeral for the number.

1. 16 XVI 2. 23 XXIII 3. 54 LIV 4. 89 LXXXIX 5. 245 CCXLV

*Ex. 6–10. Write a base-five numeral for the number.

6. 7 12_{five} 7. 18 33_{five} 8. 25 100_{five} 9. 55 210_{five} 10. 131 1011_{five}

Ex. 11–15. Write an exponent form for the number.

11. 25 5^2 12. 125 5^3 13. 32 2^5 14. 100 10^2 15. 50 − 1 7^2

Ex. 16–20. Rename with a fraction in simplest form.

16. 3.5 $\frac{7}{2}$ 17. $1\frac{2}{3}$ $\frac{5}{3}$ 18. 0.03 $\frac{3}{100}$ 19. 3 ÷ 8 $\frac{3}{8}$ 20. 0.25 $\frac{1}{4}$

Ex. 21–25. Rename with a mixed form.

21. $\frac{20}{3}$ $6\frac{2}{3}$ 22. 23.5 $23\frac{1}{2}$ 23. 18 ÷ 15 $1\frac{1}{5}$ 24. 1.003 $1\frac{3}{1,000}$ 25. $\frac{348}{15}$ $23\frac{1}{5}$

Ex. 26–30. Rename with a decimal.

26. $3\frac{7}{100}$ 3.07 27. $45\frac{2}{50}$ 45.04 28. $\frac{312}{100}$ 3.12 29. 15 ÷ 10 1.5 30. $5\frac{3}{4}$ 5.75

Ex. 31–32. Rename the members of the set with decimals and then show the numbers in order from least to greatest.

31. $\{\frac{7}{2}, 3\frac{3}{4}, 3\frac{7}{10}, 3\frac{4}{5}\}$ $\{3.5, 3.7, 3.75, 3.8\}$ or $\{\frac{7}{2}, 3\frac{7}{10}, 3\frac{3}{4}, 3\frac{4}{5}\}$ 32. $\{\frac{1}{2}, \frac{24}{50}, \frac{2}{5}, \frac{11}{100}\}$ $\{0.11, 0.4, 0.48, 0.5\}$ or $\{\frac{11}{100}, \frac{2}{5}, \frac{24}{50}, \frac{1}{2}\}$

Ex. 33–34. Rename with fractions showing the same denominator and then show in order from least to greatest.

33. $\{2\frac{1}{2}, 2.3, \frac{9}{4}\}$ $\{\frac{45}{20}, \frac{46}{20}, \frac{50}{20}\}$ or $\{\frac{9}{4}, 2.3, 2\frac{1}{2}\}$

34. $\{0.23, \frac{1}{5}, \frac{3}{20}\}$ $\{\frac{15}{100}, \frac{20}{100}, \frac{23}{100}\}$ or $\{\frac{3}{20}, \frac{1}{5}, 0.23\}$

Remembering about Rounding Numbers

Resurvey [W]

Ex. 1–5. Round to the nearest ten; to the nearest thousand.

1. 23,698 23,700; 24,000 2. 33,608 33,610; 34,000 3. 870,806 870,810; 871,000 4. 987 990; 1,000 5. 1,009 1,010; 1,000

Ex. 6–10. Round to the nearest hundredth; to the nearest whole number.

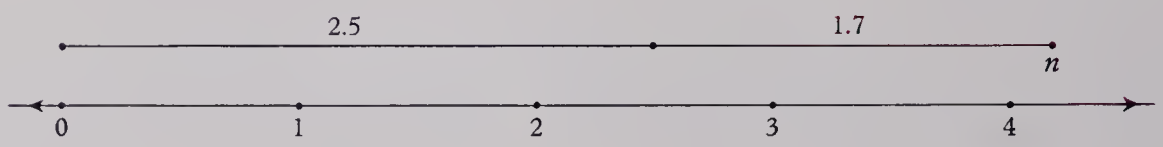
6. 4.046 4.05; 4 7. 36.809 36.81; 37 8. 0.755 0.76; 1 9. 0.9993 1.00; 1 10. $15\frac{367}{1,000}$ 15.37; 15

Adding with Decimals

Resurvey [O]

1. It is about 1.7 miles from City Park to the scout camp. When the boys left the park, the cyclometer on Jim's bicycle registered 2.5 miles. About what should it register when he arrives at camp?

a. Explain how the number-line picture below shows the relationship expressed in the problem. It shows the addends-sum relationship.



A

$$n = 2.5 + 1.7$$
$$= \frac{25}{10} + \frac{17}{10}$$
$$= \frac{42}{10}$$
$$n = 4\frac{2}{10}, \text{ or } 4.2$$

B

Add

$$\begin{array}{r} {\scriptstyle 1} \\ 2.5 \\ 1.7 \\ \hline 4.2 \end{array}$$

b. Box A. To solve $n = 2.5 + 1.7$ we can rename 2.5 and 1.7 with fractions. Explain the work shown. Key idea: The decimals are renamed with fractions, the work is done, and the sum is renamed with decimal.

*c. Instead of renaming with fractions as in box A, we can work as in box B.

Add tenths: $7 + 5 =$ _?_[12] Rename 12 tenths as 1 one + 2 tenths. Write a 2 in tenth's place in the numeral for the sum and write a 1 in the column that shows ones.

Add ones: $1 + 2 + 1 =$ _?_[4] Write a 4 in one's place in the numeral for the sum.

d. When Jim arrives at camp, the cyclometer on his bicycle should register about _?_[4.2] miles.

* Point out that we can add tenths and ones just like we add ones and tens.

Teaching Pages 162 and 163

Pupil's Objective

To resurvey addition of rational numbers named with decimals.

Background

Finding the sum of two numbers named with decimals is essentially the same as finding the sum of two whole numbers. The addition of 236 and 123 may be compared with the addition of 2.36 and 1.23 by the use of expanded forms.

$$\begin{aligned} 236 &= 2 \text{ hundreds} + 3 \text{ tens} + 6 \text{ ones} \\ \underline{123} &= \underline{1 \text{ hundred} + 2 \text{ tens} + 3 \text{ ones}} \\ & 3 \text{ hundreds} + 5 \text{ tens} + 9 \text{ ones} = 359 \end{aligned}$$

$$\begin{aligned} 2.36 &= 2 \text{ ones} + 3 \text{ tenths} + 6 \text{ hundredths} \\ \underline{1.23} &= \underline{1 \text{ one} + 2 \text{ tenths} + 3 \text{ hundredths}} \\ & 3 \text{ ones} + 5 \text{ tenths} + 9 \text{ hundredths} = 3.59 \end{aligned}$$

Although the use of the expanded form is helpful in explaining addition with decimals, the work may simply be written as

$$\begin{array}{r} 2.36 \\ \underline{1.23} \\ 3.59 \end{array}$$

The procedure for adding whole numbers is also followed when adding with decimals when the sum of the numbers shown in one of the places is expressed by a 2-place numeral. When adding 573 and 254, the sum of the numbers shown in the ten's place is shown by a 2-place numeral.

$$\begin{aligned} 573 &= 5 \text{ hundreds} + 7 \text{ tens} + 3 \text{ ones} \\ \underline{254} &= \underline{2 \text{ hundreds} + 5 \text{ tens} + 4 \text{ ones}} \\ & 7 \text{ hundreds} + 12 \text{ tens} + 7 \text{ ones} \end{aligned}$$

This sum is renamed:

$$8 \text{ hundreds} + 2 \text{ tens} + 7 \text{ ones}$$

and the work is written:

$$\begin{array}{r} 573 \\ \underline{254} \\ 827 \end{array}$$

Applying this same procedure to addition with decimals, we have

$$\begin{aligned} 5.73 &= 5 \text{ ones} + 7 \text{ tenths} + 3 \text{ hundredths} \\ \underline{2.54} &= \underline{2 \text{ ones} + 5 \text{ tenths} + 4 \text{ hundredths}} \\ & 7 \text{ ones} + 12 \text{ tenths} + 7 \text{ hundredths} \end{aligned}$$

The sum is renamed as:

$$8 \text{ ones} + 2 \text{ tenths} + 7 \text{ hundredths}$$

and the work is written:

$$\begin{array}{r} 5.73 \\ \underline{2.54} \\ 8.27 \end{array}$$

When adding whole numbers, it is essential that the digits in each respective place be written one under the other. Using the vertical form, the addition of 92; 6; 103; and 1,234 is written

$$\begin{array}{r} 92 \\ 6 \\ 103 \\ \underline{1{,}234} \end{array}$$

The digits naming the addends are aligned according to their respective place values with the digits in one's place being used as a guide. This same idea is applied when adding with decimals using the vertical form. The addition of 0.92, 0.6, 103, and 12.34 would be written in vertical form as

$$\begin{array}{r} 0.92 \\ 0.6 \\ 103 \\ \underline{12.34} \end{array}$$

In aligning the digits according to their respective place values, the decimal point is used as a guide.

To find the sum of two rational numbers named with decimals, the addends may first be expressed with fractions, the sum of the fraction numbers determined, and this sum then expressed with a decimal. However, addition with decimals in vertical form is usually the preferred method.

Pre-Book Lesson

Review addition of whole numbers using expanded forms, emphasizing the importance of aligning digits according to their respective place values using one's place as a guide. The addition is performed by finding the sum of the numbers shown in each respective place. It is necessary to rename the sum shown by digits in any one column if the sum is named by a 2-place numeral.

Using the Text Pages

• Ex. 1. Make sure pupils understand what "cyclometer" means. When finding the sum of two measures, the unit of precision should be the same for both measurements. If the unit of precision for one measurement is $\frac{1}{10}$ inch, then the unit of precision of the other measurement should also be $\frac{1}{10}$ inch. Make sure pupils realize the relevance of this to Ex. 1 where the measures are expressed with decimals.

• Ex. 2. Have the pupils decide which of the two methods, the one shown in box A or the one shown in box B, seems to be the easier method. You may wish to use the expanded form as shown in the Background to help pupils understand the rationalization underlying the addition.

• Ex. 4. Use of the expanded form may be helpful.

• Ex. 5, 19, and 20. Emphasize the importance of aligning the digits according to their respective place values using the decimal point as a guide.

Individualizing Instruction

• If the sum of two measures is to be determined, ask the *more capable pupils* if the measurements should first be expressed with the same unit of precision. Ask why the work in finding the sum of 19.1, 3.27, and 103.294 may be written

$$\begin{array}{r} 19.100 \\ 3.270 \\ \underline{103.294} \\ 125.664 \end{array}$$

but if the addends are measures, the work should be written as

$$\begin{array}{r} 19.10 \\ 3.27 \\ \underline{103.29} \\ 125.66 \end{array}$$

and then rounded to the nearest tenth.

Let these pupils (a) determine place values of the first three places to the right of the decimal point in a base-five numeral ($\frac{1}{5}$, $\frac{1}{25}$, $\frac{1}{125}$); (b) express a base-five numeral such as 3.412_{five} with a base-ten mixed form; and (c) find the sum of two numbers shown with base-five numerals and have them express this sum with both a base-five numeral and a base-ten mixed form. As an example:

$$\begin{array}{r} 3.412_{\text{five}} \\ \underline{2.314_{\text{five}}} \\ 11.231_{\text{five}} \end{array}$$

$$\begin{aligned} 11.231_{\text{five}} &= (1 \times 5) + (1 \times 1) + (2 \times \tfrac{1}{5}) + \\ &\qquad (3 \times \tfrac{1}{25}) + (1 \times \tfrac{1}{125}) \\ &= 5 + 1 + \tfrac{2}{5} + \tfrac{3}{25} + \tfrac{1}{125} \\ &= 6 + \tfrac{50}{125} + \tfrac{15}{125} + \tfrac{1}{125} \\ &= 6\tfrac{66}{125} \end{aligned}$$

• Give *all pupils* practice adding with decimals that do not have the same number of places.

• *Slower learners.* Give these pupils help in renaming the sums with decimals and in remembering to align numerals according to their place value using the decimal point as a guide.

2. At the board, solve $n = 73.8 + 68.3$ first as in box A and then as in box B. See below.

C
Add
1.35
7.83
?.?8
9 1

3. Box C. To add 1.35 and 7.83, we first add the numbers shown in ?-hundredth's place. Give the missing numerals.

4. Explain how adding with decimals is similar to adding with numerals for whole numbers. Adding tenths, hundredths, and thousandths is like adding hundreds, tens, and ones.

5. At the board, find the number represented by n.

a. $n = 1.23 + 5.75$ 6.98 **b.** $n = 5.64 + 2.28$ 7.92 **c.** $n = 26.79 + 13.83$ 40.62

6. Is it true that $5.27 + 3.89 = 5\frac{27}{100} + 3\frac{89}{100} = \frac{527}{100} + \frac{389}{100}$? Yes

At the board, find $5.27 + 3.89$ by adding in vertical form with

a. decimals. 5.27 / 3.89 / 9.16 **b.** mixed forms. See left. **c.** fractions. 527/100, 389/100

6. b. $5\frac{27}{100}$, $3\frac{89}{100}$ / $8\frac{116}{100}$, or $9\frac{4}{25}$

[W] Ex. 7–11. Add and then check your answer by renaming the addends with decimals and adding again. 916/100, or $9\frac{4}{25}$

7. $8\frac{8}{10}$, or $8\frac{4}{5}$ 8. $41\frac{32}{100}$, or $41\frac{8}{25}$

7. $5\frac{1}{10}$, $3\frac{7}{10}$ — 5.1 + 3.7 = 8.8

8. $14\frac{3}{100}$, $27\frac{29}{100}$ — 14.03 + 27.29 = 41.32

9. $\frac{29}{10}$, $\frac{43}{10}$ — 2.9 + 4.3 = 7.2; $\frac{72}{100}$, or $7\frac{1}{5}$

10. $7\frac{8}{100}$, $\frac{3}{100}$ — 7.08 + 0.03 = 7.11; $7\frac{11}{100}$

11. $\frac{7}{100}$, $\frac{13}{100}$ — 0.07 + 0.13 = 0.20; $\frac{20}{100}$, or $\frac{1}{5}$

Ex. 12–16. Copy and add.

12.	13.	14.	15.	16.
0.3	0.03	1.57	14.93	153.94
0.8	0.78	3.62	3.88	5.07
1.1	0.81	5.19	18.81	159.01

Ex. 17–22. Find the number represented by n.

17. $n = 2\frac{5}{10} + 3\frac{6}{10}$ $6\frac{1}{10}$ 18. $n = \frac{23}{10} + \frac{57}{10}$ 8 19. $n = 15.74 + 23$ 38.74

20. $n = 14 + 9.58$ 23.58 21. $n = 13.62 + 0.92$ 14.54 22. $n = 1{,}062.57 + 1.37$ 1,063.94

Ex. 23–27. Copy and add.

23.	24.	25.	26.	27.
6.3	17.28	8.03	27.89	235.01
7.4	13.56	9.60	7.39	40.07
5.8	14.04	7.37	0.09	18.07
19.5	44.88	25.00	35.37	293.15

28. One year, the high-school record for the high jump was 6 ft. 8 in. while the world record was 7 ft. $5\frac{3}{4}$ in. How much higher was the world record than the high-school record? $9\frac{3}{4}$ in.

♦ **Extra Examples.** Sets 76–78.

2. $n = 73.8 + 68.3$

$= \frac{738}{10} + \frac{683}{10}$

$= \frac{1{,}421}{10}$

$= 142\frac{1}{10}$, or 142.1

11
73.8
68.3
142.1

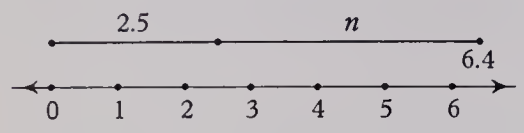
* If necessary, reexplain and reemphasize the importance of aligning the digits according to place value using the decimal point as a guide.

Subtracting with Decimals

Resurvey [O]

1. When Jim left City Park, the cyclometer on his bicycle registered 2.5 miles. When he returned home, it registered 6.4 miles. How far had Jim traveled?

A



$2.5 + n = 6.4$

a. Box A. Explain how both the number-line picture and the mathematical sentence show the relationship expressed in the problem. They show the addends-sum relationship.

b. Box B. In $2.5 + n = 6.4$, does n represent the sum or an addend? addend To find the number represented by n, we subtract the known addend from the sum. Explain the work shown. Key idea: The decimals are renamed with fractions, the work is done, and the answer is renamed with a decimal.

B

$2.5 + n = 6.4$

$n = 6.4 - 2.5$

$= \frac{64}{10} - \frac{25}{10}$

$= \frac{39}{10}$, or 3.9

c. Instead of renaming with fractions, we can subtract as shown in box C.

C

Subtract

$\begin{array}{r} 5\ \textcircled{14} \\ \not{6}.\not{4} \\ 2.5 \\ \hline 3.9 \end{array}$

Subtract tenths: Can we subtract 5 tenths from 4 tenths? No Can we rename 6.4 as 5 ones + 14 tenths? Yes $14 - 5 =$ 9, so we write a 9 in tenth's place.

Subtract ones: $5 - 2 =$ 3, so we write a 3 in one's place.

d. What is the answer for the problem? 3.9 miles

*2. On the board, copy and subtract. Try to work Ex. d–e *without* writing small numerals such as those shown in color in box C.

a.	b.	c.	d.	e.
3.7	0.43	0.08	3.72	13.21
2.3	0.25	0.02	0.47	7.68
1.4	0.18	0.06	3.25	5.53

[W]

Ex. 3–12. Copy and subtract.

3.	4.	5.	6.	7.
0.94	5.06	17.40	4.68	13.24
0.35	0.01	3.51	0.77	10.72
0.59	5.05	13.89	3.91	2.52
8.	**9.**	**10.**	**11.**	**12.**
9.34	7.63	7.00	235.7	7.62
0.84	3.63	3.55	200.8	5.37
8.50	4.00	3.45	34.9	2.25

▲ **Reteaching.** Set 13. ◆ **Extra Examples.** Sets 79–80.

Teaching Page 164

Pupil's Objective

To resurvey subtraction of rational numbers named with decimals.

Background

Subtraction of rational numbers named with decimals is an extension of the concept of subtraction of whole numbers. In subtracting with decimals, the digits of the numerals are aligned according to their respective place values, using the decimal point as a guide. The subtraction 7.89 − 5.31 may be explained with the use of expanded forms:

$$\begin{aligned} 7.89 &= 7 \text{ ones} + 8 \text{ tenths} + 9 \text{ hundredths} \\ \underline{5.31} &= \underline{5 \text{ ones} + 3 \text{ tenths} + 1 \text{ hundredth}} \\ & \quad 2 \text{ ones} + 5 \text{ tenths} + 8 \text{ hundredths} = 2.58 \end{aligned}$$

If a number in the sum is less than the number in the same place in the known addend, the sum will have to be renamed so that the subtraction may be performed. This step is necessary in performing the subtraction 7.47 − 4.85:

$$\begin{aligned} 7.47 &= \overset{6}{\cancel{7}} \text{ ones} + \overset{14}{\cancel{4}} \text{ tenths} + 7 \text{ hundredths} \\ \underline{4.85} &= \underline{4 \text{ ones} + 8 \text{ tenths} + 5 \text{ hundredths}} \\ & \quad 2 \text{ ones} + 6 \text{ tenths} + 2 \text{ hundredths} = 2.62 \end{aligned}$$

The work is written as

$$\begin{array}{r} \overset{6}{\cancel{7}}\,\overset{(14)}{\cancel{4}}7 \\ 4.85 \\ \hline 2.62 \end{array} \quad \text{or more simply as} \quad \begin{array}{r} 7.47 \\ 4.85 \\ \hline 2.62 \end{array}$$

In the subtraction 1.95 − 0.415, there is no digit in thousandth's place for the sum 1.95, but there is a digit in thousandth's place for the known addend. The sum 1.95 may be renamed 1.950 because $\frac{195}{100} = \frac{1,950}{1000}$, and the subtraction may be shown

$$\begin{array}{r} 1.950 \\ 0.415 \\ \hline 1.535 \end{array}$$

Pre-Book Lesson

Review subtraction of whole numbers using expanded form. Emphasize the importance of aligning the digits according to their respective place values, using one's place as a guide; performing the subtraction by subtracting the number in the known addend from the number in the sum in each respective place; and of renaming the sum when a digit in the numeral for the known addend names a number greater than that in the sum.

Using the Text Page

• Ex. 1. Point out how subtraction of rational numbers expressed with decimals may be considered as an extension of subtraction of whole numbers. Point out the importance of aligning the digits according to their place values, using the decimal point as a guide.

• Ex. 2. Before proceeding with this example, you may use material from the Background showing work with expanded forms and renaming.

Individualizing Instruction

• Give *all pupils* an incomplete magic square such as the following. Have them insert decimals in the blank boxes so that the sum of the numbers shown in each row, each column, and along each diagonal equals the same number.

(*3.12*)	2.56	2.96
2.72	2.88	(*3.04*)
2.80	(*3.20*)	2.64

• Give *all pupils* several terms of an arithmetic progression such as 1.17, 1.55, 1.93, . . . and ask them to determine the pattern (in this example, 1.55 is 0.38 greater than 1.17, 1.93 is 0.38 greater than 1.55). Now have them name the next two numbers in the series. This gives them experience in both addition and subtraction with decimals.

Teaching Page 165

Pupil's Objective

To solve problems involving addition or subtraction with decimals having digits in tenth's and hundredth's places.

Background

In deciding which operation, addition or subtraction, must be used when solving a problem, the following should be considered:

(a) Addition is required to find how many in all, or to find the resulting quantity when a given quantity is increased.

(b) Subtraction is required to find by how many a quantity is increased or decreased when the original and final quantities are given.

(c) Subtraction is required to find the difference between two quantities.

Pre-Book Lesson

• Ask for suggestions for determining whether addition or subtraction is the proper operation to use in solving a problem. Use material from the Background to establish a good reasoning procedure.

• Let some pupils make up problems and have others tell whether addition or subtraction should be used in the solution and why.

Using the Text Page

Ex. 1–5. For each of these exercises ask, "Is the sum given in the problem, or is a sum to be found?" If a sum is to be found, addition is necessary. If the sum is given and a missing addend is to be determined, subtraction is necessary.

Make sure pupils remember to express only measures in the mathematical sentences for the problems.

Individualizing Instruction

Give the *slower learners* verbal problems without numbers, such as:

(a) If you know the weight of an empty freight car and the weight of the lumber being loaded into the freight car, how do you find the weight of the loaded freight car?

(b) If you know the amount of rainfall for the month of July and the amount of rainfall for August, how do you find how much greater or less the rainfall was in August than in July?

These pupils may need assistance in reading the problems so as to be able to determine which operation to use for the solution and which data is pertinent.*

* See item 6, page xix. The relationships in the problem might be visualized more easily through use of a number-line picture.

NOTES

Add or Subtract?

Tenths and hundredths [O]

For each of Ex. 1–5, tell what operation or operations to use to find the answer for the problem.

1. When the Fraser family went to Rocky Point Park for an all-day picnic, they drove 4.9 miles to a store and then 18.7 miles further to the beach. That was a total of how many miles? Addition; n = 4.9 + 18.7; 23.6

2. In making sandwiches for the picnic, Mrs. Fraser used 7.5 oz. of tuna and 2.5 oz. of chopped olives. How many ounces of tuna and chopped olives did she use in all? Addition; n = 7.5 + 2.5; 10

3. Paul and Ed asked their father to bring a stop watch to time their swimming races. Paul swam to Rocky Point in 56.3 seconds. Ed's time was 48.6 seconds. Ed's time was how much less than Paul's? Subtraction; n = 56.3 − 48.6; 7.7 seconds

4. The charts at the park showed 3.17 in. of rainfall during June and 2.86 in. during July. There were how many more inches of rainfall during June than during July? Subtraction; n = 3.17 − 2.86 0.31 in.

5. In returning home, the Frasers drove 2.8 miles to a post office and then 22.6 miles to their house. Was the return trip longer or shorter than the trip to the beach (Ex. 1)? longer How much longer or shorter? Addition and Subtraction; n = (2.8 + 22.6) − 23.6; 1.8 miles

[W]

Now write and solve a mathematical sentence to find the answer for each of Ex. 1–5.

* When renaming in order to subtract 9 thousands, rename 600 hundredths as 599 hundredths and 10 thousandths. This is shown as 5 99 (14) 256.004

A	B	C	D
Add	Add	Subtract	Subtract
2.062	?1 11 3.2673	16.537	3 (11) 5.0041
1.325	0.9159	7.324	0.0027
?.??7	?.?832	?.213	5.00?4
3 38	4 1	9	1

More Adding and Subtracting with Decimals

Thousandths, ten-thousandths [O]

1. Box A. To find the sum of 2.062 and 1.325, we first add the numbers shown in _?_ thousandth's place. Give the missing numerals.

2. The fourth place to the right of the decimal point is _?_ ten-thousandth's place. Give the missing numerals for the work in box B.

3. Box C. To subtract 7.324 from 16.537, we first work in the column that shows _?_ thousandth's. Give the missing numeral and explain the work shown.

4. Box D. Give the missing numeral. Why was 5.0041 renamed 5 ones + 3 thousandths + 11 ten-thousandths? Key idea: To subtract 4 ten-thousandths

5. Work Ex. a–e on the board. Try to work Ex. d–e without writing small numerals such as those shown in the boxes.

a. Add	b. Subtract	c. Add	d. Subtract	e. Subtract
5.603	233.1057	5.623	0.524	15.0205
1.305	16.0024	17.598	0.097	1.0029
6.908	217.1033	23.221	0.427	14.0176

[W]

Ex. 6–10. Write a decimal for the number.

6. $8\frac{23}{1,000}$ 8.023 7. $17\frac{851}{1,000}$ 17.851 8. $3\frac{53}{10,000}$ 3.0053 9. $\frac{271}{10,000}$ 0.0271 10. $23\frac{1}{1,000}$ 23.001

Ex. 11–20. Copy and add. Then copy and subtract.

11. 8.356	12. 13.027	13. 1.0057	*14. 256.004	15. 15.3289
2.472	6.325	0.0098	3.999	1.3077
10.828; 58.84	19.352; 6.702	1.0155; 0.9959	260.003; 252.005	16.6366; 14.0212
16. 6.239	17. 7.083	18. 14.0082	19. 8.7013	20. 230.004
1.007	0.095	7.3422	0.0888	15.328
7.246; 5.232	7.178; 6.988	21.3504; 6.6660	8.7901; 8.6125	245.332; 214.676

♦ **Extra Examples.** Sets 81–84.

Teaching Page 166

Pupil's Objective

To add and subtract rational numbers named with decimals having digits in thousandth's and ten-thousandth's places.

Background

Adding and subtracting with decimals having digits in thousandth's and ten-thousandth's places is a simple extension of adding and subtracting with decimals having digits only through hundredth's place. After the digits have been aligned aeeording to place values using the decimal point as a guide, the addition or subtraction with decimals is almost identieal to the addition or subtraction of whole numbers.

Pre-Book Lesson

- Review place values in decimals through the ten-thousandth's place.
- Have pupils add and subtract with decimals having digits through hundredth's place, using the expanded form. Then ask whether the same method could be used with decimals having digits in thousandth's and ten-thousandth's places. Have them work several examples of this type.

Using the Text Page

- Ex. 14, 18, and 20. Sometimes pupils have difficulty with subtraetion exereises when the sum is expressed with two or more consecutive zeros. Be sure the pupils can perform this subtraction. Give additional exercises if needed. For pupils who experience difficulty with such subtractions, it may be well to have them write the small numerals such as those shown in the boxes as an aid in renaming the sum, or they may use the expanded form.
- Ex. 11–20. Have pupils check their answers to addition by applying the Commutative Property of Addition.
- Ex. 11–20. Have pupils check their subtractions by addition.

Individualizing Instruction

- *More capable pupils* may express with a base-ten mixed form the number named with a base-five numeral such as 3.1243_{five}. The value of each place is $\frac{1}{5}$ of the value of the place to its left. Thus, $3.1243_{\text{five}} = (3 \times 1) + (1 \times \frac{1}{5}) + (2 \times \frac{1}{25}) + (4 \times \frac{1}{125}) + (3 \times \frac{1}{625})$, or $(3) + (\frac{1}{5}) + (\frac{2}{25}) + (\frac{4}{125}) + (\frac{3}{625})$. Have them perform addition and subtraction using sueh numerals.
- Give *all pupils* addition and subtraction exercises with decimals which do not all have the same number of places to the right of the decimal point, such as:

(a) $9.32 + 17.5146 + 160.1 + 18$
(b) $17.3 - 15.9973$

Pupil's Objective

To extend the concept of the addends-sum relationship to numbers named with decimals.

Background

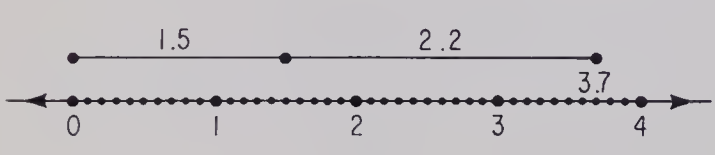
The addends-sum relationship for rational numbers is the same as that for whole numbers. This relationship may be expressed as:

$$\text{If } c = a + b,$$
$$\text{then } c - a = b,$$
$$\text{and } c - b = a.$$

Addition is commutative, so the first equation may also be expressed as $c = b + a$.

The number line was used in explaining the meaning of the addends-sum relationship of whole numbers, and it may also be used in explaining the addends-sum relationship of rational numbers named with decimals.

The sentence $3.7 = 1.5 + 2.2$ may be shown by the use of a number-line picture as follows:



If two addends, 1.5 and 2.2, are given, the relationship may be expressed as:

$$n = 1.5 + 2.2,$$
$$\text{or } n = 2.2 + 1.5.$$

If one addend (1.5) and the sum (3.7) are given, the relationship may be expressed as $3.7 = 1.5 + n$. If the addend (2.2) and the sum (3.7) are given, the relationship may be expressed as $3.7 = n + 2.2$. If both addends are given, the sum is found by addition. If one addend and the sum are given, the other addend is found by subtracting the given addend from the sum:

$$3.7 = 1.5 + n \qquad 3.7 = n + 2.2$$
$$n = 3.7 - 1.5 \qquad n = 3.7 - 2.2$$
$$= 2.2 \qquad = 1.5$$

Pre-Book Lesson

- Review expressing the addends-sum relationship of whole numbers in as many ways as possible. Refer to page 30.
- Extend a chart showing our place-value system to the right of the decimal point to ten-thousandth's place. List several decimals on the board and have pupils use the chart to give the place value for each digit. Draw a number-line picture on the board and have pupils tell where certain decimals would appear in relation to the whole numbers shown.

Using the Text Page

- Ex. 1–6. A number-line picture as shown in the Background section may be used to show this relationship.
- Ex. 7–20. Have the pupils check addition by applying the Commutative Property of Addition, and have them check subtraction with addition.
- Ex. 21. Explain the meaning of *m.p.h.* as meaning *miles per hour*. It is a unit of measurement for time and a unit of measurement for distance used in conjunction with one another. Pupils still operate on the measures only.

Individualizing Instruction

- Have *all pupils* make up and solve problems that involve the addends-sum relationship expressed with decimals.
- *More capable pupils* may wish to devise problems where it is necessary to express the addends-sum relationship with decimals.

In Ex. 21, they may express the winning speeds in meters and kilometers.

The Addends-Sum Relationship

[o]

1. Do the four sentences in box A illustrate the addends-sum relationship? Yes What is the sum? 3.7 What are the addends? 1.5 and 2.2

2. Box B. In $n - 0.8 = 3.3$, does n represent the sum or an addend? sum Explain the work shown.

3. When the addends are known, we add to find the sum.

4. Box C. In $2.34 - n = 1.63$, does n represent the sum or an addend? addend Explain the work shown.

5. When the sum and one of two addends are known, we subtract to find the other addend.

6. For each of Ex. 7–20, tell whether the letter represents the sum or an addend. 7. sum; 8. sum; 9. sum; 10. sum; 11. addend; 12. sum; 13. sum; 14. addend; 15. addend; 16. addend; 17. sum; 18. addend; 19. addend; 20. addend

A

$3.7 = 1.5 + 2.2$
$3.7 - 1.5 = 2.2$
$3.7 - 2.2 = 1.5$
$3.7 = 2.2 + 1.5$

B

$n - 0.8 = 3.3$
$n = 3.3 + 0.8$
$= 4.1$

C

$2.34 - n = 1.63$
$n = 2.34 - 1.63$
$= 0.71$

[w]

Ex. 7–14. Find the number represented by the letter.

7. $n = 2.65 + 7.98$ 10.63
8. $3.776 + 0.076 = x$ 3.852
9. $n - 34.05 = 4.88$ 38.93
10. $n = 34.82 + 346.09$ 380.91
11. $34.660 - 3.009 = x$ 31.651
12. $x = 3.8843 + 0.0078$ 3.8921
13. $x - 4.700 = 16.442$ 21.142
14. $356.72 - n = 56.99$ 299.73

Ex. 15–20. Solve. Rename the whole number with a decimal if it will help you. For example, in Ex. 15, 23 may be renamed 23.00 before subtracting.

15. $n = 23 - 16.82$ 6.18
16. $478 - n = 3.78$ 474.22
17. $0.889 = x - 33$ 33.889
18. $n = 17 - 8.7794$ 8.2206
19. $7.98 - 3 = x$ 4.98
20. $289 - n = 34.78$ 254.22

21. One year the winning speed for the "Indianapolis 500" was 114.820 m.p.h. Nineteen years later the winning speed was 150.868 m.p.h. How much faster was the later winning speed? 36.048 m.p.h.

◆ **Extra Examples.** Set 85.

* The only prime number between 20 and 25 is 23.

Without Paper and Pencil

[o]

1. The mathematical sentence $2.68 + 3.02 = 3.02 + 2.68$ is an illustration of the Commutative Property of Addition.

2. The sentence $(5.1 + 0.8) + 3.2 = 5.1 + (0.8 + 3.2)$ is an illustration of the Associative Property of Addition.

Ex. 3–8. Give the number represented by n.

3. $n + 0.7 = 0.7 + 3.67$ 3.67
4. $36 + 3.88 = n + 36$ 3.88
5. $8 + (3.4 + 5.6) = (8 + n) + 5.6$ 3.4
6. $3.804 + 0 = n$ 3.804
7. $28.9 + (5 - 5) = n$ 28.9
8. $34.8256 = 34 + n$ 0.8256

Ex. 9–14. Tell whether the sentence is *true* or *false*.

9. $2 + 1.4 < 3.5$ T
10. $8 - 2.2 > 6$ F
11. $1.3 + 5.8 > 7$ T
12. $0.8 + 0.5 < 0.7 + 0.5$ F
13. $0.6 + 0.4 = 0.10$ F
14. $4.38 + 5.45 = (4 + 5) + (0.38 + 0.45)$ T

Can You Follow Directions?

[w]

1. Multiply 367 by 87. 31,929
2. Add \$758.93 and \$84.77. \$843.70
3. Subtract 4.672 from 15.881. 11.209
4. Add 34,707 and 4,796. 39,503
5. Multiply \$43.77 by 15. \$656.55
6. Multiply 702 by 735. 515,970
7. Subtract 4.8354 from 37. 32.1646
8. Add 0.076 and 0.883. 0.959
9. Divide 6,417 by 31. 207
10. Subtract \$4.89 from \$10. \$5.11
11. Add $4\frac{1}{2}$ to the sum of $3\frac{3}{4}$ and $7\frac{1}{3}$. $15\frac{7}{12}$
12. Subtract $13\frac{1}{5}$ from the sum of $34\frac{2}{5}$ and $15\frac{1}{3}$. $36\frac{8}{15}$
13. Subtract the sum of 5 and 0.67 from the sum of 3.8 and 7.6. 5.73
14. Multiply 637 by the sum of 37 and 114. 96,187
* 15. Multiply 200 by the prime number that is greater than 20 and less than 25. 4,600
16. Find the sum of 345, 607, 17, 589, and 43. 1,601

Teaching Page 168

Pupil's Objectives

(a) To apply the Commutative Property of Addition and the Associative Property of Addition to solving mathematical sentences with decimals; and (b) to review following directions in exercises involving addition, subtraction, multiplication, or division.

Background

The operation addition is both commutative and associative. The sum of any number of addends is the same regardless of their order, or the way in which they are grouped. By means of these exercises, the pupils review these properties of addition and learn to solve mathematical sentences using these properties.

The purpose of the exercises on the lower half of the page is to practice following directions in addition, subtraction, multiplication, and division. Pupils will usually encounter less difficulty in working division or subtraction examples. One of the reasons for this is that while addition and multiplication are both associative and commutative, division and subtraction are not.

When performing subtraction or division with non-negative numbers only, the pupils can usually remember to subtract the lesser number from the greater and to divide the greater number by the lesser. However, in more advanced work when these operations are performed on positive and negative numbers, this procedure will no longer apply. The pupil should now begin to recognize correctly known and unknown addends and sums, and known and unknown factors and products regardless of the name used to identify them.

Pre-Book Lesson

• Review properties of addition in the set of whole numbers. As the properties are reviewed, have pupils write mathematical sentences on the board to illustrate them. Have pupils explain why subtraction and division are non-commutative and non-associative.

• You may wish to use these exercises as a quiz. If so, have the review prior to opening the text. *More capable pupils* may work Extra Activities or Supplementary Activities upon completing their work.

Using the Text Page

• Ex. 3–4. It is intended that these mathematical sentences be solved applying the Commutative Property of Addition, and not by applying the concept of the addends-sum relationship.

• Ex. 14. Point out that this is an application of the Commutative Property of Addition.

Individualizing Instruction

• Ask the *more capable pupils* if the sum of any two rational numbers shown with decimals is always a rational number. (It is.) Have them explain.

• Have *all pupils* work examples such as the following:

(1) subtract $7\frac{1}{3}$ from 9.7

(2) from 10.3 subtract $4\frac{1}{4}$

(3) add $6\frac{3}{4}$, 17, and 5.371

• If the exercises have been used as a quiz, group together *all pupils* showing the same weaknesses and provide reteaching exercises which may be selected at random from previous pages.

Teaching Page 169

Pupil's Objectives

(a) To solve problems that require adding or subtracting with decimals for their solution; and (b) to realize that such problems are encountered in daily living experiences.

Background

These exercises not only give the pupil some experience at solving verbal problems that require adding or subtracting with decimals, but they show the practical use for decimals and the practical use for the metric system of measurement.

You may wish to refer to the Background sections for previous pages involving operations with decimals and expressing measures in decimal form.

Pre-Book Lesson

• As motivation for this lesson, ask pupils if any of them are considering becoming nurses or doctors. Ask if any pupil has ever been a patient in a hospital to find if they have had an encounter with decimals in this situation. Ecplain how the metric system and decimals are used in these situations.

• Review briefly adding and subtracting with decimals and expressing measurements with metric units.

Using the Text Page

• Ex. 1–5. Where practical, have the pupils express the relationship among the data in the problem with a mathematical sentence before performing any calculations. Remind pupils to check which procedure to use before solving the problems. Many do not examine the data sufficiently to determine whether to add or subtract.

• Ex. 2. The term *muslin* should be clarified so that everyone understands the meaning.

Individualizing Instruction

• Have *all pupils* suggest other situations in which adding or subtracting with decimals might be required. Remember, our money system is a decimal system.

These pupils may report to the class examples of where they or their parents use decimals in their daily work.

• The *slower learners* may have to be reminded to solve these problems carefully step by step:

(a) Determine whether a sum is missing or an addend is missing.

(b) If the sum is missing, addition is indicated; if an addend is missing, subtraction is indicated.

(c) Decide which data is pertinent and which is unnecessary for the solution of the problem.

(d) Express the addends-sum relationship in the form of a mathematical sentence.

(e) Perform the necessary operation.

• *More capable pupils* may refer to a science book or scientific journal and, using information found in these sources, or other hypothetical information, make up problems which involve operations with decimals and which express measurements in the metric system. They may try out the problems on each other.

Using Decimals in a Hospital

A. and S. with decimals [W]

JoAnn is a student nurse. Here are some of her problems. See if you can solve them.

1. When a patient's temperature is 101.8°, how many degrees above the normal body temperature of 98.6° is it? 3.2

2. To make a narrow bandage, JoAnn cut a strip of muslin 1.5 inches wide from a piece of muslin 6.2 inches wide. How wide was the piece that was left? 4.7 inches

3. For dinner one of the patients ate 13.5 oz. of meat soup, 5.8 oz. of beefsteak, 1.2 oz. of mashed potato, and a 1.0-oz. roll. How many ounces of food did that patient eat? 21.5

4. One of JoAnn's textbooks contained the following information about vitamin C and fruit juices.

Fruit Juice Needed for One Day's Supply of Vitamin C

Juice	Orange	Grapefruit	Pineapple	Tomato
Cups	0.375	0.375	1.750	0.875

a. How many cups less of orange juice than pineapple juice are needed for one day's supply of vitamin C? 1.375

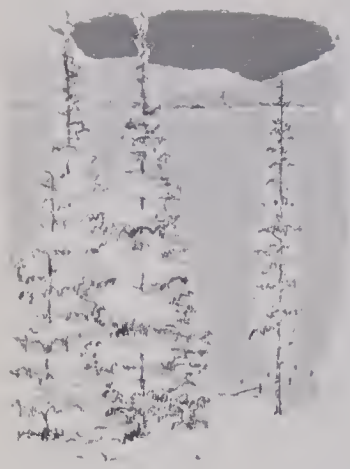
b. How many cups more of tomato juice than grapefruit juice is needed for one day's supply of vitamin C? 0.5

* **5.** Medicine is sometimes weighed using a metric unit of weight called a *gram* (gm.). A 0.016-gm. pill weighs how much less than a 0.324-gm. pill? 0.308 grams

* 28 grams is about equal in weight to 1 ounce.

Estimating The Answer

[O]



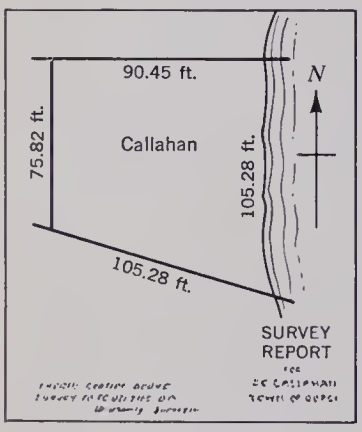
1. Mr. Callahan's lot at Spruce Lake has 105.28 ft. of water frontage. The lot next to his has 81.75 ft. of water frontage and is for sale. If he buys that lot, he will own how many feet of water frontage?

a. To estimate the answer, we may round each number to the nearest whole number. Rounded to the nearest whole number, 105.28 is 105 and 81.75 is 82. 105 + 82 = 187, so 187 ft. is an estimate of the answer.

b. At the board, find the correct answer for the problem. 187.03 ft.

For each of Ex. 2–8, give an estimate of the correct answer for the problem by rounding each number to the nearest whole number. Do not work the problems yet.

2. Mr. Masten owns 93.87 ft. of water frontage. That is how many feet less of water frontage than is owned by Mr. Callahan (Ex. 1)? 11; 11.41 ft.



3. The dimensions of Mr. Callahan's lot are shown at the left. What is the perimeter of the lot? 376 ft.; 376.83 ft.

4. How many feet of fencing material will Mr. Callahan need to build a fence along the two shorter sides of his lot? 166; 166.27

5. The sum of two numbers is 12.369. If one of the addends is 3.897, what is the other addend? 8; 8.472

6. Find the number represented by n in the sentence

$$32.7005 = 15.6238 + n.$$ 17; 17.0767

7. Find the perimeter of a triangle whose sides are 15.4 yd., 23.8 yd., and 27.1 yd. 66 yd.; 66.3 yd.

8. Find the perimeter of a rectangle that is 12.26 ft. wide and 23.79 ft. long. 72 ft.; 72.1 ft.

[W]

Now find the correct answers for Ex. 2–8.

Teaching Page 170

Pupil's Objective

To learn to estimate the results when adding or subtracting with decimals by rounding the number shown to the nearest whole number before performing the operation.

Background

One of the steps suggested in the procedure for solving verbal problems is to estimate the answer. The estimated answer can then be compared with the calculated answer to determine the reasonableness of the calculated answer. Estimating answers is particularly important in multiplying or dividing with decimals. Estimating the answer can be very helpful in determining if a decimal point has been placed correctly in the final answer. There is less likelihood of the decimal point being misplaced in addition and subtraction than in multiplication or division. However, estimating by rounding is still excellent preparation for estimating products and quotients in later work.

Pre-Book Lesson

Review rounding whole numbers and numbers shown with mixed forms, and decimals. List several numerals for numbers to be rounded and several problems on the board, and have pupils estimate the answers.

Using the Text Page

• Ex. 1–4. Make sure pupils understand the meaning of the terms *lot*, *frontage*, and *water frontage*. Discuss briefly ownership of summer homes or land to bring to mind any familiarity with the topic pupils might have.

• Ex. 2–8. In estimating answers, pupils should perform the addition or the subtraction of the rounded numbers mentally if possible. However, some pupils may have to use pencil and paper in estimating a sum such as in Ex. 7. The best time to perform the estimation is usually after the relationship in the problem has been expressed with a mathematical sentence.

• Ex. 3 and 7. Ask pupils for the name of the geometric figures represented.

Individualizing Instruction

• Ask *more capable pupils* for the maximum difference between the estimated sum of two numbers and their calculated sum if:

(a) both addends were rounded to the next greater whole number. (1)

(b) one addend was rounded to the next greater whole number and the other addend was rounded to the next lesser whole number. (0.5)

(c) both addends were rounded to the next lesser whole number. (1)

These pupils may also express the measurements in Ex. 2–8 in suitable metric units and compare the answers from computations with these measures to those obtained from measures in English units.

• Give *all pupils* exercises in estimating sums when three addends are involved.

Extend this concept of estimating answers to subtraction for these pupils.

• *Slower learners* may need a brief review of rounding numbers expressed with decimals prior to setting up mathematical sentences to solve the problems.

Teaching Page 171

Pupil's Objectives

(a) To learn the use of the thimble scale on a micrometer; and (b) to gain experience adding with decimals.

Background

Each labeled unit on the barrel scale represents 0.100 in. Each unlabeled unit represents 0.025 in. One complete turn on the thimble moves it 25 units and increases the measurement by 0.025 in. This means each of the 25 units represents 0.001 in. The sum of the numbers represented by the barrel scale and the thimble scale readings is equal to the measure of the dimension of the object being measured. If the barrel reading is 0.375 in. and the thimble reading is 0.017 in., the measure of the dimension of the object being measured is 0.375 + 0.017 or 0.392 in inches.

Pre-Book Lesson

• Again bring to class a machinist's micrometer or the cardboard micrometer which you may have made. Make sure pupils are able to identify and understand the function of each part, its movement, and corresponding scales.

• Briefly review the work on pages 148 and 149 in reading a micrometer when the thimble reading is 0. As a preparation for the introduction to reading the thimble scale, ask:

(a) Each labeled unit on the barrel scale represents what part of an inch? (0.100)

(b) Each unlabeled unit on the barrel scale represents what part of an inch? (0.025)

(c) How far does the thimble move along the barrel as it is rotated one complete turn? (0.025 in.)

(d) How many units are there on the thimble scale? (0.025)

(e) If one complete turn of the thimble moves the thimble 0.025 in. along the barrel scale and if there are 25 units on the thimble scale, what must each unit on the timble scale represent? (0.001 in.)

Using the Text Page

• Ex. 1 and 2. The thimble is scaled in thousandths of an inch with a numeral next to every fifth mark. Pupils are to read the scale "up" from the last numeral, letting each mark represent 0.001 in.

• Ex. 3. If there are 25 units on the thimble scale, each 0.001 in., then one complete rotation passes through all 25 units or 0.025 in.

• Ex. 4. Explain that the measure of the distance being measured is found by adding the measures shown by the barrel reading and the thimble reading.

• Ex. 5. You may wish to record several barrel and thimble readings on the board and, using the micrometer, have pupils find the total for the dimension being measured.

• Ex. 6–9. Before the pupils begin the written exercises, be sure they understand that the only time the thimble scale is right on a mark on the barrel scale is when the thimble scale reads 0. In this instance, only a reading of the barrel scale is necessary. When the thimble scale reading is greater than 0, the end of the thimble will be between two marks on the barrel scale, and the barrel reading will be the last mark clearly visible. In this instance a reading of both barrel and thimble scales is necessary.

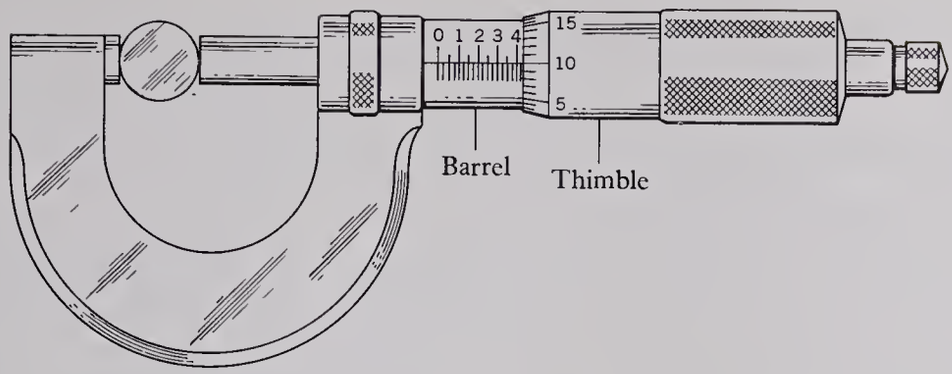
Individualizing Instruction

• Ask the *more capable pupils* what the barrel reading and the scale reading would be when measuring bolts with diameters such as 0.876 in.; 0.456 in.; and 0.095 in.

• Impress upon the *slower learners* that micrometers are used in a large number of occupations not requiring a college education, and that knowledge of how to use a micrometer and how to work with decimals is important to many people who do not go into higher education.

Barrel Reading: 0.425 in.

Thimble Reading: 0.010 in.



Add
0.425
0.010
0.435
Diameter of bolt is 0.435 in.

More Work with a Micrometer

A. with decimals [O]

1. On pages 148–149, you learned how to read a micrometer when the thimble shows "0." Give the reading for Fig. 1. 0.650 in.

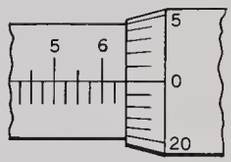


Fig. 1

2. Each unit on the thimble scale represents 0.001 in. Fig. 2 shows 3 units on the thimble scale (read *up*) so the thimble reading is 3×0.001, or 0.003 in. Give the thimble reading for Fig. 3. 0.0017

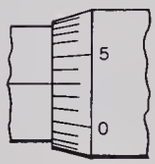


Fig. 2

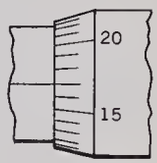


Fig. 3

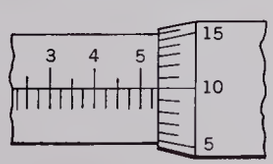
3. One complete turn of the thimble moves the end of the thimble 0.025 in. Explain why there are 25 units shown on the thimble scale. Since one complete turn of the thimble is 0.025, we partition the thimble into 25 units so that each unit is 0.001.

4. The micrometer shown above is being used to measure the diameter of a steel bolt. The barrel reading is 0.425 in. and the thimble reading is 0.010 in. We add to find the diameter of the bolt. Explain the work in the box. The sum of the barrel and thimble readings is found.

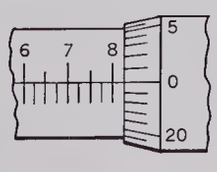
5. At the board, find the measurement when the barrel reading is 0.375 in. and the thimble reading is 0.014 in. 0.389 in.

[W]

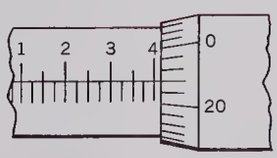
Give the measurement for each of Ex. 6–9.

6. 

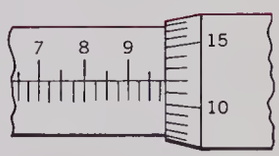
0.535 in.

7. 

0.825 in.

8. 

0.422 in.

9. 

0.987 in.

How Well Do You Remember?

[W]

Ex. 1–3. Write a decimal for the number.

1. 7 ones + 3 tenths + 8 hundredths + 4 thousandths 7.384

2. Forty-two and one hundred fifty-six thousandths 42.156

3. Eight thousand six hundred twenty ten-thousandths 0.8620

4. Using 2.5 and 8.3 for the addends and n for the sum, write four sentences to illustrate the addends-sum relationship. $n = 2.5 + 8.3$; $n - 2.5 = 8.3$; $n - 8.3 = 2.5$; $n = 8.3 + 2.5$

Ex. 5–9. Round to the nearest tenth; to the nearest hundredth.

5. 4.708 4.7; 4.71 6. 0.993 1.0; 0.99 7. 23.0926 23.1; 23.09 8. 0.099 0.1; 0.10 9. 456.807 456.8; 456.81

10. Copy and complete the following table.

$0.5 = \frac{5}{10} = \frac{1}{2}$	$0.2 = \frac{?2}{?10} = \frac{?1}{?5}$	$0.125 = \frac{125}{1{,}000} = \frac{1}{8}$
	$0.4 = \frac{?4}{?10} = \frac{?2}{?5}$	$0.375 = \frac{?375}{1{,}000} = \frac{?3}{?8}$
$0.25 = \frac{?25}{100} = \frac{?1}{?4}$	$0.6 = \frac{?6}{?10} = \frac{?3}{?5}$	$0.625 = \frac{?625}{1{,}000} = \frac{?5}{?8}$
$0.75 = \frac{?75}{100} = \frac{?3}{?4}$	$0.8 = \frac{?8}{?10} = \frac{?4}{?5}$	$0.875 = \frac{?875}{1{,}000} = \frac{?7}{?8}$

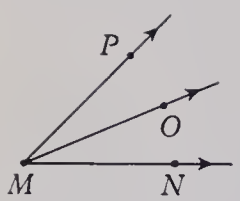


Fig. 1

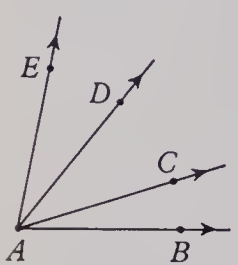


Fig. 2

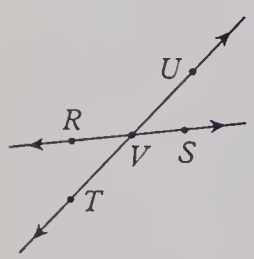


Fig. 3

Try These Exercises

Enrichment [W]

In Fig. 1, angles NMO and OMP have a common side (ray MO) and a common vertex (point M), and the interiors for the two angles have no points in common. Such angles are called *adjacent angles*.

Two angles that have a common side and a common vertex and whose interiors have no points in common are called adjacent angles.

1. Fig. 2. Name four pairs of adjacent angles. $\angle EAD$ and $\angle DAC$; $\angle EAD$ and $\angle DAB$; $\angle EAC$ and $\angle CAB$; $\angle DAC$ and $\angle CAB$

2. Fig. 2. Are angles BAD and BAC adjacent? No

3. Fig. 3. Are angles BAC and DAE adjacent? No

4. Fig. 3. Name two pairs of non-adjacent angles. $\angle RVT$ and $\angle UVS$; $\angle RVU$ and $\angle SVT$

Teaching Page 172

Pupil's Objectives

(a) To review some concepts related to decimals, including renaming from a decimal to a fraction in simplest form; and (b) to study, as an enrichment topic, the meaning of adjacent angles.

Background

The purpose of this page is to determine if there are any weaknesses in several concepts related to decimals.

The enrichment topic on this page pertains to the meaning of *adjacent angles*. Adjacent angles are any two angles that have a common ray, a common vertex, and whose interiors have no point in common.

Pre-Book Lesson

You will probably wish to proceed directly with the material on the text page without any oral work or Pre-Book Lesson. Give instructions for the use of the review exercises and the enrichment section.

Using the Text Page

• Ex. 10. The pupil is to first rename the number with a fraction and then rename with a fraction in simplest form.

• Ex. 1. The questions in the enrichment exercises may be assigned without any help or explanation of any kind on the part of the teacher. Later the teacher can explain the concept of adjacent angles for those who are unable to master the concept without help.

Individualizing Instruction

• *More capable pupils* may be given decimals such as: 0.0625 0.1875 0.3125 0.4375 0.5625 0.6875 0.8125 and be asked to rename each with a fraction in simplest form.

They may also work Extra Activities or Supplementary Activities when they have completed their work.

• *All pupils* making errors in answering the questions at the top of the page may restudy appropriate pages in the chapter and do Extra Practice Sets from the back of the book.

• Have *all pupils* complete the table below similar to the one on pupil's page 149 as a review and a practical application of work with decimals. If a micrometer is available, you may wish to have pupils use it during the completion of this activity. The first one is done for you and answers are given in parentheses.

Micrometer Reading	*Barrel Scale*		*Thimble Scale*
	Labeled Units	*Unlabeled Units*	
0.475 in.	4	3	0
0.630 in.	6	1	(5) _ ? _
(0.365) _ ? _	3	2	15
0.233	(2) _ ? _	1	8
0.493	(4) _ ? _	3	(18) _ ? _
0.575	(5) _ ? _	(3) _ ? _	0

Pupil's Objectives

(a) To review the concept of a region; and (b) to recognize the need for a way to measure the size of a region.

Background

The purpose of the exercises on this page is to review the concept of regions as preparation for an extension to the concept of area on the next page.

The following definitions can be used as references for teaching the concepts on this page:

(a) A *curve* is a set of points in a plane which may be represented by a drawing made without lifting the pencil from the paper.

(b) A *closed curve* is a set of points in a plane which may be represented by a drawing that starts and stops at the same point. It has no end points.

(c) A *simple closed curve* is a set of points in a plane which may be represented by a drawing that starts and stops at the same point, no dot being touched twice except the one representing the starting point. Each of the simple closed curves separates a plane into three sets of points: (1) the set of points belonging to the curve, (2) the set of points in the interior for the curve, and (3) the set of points in the exterior for the curve.

(d) A *region* is the union of a simple closed curve and its interior. It is usually named for the curve which forms its boundary. The size of a region is called its *area*.

Pre-Book Lesson

• The teacher may wish to use exercises selected at random from previous pages dealing with curves, closed curves, simple closed curves, and so on. You may also discuss the material from the Background.

• Draw on the chalkboard a model of a wheat field in the shape of a square region, the length of each side being 6 inches. Draw a model of a rectangular region, 1 inch by 11 inches, to represent another field of wheat. Ask the pupils in what way the dimensions of the two wheat fields are similar (their perimeters are the same). Ask which field is probably larger. The point to make is that although the boundaries enclosing the two fields have the same perimeters, the field that is in the shape of a square is larger than the other. The perimeter of a rectangle enclosing a region is not a measure of the size of the region. We need some other way to measure the size of a region.

For a given perimeter, a region shaped like a square will have the maximum area.

Using the Text Page

• Ex. 1–4. These examples should refresh the pupils' memories as to the meaning of region.

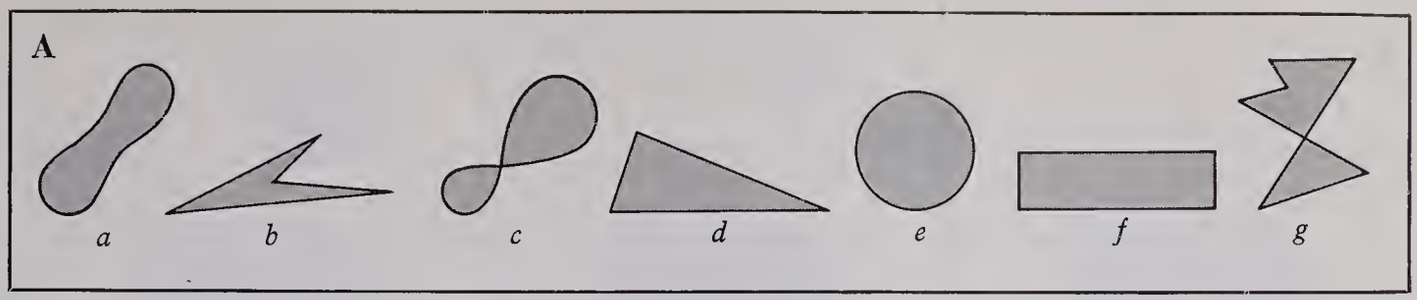
• Ex. 3. To eliminate vocabulary difficulties, make sure the pupils understand that "quadrangular region" should be associated with a quadrilateral and its interior.

• Ex. 5 and 6. Ex. 5 involves the relative size of two regions, and Ex. 6 should cause the pupils to recognize the need for a way to measure the size of a region. The sizes of the regions represented in box B are so nearly the same that it is impossible to tell by observation alone the differences among those sizes.

Individualizing Instruction

• Ask the *more capable pupils* for their explanation of the *size* of a region.

• Have *all pupils* use paper to make a model of a square region, the length of each side being 5 inches, and a model of a rectangular region 2 inches by 12 inches. Have them determine which is the larger region by cutting the model of the rectangular region into smaller squares and trying to fit the pieces on the model for the square region.



Remembering about Regions

[O]

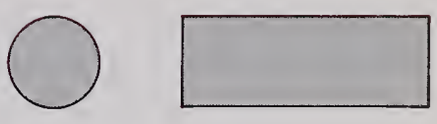
1. Box A. Is each curve shown a closed curve? Yes Give a definition of a closed curve. Key idea: A curve which can be shown with a drawing that starts and ends at the same place.

2. Which curves shown in box A are simple closed curves? a, b, d, e, f Which are not? c, g Give a definition of a simple closed curve. Key idea: A closed curve which does not intersect itself.

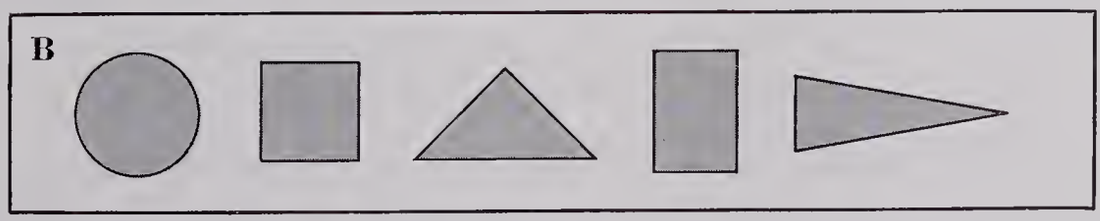
3. The union of a simple closed curve and its interior is called a region. Define a polygonal region; a circular region; a quadrangular region; a triangular region; a rectangular region. The union of a polygon and its interior; a circle and its interior; a quadrilateral and its interior; a triangle and its interior; a rectangle and its interior.

4. Name some things in your classroom that are models of polygonal regions; circular regions; rectangular regions. Answers will vary.

5. For the regions pictured, does the rectangular region appear to be larger than the circular region? Yes



6. Does one of the regions shown in box B appear to be larger than the other regions or do they all appear to be the same size? They all appear to be the same size. Do you see that we need some way to measure the size of a region? Yes



[W]

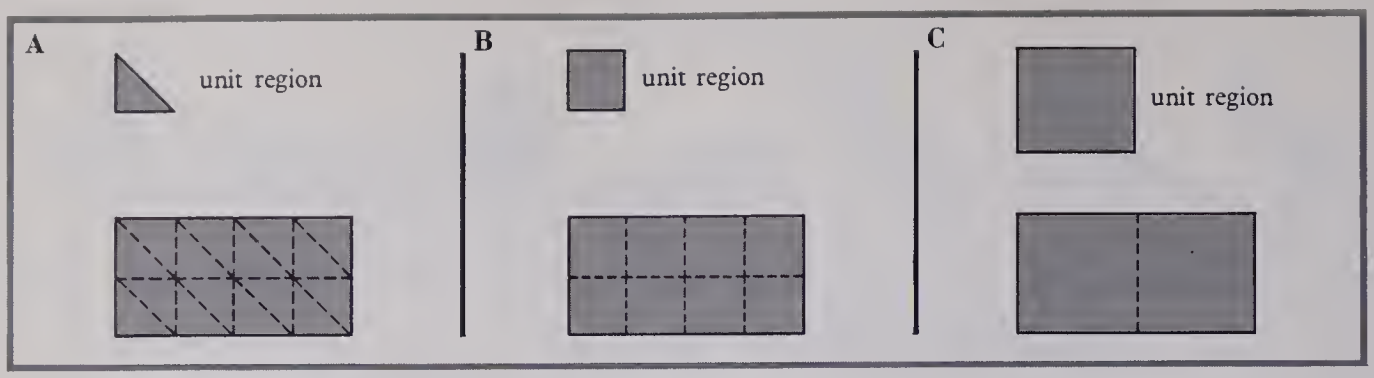
7. Draw five pictures of curves that are not closed. Sample answers:

8. Draw five pictures of simple closed curves.

9. Draw and lightly shade a model of

a. a square region. b. a circular region.

c. a region that is not a polygonal region.



The Area of a Region

Resurvey [O]

1. The *area* of a region is the size of the region. To measure the area of a region we use, for a unit of measurement, a unit region called a unit of area. In which of boxes A–C above is the unit of area a triangular region? A a square region? B and C

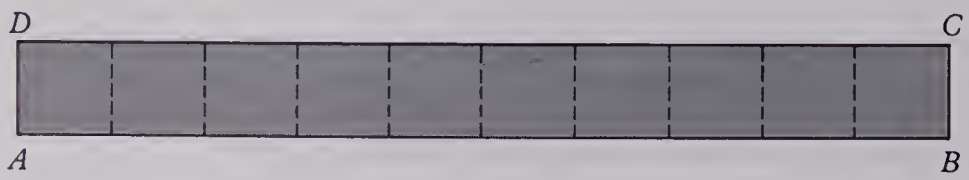
2. Box A. The large rectangular region is shown partitioned into how many unit regions? 16 Then the area of the rectangular region is _?_ 16 unit regions.

3. In box B, what is the area of the rectangular region in terms of the unit region shown? 8 unit regions In box C, the same rectangular region is shown. What is its area in terms of the unit region shown in that box? 2 unit regions

4. Standard unit regions are usually square regions. The square region shown at the left is a standard unit called a *square centimeter* (sq. cm.). Its area is 1 sq. cm. Square foot, square yard, and square mile are some other standard units of area. Describe a region whose area is 1 sq. ft.; 1 sq. yd.; 1 sq. mi.
The union of a square with sides measuring 1 (foot, yard, mile) and its interior.

5. Name some models of regions in your classroom that seem to have areas greater than 1 sq. ft.; less than 1 sq. ft. Answers will vary.

6. Region *ABCD*, shown below, appears to be partitioned into how many square centimeters? 10 Then its area is _?_ 10 sq. cm.



Teaching Pages 174 and 175

Pupil's Objectives

(a) To review area of a region as the size of a region; and (b) to find areas by partitioning regions into unit regions.

Background

The pupils have learned to determine the measure of a segment by partitioning it into standard unit segments. The measure of the segment is the number of such standard unit segments into which the given segment can be partitioned.

The area of a region is determined in a similar way. To determine the area of a region, it is partitioned into smaller regions each being a standard unit of area with equal measure. The measure of the area is the number of such standard unit regions into which the given region can be partitioned.

In defining standard unit area, it was found that unit regions in the shape of a square are the most convenient. For each of the standard units of length (inch, foot, mile, meter, and so forth) there is a standard unit of area defined as a square region the length of each side being 1 standard unit of length. Some examples of standard units of area are the square inch (sq. in.), square foot (sq. ft.), square yard (sq. yd.), square mile (sq. mi.), square centimeter (sq. cm.), and square meter (sq. m.).

A square inch in not a *square*; it is a *square region*. While its area (measurement) is defined as being 1 square inch, the measure of its area is 1 in square inches.

A *rectangular region* has the property of area. It is important to realize that the question "What is the area of rectangle *ABCD*?" may be meaningless. A rectangle per se cannot be partitioned into unit regions. A rectangle is only partitioned into segments because its only dimension is length. We call this length the perimeter of the rectangle.

The measure of the area of any region is the number of standard unit regions into which the region can be partitioned. This concept is a simple intuitive approach. The question may arise as to the difficulty of finding the area of a circular region when the standard unit regions are square regions. The area of a circle will be dealt with later.

Pre-Book Lesson

- Ask for a description of how we can determine the measure of a segment and of an angle. Ask for suggestions as to how a system of units of measurement for area might be developed. Discuss material from the Background at this point.
- Review the ideas of regions interior and exterior to a geometric figure.

Using the Text Pages

- Ex. 4. Ask for possible reasons why square regions were selected as standard unit regions instead of regions with other shapes such as triangular or rectangular regions. Ask for other possible standard units of area including units in the metric system.
- Ex. 6. Point out that a rectangle does not have the property of area, but a region has the property of area. Make sure pupils understand the definitions and meanings of the geometric figures involved.
- Ex. 7. Question the pupils to be sure they remember what is meant by congruent regions. Models of congruent regions may prove useful with those pupils experiencing difficulty here. Make sure the pupils understand the meaning of the word *dimension*.

Individualizing Instruction

- Have *all pupils* make a model of a rectangular region 6 inches by 2 inches. Have them define arbitary unit regions such as a right triangle whose legs are each 2 inches in length; a square, each of whose sides is 2 inches in length; and a rectangle 1 inch by $\frac{1}{2}$ inch. Name them A-unit, B-unit, and C-unit respectively. Have the pupils determine the area of the given region in terms of each of these units.

All pupils can be given a rectangular region 1 centimeter by 4 inches, and asked to discuss the difficulty of finding its area because of the different units of measurement used in expressing the lengths of its sides.

• The *slower learners* may make models of standard units cut from stiff paper or cardboard and use them in partitioning regions to find the area of the regions.

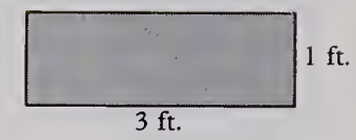
List on the chalkboard, as the pupils suggest them, various standard units of area. For each standard unit listed, ask the pupils for the names of things that would most likely be measured in this unit of measurement.

• An unusual standard unit of measurement of area is the *acre* which is defined as being an area equivalent to 160 square rods. Have the *more capable pupils* look up in a reference book the meaning of *acre* and give a report to the class. Have them compare the relative size of an acre to other standard units such as the square mile. Have pupils describe the area of a familiar region in terms of acres.

NOTES

The measure of the area of a region is equal to the number of unit regions into which the given region is partitioned.

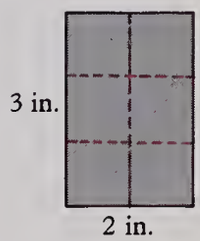
7. Give the dimensions of the rectangular region shown at the right. 3 ft., 1 ft. Can this region be partitioned into 3 congruent square regions each having an area of 1 sq. ft.? Yes Then what is the area of the region shown? 3 sq. ft.



8. Give the area of a rectangular region if its dimensions are

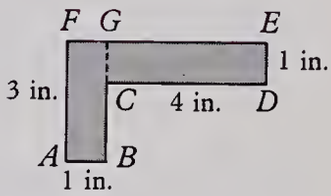
a. 5 in. by 1 in. 5 sq. in. **b.** 9 yd. by 1 yd. 9 sq. yd.

9. At the right, a rectangular region 3 in. by 2 in. is shown partitioned into two smaller congruent rectangular regions each with dimensions _?_ 3 in. by 1 in. What is the area of each of the smaller regions? 3 sq. in. Then the area of the large region is _?_ 6 sq. in.



10. On the board, show that the area of the region for Ex. 9 is 6 sq. in. by showing it partitioned into 6 congruent square regions each having an area of 1 sq. in. See diagram.

11. At the right, region *ABCDEF* is shown partitioned into two rectangular regions; region *ABGF* and region CDEG _?_. Give the areas of these two regions. 3 sq. in.; 4 sq. in. Then the area of region *ABCDEF* is _?_ 7 sq. in. Why? 3 + 4 = 7



[W]

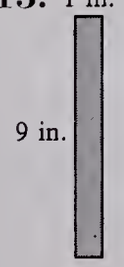
12. Write whether we would use a *unit of length* or a *unit of area* to indicate

a. the height of a boy. Unit of length **b.** the perimeter of a garden. Unit of length

c. the amount of material needed to make a tent. Unit of area

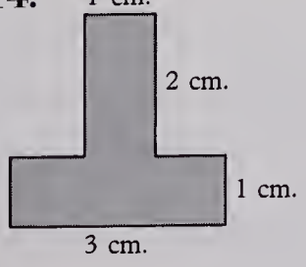
Ex. 13–16. Find the area of the region.

13.



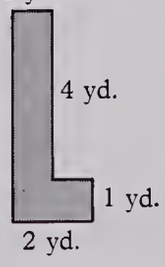
9 sq. in.

14.



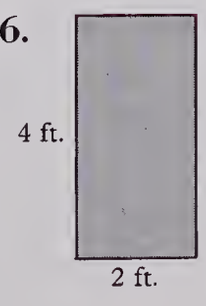
5 sq. cm.

15.



6 sq. yd.

16.

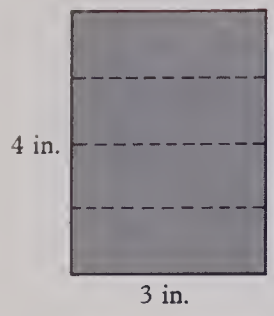


8 sq. ft.

* Review that "congruent" means "same size and shape."

The Area of a Rectangular Region

$A = l \times w$ [O]



* 1. A rectangular region 4 in. by 3 in. can be partitioned into 4 congruent rectangular regions each having an area of _?_ 3 sq. in. Then the measure of the region is $3 + 3 + 3 + 3$, or _?_ 4 $\times$ 3, or _?_ 12 in square inches. Can you give a short way to find the area of such a region? Find the product of the measures of its length and width.

We can find the measure of the area of a rectangular region by finding the product of the measures of its length and width. If A represents the measure of the area, and l and w represent the measures of the length and width, then

$$A = l \times w.$$

$A = l \times w$
$= 15 \times 8$
$= 120$

2. Explain the work in the box for finding the area of a rectangular region 15 in. by 8 in. The measures of length and width are multiplied. The area is _?_ 120 sq. in.

3. At the board, find the area of a rectangular region if its dimensions are a. 27 in. by 18 in. 486 sq. in. b. 37 mi. by 8 mi. 296 sq. mi.

4. To find the area of a rectangular region 2 ft. by 3 in., we may first rename 2 ft. as _?_ 24 in. Then the measure of the area in square inches is _?_ 24 $\times$ 3, or _?_ 72.

5. Does a rectangular region 4 in. by 3 in. have the same area as a rectangular region 6 in. by 2 in.? Yes Explain. Each area is 12 sq. in.

[W]

Ex. 6–14. The length and width of a rectangular region are given. Find the area of the region.

	Length	Width		Length	Width		Length	Width
6.	7 cm.	2 cm. 14 sq. cm.	9.	18 in.	11 in. 198 sq. in.	12.	135 ft.	23 ft. 3,105 sq. ft.
7.	9 in.	7 in. 63 sq. in.	10.	5 mm.	3 mm. 15 sq. mm.	13.	320 mi.	147 mi. 47,040 sq. mi.
8.	17 ft.	14 ft. 238 sq. ft.	11.	52 mi.	23 mi. 1,196 sq. mi.	14.	275 yd.	245 yd. 67,375 sq. yd.

Ex. 15–16. The length and width of a rectangular region are given. Rename as in Ex. 4 and find the area of the region.

15. Length: 2 ft.; Width: 17 in. 408 sq. in. 16. Length: 16 ft.; Width: 4 yd. 192 sq. ft.

♦ **Extra Examples.** Set 86. ● **Extra Activity.** Set 168.

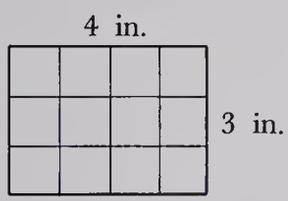
Teaching Page 176

Pupil's Objective

To learn that the measure of the area of a rectangular region is equal to the product of the measure of the length and the measure of the width.

Background

To find the area of a rectangular region, the region is partitioned into squares, each of which is a standard unit region such as the square inch. To find the area of a rectangular region whose length is 4 inches and whose width is 3 inches, the original region is partitioned into square regions with sides 1 inch in length.



The squares can then be counted in order to discover that the area is 12 square inches.

When both dimensions are expressed in the same unit of measurement, the measure of the area for a rectangular region is equal to the product of the measures of the length and width.

Pre-Book Lesson

Have the pupils partition a 2 inch by 5 inch rectangular region into square inches. Ask them for four different ways of determining the number of squares into which the region has been partitioned. The four ways are: (1) counting the squares, (2) determining the sum $2 + 2 + 2 + 2 + 2$, (3) determining the sum $5 + 5$, (4) determining the product 2×5.

Using the Text Page

• Ex. 1–2. Use material from the Background in a discussion of partitioning a region into other rectangular regions or square regions. The number of rectangular or square regions can be determined by counting. Compare the number of square units with the number of units in the length and width. Emphasize that multiplication is an operation on numbers, and that only the measures of the length and width are multiplied. In this formula $A = l \times w$, A, l, and w do not represent area, length and width. They represent the measure of the area, the measure of the length, and the measure of the width respectively.

• Ex. 4. Ask why the measures must be expressed in the same units.

• Ex. 6–14. Make sure pupils express their measurements in square units.

• Ex. 15–16. The measurement in the larger unit should be renamed in the smaller unit. If the measurements are named in the larger unit, determining the area will require multiplication with fractions and mixed forms which has not yet been resurveyed.

Individualizing Instruction

• *More capable pupils* may determine by experimentation how the area of a rectangular region changes when: the width is doubled but the length is left unchanged; the length is doubled but the width is left unchanged; both the length and the width are doubled.

• *All pupils* may select some arbitrary perimeter for a rectangular region such as 16 inches, and determine by experimentation the dimensions of a rectangular region of this perimeter which has the greatest area. They should discover that a square region will have the greatest area.

Lead these pupils to devise a special formula for finding the area of a square region ($A = s^2$). Bring attention to the properties of a square, stressing that all four sides have the same measurement.

• If *slower learners* are having difficulty renaming one of the measurements, illustrate on the chalkboard the rectangular region in Ex. 4. Partition the length (2 ft.) into 24 inch segments. Partition the region into units 1 inch square. Discuss the necessity of having the length and width in terms of the same unit of measurement before finding the measure of the area. Emphasize the reasons why area is expressed in square units.

Pupil's Objective

To review renaming the measurement of an area with an equivalent measurement expressed in a larger or a smaller unit.

Background

Each of the standard units of measurement of area is a region. Each can be partitioned into a number of smaller standard unit regions. For example, 1 square yard, 9 square feet, and 1,296 square inches are all measurements for the same area. In the metric system, 1 square meter, 100 square decimeters, 10,000 square centimeters, and 1,000,000 square millimeters are measurements for the same area. Notice the ease of renaming measurements in the metric system.

The relationship of the measure of an area expressed in small units to the measure of the same area expressed in larger units, and the number of small units equivalent to one larger unit may be written:

(Measure of area expressed in small unit) = (Measure of area expressed in larger unit) × (Number of small units equivalent to 1 larger unit)

This factors-product relationship may also be expressed as:

(Measure of area expressed in large unit) = (Measure of area expressed in smaller unit) ÷ (Number of smaller units equivalent to 1 larger unit)

Pre-Book Lesson

Have the pupils find equivalent measurements with a larger or smaller unit such as when expressing 36 inches as feet or 2 meters as centimeters. It will be more difficult for pupils to visualize an equivalence between 9 square feet and 1 square yard than it is for them to visualize an equivalence between 3 feet and 1 yard.

Using the Text Page

• Ex. 3–4. After working a few examples, try to have pupils state a generalization for expressing equivalent measurements of area using larger or smaller units. Write any suggestions on the board so that pupils may more easily understand. Use material from the Background if necessary.

Point out that when we rename a measurement of area with smaller units, we multiply by the number of smaller units equivalent to one larger unit. When we rename with larger units, we divide by the number of smaller units equivalent to one larger unit.

• Ex. 6. Pupils must rename one of the measurements so that they are expressed in square feet or in square yards.

• Ex. 7. The area may be found in square feet and then converted to square yards. This is preferable to trying to express the length and width in yards and then multiplying the measures, as using yards involves multiplying with mixed forms.

• Ex. 14. The measurement 4 yd. and 2 yd. may be expressed as 12 ft. and 6 ft., and then the area would be found by multiplying the measures 12 and 6; or, the measures 4 and 2 in yards may be multiplied and the resulting number of square yards would be renamed as square feet. Either of these methods is preferable to expressing the given measurements 17 ft. and 5 ft. in yards and multiplying the measures since these would be expressed in mixed form.

Individualizing Instruction

• Ask *all pupils* to describe the advantages of the metric system of units of area over the use of English units of area when it comes to expressing a measurement as an equivalent measurement with a larger or smaller unit.

• Have the *slower learners* make a model of a square foot and then have them partition it into 144 square inches. Have them make a model of a square yard and partition it into 9 square feet. Display these in the classroom. Making these models before the written work may help them visualize the comparison between equivalent measurements in square inches, square feet, and square yards.

* You may need to draw models and partition them for pupils to understand why the number of square feet equal to 1 square yard is not the same as the number of feet equal to 1 yard.

Renaming Measurements of Area

Resurvey [O]

*1. A square region with each side 1 ft. has an area of _?_ (1) sq. ft. Can we rename 1 ft. as 12 in.? (Yes) Then can the area of the square region also be expressed as 144 sq. in.? (Yes) To show that these are two measurements for the same area, we write, *1 sq. ft. = 144 sq. in.*

2. On the board, copy and complete Ex. a–d.

a. 1 sq. yd. = _?_ (9) sq. ft.
b. 1 sq. yd. = _?_ (1,296) sq. in.
c. 1 sq. m. = _?_ (10,000) sq. cm.
d. 1 sq. m. = _?_ (100) sq. dm.

3. 9 sq. ft. = 1 sq. yd., so to rename 72 sq. ft. in terms of square yards, we divide 72 by _?_ (9). 72 sq. ft. = _?_ (8) sq. yd.

4. If a measurement is given in terms of square yards, tell how to rename the measurement in terms of square feet. (Multiply the number of square yards by 9.)

6 sq. yd. = _?_ (54) sq. ft. 10 sq. yd. = _?_ (90) sq. ft.

5. On the board, copy and complete Ex. a–d.

a. 153 sq. ft. = _?_ (17) sq. yd.
b. 432 sq. in. = _?_ (3) sq. ft.
c. 49 sq. ft. = _?_ ($5\frac{4}{9}$) sq. yd.
d. 15 sq. ft. = _?_ (2,160) sq. in.

6. Which region is larger, a region with an area of 3 sq. yd. or a region with an area of 29 sq. ft.? (29 sq. ft. region) How much larger? (2 sq. ft.)

7. A badminton court is rectangular. It is 44 ft. by 20 ft. What is the area of the court in terms of square feet? (880 sq. ft.)

[W]

Ex. 8–13. Copy and complete.

8. 1 sq. ft. = _?_ (144) sq. in.
9. 15 sq. ft. = _?_ (2,160) sq. in.
10. 250 sq. in. = _?_ ($1\frac{53}{72}$) sq. ft.
11. 207 sq. ft. = _?_ (23) sq. yd.
12. 21 sq. ft. = _?_ ($2\frac{1}{3}$) sq. yd.
13. 3 sq. m. = _?_ (30,000) sq. cm.

14. The area of a rectangular region 17 ft. by 5 ft. is how many square feet greater than the area of a rectangular region 4 yd. by 2 yd.? (13 sq. ft.)

♦ **Extra Examples.** Set 87.

Problem-Solving

[O]

1. Mr. Topper is remodeling a room that has a rectangular floor 21 ft. by 15 ft. He can buy tile at 19¢ per square foot or linoleum at $2.00 per square yard. To cover the floor, which would cost less and how much less?

A

$n = (21 \times 15) \times 19$
$= 315 \times 19$
$= 5{,}985$

Cost of tile:
$59.85

B

$n = (315 \div 9) \times 2$
$= 35 \times 2$
$= 70$

Cost of linoleum:
$70.00

a. Box A. To find what the cost in cents of tile would be, we multiply the product of 21 and ~~_?~~ 15 by ~~_?~~ 19. Why? To get the number of square feet of floor and the total cost

b. Explain each step in box A. The number of square feet of floor is multiplied by 19. What would be the cost of the tile in terms of dollars and cents? $59.85

c. Box B. To find what the cost of linoleum would be, we first must find how many square yards of linoleum would be needed. Explain the work shown. The number of square yards of floor (315 ÷ 9) is multiplied by 2.

d. What would be the cost of the linoleum? $70.00

e. For covering the floor, which would cost less, tile or linoleum? tile How much less? $10.15

[W]

2. How much would wall-to-wall carpeting cost for a rectangular floor 18 ft. by 15 ft. if carpeting cost $8.00 per square yard? $240.00

Teaching Pages 178 and 179

Pupil's Objective

To solve problems pertaining to area including problems requiring more than one step.

Background

Previously, the pupils have been encouraged to solve two-step problems by expressing both steps in a single mathematical sentence. Sometimes, it is preferable to use two separate mathematical sentences. This is particularly true when different units of measurement are given in a problem or when solving the problem requires more than two operations.

When determining which of two given floors has the greater area, one floor being 9 ft. by 13 ft. and the other being 6 yd. by 7 yd., it is best to use two mathematical sentences because of the two different units of measurement involved. If the pupil were encouraged to express the relationship in the problem as a single mathematical sentence, he might write it as

$$n = (9 \times 12) - (6 \times 7)$$

which is wrong because a measure in square yards is being subtracted from a measure in square feet. This is meaningless. Instead, the pupil should be encouraged to write two mathematical sentences, expressing the answer to each in proper units:

$n = 9 \times 13$
$= 117$

The measure of the area of one floor is 117 in sq. ft.

$n = 6 \times 7$
$= 42$

The measure of the area of the other floor is 42 in sq. yd.

The pupil should then realize that he cannot subtract a measure expressed in square yards from a measure expressed in square feet. To subtract, one of the measurements must first be expressed in terms of units of the other measurement.

42 (in sq. yd.) $= 9 \times 42$ or, 378 in sq. ft.

The difference between the measures of the larger area and the smaller area can then be determined by subtracting:

$$n = 378 - 117$$
$$= 261$$

The area of the larger floor is 261 sq. ft. greater than the area of the smaller floor.

Even if the measurements given in a problem are all expressed in the same unit, it is often best to use two mathematical sentences. For example, in finding how much more it would cost to tile a floor 9 ft. by 14 ft. than to tile a floor 7 ft. by 13 ft. when the cost of tile is 25¢ a square foot, the relationship may be expressed as:

$$n = [25 \times (9 \times 14)] - [25 \times (7 \times 13)]$$

For most pupils this expression is too involved. It would be better to use two mathematical sentences, $n = 25 \times (9 \times 14)$ and $n = 25 \times (7 \times 13)$, solve each of them, and subtract the results.

If a problem contains measurements expressed in two different units, it is necessary to express one of the measurements in terms of the unit of the other measurement. The pupil has a choice either to name the measurement expressed in the smaller unit in terms of the larger unit or to name the measurement expressed in the larger unit in terms of the smaller unit.

One method is just as correct as the other.

However, when a measurement expressed in smaller units is changed to larger units, the measure is likely to be shown by a fraction or mixed form. Because multiplying with fractions and mixed forms has not yet been resurveyed, it may be best to encourage pupils to express measurements in the smaller units when changing measurements is necessary.

Pre-Book Lesson

• Give the pupils exercises for which it would be better to express the relationship with two or more mathematical sentences rather than one. For example, "Find how much greater the area of a rectangular region 3 yd. by 4 yd. is than a region 5 ft. by 7 ft."

• Review the basic steps necessary for efficient problem-solving, such as: careful reading; finding what information is given; finding what information is sought; and finding the relationship between the two.

• Review the methods for changing measurements from one unit of measurement to another. Ask pupils why it is meaningless to operate with measures that have different units of measurement.

Using the Text Pages

• Ex. 1. Point out the reason for using two mathematical sentences instead of trying to express the relationship with a single mathematical sentence.

• Ex. 2. Have the pupils solve Ex. 2 by first changing the given dimensions to equivalent measurements in yards and finding the area. Then have them find the area in square feet and change this measurement to square yards. Have them compare these methods and determine the conditions under which one method is more advisable than the other.

• Ex. 4. Pupils may express the factors-product relationship $A = l \times w$ as $l = A \div w$ and as $w = A \div l$.

• Ex. 5. Lead the pupils to see that the garden may be separated into two rectangular regions, and that enough information is given so that the length and width of each rectangle can be found. The sum of the measures of the areas of the two regions will be the measure of the area of the garden.

Individualizing Instruction

• Encourage *all pupils* to bring to class examples of optical illusions for use during a discussion of the enrichment section.

• *Slower leaners* who have reading difficulties may be assisted while other pupils are doing the exercises above. They may benefit by a discussion of the information given and the question expressed in the problem before expressing mathematical sentences.

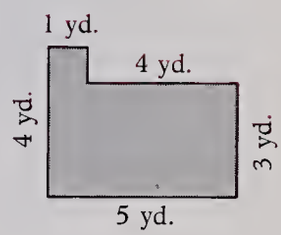
NOTES

* Point out that optical illusions of this same type are used to advantage in architecture, clothing design, art, and so on.

3. A card table is square, 30 in. on a side. What is the area of the playing surface? 900 sq. in.

4. The area of a rectangular lot is 1,476 sq. ft. The length of the lot is 82 ft. Find the width. 18 ft.

5. The shape of Mrs. Morgan's rock garden is shown at the right. **a.** What is the perimeter of the garden? 18 yd. **b.** What is the area of the garden? 16 sq. yd.



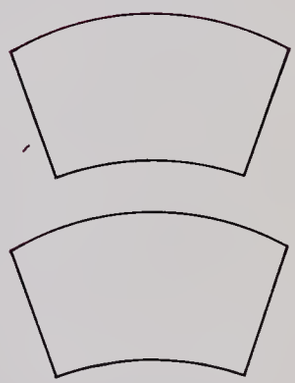
6. The area of a region is $23\frac{1}{3}$ sq. yd. That is how many more square yards than $17\frac{1}{2}$ sq. yd.? $5\frac{5}{6}$ sq. yd.

7. How many more square centimeters is 23.67 sq. cm. than 15.84 sq. cm.? 7.83 sq. cm.

8. A tennis court is rectangular. For singles, the court is 78 ft. by 27 ft. and for doubles, the court is 78 ft. by 36 ft. How much larger is the court for doubles than for singles? 702 sq. ft.

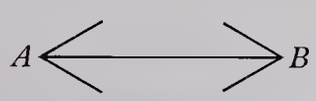
Just for Fun

Enrichment [W]

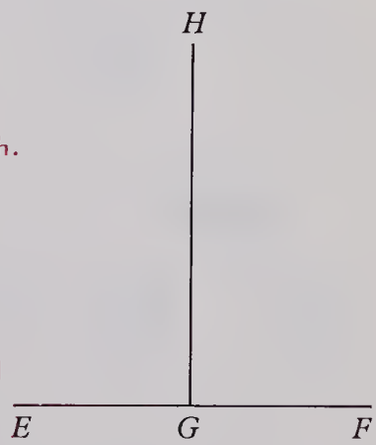


* **1.** The two regions shown at the right are the same shape and size so they have the same area. However, to many observers, the lower model appears to be larger than the other. This is an illustration of an *optical illusion.* Cut models for two such regions from cardboard and place one above the other as shown. As you slowly interchange them, does it appear that the area of one model increases while the area of the other decreases? Yes

2. Which segment, AB or CD, appears to be longer? $\overline{CD}$ Measure both segments with your ruler. They are the same length.



3. Which segment, EF or GH, appears to be longer? $\overline{GH}$ Measure both segments with your ruler. They are the same length.



Continent	Area
Africa	11,500,000 sq. mi.
Antarctica	5,300,000 sq. mi.
Asia	16,900,000 sq. mi.
Australia	2,950,000 sq. mi.
Europe	3,750,000 sq. mi.
North America	9,300,000 sq. mi.
South America	6,800,000 sq. mi.

Comparing Areas

[W]

1. The area of North America is how many square miles greater than the area of a. South America? 2,500,000 sq. mi. b. Australia? 6,350,000 sq. mi.

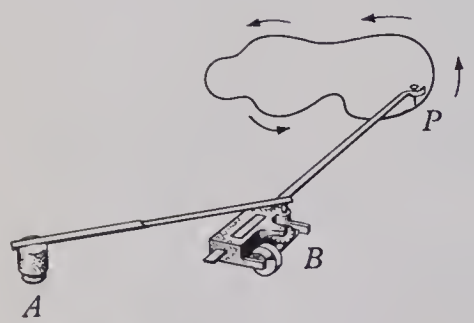
2. Which continent has the greatest area? Asia

3. How much greater is the area of the Pacific Ocean (63,801,668 sq. mi.) than the area of the Atlantic Ocean (31,839,306 sq. mi.)? 31,962,362 sq. mi.

4. Name the other two major oceans of the world. Indian, Arctic

An Instrument for Finding Area

Enrichment [W]



The instrument shown at the left is called a *planimeter*. It can be used to find the area of a model of a region when the region is enclosed by a simple closed curve. Holding A fixed, P is moved along the boundary starting at one point and finishing at that point. The disk (B) rotates and gives the area of the region.

Fig. 1

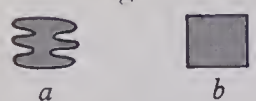


Fig. 2

1. By using a planimeter, the area of a region like the one shown in Fig. 1 was found to be 25 sq. in. That region was how much larger than a rectangular region 3 in. by 7 in.? 4 sq. in.

2. Fig. 2. Which region, *a* or *b*, seems to be the larger region? Region b

Teaching Page 180

Pupil's Objective

To compare the sizes of areas of different portions of the earth's surface.

Background

Finding the area of a portion of the earth's surface is usually much more complicated than determining the area of a region bounded by some geometric figure such as a rectangle or a square. The boundary lines of most states, countries, continents, and oceans do not form a polygon. They are usually very irregular in shape.

One method of determining the size of the area for a portion of the earth's surface is to obtain a scale drawing or map of the region and use a planimeter, described in the enrichment exercises at the bottom of the page. However, it is impossible to represent accurately with a flat map a portion of the surface of the earth. The earth is like a sphere while the map is flat. A portion of the earth's surface 6 miles long and 4 miles wide has, because of its curvature, a greater area than that of a rectangle with this same length and width.

The question arises as to the meaning of area when referring to the surface of the earth. Is it to be assumed that the earth is very smooth and flat? If not, a region 3 miles long and 4 miles wide will have a greater area if it is a mountainous region than if it is a flat plain.

Although all of these problems pertaining to measuring the surface of the earth can be quite fascinating, pupils should only deal with comparing areas. It is important, however, that the teacher be aware of these problems when discussing areas of regions of the earth's surface.

Pre-Book Lesson

Using a globe or maps, question the pupils as to relative sizes of states, countries, continents, and oceans. Assign the top of the page to the entire class. Discuss the enrichment section with *all pupils*. If the situation permits, you may have *all pupils* work the enrichment section as well.

Using the Text Page

When doing the enrichment exercises at the bottom of the page, some pupils may wonder how the planimeter actually determines the measure of a model of a region enclosed by a simple closed curve. Finding the measure of the areas of such regions requires a knowledge of mathematics far beyond this grade level. Let it be sufficient to say that as the disk is rotated, it indicates the area of the region.

You may wish to discuss the material from the Background with *more capable pupils* and assign as a research project, the investigation of methods of measuring the earth's surface. Reference to material from *The National Geographic Magazine* and other general reference works would be valuable. You may also have them work Extra Activities when they have completed work on the page.

Individualizing Instruction

Ask the *more capable pupils* to point out differences between a plane region and a portion of the earth's surface.

Teaching Pages 181, 182, and 183

Pupil's Objective

To take the regular end-of-chapter tests including (a) Test of Information and Meaning 4; (b) Diagnostic Test 4; (c) Problem Test 4; and (d) Computation Test 4.

Background

These tests provide a measurement of each pupil's mastery of the concepts presented in this chapter, and they identify pupils' weaknesses in concepts and skills so that these weaknesses may be remedied with individual instruction and practice. The teacher may wish to refer to previous Background sections prior to a general chapter review with pupils.

Pre-Book Lesson

- Instructions for using the text pages and use of paper should be given at this time. Suggest to *more capable pupils* that they may work the Supplementary Activities after they have completed the tests.
- You may wish to have a brief review prior to each test. In that case, select examples related to those on the test pages so that glaring weaknesses may be rectified prior to the test.

Using the Text Pages

- These tests may be given in any order, but no more than two should be given in any one day.
- Pupils should be encouraged to check their work wherever practical. Addition and multiplication may be checked by using the commutative property; subtraction may be checked by addition; and division may be checked by multiplication.
- Pupils should be encouraged to write neatly. The teacher may have them arrange their work in such a way as to facilitate scoring.
- Pupils should be reminded before taking Problem Test 4 to (a) estimate the answer before performing computations; (b) express the relationship in the problem with a mathematical sentence where practical; (c) show all computation on the test paper; (d) compare the estimated answer with the calculated answer to determine the reasonableness of the calculated answer; and (e) express the answer in appropriate units where necessary.

Individualizing Instruction

- After the papers have been scored, they should be immediately returned to *all pupils*. These pupils are to correct the errors, and do additional exercises and problems of the type in which they made the errors.
- *Slower learners* may benefit by working Extra Examples Sets and referring back to the original text page where certain material in which they are weak was presented.
- *More capable pupils* may do Supplementary Activities related to this material or material previously taught.

Table of Per Cents for Chapter 4 Scores

Problem Test		Computation Test			
Score	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*
1	13	1	5	11	55
2	25	2	10	12	60
3	38	3	15	13	65
4	50	4	20	14	70
5	63	5	25	15	75
6	75	6	30	16	80
7	88	7	35	17	85
8	100	8	40	18	90
		9	45	19	95
		10	50	20	100

Do You Understand?

Test of Information and Meaning 4

Ex. 1–3. Copy and complete by writing $=$ or $<$ or $>$.

1. $0.56 \; \_?\_ \; 0.6$ (answer: $\leq$)
2. $0.125 \; \_?\_ \; \frac{1}{8}$ (answer: $=$)
3. $2\frac{5}{8} \; \_?\_ \; 2.5$ (answer: $>$)

Ex. 4–9. Copy and complete.

4. $28 \text{ in.} = \_?\_ \text{ ft. } \_?\_ \text{ in.}$ (answer: 2, 4)
5. $3 \text{ T. } 1{,}450 \text{ lb.} = \_?\_ \text{ lb.}$ (answer: 7,450)
6. $5 \text{ km.} = \_?\_ \text{ m.} = \_?\_ \text{ cm.}$ (answer: 5,000; 500,000)
7. $23 \text{ da.} = \_?\_ \text{ wk. } \_?\_ \text{ da.}$ (answer: 3, 2)
8. $75 \text{ qt.} = \_?\_ \text{ gal. } \_?\_ \text{ qt.}$ (answer: 18, 3)
9. $7 \text{ cm.} = \_?\_ \text{ dm.} = \_?\_ \text{ mm.}$ (answer: 0.7, or $\frac{7}{10}$; 70)

Ex. 10–13. Name the unit of precision and give the greatest possible error.

10. $3\frac{1}{4}$ in. ($\frac{1}{4}$ inch, $\frac{1}{8}$ inch)
11. $7\frac{0}{8}$ ft. ($\frac{1}{8}$ foot, $\frac{1}{16}$ foot)
12. 12 cm. 47 mm. (1 millimeter, $\frac{1}{2}$ millimeter)
13. $17\frac{0}{10}$ mm. ($\frac{1}{10}$ millimeter, $\frac{1}{20}$ millimeter)

Ex. 14–16. Tell whether n represents the sum or an addend and then solve.

14. $n = 2.68 - 1.73$ (addend; 0.95)
15. $5.5 - n = 3.68$ (addend; 1.82)
16. $n - 7.829 = 3.457$ (sum; 11.286)

17. List within braces the members of $\{\frac{7}{10}, \frac{3}{4}, 0.66, \frac{5}{8}\}$ which make $n - 0.55 > 0$ a true mathematical sentence. $\{\frac{7}{10}, \frac{3}{4}, 0.66, \frac{5}{8}\}$

18. Find the area of a rectangular region 17 yd. by 13 yd. 221 sq. yd.

Ex. 19–20. Copy and add. Ex. 21–22. Copy and subtract.

19.

yd.	ft.
18	2
82	2
100	4; 101 yd. 1 ft.

20.

m.	cm.
3	79
14	38
17	117; 18 m. 17 cm.

21.

gal.	qt.
14	2
9	3
4	3; 4 gal. 3 qt.

22.

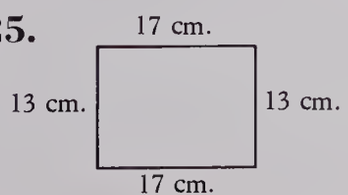
ft.	in.
5	8
4	11
	9; 9 in.

Ex. 23–24. Copy and complete.

23. $2 \text{ sq. ft.} = \_?\_ \text{ sq. in.}$ (answer: 288)
24. $23 \text{ sq. ft.} = \_?\_ \text{ sq. yd.}$ (answer: $2\frac{5}{9}$)

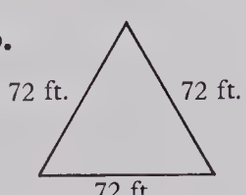
Ex. 25–27. Use the appropriate sentence, $P = 2 \times (a + b)$ or $P = n \times s$, to find the perimeter. Show your work.

25.



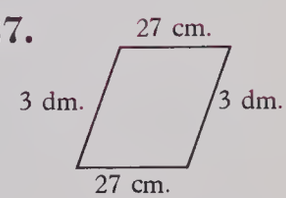
$P = 2 \times (13 + 17)$
$= 2 \times 30$
$= 60$
60 cm.

26.



$P = 3 \times 72$
$= 216$
216 ft.

27.



$P = 2 \times (30 + 27)$
$= 2 \times 57$
$= 114$
114 cm. or 11.4 dm.

Do You Make Mistakes?

Diagnostic Test 4

	a	b	c	d	Study Pages	Practice: Use Sets
1. Add	15.4 23.8 39.2	4.89 13.29 18.18	5.67 0.64 0.07 6.38	25.78 3.27 23.86 52.91	162–163	76–77
2. Subtract	7.9 3.7 4.2	21.07 1.58 19.49	637.23 0.99 636.24	90.90 0.88 90.02	164	79–80
3. Add	2.668 3.907 6.575	13.9082 4.5506 18.4588	285.0073 0.0803 285.0876	80.8077 0.9948 81.8025	166	81–82
4. Subtract	7.805 0.404 7.401	22.1072 13.8066 8.3006	59.6000 0.0778 59.5222	0.0687 0.0099 0.0588	166	83–84

Can You Solve Problems?

Problem Test 4

Let n represent the number which will answer the question in the problem. Write a mathematical sentence for the problem and then solve to find the answer for the problem.

1. Joe attends a metal-working class where certain measurements are made to the nearest $\frac{1}{100}$ inch. Joe recorded the length of a rectangular steel plate as 21.35 in. and the width as 15.75 in. The length of the plate was how much greater than the width? ∨

$n = 21.35 - 15.75$; 5.60 in.

2. What is the perimeter of the rectangular steel plate (Ex. 1)? ∨

$n = 2 \times (21.35 + 15.75)$; 74.20 in.

3. For a seafood dish, Mrs. Henry used $1\frac{1}{2}$ lb. of cooked haddock, 7.5 oz. of lobster meat, 6.5 oz. of crabmeat, and 5.5 oz. of shrimp. How much seafood did she use in all? ∨

$n = 24 + 7.5 + 6.5 + 5.5$; 43.5 oz.

Supplementary Activities

Some of these activities may be assigned to pupils who finish the end-of-chapter tests early, some may be assigned to the rest of the class while the teacher is giving help to those who need additional attention, and all may be used for review and enrichment at any time.

• Give *more capable pupils* a list of measurements such as the following and have them fill in the blanks. Only those measurements are selected for which each larger unit is equivalent to 12 of the corresponding smaller units. Then have them express the measurements as base-twelve numerals:

3 feet = _ ? _ inches 7 gross = _ ? _ dozen
14 years = _ ? _ months 13 dozen = _ ? _ units

3_{twelve} feet = $3_{twelve} \times 10_{twelve}$, or $(30)_{twelve}$ inches
12_{twelve} years = $12_{twelve} \times 10_{twelve}$, or $(120)_{twelve}$ months
7_{twelve} gross = $7_{twelve} \times 10_{twelve}$, or $(70)_{twelve}$ dozen
11_{twelve} dozen = $11_{twelve} \times 10_{twelve}$, or $(110)_{twelve}$ units

The purpose of this is to show that converting to an equivalent measurement with a smaller or larger unit which has the ratio 1:12 only requires the annexing of zeros when the numerals are base-twelve numerals. This is very similar to converting measurements using base-ten numerals in the metric system.

• Give pupils a blank or a partially completed 5 × 5 magic square and have them fill in the blanks with fractions, mixed forms, or decimals so that the sum of the numbers shown in each row, in each column, and along each diagonal are equal. A magic square can be made by representing the numbers of an arithmetic progression in the blanks in the order indicated below. The first number may be arbitrary, but each number after that must be a fixed amount greater or less than the preceding.

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

• Have the pupils show a square 16 inches by 16 inches and draw lines to partition it into two congruent triangular regions and two congruent trapezoidal regions as shown in Fig. 1.

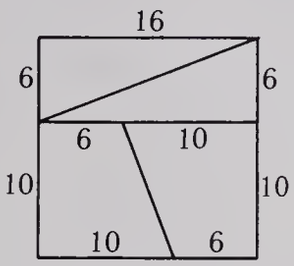


Fig 1

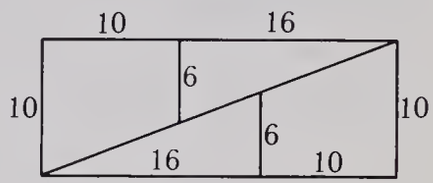


Fig. 2

Then have them cut out the models for the regions and reassemble them to form a rectangular region such as that shown in Fig. 2. The measure of the area of the rectangular region is 10 × 26 in square inches, or 260 in square inches. However, the measure of the area of the original square region is only 16 × 16 or 256 in square inches. Where did the other 4 square inches come from? (Actually, when the pieces are put together, they do not fit exactly so a true rectangular region cannot be formed.)

• Ask if there are any pupils willing to do some library research on *time*, what we mean when we say that time "passes," and how we actually measure time. Ask why, when the metric system was developed, no metric units of measurement were established for measuring time. (All the present units of time are defined in terms of the earth's movement in relation to the stars.)

• Those pupils who enjoy working with numerals is bases other than ten may do the following:

(a) Express the number named by 1.1101_{two} as a base-ten numeral. ($1\frac{13}{16}$, or 1.8125)

(b) Express the number named by 3.456_{seven} as a base-ten numeral. ($3\frac{237}{343}$, or 3.691)

(c) Express the sum of 1.23_{four} and 2.22_{four} first as a base-four numeral and then as a base-ten numeral. (10.11_{four}, $4\frac{5}{16}$ or 4.31)

(d) Express the answer to the subtraction $3.421_{six} - 1.255_{six}$ first as a base-six numeral and then as a base-ten numeral. (2.122_{six}, $2\frac{25}{108}$ or 2.231)

• Have pupils refer to page 148 and review the material on micrometers. Let them research and try to find models of other similar types of measuring devices such as Vernier's calipers, Jo-blocks, and oscilloscopes. Each of these devices measures indirectly a property of an object and this point might well be brought out in a general discussion.

• Have the pupils assume that the metric system, with its prefixes *milli*, *centi*, *deci*, and *kilo* are to be used for the measurement of time with the *day* being the basic unit. Ask them to name the measurement in Column B that most closely approximates the measurement in Column A.

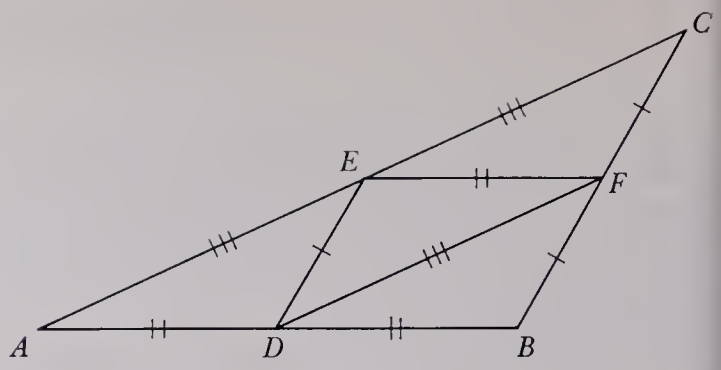
Column A		Column B
1 kiloday	(3 years)	$2\frac{1}{2}$ hours
1 milliday	($1\frac{1}{2}$ minutes)	5 hours
1 deciday	($2\frac{1}{2}$ hours)	15 minutes
1 centiday	(15 minutes)	5 minutes
		12 years
		3 years
		$1\frac{1}{2}$ minutes

• The weight of a diamond is usually expressed in terms of a unit called a *carat*. Ask if someone would like to look up and report to the class on the definition and use of *carat* as a unit of measurement of weight.

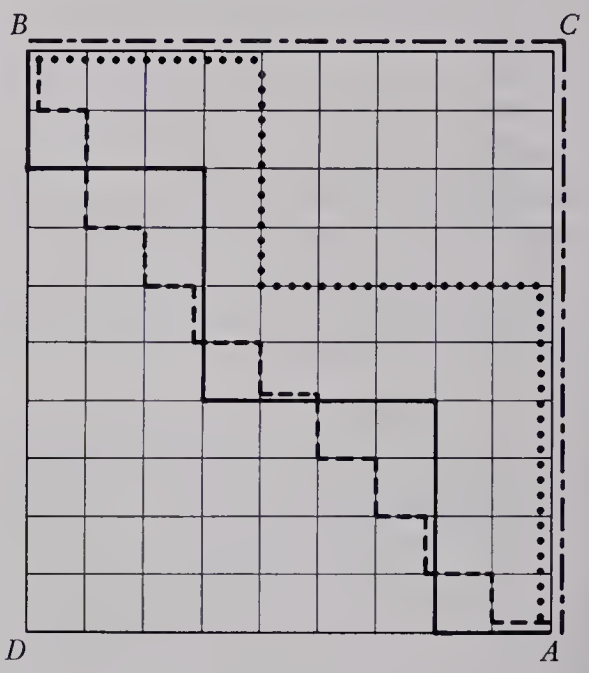
• The use of number patterns is sometimes helpful in giving the pupils practice in adding and subtracting with fractions, mixed forms, and decimals. Give the pupils the first few numbers of an arithmetic sequence and have them determine the next two numbers in the series. For example:

(a) 3.4, 4.6, 5.8, . . . (7.0, 8.2)
(b) $5\frac{1}{2}$, $4\frac{1}{4}$, 3, . . . ($1\frac{3}{4}$, $\frac{1}{2}$)
(c) 8, $8\frac{3}{4}$, $7\frac{1}{2}$, . . . ($8\frac{1}{4}$, 7)

• All segments marked /// have the same measure in inches. This also holds true for segments marked // and /. In the figure at the top of the next column, $m(\overline{AB}) = 10$, $m(\overline{BC}) = 8$ and $m(\overline{AC}) = 14$. Have pupils find what length of string is required to cover all the segments pictured.



• If the earth's atmosphere exerts a pressure of 14.7 pounds on every square inch of surface that it contacts, what pressure would be exerted on a rectangular box if each side has a width of 6 inches and a height of 4 inches? (6 sides with a total measure of area of 168 in square inches means a total pressure of 2,469.6 pounds.)



• Try this with *all pupils*. You want to go from point *A* to point *B*, but you must follow the segments. What is the shortest path? (They are all the same.) Guess before you try the following questions. Let the measure of length for each side of each small square be 1 unit. What is the measure of length in units of the path shown by: a. - - - b. · · · c. — d. – · – · – ·(Each path is 19 units long.)

4. Joe's sister, June, works at the Fabric Center. Each time June cuts off a piece of yard goods from a bolt, or roll, of material, she has to record the number of yards of material that are left on the bolt. If the last record is 30.75 yd. and June cuts off 3.85 yd., what does she record? $n = 30.75 - 3.85$; 26.90 yd.

5. What is the perimeter of a quadrilateral whose sides are 4.637 in., 4.6 in., 4.898 in., and 5.402 in.? $n = 4.637 + 4.6 + 4.898 + 5.402$; 19.537 in.

6. Find the area of a rectangular region 56 yd. by 34 yd. $n = 56 \times 34$; 1,904 sq. yd.

7. 23.86 is how much greater than the sum of 3.78 and 0.94? $n = 23.86 - (3.78 + 0.94)$; 19.14

8. The area of a rectangular region 13 cm. by 5 cm. is how many square centimeters less than the area of a square region 10 cm. on a side? $n = (10 \times 10) - (13 \times 5)$; 35

How Well Can You Compute?

Computation Test 4

Ex. 1–5. Subtract the number from 51.6.

1. 35 16.6 2. 18.9 32.7 3. 30.78 20.82 4. $20\frac{3}{4}$ 30.85 5. 0.4908 51.1092

Ex. 6–10. Add.

6.	7.	8.	9.	10.
8.937	2,738.93	$37.98	3.705	78.9
5.508	552.97	5.78	0.883	34
14.445	3,291.90	14.75	6.669	4.999
		$58.51	11.257	117.899

Ex. 11–20. Find the number represented by the letter.

11. $25.5 + n = 48.3$ 22.8
12. $n = 8.54 - 2.67$ 5.87
13. $x = 35 - 17.88$ 17.12
14. $78 - x = 3.442$ 74.558
15. $3.56 + 5.77 + 8.92 = n$ 18.25
16. $n = 4.81 - (3.72 + 1.01)$ 0.08
17. $n = 34.837 - 28.993$ 5.844
18. $0.0594 - 0.0088 = x$ 0.0506
19. $(352 \div 11) - 31.55 = n$ 0.45
20. $n = 7 - (3.555 + 2.073)$ 1.372

5

Multiplying a Rational Number

Resurvey [O]

Bill invited four of his friends to stop at his house after school to see his stamp collection. After they had seen the stamps, Bill's mother offered the boys some pie and milk. She cut an apple pie into 8 pieces of the same size.

1. Each of the eight pieces of apple pie was what part of the whole pie? $\frac{1}{8}$

2. Each of the five boys ate one piece of apple pie. In all, that was what part of the pie? $\frac{5}{8}$

a. At the board, use addition to find the answer. Did you find the sum of five $\frac{1}{8}$'s? Yes $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \_?\frac{5}{8}\_$

b. To solve a mathematical sentence such as $n = 6 + 6 + 6$, we can add, but since the addends are equal, can we also solve by multiplying? Yes $n = 3 \times \_?6\_$, or $\_?18\_$.

c. Since the addends are equal, $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$ can be expressed as $\_?5\_ \times \frac{1}{8}$. Since you found the sum of five $\frac{1}{8}$'s by adding, you know that $5 \times \frac{1}{8} = \_?\frac{5}{8}\_$.

Overview—Chapter 5

• The work in Chapter 5 pertains mainly to multiplying and dividing rational numbers expressed with fractions and mixed forms. Other topics presented include properties involved in the multiplication of rational numbers; renaming factors; multiplication of rational numbers with more than two factors; identity element for multiplication; differentiating between multiplication and division; reciprocals; and division of rational numbers using the common-denominator method and the reciprocal method. The last part of the chapter involves comparing numbers and quantities by the use of ratios and related work in reading and making scale drawings and maps.

• **Problem-solving.** Problems involving an interesting variety of topics are presented in this chapter. They require multiplication and division of rational numbers and deal with measurements in both the English and metric systems in social and school situations, metallurgy, and the measurement of electrical energy. In some cases, the pupils must differentiate among operations to be used. The use of scale drawings in science and of ratios in reading a road map are also presented.

• **Maintenance.** Cumulative review for material presented earlier in the text and summary exercises for concepts extended within the chapter are presented. Topics include addition and subtraction of rational numbers in fraction form, decimal form, and in mixed form and their use in mathematical sentences; addition and subtraction of greater numbers shown with decimals; multiplication with fractions; division with fractions; true and false mathematical sentences and inequalities; finding perimeters of polygons and areas of rectangular regions; measure of an angle in degrees; precision in measurement; factoring; ratios; renaming numbers as a sum of two primes and as a product of primes; exponent forms; division of rational numbers by the common denominator method and the reciprocal method; and union and intersection of sets.

• **Enrichment.** Although there are suggestions for enrichment in the lesson plans, there is a special topic on page 219 dealing with rational numbers shown in exponent form. The sentence $\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$ is a generalization for this concept. The Supplementary Activities at the end of the chapter suggest many additional topics for enrichment.

• **Tests.** Any of the practice, summary, or review exercises may be used as tests. The end-of-chapter tests include such topics as inequalities; division and multiplication of rational numbers in fraction and mixed forms; fractions and mixed forms in mathematical sentences; problems involving rational numbers, measurement of angles in degrees, fractions in simplest form, ratios, scale drawings, perimeter, and area of a rectangular region.

Teaching Pages 184 and 185

Pupil's Objective

(a) To review the meaning of multiplying a rational number by a whole number; (b) to relate multiplication to finding the sum of a number of equal addends and (c) to develop an algorithm for multiplication of rational numbers.

Background

Multiplying a whole number by a whole number can be used instead of adding several equal addends. 3×4 is equal to $4 + 4 + 4$. This same principle applies to multiplying a rational number by a whole number.

$$3 \times \tfrac{2}{5} = \tfrac{2}{5} + \tfrac{2}{5} + \tfrac{2}{5}$$

When adding equal rational numbers, the numerator of the sum is always the sum of the numerators of the addends.

$$\tfrac{2}{5} + \tfrac{2}{5} + \tfrac{2}{5} = \tfrac{2+2+2}{5}$$

Because the addends are equal, multiplication can be used instead of addition.

$$3 \times \tfrac{2}{5} = \tfrac{3 \times 2}{5}$$
$$= \tfrac{6}{5}, \text{ or } 1\tfrac{1}{5}$$

The numerator of the product is always the product of the given whole number and the numerator of the given rational number. The denominator of the product is always equal to the denominator of the given rational number. This can be generalized by $a \times \frac{b}{c} = \frac{a \times b}{c}$ where a stands for a whole number and $\frac{b}{c}$ stands for a rational number.

Pre-Book Lesson

Have the pupils cut a rectangular-shaped strip of paper into eight parts of the same size. Have them arrange four sets of the parts so that each set contains two of the parts. Ask, "What fraction can be used to name the number for the part of the whole strip contained in each set?" $(\frac{2}{8})$

Discuss the problem of determining the number for the part of a whole strip contained in three sets. Lead the pupils to the realization that the answer can be found by (a) counting the parts; (b) performing the addition $n = \frac{2}{8} + \frac{2}{8} + \frac{2}{8}$; and (c) performing the multiplication $n = 3 \times \frac{2}{8}$.

Using the Text Pages

• The purpose of Ex. 2 is to review the idea that the addition of equal addends may be expressed as a multiplication in which the factors are (a) the number of equal addends and (b) one of the equal addends.

• Ex. 3 develops finding the product of a whole number and a rational number by using a number-line picture. Since, to find the answer for this problem, we must find a multiple of $\frac{1}{4}$, the unit to be used is named $\frac{1}{4}$.

• Ex. 4. If the pupils have difficulty in explaining each step of the algorithm in the box, additional steps may be inserted to make the development more meaningful:

$$n = 5 \times \tfrac{1}{4}$$
$$= \tfrac{1}{4} + \tfrac{1}{4} + \tfrac{1}{4} + \tfrac{1}{4} + \tfrac{1}{4}$$
$$= \frac{1 + 1 + 1 + 1 + 1}{4}$$
$$= \tfrac{5 \times 1}{4} = \tfrac{5}{4}, \text{ or } 1\tfrac{1}{4}$$

• In each of Ex. 12–23, the whole-number factor is shown before the rational-number factor. Multiplication is commutative so the product is the same, regardless of the order in which the factors are shown. Multiplying by a rational number is introduced on page 186.

Individualizing Instruction

• The *more capable pupils* may be asked to write an expression for the product of the whole number a and the rational $\frac{b}{c}$. They may explain *why* the numerator of the product is equal to the product of the whole number and the numerator of the given rational number. Ask *why* the denominator of the product is equal to the denominator of the given rational number.

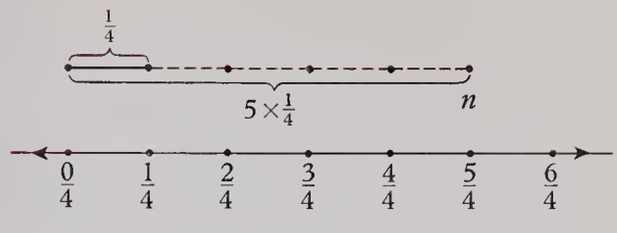
• *All pupils* may be given exercises involving a rational number expressed with various names to determine whether the product is the same regardless of the way in which the rational number is expressed. The following are some suggestions:

a. $5 \times \frac{1}{2}$ b. $5 \times \frac{11}{22}$ c. $5 \times \frac{36}{72}$

• The *slower learners* may use physical objects, such as strips of paper as models for multiplication exercises.

* Take special care to have pupils point out the related parts in the number-line picture and the product expression.

* **3.** Each of the five boys drank $\frac{1}{4}$ qt. of milk. That was how much milk in all? $1\frac{1}{4}$ qt.



a. Does $n = 5 \times \frac{1}{4}$ express the relationship in the problem? Yes The number-line picture at the right shows this relationship. Since one factor is named by a fraction showing denominator 4, the number line is shown scaled in ?'s.

b. A segment is pictured for the factor $\frac{1}{4}$. How many segments for $\frac{1}{4}$ are shown in all? 5 From the number-line scale, read the measure of the segment labeled $5 \times \frac{1}{4}$. $\frac{5}{4}$ Express the number represented by n with a mixed form; $1\frac{1}{4}$ a decimal. 1.25

c. How many quarts of milk did the boys drink in all? $1\frac{1}{4}$ qt., or 1.25 qt.

4. Another way to find $5 \times \frac{1}{4}$ is shown in the box. Explain each step shown. Key idea: The product of 5 and the numerator of $\frac{1}{4}$ is shown over the denominator of $\frac{1}{4}$.

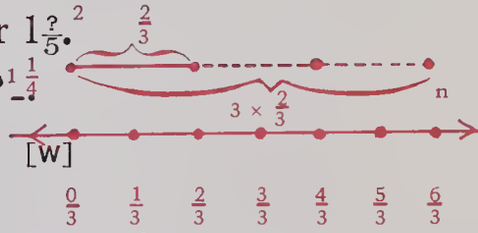
$$n = 5 \times \frac{1}{4}$$
$$= \frac{5 \times 1}{4}$$
$$= \frac{5}{4}, \text{ or } 1\frac{1}{4}$$

5. On the board, draw a number-line picture to show $3 \times \frac{2}{3}$. See right. Then solve $n = 3 \times \frac{2}{3}$ as in the box. $n = 3 \times \frac{2}{3}$, $= \frac{3 \times 2}{3}$, $= \frac{6}{3}$, or 2

6. Complete each of Ex. a–d.

a. $3 \times \frac{1}{2} = \frac{3 \times ?^{1}}{2} = \frac{?^{3}}{2}$, or $1\frac{?^{1}}{2}$. **b.** $7 \times \frac{1}{5} = \frac{7 \times 1}{?^{5}} = \frac{7}{5}$, or $1\frac{?^{2}}{5}$.

c. $8 \times \frac{3}{5} = \frac{?^{8} \times 3}{5} = \frac{?^{24}}{5}$, or $-?-^{4\frac{4}{5}}$. **d.** $2 \times \frac{5}{8} = \frac{?^{10}}{8} = \frac{?^{5}}{4}$, or $-?-^{1\frac{1}{4}}$.



Ex. 7–11. Rename with a mixed form.

7. $\frac{9}{4}$ $2\frac{1}{4}$ **8.** $\frac{24}{7}$ $3\frac{3}{7}$ **9.** $\frac{22}{10}$ $2\frac{1}{5}$ **10.** $\frac{40}{3}$ $13\frac{1}{3}$ **11.** $\frac{100}{9}$ $11\frac{1}{9}$

Ex. 12–23. As in the box, find the number represented by the letter. If the answer is named by a fraction or a mixed form, be sure the fraction is in simplest form.

12. $n = 3 \times \frac{1}{5}$ $\frac{3}{5}$ **16.** $n = 14 \times \frac{7}{8}$ $12\frac{1}{4}$ **20.** $33 \times \frac{4}{3} = n$ 44

13. $n = 11 \times \frac{2}{3}$ $7\frac{1}{3}$ **17.** $x = 37 \times \frac{5}{9}$ $20\frac{5}{9}$ **21.** $x = 16 \times \frac{2}{3}$ $10\frac{2}{3}$

14. $7 \times \frac{1}{4} = x$ $1\frac{3}{4}$ **18.** $x = 45 \times \frac{3}{4}$ $33\frac{3}{4}$ **22.** $n = 72 \times \frac{3}{5}$ $43\frac{1}{5}$

15. $y = 5 \times \frac{4}{5}$ 4 **19.** $16 \times \frac{9}{10} = n$ $14\frac{2}{5}$ **23.** $y = 75 \times \frac{3}{50}$ $4\frac{1}{2}$

24. Find the area of a rectangular region that is 6 ft. by $\frac{2}{3}$ ft. 4 sq. ft.

♦ **Extra Examples.** Sets 88–90.

*Notice that to picture $\frac{3}{4} \times 2$, we picture a segment for 2, partition it into 4 congruent segments, and then read the length of 3 of these segments from the number-line pictured below.

Multiplying by a Rational Number

[O]

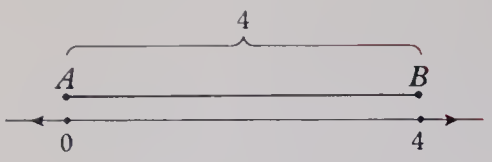


Fig. 1

1. Look at the number-line picture in Fig. 1. The measure of line segment AB is 4. How is this shown? The brace is used to indicate that $\overline{AB}$ represents 4.

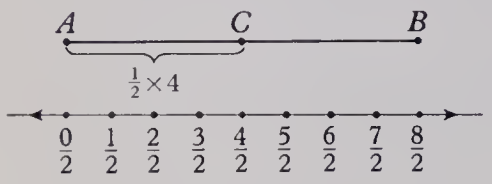


Fig. 2

2. Fig. 2. To find $\frac{1}{2}$ of $\overline{AB}$, $\overline{AB}$ is shown partitioned into two congruent segments.

a. Name these two segments. $\overline{AC}$ and $\overline{CB}$ The measure of $\overline{AB}$ is 4. What is the measure of $\overline{AC}$? 2

b. Is it true that the measure of $\overline{AC}$ is $\frac{1}{2}$ the measure of $\overline{AB}$? Yes We write, $\frac{1}{2} \times 4 = 2$.

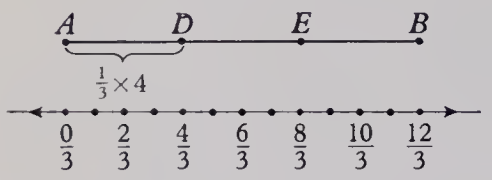


Fig. 3

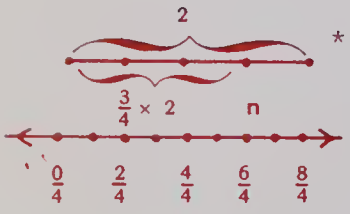
3. Fig. 3. To find $\frac{1}{3}$ of $\overline{AB}$, $\overline{AB}$ is shown partitioned into _?3_ congruent segments.

a. The measure of $\overline{AB}$ is $\frac{12}{3}$, or 4. What is the measure of $\overline{AD}$? $\frac{4}{3}$, or $1\frac{1}{3}$

b. The measure of $\overline{AD}$ is $\frac{1}{?3}$ the measure of $\overline{AB}$. $\frac{1}{3} \times 4 = \frac{?4}{3}$

$$\frac{1}{3} \times 4 = \frac{1 \times 4}{3}$$
$$= \frac{4}{3}, \text{ or } 1\frac{1}{3}$$

4. The work in the box shows how to find $\frac{1}{3} \times 4$. Explain each step shown.



* 5. On the board, draw a number-line picture for $\frac{3}{4} \times 2 = n$.

a. Show a scale of $\frac{1}{4}$'s on the number line. Why? One factor is named by a fraction showing denominator 4.

b. Partition the segment for 2 into _?4_ congruent segments.

c. How many of these segments are we considering? 3

d. What is the measure of the segment for $\frac{3}{4} \times 2$? $\frac{6}{4}$ $n = $ _?$1\frac{1}{2}$_

6. At the board, work Ex. a–d as in the box.

a. $\frac{1}{2} \times 4$ 2 **b.** $\frac{3}{4} \times 9$ $6\frac{3}{4}$ **c.** $\frac{2}{3} \times 12$ 8 **d.** $\frac{5}{6} \times 36$ 30

[W]

Ex. 7–15. Solve.

7. $n = \frac{1}{9} \times 5$ $\frac{5}{9}$
8. $n = \frac{3}{4} \times 12$ 9
9. $x = \frac{5}{6} \times 54$ 45
10. $x = \frac{6}{7} \times 8$ $6\frac{6}{7}$
11. $n = \frac{6}{7} \times 35$ 30
12. $\frac{3}{5} \times 15 = x$ 9
13. $n = \frac{5}{9} \times 75$ $41\frac{2}{3}$
14. $\frac{2}{3} \times 129 = y$ 86
15. $\frac{3}{8} \times 59 = n$ $22\frac{1}{8}$

◆ **Extra Examples.** Set 91.

Teaching Page 186

Pupil's Objective

To learn the meaning of multiplying by a rational number.

Background

The concept that $a \times \frac{b}{c} = \frac{a \times b}{c}$, where a represents a whole number and $\frac{b}{c}$ represents a rational number, was developed previously. Since multiplication is commutative, it must be true that

$$(1)\ \frac{b}{c} \times a = \frac{a \times b}{c}$$

and that

$$(2)\ \frac{b}{c} \times a = \frac{b \times a}{c}$$

To demonstrate by use of a number-line picture the multiplication $\frac{1}{a} \times b$, where $\frac{1}{a}$ represents a rational number with numerator 1 and b represents a whole number, a portion of the number-line picture whose measure is b is partitioned to show a congruent segments. The product $\frac{1}{a} \times b$ is represented by the measure of one of the pictured segments. This may be more meaningful to the pupil if the whole number b (which is a rational number) is represented by a fraction with denominator a. For example. $\frac{1}{4} \times 7 = \frac{1}{4} \times \frac{28}{4}$

If a segment whose measure is $\frac{28}{4}$ is partitioned into 4 congruent segments, the pupil should not have difficulty realizing that the measure of each of the resulting segments is $\frac{7}{4}$.

$$\frac{1}{4} \times 7 = \frac{1}{4} \times \frac{28}{4} = \frac{7}{4}, \text{ or } 1\frac{3}{4}$$

This demonstrates the principle that to multiply by $\frac{1}{4}$ produces the same result as dividing by 4.

The pupil can then be led to realize that

$$(3)\ \frac{1}{a} \times b = \frac{1 \times b}{a} = \frac{b}{a}$$

The multiplication $\frac{a}{b} \times c$ can also be demonstrated by using a number-line picture. To demonstrate $\frac{3}{4} \times 7$, a portion of the number line whose measure is 7 is partitioned to show 4 congruent segments. If $\frac{1}{4} \times 7$ is written as $\frac{1}{4} \times \frac{28}{4}$, one of the 4 congruent segments will have a measure of $\frac{7}{4}$. Since 3 of these segments are to be considered,

$$\begin{aligned} \frac{3}{4} \times \frac{28}{4} &= 3 \times \frac{7}{4} \\ &= \frac{3 \times 7}{4} = \frac{21}{4}, \text{ or } 5\frac{1}{4} \end{aligned}$$

This can also be thought of as

$$\begin{aligned} \frac{3}{4} \times 7 &= 3 \times \frac{1}{4} \times 7 \\ &= 3 \times 7 \times \frac{1}{4} \\ &= 21 \times \frac{1}{4} = \frac{21}{4}, \text{ or } 5\frac{1}{4} \end{aligned}$$

In fact, once the principle expressed in (3) has been introduced, it may be used to develop the concept of $\frac{a}{b} \times c$ as follows:

$$\begin{aligned} \frac{a}{b} \times c &= a \times \frac{1}{b} \times c \\ &= a \times c \times \frac{1}{b} \\ &= \frac{a \times c}{b} \end{aligned}$$

Pre-Book Lesson

Write on the board these mathematical sentences: $n = \frac{1}{3} \times 18$ $\quad n = \frac{1}{7} \times 20$ $\quad n = \frac{3}{5} \times 30$ Have the pupils try to make up problem situations involving the relationships described by the mathematical sentences. Then lead pupils to the realization of their need for knowing how to multiply by a rational number.

Using the Text Page

- After Ex. 3, the pupils may be given more exercises of the type (a) $\frac{1}{5} \times 7$; (b) $\frac{1}{8} \times 3$; (c) $\frac{1}{9} \times 11$, as preparation for Ex. 4. The purpose of Ex. 4 is to develop the idea $\frac{1}{a} \times b = \frac{b}{a}$. It is important to devote sufficient time to having the pupils explain each step in the box so that they will understand why $\frac{1}{a} \times b = \frac{b}{a}$ is always true.
- In Ex. 5, if the pupil understands that $\frac{1}{4} \times 2 = \frac{1}{4} \times \frac{8}{4}$, he should be able to understand that $\frac{3}{4} \times 2$ is 3 times as great as $\frac{1}{4} \times 2$.
- Ex. 6 should lead the pupil to the understanding $\frac{a}{b} \times c = \frac{a \times c}{b}$.

Individualizing Instruction

- Give *all pupils* exercises in which the factors are the same as the factors in Ex. 7–15 but with the order of the factors reversed. Have pupils compare the method on pupil's pages 184 and 185 with the method on page 186.
- Have *slower learners* cut a rectangular strip of paper to show six congruent regions. Ask them how they would solve the problem of finding $\frac{2}{3}$ of the strip, using the method described in Ex. 5.

Pupil's Objective

To apply the Commutative Property and the Associative Property of Multiplication to multiplication of rational numbers.

Background

On pupil's pages 184 and 185, the concept

$$(1)\ a \times \tfrac{b}{c} = \tfrac{a \times b}{c}$$

was developed. On page 186, the concept

$$(2)\ \tfrac{b}{c} \times a = \tfrac{b \times a}{c}$$

was developed. The numerator of the product in equation (2) may also be expressed as "$a \times b$" because a and b represent whole numbers and multiplication is commutative for the set of whole numbers. Therefore,

$$(3)\ \tfrac{b}{c} \times a = \tfrac{a \times b}{c}$$

From equation (1) and equation (3) we can see that

$$(4)\ a \times \tfrac{b}{c} = \tfrac{b}{c} \times a$$

When multiplying a whole number and a rational number, multiplication is commutative and the order of the factors does not affect the product.

To determine whether multiplication involving whole numbers and a rational number is associative, we compare the results of the multiplication $(a \times b) \times \frac{c}{d}$ with the results of the multiplication $a \times (b \times \frac{c}{d})$. In each of these multiplications, the factors are the same and the factors are expressed in the same order, but the grouping of the factors is different.

$$(5)\ (a \times b) \times \tfrac{c}{d} = \tfrac{a \times b \times c}{d}$$
$$(6)\ a \times (b \times \tfrac{c}{d}) = a \times \tfrac{b \times c}{d} = \tfrac{a \times b \times c}{d}$$

This illustrates use of the Associative Property of Multiplication, which may be generalized as, "If three numbers shown in a specified order are to be multiplied, you may multiply the third by the product of the first two or you may multiply the product of the last two by the first."

Pre-Book Lesson

• Review the meaning of *commutative* by asking which of these activities are commutative:

a. Putting on your shirt and putting on your sweater.

b. Opening a door and walking through a doorway.

c. Putting on your hat and putting on your coat.

Ask "Can the measure for the area of a rectangular region whose length is 2 units and width $\frac{3}{4}$ unit be expressed as $\frac{3}{4} \times 2$? Can it also be expressed as $2 \times \frac{3}{4}$?"

• Review the meaning of *associative* by asking

a. "When multiplying three numbers, how many numbers can be multiplied at one time?"

b. "Does it matter whether the first two numbers or the second and third are multiplied first?"

Using the Text Page

• Pupils should understand that finding examples of multiplication in which the order of the factors does not affect the product does not prove that multiplication is commutative. Ex. 1 is simply one illustration of the use of this property.

• In Ex. 4, the order of the factors has been changed but the grouping has not been. Therefore, this illustrates use of the Commutative Property of Multiplication. In an example such as $(8 \times \frac{3}{50}) \times 2 = (8 \times 2) \times \frac{3}{50}$, the order of the factors has been changed and also the grouping of the factors has been changed. This illustrates application of both the Commutative Property and Associative Property of Multiplication.

Individualizing Instruction

• Ask *all pupils* which is the easier:

a. $(2 \times 9) \times \frac{5}{6}$ or $2 \times (9 \times \frac{5}{6})$?
b. $(5 \times \frac{3}{4}) \times 28$ or $5 \times (\frac{3}{4} \times 28)$?
c. $(\frac{7}{8} \times 9) \times 16$ or $(\frac{7}{8} \times 16) \times 9$?

Have them justify each choice.

• Have the *more capable pupils* compare the product of $a \times \frac{b}{c}$ with the product $\frac{b}{c} \times a$. How does this show that multiplication is commutative? Have them compare the product $(a \times b) \times \frac{c}{d}$ with the product $a \times (b \times \frac{c}{d})$. How does this show that multiplication is associative?

* Encourage pupils to include "of multiplication" or "of addition" when referring to commutative or associative property. That is, the operation should be identified.

Remembering Properties of Multiplication

Properties for rational numbers [O]

1. At the board, find $2 \times \frac{3}{7}$. $\frac{6}{7}$ Then find $\frac{3}{7} \times 2$. $\frac{6}{7}$ Is $2 \times \frac{3}{7} = \frac{3}{7} \times 2$ a true mathematical sentence? Yes This is an illustration of the Commutative Property of Multiplication.

2. Complete Ex. a–c so that each will be an illustration of the Commutative Property of Multiplication.

a. $\frac{3}{4} \times 5 = \frac{5}{} \times \frac{3}{4}$ b. $6 \times \frac{7}{15} = \frac{7}{15} \times 6$ c. $37 \times \frac{5}{12} = \frac{5}{12} \times 37$

3. Explain the work in boxes A and B. Is $(2 \times 3) \times \frac{2}{7} = 2 \times (3 \times \frac{2}{7})$ a true mathematical sentence? Yes This is an illustration of the Associative Property of Multiplication.

A	$(2 \times 3) \times \frac{2}{7} = 6 \times \frac{2}{7}$ $= \frac{6 \times 2}{7}$ $= \frac{12}{7}$, or $1\frac{5}{12}$
B	$2 \times (3 \times \frac{2}{7}) = 2 \times \frac{6}{7}$ $= \frac{2 \times 6}{7}$ $= \frac{12}{7}$, or $1\frac{5}{12}$

* 4. Which property does the mathematical sentence $(8 \times \frac{3}{50}) \times 2 = 2 \times (8 \times \frac{3}{50})$ illustrate—the Commutative Property of Multiplication or the Associative Property? Explain. It shows $\frac{24}{50} \times 2 = 2 \times \frac{24}{50}$ [W]

Ex. 5–8. Give the number represented by n by thinking about properties of multiplication.

5. $n \times \frac{2}{3} = \frac{2}{3} \times 59$ 59 6. $(15 \times \frac{1}{7}) \times n = 15 \times (\frac{1}{7} \times 34)$ 34

7. $708 \times \frac{5}{6} = n \times 708$ $\frac{5}{6}$ 8. $10 \times (15 \times \frac{7}{15}) = (15 \times n) \times 10$ $\frac{7}{15}$

Ex. 9–14. Solve.

9. $n = \frac{3}{4} \times 27$ $20\frac{1}{4}$ 10. $y = 10 \times (15 \times \frac{7}{15})$ 70 11. $n = 15 \times \frac{3}{5}$ 9

12. $x = \frac{5}{6} \times 12$ 10 13. $n = (6 \times \frac{4}{3}) \times 12$ 96 14. $y = \frac{3}{2} \times 7$ $10\frac{1}{2}$

Ex. 15–19. Rename with a mixed form.

15. $\frac{14}{5}$ $2\frac{4}{5}$ 16. $\frac{67}{15}$ $4\frac{7}{15}$ 17. $\frac{90}{35}$ $2\frac{4}{7}$ 18. $\frac{100}{17}$ $5\frac{15}{17}$ 19. $\frac{68}{22}$ $3\frac{1}{11}$

Ex. 20–24. Rename with a fraction in simplest form.

20. $\frac{142}{88}$ $\frac{71}{44}$ 21. $3\frac{1}{5}$ $\frac{16}{5}$ 22. $10\frac{3}{4}$ $\frac{43}{4}$ 23. $23\frac{5}{9}$ $\frac{212}{9}$ 24. $13\frac{2}{3}$ $\frac{41}{3}$

Ex. 25–27. Solve.

25. $n = 6\frac{3}{4} + 4\frac{2}{5} + 5\frac{1}{2}$ $16\frac{13}{20}$ 26. $x = \frac{2}{3} \times 18$ 12 27. $y = (\frac{3}{8} + \frac{1}{12}) \times 16$ $7\frac{1}{3}$

Renaming Factors

[O]

1. Mrs. Walther bought three $1\frac{2}{3}$-lb. packages of cheese at the store. How many pounds of cheese did she buy in all?

a. Explain how the mathematical sentence $n = 3 \times 1\frac{2}{3}$ shows the relationship expressed in the problem. It expresses the factors-product relationship.

b. Box A. The number named by the mixed form $1\frac{2}{3}$ has been renamed $\frac{?}{3}$. ^{5} Explain the work shown. Key idea: The number is renamed with a fraction and multiplied by 3.

c. How much cheese did Mrs. Walther buy? 5 lb.

A

$$n = 3 \times 1\tfrac{2}{3}$$
$$= 3 \times \tfrac{5}{3}$$
$$= \tfrac{3 \times 5}{3}$$
$$n = \tfrac{15}{3}, \text{ or } 5$$

B

$$n = 2\tfrac{1}{2} \times 7$$
$$= \tfrac{5}{2} \times 7$$
$$= \tfrac{5 \times 7}{2}$$
$$n = \tfrac{35}{2}, \text{ or } 17\tfrac{1}{2}$$

2. Use addition to find the answer for Ex. 1. $1\frac{2}{3} + 1\frac{2}{3} + 1\frac{2}{3} = 3\frac{6}{3}$, or 5

3. The distance from Mrs. Walther's house to the store is $2\frac{1}{2}$ times the distance from her house to the post office. If the post office is 7 miles from her house, how far is the store from her house?

a. Explain the work in box B. Key idea: $2\frac{1}{2}$ is renamed with a fraction and then the product is found.

b. What is the answer for the problem? $17\frac{1}{2}$ miles

[W]

Ex. 4–12. Solve as in box A or box B.

4. $n = 3 \times 2\frac{1}{5}$ $6\frac{3}{5}$
5. $n = 4 \times 3\frac{2}{3}$ $14\frac{2}{3}$
6. $n = 1\frac{1}{7} \times 5$ $5\frac{5}{7}$
7. $x = 4 \times \frac{5}{6}$ $3\frac{1}{3}$
8. $16 \times 1\frac{1}{2} = n$ 24
9. $7 \times 7\frac{1}{3} = x$ $51\frac{1}{3}$
10. $x = 3 \times 2\frac{11}{12}$ $8\frac{3}{4}$
11. $6\frac{3}{5} \times 7 = y$ $46\frac{1}{5}$
12. $y = 4 \times 3\frac{7}{12}$ $14\frac{1}{3}$

◆ **Extra Examples.** Set 92.

Teaching Page 188

Pupil's Objective

To discover how to find the product of a whole number and a rational number named in mixed form.

Background

If a represents a whole number and $\frac{b}{c}$ represents a rational number named with a fraction, then

$$(1)\ a \times \frac{b}{c} = \frac{a \times b}{c}$$

and

$$(2)\ \frac{b}{c} \times a = \frac{b \times a}{c}$$

This is true when $\frac{b}{c}$ represents any rational number, including a rational number whose numerator is greater than its denominator. In multiplications involving use of a mixed form, the factor may be renamed from a mixed form to a fraction and the multiplication performed applying the principles stated in (1) and (2) above.

Pre-Book Lesson

Ask the pupils for suggestions as to how to work the multiplication $5 \times 2\frac{1}{3}$ applying the principles introduced on the previous pages.

Using the Text Page

• The oral work for Ex. 1–3 is designed to develop the concept that the multiplication explained on pages 184–187 can be extended to work with mixed forms simply by renaming from the mixed form to a fraction, and proceeding with the multiplication.

• Ex. 4–12 should indicate whether your pupils understand how to rename from a mixed form to a fraction and multiply as they have been taught on previous pages.

Individualizing Instruction

• Give the *more capable pupils* examples involving multiplication using a mixed form in which there is an error. Ask them to locate the error.

1. $3 \times 4\frac{1}{2} = 3 \times 4 \times \frac{1}{2}$
$= 12 \times \frac{1}{2}$
$= \frac{12}{2}$, or 6

2. $5\frac{2}{3} \times 9 = n$
$\frac{13}{3} \times 9 = n$
$\frac{13 \times 9}{3} = n$
$\frac{117}{3} = n$
$39 = n$

3. $5 \times 1\frac{1}{6} = 5 \times \frac{7}{6}$
$= \frac{5+7}{6}$
$= \frac{12}{6}$, or 2

4. $6\frac{3}{4} \times 5 = \frac{22}{4} \times 5$
$= \frac{22 \times 5}{4}$
$= \frac{110}{4}$, or $27\frac{1}{2}$

• Have *all pupils* think of problem situations containing relationships which could be expressed by the mathematical sentences in Ex. 4–12.

• Check on the *slower learners* to see if they are making errors in renaming from mixed forms to fractions. Review and give them additional practice exercises if necessary.

Teaching Page 189

Pupil's Objective

To learn to use the distributive property in finding the product of a whole number and a rational number named in mixed form.

Background

On text page 188, the pupils were introduced to the method of multiplying a whole number and a rational number expressed in mixed form by renaming from the mixed form to a fraction. There is another method of performing this multiplication. The mixed form may first be renamed to show the sum of a whole number and a rational number. The multiplication is then performed by making use of the principle that multiplication is distributive over addition.

If a represents a whole number and $b\frac{c}{d}$ represents a rational number named in mixed form, the multiplication $a \times b\frac{c}{d}$ may be performed by first renaming the mixed form to show the sum $b + \frac{c}{d}$:

$$(1)\ a \times b\tfrac{c}{d} = a \times (b + \tfrac{c}{d})$$

Applying the distributive property, we have

$$= (a \times b) + (a \times \tfrac{c}{d})$$
$$= (a \times b) + (\tfrac{a \times c}{d})$$

The work in box B on the text page shows how the computation may be shown, using $4 \times 3\frac{1}{3}$ as an example.

The final product is the sum of two products: (1) the product of the whole-number factor and the whole number shown in the mixed form and (2) the product of the whole-number factor and the rational number shown by the fraction in the mixed form. Box C and box D on the text page show how the work may be written in vertical form. This vertical form is usually easier to use than the method shown on pupil's page 188 when the factors are greater numbers.

Pre-Book Lesson

Review the use of the distributive property with whole numbers: $a \times (b + c) = (a \times b) + (a \times c)$. If pupils have any difficulty understanding the Distributive Property, an illustration such as the following may help:

$$\begin{aligned} 3 \times (4 + 5) &= (4 + 5) + (4 + 5) + (4 + 5) \\ &= 4 + 4 + 4 + 5 + 5 + 5 \\ &= (3 \times 4) + (3 \times 5) \\ &= 12 + 15 = 27 \end{aligned}$$

The steps shown above give the distributive property more meaning, but pupils should be able to apply the principle that $a \times (b + c) = (a \times b) + (a \times c)$ without going through the steps.

Using the Text Page

- Boxes A, B, C, and D illustrate four methods of writing the work for $4 \times 3\frac{1}{3}$. To minimize the possibility of confusion, the pupils should be given practice with using each of these methods before going on to the next method.
- Have the pupils perform Ex. 11, 12, 16–18 also by the method shown in box A, and then ask them if they can see any advantage in using the method shown in box C or the method shown in box D.

There is a danger that some pupils may write

(1)		as	(2)	
	35			35
	$8\frac{2}{7}$			$8\frac{2}{7}$
	10			10
	280			2 80
	290			2,810

Watch for this and have the pupils explain why the work shown in (2) is wrong.

- For Ex. 7–18, you may want pupils to show (beside the corresponding partial product shown) the work for finding each partial product.

Individualizing Instruction

- *More capable pupils* may explain why in box B $4 \times (3 + \frac{1}{3})$ is not equal to $4 \times 3 + \frac{1}{3}$.
- Have *all pupils* suggest problems that can be solved by the methods shown on this page.
- If any of the *slower learners* have difficulty with one or more of the methods, have them determine which one is easiest for them and have them concentrate on using that method.

Using the Distributive Property

[o]

1. Explain the work in box A for finding $4 \times 3\frac{1}{3}$. Key idea: $3\frac{1}{3}$ is renamed with a fraction and multiplied by 4.

2. In box B, $3\frac{1}{3}$ has been renamed $3 + \frac{?}{3}$. $4 \times (3 + \frac{1}{3}) = (\_?\_ \times 3) + (\_?\_ \times \frac{1}{3})$ This is an illustration of the $\_?\_$ Property. Explain each step shown. 1 4 Distributive

3. Box B. $4 \times 3\frac{1}{3}$ can be expressed as $(4 \times 3) + (4 \times \frac{1}{3})$. Could it also be expressed as $(4 \times \frac{1}{3}) + (4 \times 3)$? Yes What property permits this? Commutative Property of Addition

4. Study the work in box C. Which step shows $4 \times \frac{1}{3}$? 1 4×3? 2 Does step (3) show the sum of these two numbers? Yes

5. In box D, which step shows $\frac{1}{3} \times 4$? 1 3×4? 2 Is the sum of these two numbers the same as the sum of $4 \times \frac{1}{3}$ and 4×3? Yes Why? Key idea: They name the same numbers.

A

$$4 \times 3\tfrac{1}{3} = 4 \times \tfrac{10}{3}$$
$$= \tfrac{40}{3}, \text{ or } 13\tfrac{1}{3}$$

B

$$4 \times 3\tfrac{1}{3} = 4 \times (3 + \tfrac{1}{3})$$
$$= (4 \times 3) + (4 \times \tfrac{1}{3})$$
$$= 12 + \tfrac{4}{3}$$
$$= 12\tfrac{4}{3}, \text{ or } 13\tfrac{1}{3}$$

C

Multiply

$3\frac{1}{3}$

4

$(4 \times \frac{1}{3}) = 1\frac{1}{3}$ (1)

$(4 \times 3) = 12$ (2)

$13\frac{1}{3}$ (3)

6. At the board, find $7 \times 2\frac{1}{2}$ first as in box B and then as in box C. Then commute the factors and work as in box D. See below.

D

Multiply

4

$3\frac{1}{3}$

$1\frac{1}{3}$ (1)

12 (2)

$13\frac{1}{3}$ (3)

[w]

Ex. 7–18. Copy and multiply as in box C or box D.

7. $2\frac{1}{2}$	8. $3\frac{2}{5}$	9. $3\frac{4}{5}$	10. $4\frac{2}{3}$	11. $2\frac{1}{7}$	12. $5\frac{2}{3}$
$\frac{4}{10}$	$\frac{5}{17}$	$\frac{2}{7\frac{3}{5}}$	$\frac{6}{28}$	$\frac{14}{30}$	$\frac{15}{85}$
13. 4	14. 6	15. 12	16. 30	17. 35	18. 48
$8\frac{1}{3}$	$5\frac{1}{2}$	$6\frac{2}{3}$	$23\frac{1}{5}$	$8\frac{2}{7}$	$5\frac{5}{12}$
$33\frac{1}{3}$	33	80	696	290	260

Ex. 19–24. Find the number represented by n.

19. $n = 38 \times 5\frac{1}{2}$ 209 **20.** $n = 160 \times 10\frac{3}{8}$ 1,660 **21.** $n = 6\frac{7}{15} \times 45$ 291

22. $\frac{3}{5} \times 25 = n$ 15 **23.** $n = 5\frac{1}{4} \times 200$ 1,050 **24.** $3\frac{1}{7} \times 10 = n$ $31\frac{3}{7}$

♦ **Extra Examples.** Sets 93–95.

6. $7 \times 2\frac{1}{2} = 7 \times (2 + \frac{1}{2})$
$= (7 \times 2) + (7 \times \frac{1}{2})$
$= 14 + \frac{7}{2}$
$= 14\frac{7}{2}$, or $17\frac{1}{2}$

$2\frac{1}{2}$ $\quad$ 7
7 $\quad$ $2\frac{1}{2}$
$3\frac{1}{2}$ $\quad$ $3\frac{1}{2}$
14 $\quad$ 14
$17\frac{1}{2}$ $\quad$ $17\frac{1}{2}$

*Notice that for Ex. 1, the question "how many hours" is answered "$10\frac{1}{2}$." If the question had been "how long," the answer would have been "$10\frac{1}{2}$ hours."

Solving Problems

[O]

*For each of Ex. 1–10, give a mathematical sentence which could be used to find the answer for the problem.

1. During one week, Alan studied his homework for $1\frac{3}{4}$ times as many hours as John. If John studied his homework for 6 hours that week, for how many hours did Alan study? $n = 1\frac{3}{4} \times 6$; $10\frac{1}{2}$

2. It took Alan 2 hours to read the science book assigned for that week. John read the same book in $\frac{3}{4}$ the time that it took Alan. How many hours did it take John to read the book? $n = 2 \times \frac{3}{4}$; $1\frac{1}{2}$

3. The Sewing Club is making aprons to sell at their annual bazaar. If each apron takes $1\frac{3}{8}$ yd. of material, how many yards are needed for 56 aprons? $n = 1\frac{3}{8} \times 56$; 77

4. Cardboard boxes for the aprons cost $2\frac{1}{2}$¢ each. What would be the total cost for 56 boxes? $n = 56 \times 2\frac{1}{2}$; 140¢, or \$1.40

5. Miss Young said that $\frac{7}{8}$ of the class had perfect attendance for the first two months of school. If the number of pupils enrolled was 40, how many had perfect attendance? $n = \frac{7}{8} \times 40$; 35

6. If Bob shells $2\frac{3}{4}$ lb. of walnuts per hour, how many pounds of walnuts can he shell in 6 hours? $n = 2\frac{3}{4} \times 6$; $16\frac{1}{2}$

7. If each ping-pong ball weighs $\frac{2}{25}$ oz., what is the weight of a dozen ping-pong balls? $n = \frac{2}{25} \times 12$; $\frac{24}{25}$ oz.

8. The weight of four $6\frac{1}{2}$-oz. softballs is how much greater than the weight of four $5\frac{1}{8}$-oz. baseballs? $n = \left(4 \times 6\frac{1}{2}\right) - \left(4 \times 5\frac{1}{8}\right)$; $5\frac{1}{2}$ oz.

9. The sum of $3\frac{1}{2}$ and $5\frac{1}{3}$ is how much greater than the product of 4 and $2\frac{1}{5}$? $n = \left(3\frac{1}{2} + 5\frac{1}{3}\right) - \left(4 \times 2\frac{1}{5}\right)$; $\frac{1}{30}$

10. The product of 24 and $6\frac{5}{6}$ is how much greater than the sum of $3\frac{5}{8}$ and $40\frac{3}{4}$? $n = \left(24 \times 6\frac{5}{6}\right) - \left(3\frac{5}{8} + 40\frac{3}{4}\right)$; $119\frac{5}{8}$

[W]

Now work each of Ex. 1–10.

◆ **Extra Problems.** Set 144.

Teaching Page 190

Pupil's Objective

To solve problems involving a factors-product relationship requiring multiplication of a whole number and a rational number that is not a whole number.

Background

Most of the problems on this page involve a factors-product relationship requiring the multiplication of a whole number and a rational number expressed either with a fraction or with a mixed form.

One of the steps in the problem-solving procedure is to estimate the answer. In estimating the answers to these problems, the pupils must be aware that when multiplying a whole number and a rational number that is not a whole number, the product will be (a) greater than the whole number when the other factor is greater than 1; (b) less than the whole number when the other factor is less than 1.

Pre-Book Lesson

• Review expressing with mathematical sentences factors-product relationships which require multiplication for their solution. Have the pupils suggest various ways of recognizing such a relationship in a problem.

• Ask how the product of a whole number and a rational number compares with the whole number when the rational number is (a) greater than 1; (b) less than 1; (c) equal to 1.

Using the Text Page

• Ex. 1–7 contain a factors-product relationship requiring multiplication.

• Ex. 8–9 contain an addends-sum relationship requiring subtraction, but the sum and known addend are themselves products or sums.

• If pupils have difficulty with Ex. 10, they may be given preliminary exercises which develop gradually the concept to be used in the problem. Ask,

a. What operation is used to find how much greater one number is than another?

b. How much greater than $3\frac{3}{4}$ is $17\frac{1}{2}$?

c. How much greater than $17\frac{1}{2}$ is the sum of $1\frac{1}{2}$ and $2\frac{3}{4}$? (It isn't.)

d. The product of 2 and $7\frac{1}{2}$ is how much greater than the sum of $2\frac{5}{6}$ and $1\frac{1}{5}$?

Individualizing Instruction

• Have the *more capable pupils* first estimate the answer for each of the problems before solving the mathematical sentence.

• Show on the board factors-product relationships involving a whole number and a rational number requiring multiplication, and have *all pupils* suggest problem situations described by these relationships.

• Give the *slower learners* meaningful problems which they might encounter in real-life situations, such as "If a recipe requires $2\frac{3}{4}$ cups of flour, how many cups of flour will be needed to make a double recipe?"

• Check the work of *all pupils* who obtain wrong answers for the problems to see if their error is in (a) determining the relationship expressed in the problem; (b) expressing the relationship as a mathematical sentence; (c) performing the operation required to solve the mathematical sentence. Some pupils may need more practice in multiplying a whole number and a rational number.

Teaching Page 191

Pupil's Objectives

(a) To have mixed computational practice with whole numbers and rational numbers expressed with fractions and decimals; (b) to practice solving mathematical sentences requiring addition or subtraction; and (c) to use the concept of multiplying a whole number and a mixed form to find the area of a rectangular region.

Background

Maintaining proficiency in computational skills requires frequent practice. The exercises on this page provide practice in adding and subtracting whole numbers and rational numbers expressed with fractions and decimals, and provide practice in multiplying and dividing whole numbers.

The exercises at the bottom of the page not only provide a review of finding areas of rectangular regions, but also permit the pupils to apply knowledge of multiplying a whole number and a rational number in finding the measures.

The exercises on this page may reveal weaknesses of some pupils in certain areas of computation which should be corrected with remedial work.

Pre-Book Lesson

- Review how to solve addends-sum relationships emphasizing how to determine whether addition or subtraction is required. That is, if the sum is unknown, add; but if an addend is unknown, subtract.
- Review meaning of rectangle, the measures of length and width of a rectangle, rectangular region, and how to find the area of a rectangular region.
- Include a brief review of the standard units of measurement of length and area, both English and metric units.

Using the Text Page

- One example of each type of exercise on this page may be used for oral work at the board before beginning the written work.
- The *slower learners* may be questioned on how to solve each type of exercise, to determine whether they need further review before beginning the written work.
- Have the pupils explain why it is necessary in Ex. 8–10 at the bottom of the page to rename one of the measurements. Have them suggest possibilities of renaming one or both of the measurements.
- All or part of these exercises may be used for testing to diagnose weaknesses.

Individualizing Instruction

Have *all pupils* restudy appropriate pages in this or previous chapters if certain weaknesses are apparent.

To Keep in Practice

[W]

Ex. 1–5. Copy and add.

1.	2.	3.	4.	5.
$35\frac{1}{2}$	$489\frac{4}{5}$	23.78	2.8873	\$45.88
$15\frac{3}{4}$	$63\frac{2}{3}$	4.77	15.0556	6.89
$51\frac{1}{4}$	$553\frac{7}{15}$	28.55	17.9429	3.75
				\$56.52

Ex. 6–10. Copy and subtract.

6.	7.	8.	9.	10.
2,784	$23\frac{5}{6}$	$508\frac{2}{7}$	23.607	\$950.75
1,774	$7\frac{2}{3}$	$66\frac{2}{3}$	8.993	14.88
1,010	$16\frac{1}{6}$	$441\frac{13}{21}$	14.614	\$935.87

Ex. 11–13. Multiply.

11.	12.	13.
352	1,497	\$5.98
38	18	16
13,376	26,946	\$95.68

Ex. 14–15. Divide.

14. $13\overline{)3{,}861}$ 297, R0

15. $47\overline{)36{,}782}$ 782, R28

Ex. 16–21. Solve.

16. $n = 23.67 + 34.78$ 58.45

17. $7.89 - n = 3.88$ 4.01

18. $x - 35.908 = 5.333$ 41.241

19. $n - 3\frac{5}{6} = 41\frac{1}{2}$ $45\frac{1}{3}$

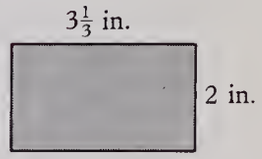
20. $x - 44 = 23\frac{2}{3}$ $67\frac{2}{3}$

21. $n = 23\frac{11}{16} - (3 + \frac{7}{8})$ $19\frac{13}{16}$

Finding Areas

Resurvey [O]

1. Give the dimensions of the rectangular region shown at the right. $3\frac{1}{3}$ in. and 2 in. To find the area of the region in terms of square inches, we multiply _?_ $3\frac{1}{3}$ by _?_ 2. At the board, find the area of this region. $6\frac{2}{3}$ sq. in.



[W]

Ex. 2–10. The length and width of a rectangular region are given. Find the area of the region. Remember to show the unit of area. You will have to rename one of the measurements in each of Ex. 8–10.

	Length	Width		Length	Width		Length	Width
2.	$23\frac{1}{2}$ in.	5 in. $117\frac{1}{2}$ sq. in.	5.	34 mi.	$4\frac{1}{6}$ mi. $141\frac{2}{3}$ sq. mi.	8.	13 ft.	4 in. $4\frac{1}{3}$ sq. ft., or 624 sq. in.
3.	$18\frac{2}{3}$ cm.	9 cm. 168 sq. cm.	6.	100 m.	$37\frac{3}{8}$ m. $3{,}737\frac{1}{2}$ sq. m.	9.	5 ft.	$1\frac{1}{2}$ yd. $2\frac{1}{2}$ sq. yd., or $22\frac{1}{2}$ sq. ft.
4.	24 yd.	$15\frac{3}{4}$ yd. 378 sq. yd.	7.	$41\frac{3}{5}$ in.	20 in. 832 sq. in.	10.	56 m.	31 cm. 17.36 (or $17\frac{9}{25}$) sq. m., or 173,600 sq. cm.

* The region model is effective for picturing products less than 1. The number-line picture, although slightly more complicated, is basically the same regardless of the size of the product.

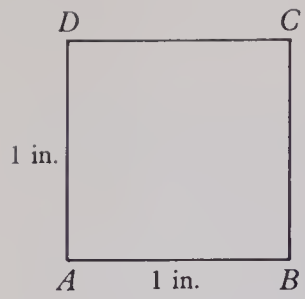


Fig. 1

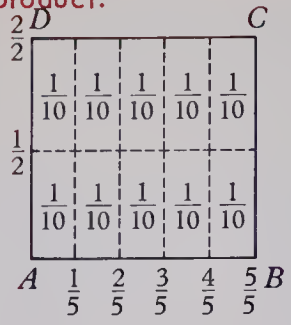


Fig. 2

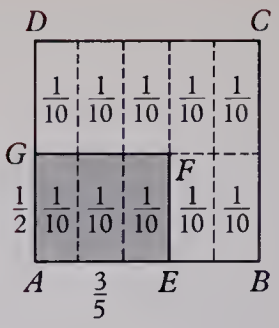


Fig. 3

Products of Rational Numbers

Resurvey [O]

1. Give the dimensions of the square region shown in Fig. 1. 1 in. by 1 in. The area of region $ABCD$ is _?_ 1 sq. in. Fig. 2 and Fig. 3 show how to use a region to find $\frac{1}{2} \times \frac{3}{5}$.

2. Fig. 2. Since the factor $\frac{1}{2}$ has a denominator of 2, side AD is shown partitioned into _?_ 2 congruent segments. Side AB is shown partitioned into _?_ 5 congruent segments. The factor $\frac{1}{5}$ has a denominator of 5. Why? ^ Into how many congruent regions has region $ABCD$ been partitioned? 10 The measure of the area of region $ABCD$ is 1, so the measure of each small region is _?_ $\frac{1}{10}$

3. Fig. 3. Is region $AEFG$ the union of three of the smaller regions? Yes ^ The measure of region $AEFG$ is _?_ $\frac{3}{5} \times \frac{1}{10}$, or $\frac{?}{10}$. 3

*4. The length of region $AEFG$ is $\frac{3}{5}$ in. and its width is _?_ $\frac{1}{2}$ in., so a product expression for the measure of its area is _?_ $\frac{3}{5} \times$ _?_ $\frac{1}{2}$. From Ex. 3, we know that the measure of the area is $\frac{3}{10}$, so it must be true that $\frac{3}{5} \times \frac{1}{2} = \frac{?}{10}$. 3

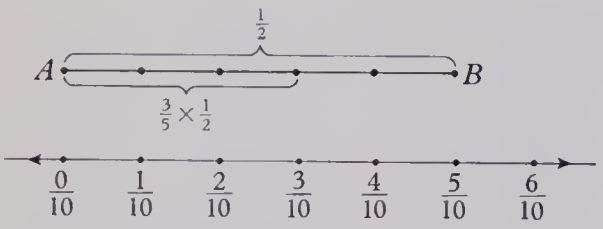
5. The number-line picture below shows $\frac{3}{5} \times \frac{1}{2}$.

a. Since the denominators are 5 and 2, the number line is shown scaled in _?_ $\frac{1}{10}$'s What is the measure of $\overline{AB}$? $\frac{1}{2}$

b. Think of $\overline{AB}$ as a unit. To find $\frac{3}{5}$ of $\overline{AB}$ we partition $\overline{AB}$ into _?_ 5 segments.

c. The 3 in $\frac{3}{5}$ indicates that we are considering _?_ 3 of the congruent segments.

d. The measure of the segment for $\frac{3}{5} \times \frac{1}{2}$ is _?_ $\frac{3}{10}$ $\frac{3}{5} \times \frac{1}{2} =$ _?_ $\frac{3}{10}$



Teaching Pages 192 and 193

Pupil's Objectives

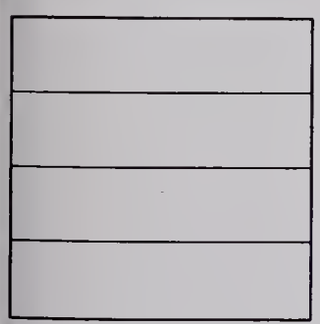
(a) To review the meaning of the product of two rational numbers; and (b) to have practice in multiplying two rational numbers.

Background

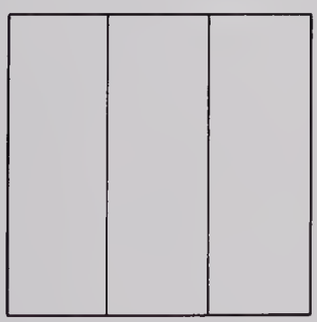
If $\frac{a}{b}$ and $\frac{c}{d}$ represent two rational numbers, their product is represented by $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$. To give the multiplication of rational numbers meaning for the pupil, you may use various models. One such model is the square-unit region. To represent the multiplication of two rational numbers expressed with fractions, a square-unit region is shown partitioned horizontally according to the denominator of the multiplier (shown by the first fraction—the one on the left) and vertically according to the denominator of the rational number being multiplied (shown by the second fraction—the one on the right). The number of resulting parts will represent the denominator of the product of the two rational numbers.

To use a model of a square-unit region for the multiplication $\frac{3}{4} \times \frac{2}{3}$, the region is shown partitioned horizontally according to the denominator of the multiplier, $\frac{3}{4}$ (see A);

A

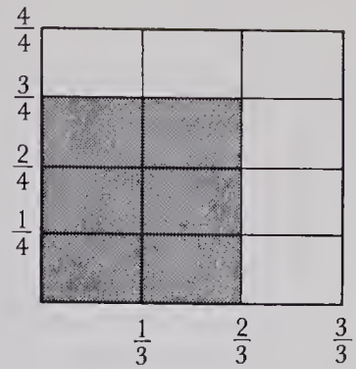


B



and then vertically according to the denominator of the second factor, $\frac{2}{3}$ (see B). The region is now shown partitioned into 12 congruent regions, the area of each being $\frac{1}{12}$ in square units. The product for $\frac{3}{4} \times \frac{2}{3}$ can be suggested by showing the area of a region whose length is $\frac{3}{4}$ unit and whose width is $\frac{2}{3}$ unit (see C).

C



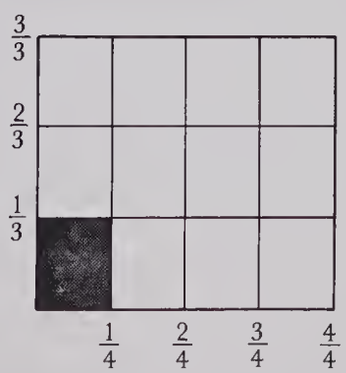
From the diagram it can be seen that the area of such a rectangular region is $\frac{6}{12}$ unit. Therefore, $\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$, or $\frac{1}{2}$.

If a unit region is partitioned vertically according to the denominator of one fraction and horizontally according to the denominator of the other fraction, the number of congruent regions formed by the partitioning will always be equal to the product of the two denominators. In the multiplication $\frac{a}{b} \times \frac{c}{d}$, the denominator will always be equal to the number represented by $b \times d$.

If the product represented by $\frac{a}{b} \times \frac{c}{d}$ is suggested by the area of a region whose length is $\frac{a}{b}$ of a vertical unit and whose width is $\frac{c}{d}$ of a horizontal unit, the length of the resulting region will be a vertical segments and the width c horizontal segments. The area will be $a \times c$ subdivisions, each of which has an area of $\frac{1}{b \times d}$ unit region. Therefore, $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$.

Pre-Book Lesson

• Have pupils draw a model of a square foot on the board. Have them partition it vertically to show 4 congruent regions and then horizontally to show 3 congruent regions.



Ask them to find the area of the shaded rectangle whose length is $\frac{1}{3}$ ft. and whose width is $\frac{1}{4}$ ft.

• Use this same procedure to have pupils find the area of a rectangle whose dimensions are (a) length $\frac{1}{6}$ ft., width $\frac{1}{12}$ ft.; (b) length $\frac{1}{2}$ ft., width $\frac{1}{4}$ ft.; (c) length $\frac{1}{3}$ ft., width $\frac{1}{6}$ ft. Lead pupils to the discovery that $\frac{1}{a} \times \frac{1}{b} = \frac{1}{a \times b}$.

Using the Text Pages

• After Ex. 4, in order to reinforce the concept presented by Ex. 1–4, give more examples in using a model of a square-unit region for these multiplications: (a) $\frac{2}{3} \times \frac{3}{4}$; (b) $\frac{2}{5} \times \frac{5}{6}$; (c) $\frac{3}{5} \times \frac{5}{8}$. Some of the pupils already may be able to see that the multiplication represented by $\frac{a}{b} \times \frac{c}{d}$ is always equal to $\frac{a \times c}{b \times d}$.

• In Ex. 5–6, have pupils realize that $\frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$ and that $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$, arrived at through the use of number-line pictures, can also be verified by use of models of unit regions. Make sure that pupils understand about the unit to be shown on the number-line picture. Since the basic unit of the multiplier (Ex. 5) is $\frac{1}{5}$ for $\frac{3}{5} \times \frac{1}{2}$ ($\frac{1}{4}$ for $\frac{3}{4} \times \frac{1}{2}$ in Ex. 6), $\frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$ so the product will be a multiple of $\frac{1}{10}$. Pupils, therefore, will partition the number-line picture to show $\frac{1}{10}$'s ($\frac{1}{8}$'s for Ex. 6).

• In Ex. 7, $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$ can be given greater meaning for the *more capable pupils* by asking questions like the following:

a. In finding the product for $\frac{a}{b} \times \frac{c}{d}$ by use of a square-unit region, into how many regions will the square-unit region be partitioned? $(b \times d)$

b. What will be the area of each of the small regions shown? ($\frac{1}{b \times d}$ square unit)

c. If the area of a rectangular region whose length is $\frac{a}{b}$ unit and whose width is $\frac{c}{d}$ unit is used to suggest $\frac{a}{b} \times \frac{c}{d}$, this region will be composed of how many of the small regions? $(a \times c)$

d. If the rectangular region is composed of $a \times c$ small regions each of which has an area of $\frac{1}{b \times d}$ square unit, what will be the area of the rectangular region? ($\frac{a \times c}{b \times d}$)

• In connection with Ex. 25, ask if anyone knows what this illustrates. It illustrates that multiplication distributes over subtraction.

Individualizing Instruction

• *All pupils* may be asked if they can explain why the product for each of Ex. 10–19 is less than either of the two factors.

• Suggest to the *more capable pupils* that they attempt to extend the concept of multiplying rational numbers to those rational numbers where the numerator is greater than the denominator and try to verify their answers by the use of square-unit regions.

• Have *all pupils* suggest real-life situations that require multiplication of rational numbers—such as cutting in halves a recipe that calls for $\frac{3}{4}$ of a cup of sugar.

• If there are any pupils who have difficulty understanding that $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$, the following may be of some help, using either numerals or letters:

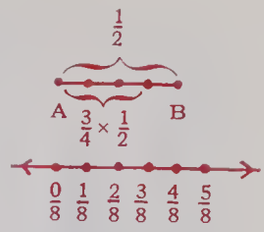
$$\begin{aligned} \frac{a}{b} \times \frac{c}{d} &= a \times \frac{1}{b} \times c \times \frac{1}{d} \\ &= a \times c \times \frac{1}{b} \times \frac{1}{d} \\ &= (a \times c) \times \frac{1}{b \times d} \\ &= \frac{a \times c}{b \times d} \end{aligned}$$

Using numerals:

$$a = 2;\ b = 3;\ c = 4;\ d = 5$$
$$\begin{aligned} \frac{2}{3} \times \frac{4}{5} &= 2 \times \frac{1}{3} \times 4 \times \frac{1}{5} \\ &= 2 \times 4 \times \frac{1}{3} \times \frac{1}{5} \\ &= (2 \times 4) \times \frac{1}{3 \times 5} \\ &= \frac{2 \times 4}{3 \times 5} \end{aligned}$$

$$a = 9;\ b = 7;\ c = 13;\ d = 17$$
$$\begin{aligned} \frac{9}{7} \times \frac{13}{17} &= 9 \times \frac{1}{7} \times 13 \times \frac{1}{17} \\ &= 9 \times 13 \times \frac{1}{7} \times \frac{1}{17} \\ &= (9 \times 13) \times \frac{1}{7 \times 17} \\ &= \frac{9 \times 13}{7 \times 17} \end{aligned}$$

6. On the board, draw a number-line picture as in Ex. 5 to show $\frac{3}{4} \times \frac{1}{2}$. Follow the directions in Ex. a–c.



a. Since the denominators are 4 and 2, label the number line with a scale of eighths.

b. Show a segment AB for $\frac{1}{2}$ above the number-line picture. Show $\overline{AB}$ partitioned into 4 congruent segments.

c. Label the segment for the product $\frac{3}{4} \times \frac{1}{2}$. From the scale shown on the number line, the measure of the segment for $\frac{3}{4} \times \frac{1}{2}$ is $\_?\_$ $\frac{3}{8}$. $\frac{3}{4} \times \frac{1}{2} = \_?\_$ $\frac{3}{8}$

7. The work in the box shows another way to find $\frac{3}{4} \times \frac{1}{2}$. Explain the work shown. The product is found by multiplying numerators and denominators.

$$\frac{3}{4} \times \frac{1}{2} = \frac{3 \times 1}{4 \times 2} = \frac{3}{8}$$

If $\frac{a}{b}$ and $\frac{c}{d}$ represent rational numbers, then

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}.$$

8. At the board, find fractions in simplest form for Ex. a–d. Work as in the box.

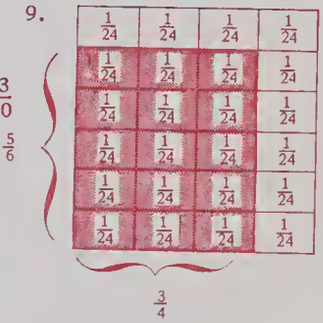
a. $\frac{5}{6} \times \frac{3}{4}$ $\frac{5}{8}$ **b.** $\frac{5}{7} \times \frac{2}{9}$ $\frac{10}{63}$ **c.** $\frac{3}{8} \times \frac{5}{12}$ $\frac{5}{32}$ **d.** $\frac{3}{10} \times \frac{11}{20}$ $\frac{33}{200}$

9. On the board, shade part of a model of a square unit region to represent $\frac{3}{4} \times \frac{5}{6}$ and then find $\frac{3}{4} \times \frac{5}{6}$ as in the box.

$\frac{3}{4} \times \frac{5}{6} = \frac{3 \times 5}{4 \times 6} = \frac{15}{24} = \frac{5}{8}$

[W]

9.



10. Represent $\frac{2}{3} \times \frac{3}{4}$ by shading part of a model of a square unit region. Name the product with a fraction in simplest form. $\frac{1}{2}$

Ex. 11–19. Solve. Express your answer in simplest form.

11. $n = \frac{2}{3} \times \frac{1}{9}$ $\frac{2}{27}$

12. $n = \frac{5}{6} \times \frac{1}{2}$ $\frac{5}{12}$

13. $n = \frac{2}{3} \times \frac{3}{8}$ $\frac{1}{4}$

14. $\frac{3}{5} \times \frac{1}{8} = n$ $\frac{3}{40}$

15. $x = \frac{1}{6} \times \frac{2}{3}$ $\frac{1}{9}$

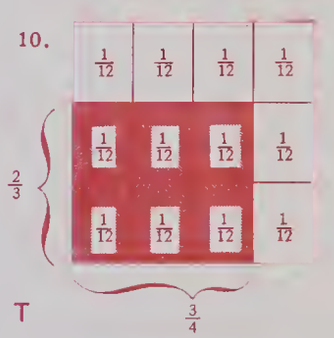
16. $\frac{3}{4} \times \frac{7}{8} = x$ $\frac{21}{32}$

17. $n = \frac{4}{5} \times \frac{5}{6}$ $\frac{2}{3}$

18. $x = \frac{5}{8} \times \frac{3}{10}$ $\frac{3}{16}$

19. $n = \frac{5}{7} \times \frac{7}{20}$ $\frac{1}{4}$

10.



Ex. 20–25. Write T or F.

20. $\frac{3}{5} = \frac{3}{10} \times 2$ T **21.** $\frac{3}{7} < \frac{2}{7} \times \frac{1}{7}$ F **22.** $\frac{5}{8} = \frac{1}{2} + (\frac{1}{16} \times 2)$ T

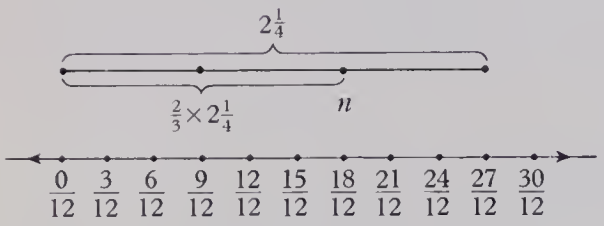
23. $5 \times \frac{4}{3} < 5 + \frac{4}{3}$ F **24.** $10 \times \frac{3}{8} < 10 + \frac{3}{8}$ T **25.** $\frac{3}{4} \times (2 - \frac{3}{8}) = (\frac{3}{4} \times 2) - (\frac{3}{4} \times \frac{3}{8})$ T

◆ **Extra Examples.** Sets 96–97.

Multiplying with Mixed Forms

[O]

Mr. Hall owns a 10-acre farm. Last year, he used $2\frac{1}{4}$ A. for tomatoes, $\frac{4}{5}$ A. for potatoes, and $1\frac{3}{4}$ A. for sweet corn. This year, he plans to use $\frac{2}{3}$ as many acres for tomatoes, $1\frac{1}{2}$ times as many acres for potatoes, and $3\frac{1}{2}$ times as many acres for sweet corn.



1. Can we use $n = \frac{2}{3} \times 2\frac{1}{4}$ to find how many acres he plans to use for tomatoes this year? Yes Does the number-line picture at the left show this relationship? Yes Explain how the number-line picture was drawn. $n = ?\frac{18}{12}$, or $1\frac{1}{2}$

Key idea: A number line scaled in $\frac{1}{12}$'s is shown and a segment measuring $2\frac{1}{4}$ is partitioned into 3 congruent segments.

A

$$n = \frac{2}{3} \times 2\frac{1}{4}$$
$$= \frac{2}{3} \times \frac{9}{4}$$
$$= \frac{2 \times 9}{3 \times 4}$$
$$= \frac{18}{12}$$
$$n = 1\frac{6}{12}, \text{ or } 1\frac{1}{2}$$

2. The work in box A shows another way to solve the mathematical sentence $n = \frac{2}{3} \times 2\frac{1}{4}$. Before multiplying, the factor $2\frac{1}{4}$ was renamed $\frac{9}{?_4}$. Explain the work.

3. In box A, $\frac{18}{12}$ was renamed $1\frac{?_6}{12}$. Instead, could $\frac{18}{12}$ first have been renamed with a fraction in simplest form and then with a mixed form? Yes $\frac{18}{12} = \frac{3}{?_2} = 1\frac{?_1}{2}$

4. At the board, solve each of Ex. a–c as in box A.

a. $n = \frac{4}{5} \times 2\frac{2}{3}$
$= \frac{4}{5} \times \frac{8}{3}$
$= \frac{4 \times 8}{5 \times 3}$
$= \frac{32}{15}$
$n = 2\frac{2}{15}$

b. $n = \frac{1}{7} \times 5\frac{1}{5}$
$= \frac{1}{7} \times \frac{26}{5}$
$= \frac{1 \times 26}{7 \times 5}$
$n = \frac{26}{35}$

c. $n = \frac{3}{8} \times 3\frac{1}{4}$
$= \frac{3}{8} \times \frac{13}{4}$
$= \frac{3 \times 13}{8 \times 4}$
$= \frac{39}{32}$
$n = 1\frac{7}{32}$

Teaching Pages 194 and 195

Pupil's Objectives

(a) To learn to multiply with mixed forms; and (b) to have practice solving mathematical sentences requiring multiplication of rational numbers shown in mixed form.

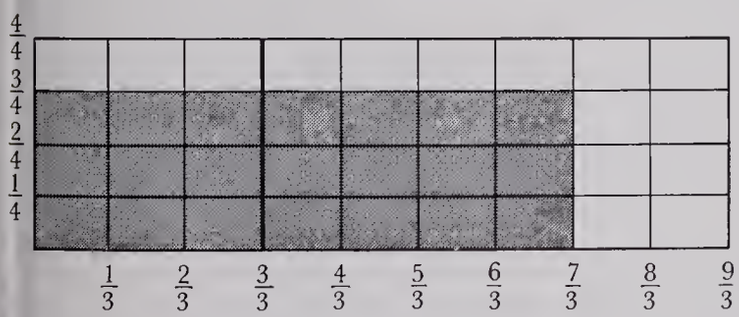
Background

The product of two rational numbers is represented by $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$, where $\frac{a}{b}$ and $\frac{c}{d}$ represent any rational numbers, including those whose numerator is greater than the denominator. To multiply with a mixed form, we first rename from the mixed form to a fraction, and then proceed with the multiplication as taught on the previous page.

(1) $\frac{2}{3} \times 1\frac{4}{7} = \frac{2}{3} \times \frac{11}{7}$
$= \frac{2 \times 11}{3 \times 7}$
$= \frac{22}{21}$, or $1\frac{1}{21}$

(2) $1\frac{2}{5} \times 6\frac{1}{2} = \frac{7}{5} \times \frac{13}{2}$
$= \frac{7 \times 13}{5 \times 2}$
$= \frac{91}{10}$, or $9\frac{1}{10}$

The use of a square-unit region as a model for showing multiplication of rational numbers may be extended to the multiplication of rational numbers named with mixed forms. The model for $\frac{3}{4} \times 2\frac{1}{3}$ would be as shown below.



Since one of the factors is greater than 2 and less than 3, it is necessary to show 3 square-unit regions for the model and to rename from the mixed form to a fraction.

$$\frac{3}{4} \times 2\frac{1}{3} = \frac{3}{4} \times \frac{7}{3}$$

Each of the square-unit regions is shown partitioned horizontally according to the denominator of one of the factors and vertically according to the denominator of the other factor. Each of the square-unit regions thus is shown partitioned into the number of smaller regions equal to the product of the denominators of the two factors, 4×3, or 12. The area of each of these smaller regions is $\frac{1}{12}$ square unit. The shaded portion in the diagram shows the rectangular region the measure of whose area is the product $\frac{3}{4} \times \frac{7}{3}$. From the diagram, it can be seen that the area consists of 3×7, or 21, smaller regions, each of whose area is $\frac{1}{12}$ square unit. Therefore, $\frac{3}{4} \times \frac{7}{3} = \frac{21}{12}$, or $1\frac{3}{4}$.

A number-line model is simpler to use for such multiplication. To use a number-line model for the multiplication $\frac{2}{3} \times 2\frac{1}{4}$, we think of the multiplication as $\frac{2}{3} \times \frac{9}{4}$ and consider that, since the basic unit of the multiplier is $\frac{1}{3}$, $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$ and so the product of $\frac{2}{3} \times \frac{9}{4}$ is a multiple of $\frac{1}{12}$. Therefore, the number-line model will be partitioned to show $\frac{1}{12}$'s and the multiplication is considered as $2 \times \frac{1}{3} \times \frac{9}{4}$, or $\frac{18}{12}$, or $1\frac{1}{2}$. The pupil's page shows a model for a multiplication of this type.

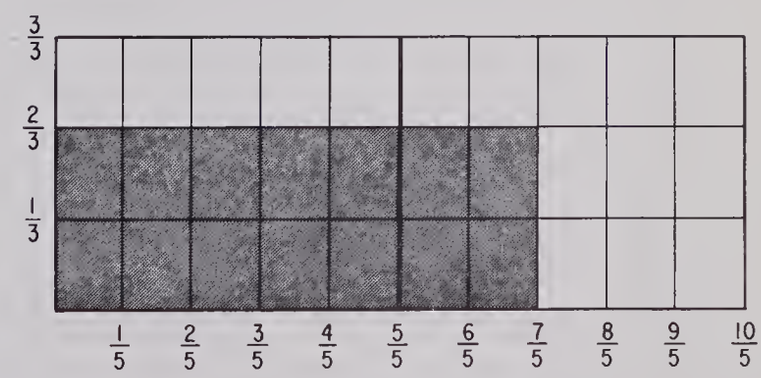
Pre-Book Lesson

• Ask for examples from real-life situations in which it would be necessary to find the product of two rational numbers shown by a fraction and a mixed form. Some possible examples are:

a. How much flour is needed to make one half of a batch of cookies if the recipe calls for $2\frac{3}{4}$ cups of flour for a whole batch?

b. A recipe calls for $\frac{2}{3}$ cup of sugar for one batch of cookies. How much sugar is needed for $2\frac{1}{2}$ batches?

• Ask what multiplication the following diagram suggests.



Using this diagram, have pupils find the product of $\frac{2}{3} \times \frac{7}{5}$. Ask if this suggests how the multiplication $\frac{2}{3} \times 1\frac{2}{5}$ might be performed.

Using the Text Pages

• For Ex. 1, the basic unit of the multiplier is $\frac{1}{3}$ and, for the multiplication $\frac{2}{3} \times 2\frac{1}{4}$, we think of $2\frac{1}{4}$ as $\frac{9}{4}$. Since $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$, the product of $\frac{2}{3} \times \frac{9}{4}$ will be a multiple of $\frac{1}{12}$ and the number-line model will be separated into $\frac{1}{12}$'s. We can think of $\frac{9}{4}$ as $\frac{27}{12}$ and find $\frac{2}{3}$ of $\frac{27}{12}$. This is twice $\frac{1}{3}$ of $\frac{27}{12}$. $\frac{1}{3}$ of 27 twelfths = 9 twelfths, so $\frac{2}{3}$ of 27 twelfths = 18 twelfths, which may be shown as $\frac{18}{12}$ or $1\frac{1}{2}$. It is not intended that the pupil use this procedure in multiplying with mixed forms, but it may give more meaning to such a multiplication.

• The pupils should be led to the realization that the concept of multiplying with mixed forms is a simple extension of

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

where $\frac{a}{b}$ and $\frac{c}{d}$ represent any two rational numbers, including those whose numerator is greater than the denominator. The pupils are not so much learning a new concept as they are extending a concept learned earlier.

Individualizing Instruction

• The *more capable pupils* may be given exercises similar to those on text page 195 and be asked to do them without using pencil and paper.

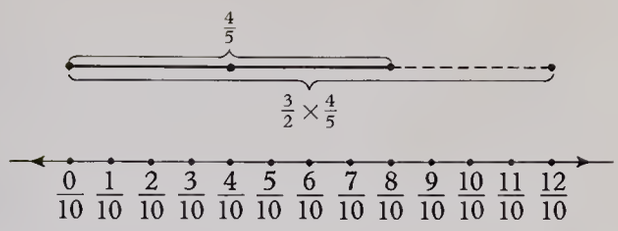
• If some of the *slower learners* have difficulty with any of these exercises, some of the *more capable pupils* may volunteer to go to the board and explain how the multiplication is shown.

• *All pupils* may be asked if they can tell how the product of two rational numbers both of which are greater than 1 compares with either of the factors. (The product is always greater than either factor.)

• Ask *all pupils* to write problems requiring the use of multiplication with mixed forms for solution.

NOTES

5. To find how many acres Mr. Hall plans to use for potatoes this year, we multiply $\frac{4}{5}$ by _?$1\frac{1}{2}$_.



a. The 2 in $1\frac{1}{2}$, or $\frac{3}{2}$, indicates that we partition the segment for $\frac{4}{5}$ into _?2_ congruent segments.

b. The 3 in $\frac{3}{2}$ indicates that we consider _?3_ such segments.

c. The measure of the segment for $\frac{3}{2} \times \frac{4}{5}$ is _?$\frac{12}{10}$_. $\frac{3}{2} \times \frac{4}{5} =$ _?$\frac{12}{10}$, or $1\frac{2}{5}$

B $n = 1\frac{1}{2} \times \frac{4}{5} = \frac{3}{2} \times \frac{4}{5} = \frac{12}{10} = 1\frac{1}{5}$

6. Explain the work in box B for finding $1\frac{1}{2} \times \frac{4}{5}$. $\frac{12}{10} = \frac{?}{5} = 1\frac{1}{5}$ $1\frac{1}{2}$ is renamed with a fraction before the product is found.

7. At the board, solve each of Ex. a–c as in box B.

a. $n = 1\frac{2}{3} \times \frac{1}{4}$ **b.** $n = 2\frac{1}{5} \times \frac{1}{3}$ **c.** $n = 4\frac{2}{7} \times \frac{2}{3}$

7. a. $= \frac{5}{3} \times \frac{1}{4} = \frac{5}{12}$ $= \frac{30}{7} \times \frac{2}{3} = \frac{60}{21} = 2\frac{6}{7}$

8. Box C. Can we use the mathematical sentence shown to find how many acres Mr. Hall will plant in sweet corn? Yes Explain the work shown. He plans to use _?$6\frac{1}{8}$_ A. for sweet corn. $3\frac{1}{2}$ and $1\frac{3}{4}$ are renamed with fractions before their product is found.

C
$$n = 3\tfrac{1}{2} \times 1\tfrac{3}{4}$$
$$= \tfrac{7}{2} \times \tfrac{7}{4}$$
$$= \tfrac{49}{8}, \text{ or } 6\tfrac{1}{8}$$

7. b. $= \frac{11}{5} \times \frac{1}{3} = \frac{11}{15}$

9. At the board, solve each of Ex. a–c as in box C.

a. $n = 7\frac{1}{2} \times 1\frac{3}{4}$ $13\frac{1}{8}$ **b.** $n = 3\frac{2}{7} \times 3\frac{1}{3}$ $10\frac{20}{21}$ **c.** $n = 2\frac{5}{8} \times 4\frac{1}{9}$ $10\frac{19}{24}$

10. Mr. Hall plans to use $1\frac{1}{2}$ A. for tomatoes, $1\frac{1}{5}$ A. for potatoes, and $6\frac{1}{8}$ A. for sweet corn. How many acres is that in all? $8\frac{33}{40}$

[W]

Ex. 11–25. Find the number represented by the letter.

11. $n = \frac{1}{3} \times 2\frac{5}{8}$ $\frac{7}{8}$ **16.** $n = 1\frac{3}{8} \times 6\frac{2}{9}$ $8\frac{5}{9}$ **21.** $n = 2\frac{1}{10} \times 2\frac{1}{4}$ $4\frac{29}{40}$

12. $n = \frac{4}{5} \times 3\frac{3}{4}$ 3 **17.** $x = 12 \times 1\frac{4}{5}$ $21\frac{3}{5}$ **22.** $x = 6\frac{2}{3} \times \frac{5}{6}$ $5\frac{5}{9}$

13. $n = 4\frac{1}{2} \times \frac{5}{8}$ $2\frac{13}{16}$ **18.** $34 \times 5\frac{1}{3} = x$ $181\frac{1}{3}$ **23.** $\frac{4}{5} \times 12\frac{2}{5} = n$ $9\frac{23}{25}$

14. $n = 3\frac{2}{3} \times \frac{3}{5}$ $2\frac{1}{5}$ **19.** $5\frac{3}{5} \times 2\frac{5}{6} = n$ $15\frac{13}{15}$ **24.** $x = 7\frac{1}{7} \times 3\frac{1}{3}$ $23\frac{17}{21}$

15. $n = 3\frac{1}{3} \times 2\frac{1}{4}$ $7\frac{1}{2}$ **20.** $14\frac{1}{2} \times 7 = x$ $101\frac{1}{2}$ **25.** $x = 17 \times 4\frac{3}{10}$ $73\frac{1}{10}$

◆ **Extra Examples.** Set 98.

* Point out that this is an algorithm – simply a way of recording with numerals. Most important is understanding the reasoning behind the algorithm.

A	B	C
$\frac{3}{5} \times \frac{2}{3} = \frac{3 \times 2}{5 \times 3} = \frac{6}{15} = \frac{2}{5}$	$\frac{3}{5} \times \frac{2}{3} = \frac{\cancel{3}^{1} \times 2}{5 \times \cancel{3}_{1}} = \frac{2}{5}$	$\frac{1}{\cancel{3}} \times \frac{2}{\cancel{3}_{1}} = \frac{2}{5}$

Multiplying with Fractions

Dividing a common factor [O]

1. Box A. Explain the work shown for finding $\frac{3}{5} \times \frac{2}{3}$. Is 3 a common factor of 6 and 15? Yes

2. Is 3 a common factor of 3×2 and 5×3? Yes Then, before multiplying, can we divide both the numerator and the denominator of $\frac{3 \times 2}{5 \times 3}$ by 3? Yes $(3 \times 2) \div 3 = (2 \times 3) \div 3 = 2 \times (3 \div 3) = 2 \times 1 = 2$ On the board, show a similar explanation for dividing the denominator by 3. $(5 \times 3) \div 3 = 5 \times (3 \div 3) = 5 \times 1 = 5$

* 3. Box B. To show the division by 3, we crossed out the numeral 3 both above the fraction bar and below the fraction bar. So, $\frac{3}{5} \times \frac{2}{3} = \frac{1 \times 2}{5 \times 1} = \frac{2}{5}$.

4. Box C. Which step in box B is done mentally in box C? The second step Is the work in box C shorter than the work in box A? Yes box B? Yes

D	E
$\frac{\cancel{4}^{1}}{\cancel{15}_{1}} \times \frac{\cancel{15}^{1}}{\cancel{16}_{4}} = \frac{1}{4}$	$n = \frac{4}{5} \times 2\frac{1}{8}$ $= \frac{\cancel{4}^{1}}{5} \times \frac{17}{\cancel{8}_{2}}$ $= \frac{17}{10}$, or $1\frac{7}{10}$

5. On the board, work Ex. a–c first as in box B and then as in box C.

a. $\frac{3}{8} \times \frac{5}{3}$ $\frac{5}{8}$ b. $\frac{2}{7} \times \frac{3}{2}$ $\frac{3}{7}$ c. $\frac{13}{17} \times \frac{17}{20}$ $\frac{13}{20}$

6. Explain the work in box D for finding $\frac{4}{15} \times \frac{15}{16}$. Key idea: Numerator and denominator were divided by their common factors. Then the product was found.

7. On the board, work Ex. a–f as in box D.

a. $\frac{2}{21} \times \frac{7}{4}$ $\frac{1}{6}$ b. $\frac{6}{35} \times \frac{15}{18}$ $\frac{1}{7}$ c. $\frac{30}{13} \times \frac{26}{27}$ $2\frac{2}{9}$

d. $\frac{3}{5} \times \frac{5}{3}$ 1 e. $\frac{7}{8} \times \frac{4}{21}$ $\frac{1}{6}$ f. $\frac{15}{16} \times \frac{5}{8}$ $\frac{75}{128}$

8. Box E. To solve $n = \frac{4}{5} \times 2\frac{1}{8}$, can we first rename $2\frac{1}{8}$ with the fraction $\frac{17}{8}$? Yes Explain the work. Key idea: After the renaming, numerator and denominator were divided by their common factor. Then the product was found.

[W]

Ex. 9–17. Solve. Work as in box E.

9. $n = \frac{3}{10} \times \frac{15}{17}$ $\frac{9}{34}$

10. $x = 8\frac{3}{8} \times \frac{24}{67}$ 3

11. $y = 3\frac{1}{2} \times \frac{2}{5}$ $1\frac{2}{5}$

12. $n = 3\frac{1}{2} \times 2\frac{2}{3}$ $9\frac{1}{3}$

13. $n = 3\frac{4}{5} \times 6\frac{2}{3}$ $25\frac{1}{3}$

14. $4\frac{2}{3} \times 1\frac{3}{5} = x$ $7\frac{7}{15}$

15. $5\frac{1}{5} \times 6\frac{1}{4} = x$ $32\frac{1}{2}$

16. $60 \times 3\frac{1}{10} = n$ 186

17. $n = \frac{2}{5} \times \frac{3}{4}$ $\frac{3}{10}$

Teaching Page 196

Pupil's Objective

To discover a way of renaming rational numbers in order to simplify the process of multiplying with fractions.

Background

Two fractions, $\frac{a}{b}$ and $\frac{c}{d}$, name the same rational number if and only if $a \times d = b \times c$. This is sometimes called the "cross-products" test. Consider $\frac{a \times c}{b \times c}$, which shows the non-zero whole-number factor c common to both numerator and denominator. Using the cross-products test, we can see that $\frac{a \times c}{b \times c} = \frac{a}{b}$ because $a \times c \times b = b \times c \times a$. This means that if the numerator and the denominator named by a fraction contain a common factor, this common factor may be divided out when the number is named in simplest form.

$$\frac{4}{5} \times \frac{3}{4} = \frac{\overset{1}{\cancel{4}} \times 3}{5 \times \underset{1}{\cancel{4}}} = \frac{3}{5}$$

In the example above, division of numerator and denominator by the common factor 4 is shown by twice striking out the numeral 4—once above the fraction bar and once below it.

Pre-Book Lesson

• Present an example in multiplication of two rational numbers for which the answer is a rational number that can be renamed in simpler form. Through careful questioning, lead the pupils to the discovery of the advantage of dividing numerator and denominator by their common factor when they are shown in a product expression rather than when they are shown in final fraction form.

• Use questions such as:

a. What is the product for $\frac{2}{3} \times \frac{3}{4}$? $(\frac{6}{12})$

b. How should this answer be expressed? (in simplest form)

c. How can this product be renamed to show it in simplest form? (Numerator and denominator are divided by their common factor.)

d. Could this dividing of numerator and denominator by their common factor have been performed before the product was shown in fraction form? (Yes)

e. What is the advantage of this procedure? (It is easier to see what the common factor is.)

Using the Text Page

• In Ex. 3, when dividing the numerator and the denominator by 3, pupils should be certain to indicate each quotient, as shown in box B, even when the quotient is 1. To emphasize this point, put on the board incorrect examples such as

a. $\frac{\cancel{5}}{\cancel{5}} \times \frac{7}{\underset{2}{\cancel{10}}} = \frac{0}{2} = 0$ b. $\frac{\cancel{5}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{5}} = 0$

and ask the pupils to describe the errors. This will point out the need for showing the quotient even when it is 1.

• Notice that on these pages the traditional term "cancel" does not appear. There is no such operation as "canceling." The mathematical operation being indicated is division.

Individualizing Instruction

• The *more capable pupils* may be given the definition of equivalent fractions in some form such as "$\frac{a}{b}$ and $\frac{c}{d}$ name the same rational number if and only if $a \times d = b \times c$." Then ask these pupils to determine if the sentence $\frac{a \times c}{b \times c} = \frac{a}{b}$ is true using the cross-products test. After they prove the sentence true, ask for the mathematical justification of dividing both numerator and denominator of a rational number by their common factor (see Background section).

• Give *all pupils* exercises such as $n = \frac{39}{49} \times \frac{7}{13}$ to be solved first by the method shown in box A and then by the method shown in box C. Using the method of box A, the product may be difficult to show in simplest form.

$$n = \frac{39}{49} \times \frac{7}{13} = \frac{39 \times 7}{49 \times 13}$$
$$= \frac{273}{637} = \text{\_ ? \_}$$

However, using the method shown in box C, the multiplication involved is much simpler:

$$n = \frac{\overset{3}{\cancel{39}}}{\underset{7}{\cancel{49}}} \times \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{13}}} = \frac{3}{7}$$

Pupil's Objective

To discover how to find the product of more than two rational numbers.

Background

Multiplication is a binary operation. To determine the product of three numbers, the product of two of the numbers is determined first and then the product of that product and the remaining factor must then be determined. Since multiplication is associative, it makes no difference whether the third factor is multiplied by the product of the first two factors shown or the product of the second and third factors shown is multiplied by the first. The result will always be the same.

$$\frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right) = \frac{a}{b} \times \left(\frac{c \times e}{d \times f}\right)$$
$$= \frac{a \times c \times e}{b \times d \times f}$$

From this we see that the numerator of the final product is the product of the numerators of the three factors, and the denominator of the final product is the product of the denominators of the three factors.

Since $\frac{a}{b} \times \frac{c}{d} \times \frac{e}{f} = \frac{a \times c \times e}{b \times d \times f}$, dividing the numerator of any one of the three factors and the denominator of any one of the three factors by a common factor is equivalent to dividing the numerator and the denominator of the final product by the common factor. The work in box C and in box D on the text page show how to apply this principle when multiplying three rational numbers.

Pre-Book Lesson

• Have the *more capable pupils* perform the multiplication represented by $\left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f}$ and the multiplication represented by $\frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right)$ and compare the results.

• Review the concept that a whole number is a rational number whose numerator is the given whole number and whose denominator is 1. The whole number a is equal to the rational number $\frac{a}{1}$, and a whole number may be renamed with a fraction. In presenting this idea, you may wish to give the pupils exercises such as finding and comparing the product for $\frac{2}{3} \times 9$ and the product for $\frac{2}{3} \times \frac{9}{1}$.

Using the Text Page

• Leave the work of Ex. 1 on the board while pupils do Ex. 2 so that they can compare the work in box A with the work on the board.

• After Ex. 2 ask the question, "If any numerator and any denominator of the three factors have a common factor, why is it all right to divide the numerator and denominator of the product by this common factor?"

• After Ex. 3 is completed, rewrite the exercise as $\frac{2}{5} \times \frac{1}{7} \times \frac{3}{10}$ to show that the order of the factors has been changed. Ask if

$$\frac{\overset{1}{\cancel{2}}}{5} \times \frac{1}{7} \times \frac{3}{\underset{5}{\cancel{10}}} = \frac{3}{175}$$

is correct. Point out that the common factor need not be contained in adjacent factors. If *any* of the numerators has a factor in common with *any* of the denominators then that will be a common factor of both the numerator and denominator of the product.

Individualizing Instruction

• Ask the *more capable pupils* to perform each of the following multiplications and to compare the three products. Ask what generalization can be made about the product of four rational numbers.

a. $\left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f} \times \frac{g}{h}$ b. $\frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right) \times \frac{g}{h}$
c. $\frac{a}{b} \times \frac{c}{d} \times \left(\frac{e}{f} \times \frac{g}{h}\right)$

• Ask *all pupils* to perform the multiplication $48 \times \frac{5}{8} \times 1\frac{5}{6} \times \frac{12}{13}$ in the four ways indicated. Ask for their opinions as to which of the four methods seems to be the easiest.

a. $(48 \times \frac{5}{8}) \times (\frac{11}{6} \times \frac{12}{13})$ b. $48 \times (\frac{5}{8} \times \frac{11}{6}) \times \frac{12}{13}$
c. $(48 \times \frac{5}{8}) \times \frac{11}{6} \times \frac{12}{13}$ d. $\frac{48 \times 5 \times 11 \times 12}{1 \times 8 \times 6 \times 13}$

• When multiplying with fractions, a *slower learner* may often fail to recognize a factor common to a numerator and to a denominator. Write on the board

$$\frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{15}}} \times \frac{6}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{10}}}{9} = \frac{24}{27}$$

This is not wrong, but the numerator and denominator still have a common factor 3.

* Point out that when pupils fail to recognize a factor common to a numerator and a denominator, they do not get a wrong answer, but simply an answer that is not in simplest form.

Finding Products

More than two factors [O]

1. At the board, find the product of $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{7}{11}$ first by finding $(\frac{1}{3} \times \frac{2}{5}) \times \frac{7}{11}$ and then by finding $\frac{1}{3} \times (\frac{2}{5} \times \frac{7}{11})$. Is $(\frac{1}{3} \times \frac{2}{5}) \times \frac{7}{11} = \frac{1}{3} \times (\frac{2}{5} \times \frac{7}{11})$ a true mathematical sentence? Yes This is an illustration of the Commutative Property of Multiplication.

2. The work in box A shows another way to find the product of $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{7}{11}$. Explain the work shown. The product of the numerators and of the denominators is found. Which step in box A is done mentally in box B? The second step

A
$$\frac{1}{3} \times \frac{2}{5} \times \frac{7}{11} = \frac{1 \times 2 \times 7}{3 \times 5 \times 11} = \frac{14}{165}$$

B
$$\frac{1}{3} \times \frac{2}{5} \times \frac{7}{11} = \frac{14}{165}$$

C
$$\frac{\overset{1}{\cancel{2}}}{5} \times \frac{3}{\underset{5}{\cancel{10}}} \times \frac{1}{7} = \frac{3}{175}$$

D
$$n = 12 \times \frac{3}{5} \times 1\frac{1}{4}$$
$$= \frac{\overset{3}{\cancel{12}}}{1} \times \frac{3}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{4}}}$$
$$= \frac{9}{1}, \text{ or } 9$$

If $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$ represent rational numbers, then

$$\frac{a}{b} \times \frac{c}{d} \times \frac{e}{f} = \frac{a \times c \times e}{b \times d \times f}.$$

3. Explain the work shown in box C for finding $\frac{2}{5} \times \frac{3}{10} \times \frac{1}{7}$. See below. For the product found in box B, do the numerator and the denominator of $\frac{14}{165}$ have any common factors? No What was the common factor in box C? 2

4. On the board, work each of Ex. a–c as in box B or box C.

a. $\frac{2}{7} \times \frac{5}{9} \times \frac{4}{3} = \frac{40}{189}$ b. $\frac{\overset{1}{\cancel{2}}}{\cancel{8}} \times \frac{\overset{1}{\cancel{4}}}{5} \times \frac{1}{\cancel{2}} = \frac{1}{10}$ c. $\frac{\overset{2}{\cancel{6}}}{7} \times \frac{1}{\cancel{3}} \times \frac{1}{5} \times \frac{2}{5} = \frac{4}{175}$

5. Box D. To solve $n = \overset{2}{\cancel{12}} \times \frac{3}{5} \times 1\frac{1}{4}$, the factors were renamed with fractions. Explain the work shown. 6. $\frac{\overset{1}{\cancel{2}}}{\cancel{9}} \times \frac{\overset{1}{\cancel{3}}}{\cancel{4}} \times \frac{\overset{11}{\cancel{22}}}{7} = \frac{11}{7}$, or $1\frac{4}{7}$

6. At the board, solve $n = \frac{2}{9} \times 2\frac{1}{4} \times 3\frac{1}{7}$ as in box D. See right.

[W]

* Ex. 7–16. Solve.

7. $n = \frac{1}{5} \times \frac{1}{4} \times \frac{2}{3}$ $\frac{1}{30}$
8. $n = \frac{7}{8} \times \frac{4}{5} \times \frac{10}{21}$ $\frac{1}{3}$
9. $n = \frac{3}{4} \times \frac{5}{8} \times 2\frac{1}{6}$ $1\frac{1}{64}$
10. $n = 2\frac{1}{4} \times \frac{2}{3} \times \frac{5}{6}$ $1\frac{1}{4}$
11. $n = \frac{5}{8} \times 1\frac{4}{5} \times 6$ $6\frac{3}{4}$
12. $n = \frac{3}{5} \times \frac{2}{9} \times \frac{15}{16} \times \frac{4}{7}$ $\frac{1}{14}$
13. $14 \times \frac{2}{7} \times \frac{3}{5} \times 5\frac{1}{3} = x$ $12\frac{4}{5}$
14. $x = 23 \times 15 \times \frac{1}{5} \times \frac{1}{3}$ 23
15. $x = 1\frac{1}{2} \times 3\frac{1}{3} \times 6 \times \frac{5}{7}$ $21\frac{3}{7}$
16. $n = 7 \times 47 \times 18 \times 1\frac{1}{6}$ 6,909

◆ **Extra Examples.** Sets 99–100.

3. A numerator and a denominator were divided by a common factor before the product was found.

How Great is the Product?

[O]

A	$1 \times \frac{2}{3} = \frac{2}{3}$ $\frac{5}{6} \times 1 = \frac{5}{6}$ $1 \times 4\frac{2}{3} = 4\frac{2}{3}$
B	$\frac{3}{4} \times 5 = 3\frac{3}{4}$ $2 \times \frac{2}{3} = 1\frac{1}{3}$ $\frac{5}{6} \times \frac{2}{7} = \frac{5}{21}$
C	$\frac{2}{3} \times 3\frac{1}{2} = 2\frac{1}{3}$ $3 \times 4\frac{1}{3} = 13$ $2\frac{2}{5} \times 1\frac{1}{3} = 3\frac{1}{5}$

1. Box A. For each sentence shown, is it true that one of the factors is 1 and that the product is equal to the other factor? Yes

> If one of the two factors is 1, then the product is equal to the other factor. Because of this, 1 is called the identity element for multiplication or the multiplicative identity.

2. Box B. For each sentence shown, is it true that one of the factors is less than 1 and that the product is less than the other factor? Yes

> If a product expression shows two factors, both greater than 0, and one factor is less than 1, then the product is less than the other factor.

3. Box C. For each sentence shown, is it true that one of the factors is greater than 1 and that the product is greater than the other factor? Yes

> If a product expression shows two factors, both greater than 0, and one factor is greater than 1, then the product is greater than the other factor.

4. Complete each of Ex. a–c with $=$ or $<$ or $>$. Explain your reasoning.

a. $4\frac{3}{4} \times 1 \_?\_ 4\frac{3}{4}$
Key idea: One factor is 1.

b. $\frac{4}{7} \times 8 \_?\_ 8$
$\frac{4}{7} < 1$

c. $4\frac{1}{2} \times \frac{2}{9} \_?\_ 4\frac{1}{2}$
$\frac{2}{9} < 1$

[W]

5. For each of Ex. 6–14, tell whether the number represented by *n is equal to* or *is less than* or *is greater than* the first factor; the second factor.

Ex. 6–14. Solve.

6. $n = 5 \times \frac{3}{4}$
7. $n = \frac{4}{5} \times 1\frac{1}{2}$
8. $n = 3\frac{3}{7} \times 1$
9. $n = 2\frac{2}{3} \times 2\frac{1}{6}$
10. $n = \frac{1}{5} \times \frac{1}{6}$
11. $n = 6 \times (\frac{3}{5} + \frac{2}{5})$
12. $n = 1 \times (5\frac{1}{5} - 3)$
13. $n = 3\frac{1}{5} \times \frac{5}{4}$
14. $n = \frac{5}{6} \times 3\frac{1}{10}$

6. Is less than; is greater than; $3\frac{3}{4}$
7. Is greater than; is less than; $1\frac{1}{5}$
8. Is equal to; is greater than; $3\frac{3}{7}$
9. Is greater than; is greater than; $5\frac{7}{9}$
10. Is less than; is less than; $\frac{1}{30}$
11. Is equal to; is greater than; 6
12. Is greater than; is equal to; $2\frac{1}{5}$
13. Is greater than; is greater than; 4
14. Is greater than; is less than; $2\frac{7}{12}$

Teaching Page 198

Pupil's Objective

To compare the product of two factors with one of the factors when the other factor is 1, less than 1, or greater than 1.

Background

If one of two factors, both greater than 0,

is	their product is
(a) 1,	equal to the other factor;
(b) less than 1,	less than the other factor;
(c) greater than 1,	greater than the other factor.

Statement (a) above is true even when the other factor is zero. For statements (b) and (c) above, it is important to emphasize that the factors must be greater than zero; because, if one of the factors is zero, the product is zero.

Another reason for emphasizing that for (b) and (c) both factors must be greater than zero is that the statement "If one of two factors is less than 1, their product is less than the other factor" is not true when both factors are negative numbers. For example, $(^{-}\frac{3}{4}) \times (^{-}\frac{1}{2}) = \frac{3}{8}$. In this example, both numbers are less than 1, but their product, being positive, is greater than either factor.

For $^{-}\frac{3}{4} \times 1\frac{1}{3} = {}^{-}1$, one of the factors is greater than 1, yet their product is less than either factor.

Pre-Book Lesson

Review the meaning of multiplying with a fraction using the number line for a model. Review that the multiplication represented by $\frac{a}{b} \times c$ means that a segment c units in length is to be partitioned into b congruent segments and a of these segments are to be considered. Ask how the product will compare with c when

(a) $a > b$	$(\frac{a}{b} \times c > c)$
(b) $a < b$	$(\frac{a}{b} \times c < c)$
(c) $a = b$	$(\frac{a}{b} \times c = c)$

Using the Text Page

- In Ex. 1, ask what fractions name the identity element for multiplication. Lead pupils to the realization that any fraction in the form $\frac{a}{a}$ where $a \neq 0$ names the identity element for multiplication in the set of rational numbers.
- After Ex. 3, have the pupils write on the board a summary of how the product of two factors compares with one of the factors when the other factor is (a) 1, (b) greater than 1, (c) less than 1.
- The exercises in Ex. 4 may be extended to

(d) $4\frac{3}{4} \times 1 \_ ? \_ 1$ (f) $4\frac{1}{2} \times \frac{2}{9} \_ ? \_ \frac{2}{9}$

(e) $\frac{4}{7} \times 8 \_ ? \_ \frac{4}{7}$

Individualizing Instruction

- Have the *more capable pupils* prove that any fraction in the form $\frac{b}{b} (b \neq 0)$ names the identity element for multiplication. The proof consists of simply showing that $\frac{a}{c} \times \frac{b}{b} = \frac{a}{c}$.
- Make up multiple choice questions for *all pupils* in which the pupil is to select the correct answer without using pencil and paper.

1. $1\frac{2}{3} \times 17$	17	$\underline{28\frac{1}{3}}$	$11\frac{1}{3}$	$8\frac{2}{3}$
2. $\frac{2}{9} \times 35$	$\underline{7\frac{7}{9}}$	$35\frac{2}{9}$	$40\frac{1}{9}$	$70\frac{1}{9}$
3. $\frac{7}{9} \times \frac{11}{12}$	$1\frac{1}{12}$	$\underline{\frac{77}{108}}$	$8\frac{5}{9}$	$5\frac{5}{12}$
4. $6\frac{7}{8} \times 4\frac{1}{5}$	4	$2\frac{7}{40}$	$3\frac{1}{8}$	$\underline{28\frac{7}{8}}$
5. $19 \times \frac{3}{4} \times \frac{4}{3}$	$18\frac{9}{16}$	20	$\underline{19}$	$\frac{3}{4}$

Pupil's Objective

To have practice in solving problems requiring multiplication of rational numbers.

Background

Each of the problems on this page involves a factors-product relationship requiring multiplication of rational numbers shown by fractions and mixed forms. A diagnosis of pupil errors in these exercises will help reveal weaknesses in problem-solving techniques and in the multiplication of rational numbers.

Pre-Book Lesson

Present to the pupils a problem requiring the multiplication of rational numbers, such as "Mr. Carlson has $2,700 in a savings account. He withdrew $\frac{2}{3}$ of his savings to make a down payment on a new car. How much was the down payment?"

In leading the class discussion on how to solve the problem, review problem-solving in general and placing emphasis on the following:

a. Determining whether the problem involves an addends-sum or a factors-product relationship

b. Expressing the relationship by a mathematical sentence

c. Determining what operation is required

d. Estimating the answer

e. Solving the mathematical sentence

f. Comparing the final answer with an estimate of the answer

Using the Text Page

• For each of the problems, have the pupils estimate the answer making use of the principles introduced on text page 198.

• For each mathematical sentence given, ask if there is another mathematical sentence that could be used to solve the problem. Pupils should be given credit for using any correct method. For example, the multiplication $22 \times 3\frac{1}{2}$ could be performed either by using the Distributive Property or by renaming $3\frac{1}{2}$ using a fraction:

a. $22 \times 3\frac{1}{2} = 22 \times (3 + \frac{1}{2})$
$= (22 \times 3) + (22 \times \frac{1}{2})$
$= 66 + 11$
$= 77$

(This may also be written in vertical form.)

b. $22 \times 3\frac{1}{2} = 22 \times \frac{7}{2}$
$= 77$

Solving the problems should then be a fairly easy task.

Individualizing Instruction

• *All pupils* may be given problems with three possible answers indicated. Pupils are to select the most probable answer by estimating. Such a problem is:

Harry Lane and George Koutsakis walked $3\frac{1}{4}$ miles per hour for $2\frac{1}{2}$ hours. How far did they walk?

(a) $5\frac{3}{4}$ mi. (b) $\underline{8\frac{1}{8}\text{ mi.}}$ (c) $27\frac{1}{4}$ mi.

• The *more capable pupils* may be given problems containing letters instead of numbers. Such a problem is

Which of the following is equivalent to the mixed form $a\frac{b}{c}$?

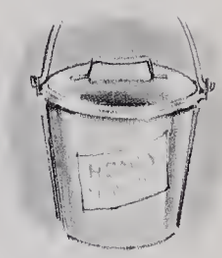
(a) $\frac{c \times a \times b}{c}$ (b) $\frac{a \times b}{c}$ (c) $\underline{\frac{(c \times a) + b}{c}}$

• Help the *slower learners* diagnose difficulties they have with solving problems and assign extra practice with problem-solving or with multiplying rational numbers, as needed.

Solving Problems

[O]

For each of Ex. 1–7, give a mathematical sentence which could be used to find the answer for the problem.



1. Last year, Mr. Parks sold twenty-two $3\frac{1}{2}$-lb. pails of honey at his market. That was how many pounds of honey in all? $n = 22 \times 3\frac{1}{2}$; 77

2. How many ounces of honey is $3\frac{1}{2}$ lb. of honey? $n = 16 \times 3\frac{1}{2}$; 56

3. How many pounds would $\frac{2}{3}$ of a $10\frac{1}{2}$-lb. cheese weigh? $n = \frac{2}{3} \times 10\frac{1}{2}$; 7

4. A box of crackers is marked "Contents, $1\frac{1}{4}$ lb." How many pounds of crackers will the box contain when it is $\frac{1}{2}$ full? $n = \frac{1}{2} \times 1\frac{1}{4}$; $\frac{5}{8}$

5. Mr. Parks feeds his dog, Rover, $1\frac{1}{4}$ cans of dog food each day. How many cans of dog food does Rover eat in one week? $n = 7 \times 1\frac{1}{4}$; $8\frac{3}{4}$

6. The large rectangular window in the front of Parks' Market is $5\frac{1}{2}$ ft. by $18\frac{3}{4}$ ft. That is how many square feet? $n = 5\frac{1}{2} \times 18\frac{3}{4}$; $103\frac{1}{8}$

7. Mr. Parks gave away $2\frac{1}{2}$ pecks of McIntosh apples and $1\frac{1}{4}$ pecks of Winesap apples each day of the 3-day apple-growers celebration. In all, how many pecks of apples was that? (Hint: How many pecks of apples did he give away each day?) $n = 3 \times (2\frac{1}{2} + 1\frac{1}{4})$; $11\frac{1}{4}$

[W]

Now find the answer for each of Ex. 1–7.

♦ **Extra Problems.** Set 145.

Using Mathematics in Metallurgy

[O]

When two or more metals are melted and mixed, an *alloy* is formed. Metal products are usually made from alloys.

1. Jeweler's gold that is marked 12K is a gold and copper alloy which is $\frac{1}{2}$ pure gold and $\frac{1}{2}$ copper. How many ounces of pure gold are there in $15\frac{1}{2}$ ounces of 12K gold?

$$\begin{aligned} n &= \tfrac{1}{2} \times 15\tfrac{1}{2} \\ &= \tfrac{1}{2} \times \tfrac{31}{2} \\ &= \tfrac{31}{4}, \text{ or } 7\tfrac{3}{4} \end{aligned}$$

a. Does $n = \frac{1}{2} \times 15\frac{1}{2}$ show the relationship expressed in the problem? Yes Explain the work in the box for solving this mathematical sentence. $15\frac{1}{2}$ is renamed with a fraction, the work is done, and the answer is renamed with a mixed form.

b. Give the answer for the problem. $7\frac{3}{4}$ ounces

2. 10K gold is a gold and copper alloy which is $\frac{5}{12}$ pure gold.

a. What part of 10K gold is copper? $\frac{7}{12}$

b. At the board, find how much pure gold there is in 72 ounces of 10K gold. Express your answer first in terms of ounces and then in terms of pounds. 30 ounces or 1 pound 14 ounces

c. How much pure gold is there in 54 ounces of 10K gold?

$22\frac{1}{2}$ ounces or 1 pound $6\frac{1}{2}$ ounces

[W]

3. Before 1934, gold coins were made and used in the United States. They were made from a gold and copper alloy which was $\frac{9}{10}$ pure gold. How many pounds of pure gold were there in 25 pounds of such coins? $22\frac{1}{2}$

Teaching Pages 200 and 201

Pupil's Objective

To have practice in solving problems requiring multiplication of rational numbers.

Background

This is one of several sets of problems devoted to a particular area of science. By studying and solving these problems, pupils are not only gaining experience in problem-solving and in maintaining computational skills, but also they are given an excellent opportunity to absorb interesting and useful scientific information.

These problems concern that area of metallurgy which pertains to the meaning and use of alloys. An alloy is the metal formed when two or more pure metals are melted and mixed. Each metal has its own individual characteristics such as density, hardness, color, brittleness, ductility (ability to be drawn into a fine wire), malleability (ability to be hammered into thin sheets), resistance to tarnishing, and strength.

A pure metal may have several characteristics which would make it ideal for a specific use and yet have one characteristic which makes it unsuitable for that use. For example, aluminum is very light and resists corrosion. These characteristics make it ideal for use in building airplanes if it were not for the fact that aluminum by itself does not possess the necessary strength. To overcome this, a small amount of copper and even smaller amounts of manganese and magnesium are added to form an alloy called *duralumin*, which has approximately the strength of steel. This alloy is ideal for many uses requiring a lightweight metal with great strength such as is needed in building airplanes.

The purpose of developing alloys is to provide a new metal which has characteristics not possessed by a pure metal. It is an interesting fact that an alloy may have a characteristic not possessed by any of the pure metals forming the alloy. Duralumin has approximately the strength of steel, yet no one pure metal used in forming this alloy has this characteristic.

The first alloy to be mentioned in the oral exercises contains gold. Gold has many desirable characteristics. It has a lovely yellow color and a soft metallic glow. Its softness makes it easy to work with and easy to shape. It is highly resistant to tarnishing of any kind. Since it is so scarce, and since it has so many desirable characteristics, it has great value. However, its softness prevents if from being useful in its pure form in instances where hardness is required, such as jewelry. To overcome this fault, other metals such as copper are added to gold to form alloys which have the hardness and strength desired.

Gold alloys are measured by *carats*. A carat is equal to one twenty-fourth part. Thus, 24-carat (24K) gold is pure gold.

The written exercises pertain to several different alloys, and the solutions of these problems require performing operations on rational numbers.

Ex. 11 gives the teacher an opportunity to discuss the Liberty Bell. This bell was purchased by the Province of Pennsylvania in 1752 for the State House in Philadelphia. It was cast in England, but broken in ringing after its arrival. It was recast in Philadelphia from the same metal with the same inscription "proclaim Liberty throughout all the land unto all the inhabitants thereof." It was rung July 8, 1776, to call citizens together to announce the adoption of the Declaration of Independence.

It rang at each successive anniversary of the adoption of the Declaration until 1835. On July 8 of that year, while tolling during the funeral solemnities for John Marshall, Chief Justice of the United States, who died in that city, the bell cracked. The Liberty Bell is no longer rung but is a treasured relic of the early days of American independence.

Pre-Book Lesson

Display articles of metallic jewelry such as decorative pins, bracelets, watches, and watchbands. Point out that each is made with an alloy that is a mixture of pure metals because pure metals have one or more characteristics which make them unsuitable for use in jewelry. Metals are melted and mixed to produce an alloy that does not have the undesirable characteristic.

Using the Text Pages

• After the oral exercises have been completed, you may wish to discuss each of the problems in the written exercises so that the pupils will understand the situation described and will understand what question is being asked.

• When they are doing the written work, have pupils use a mathematical sentence to express the relationship expressed in the problem and then solve the mathematical sentence. Having them write the mathematical sentence will help you in determining whether errors are due to analyzing the problem incorrectly or simply to making an error in computation.

Individualizing Instruction

• Give *more capable pupils* problems such as the following, and have them indicate which of the answers given is correct:

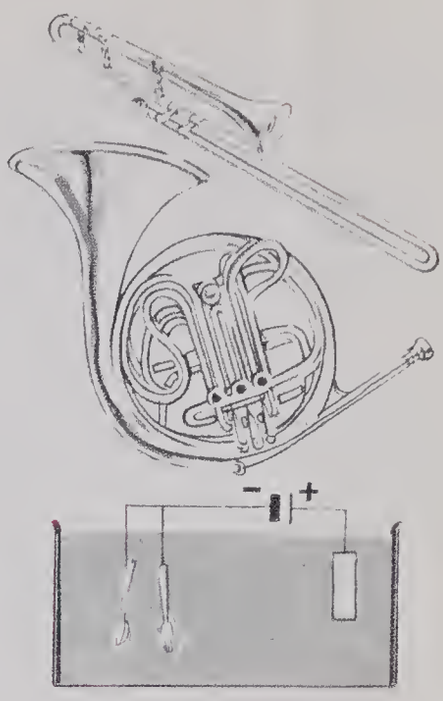
A certain alloy consists of copper and tin. If $\frac{a}{b}$ represents the fractional part of the alloy which is copper, which of the following mathematical sentences would be used to find how many pounds of tin there are in 32 pounds of alloy?

a. $n = (1 - \frac{a}{b}) \times 32$
b. $n = \frac{a}{b} \times 32$
c. $n = (1 + \frac{a}{b}) \times 32$
d. $n = \frac{a - 1}{b} \times 32$

• Have *all pupils* summarize the science they have discussed as a result of studying the problems on these two pages.

NOTES

4. Brass jewelry is often made from a copper and zinc alloy which is $\frac{4}{5}$ copper. How much copper is there in 12 pounds of such an alloy? $9\frac{3}{5}$ pounds



5. The brass horns shown at the right were made from an alloy which was $\frac{7}{10}$ copper. If one horn weighs 5 pounds and the other horn weighs 7 pounds, then together the horns contain how many pounds of copper? $8\frac{2}{5}$

6. Brass that is used for silverplating is an alloy of copper, nickel, and zinc called electroplate. Electroplate is $\frac{3}{5}$ copper and $\frac{1}{5}$ nickel. How many pounds of zinc are there in $12\frac{1}{2}$ pounds of electroplate? (Hint: What part of electroplate is zinc?) $2\frac{1}{2}$

7. If a 24-lb. bronze bell were made from an alloy which is $\frac{1}{5}$ tin, how many pounds of tin would the bell contain? $4\frac{4}{5}$

8. If an aluminum ladder contains $14\frac{3}{4}$ pounds of aluminum, how many pounds of aluminum would 12 such ladders contain? 177

9. The Gary Pen Company makes pen points from an alloy which is $\frac{1}{4}$ silver. How many pounds of silver are there in fifteen 5-lb. bars of such an alloy? $18\frac{3}{4}$

10. If $8\frac{1}{5}$ pounds of an aluminum alloy are needed to make an aluminum door, how many pounds are needed to make 20 doors? 164

11. The original Liberty Bell was cast in London, England, but cracked while being tested in Philadelphia. That bell was melted down and for every pound of the old metal, $\frac{3}{32}$ lb. of copper was added to reduce brittleness. Then the bell was recast. How many *ounces* of copper were added for each pound of the old metal? $1\frac{1}{2}$

How Well Do You Remember?

[W]

Ex. 1–2. Copy and complete.

1. $15 \times \frac{1}{7} = \frac{?}{7}$, or $2\frac{?}{7}$. $\frac{15}{7}$, or $2\frac{1}{7}$.

2. $3\frac{1}{2} \times 5\frac{2}{3} = \frac{?}{2} \times \frac{?}{3} = \frac{?}{6}$, or $\_?\_\frac{?}{6}$ $\frac{17}{2} \times \frac{17}{3} = \frac{119}{6}$, or $19\frac{5}{6}$

Ex. 3–5. Copy and complete by writing $<$ or $>$.

3. $\frac{3}{7}$ $\_?\_$ $4 \times \frac{1}{7}$ $<$

4. $2\frac{1}{5}$ $\_?\_$ $\frac{4}{5} \times \frac{3}{5}$ $>$

5. $\frac{1}{2} \times \frac{1}{3}$ $\_?\_$ $\frac{2}{3}$ $<$

Ex. 6–13. Solve.

6. $n = 4\frac{1}{2} \times 8$ 36

7. $2\frac{1}{2} \times 2\frac{1}{2} = x$ $6\frac{1}{4}$

8. $n = 3\frac{1}{4} \times 1\frac{1}{2}$ $4\frac{7}{8}$

9. $y = \frac{7}{8} \times 4\frac{1}{2}$ $3\frac{15}{16}$

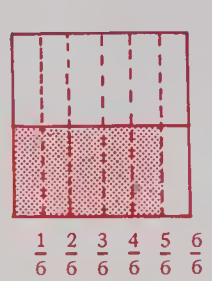
10. $n = 1\frac{1}{5} \times 4$ $4\frac{4}{5}$

11. $n = 6\frac{1}{4} \times 2\frac{1}{3}$ $14\frac{7}{12}$

12. $x = 4\frac{1}{3} \times 1\frac{2}{3}$ $7\frac{2}{9}$

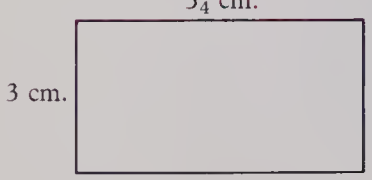
13. $\frac{3}{4} \times 2\frac{1}{5} = n$ $1\frac{13}{20}$

14. Represent $\frac{1}{2} \times \frac{5}{6}$ by shading part of a model of a square unit region.

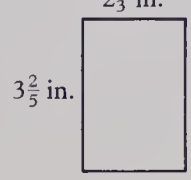
14. 

15. An angle A is partitioned into 12 smaller angles each of which measures $5\frac{1}{3}$ in degrees. Find the measure of angle A. 64

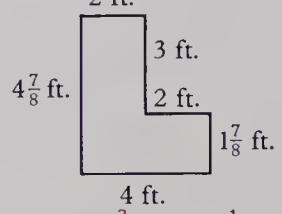
Ex. 16–19. Find the perimeter and then the area.

16. 

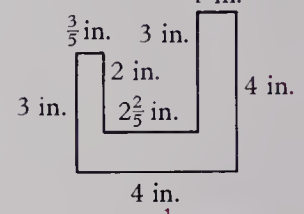
$17\frac{1}{2}$ cm.; $17\frac{1}{4}$ sq. cm.

17. 

$11\frac{7}{15}$ in.; $7\frac{14}{15}$ sq. in.

18. 

$17\frac{3}{4}$ ft.; $13\frac{1}{2}$ sq. ft.

19. 

20 in.; $8\frac{1}{5}$ sq. in.

Renaming Numbers

[W]

Ex. 1–5. Rename with a decimal.

1. $13\frac{3}{100}$ 13.03

2. $\frac{4}{5}$ 0.8

3. $2 + \frac{3}{4}$ 2.75

4. $\frac{456}{1{,}000}$ 0.456

5. $\frac{8{,}553}{10}$ 855.3

Ex. 6–10. Express as a sum of two primes.

Sample answers:

6. 8 3 + 5

7. 14 3 + 11

8. 30 7 + 23

9. 32 3 + 29

10. 100 3 + 97, 11 + 89

Ex. 11–15. Express as a product of primes.

11. 44 $2 \times 2 \times 11$

12. 27 $3 \times 3 \times 3$

13. 120 $2 \times 2 \times 2 \times 3 \times 5$

14. 135 $3 \times 3 \times 3 \times 5$

15. 222 2×111

Teaching Page 202

Pupil's Objectives

(a) To check understanding of important ideas studied to this point in Chapter 5; (b) to review renaming numbers with decimals; and (c) to review ideas pertaining to primes.

Background

The exercises at the top of the page may be used as a mid-chapter evaluation of how well the ideas pertaining to multiplying rational numbers have been learned before undertaking new work in studying dividing rational numbers. These exercises pertain to (a) multiplying rational numbers including those named with fractions and mixed forms, (b) ordering rational numbers, (c) explaining the meaning of multiplying with a rational number by showing square-unit regions, and (d) finding the perimeter of a polygon and the area of its region using English and metric units of measurement.

The exercises at the bottom of the page pertain to (a) renaming rational numbers from fractions and mixed forms to decimals, (b) expressing a whole number as a sum of two primes, and (c) expressing a whole number as a product of primes. They are intended to help the pupil maintain his skills and understandings in these areas.

Pre-Book Lesson

• Very little review should be necessary for the exercises at the top of the page. All aspects of multiplying rational numbers were developed in this chapter, and the topics of finding perimeters and areas were recently resurveyed on pages 150–153 and pages 173–179 in Chapter 4. However, it may be well to review finding areas of regions of the type shown in Ex. 18 and 19 on the text page.

• You may wish to review the topics of the type included in the exercises at the bottom of the page before assigning the written work, particularly those pertaining to prime numbers.

To review	Use exercises on page
Renaming a rational number with a decimal	161
Expressing a number as a sum of primes	67
Expressing a number as a product of primes	68, 69

Using the Text Page

• Be sure pupils are aware that there may be more than one way to find the area in Ex. 18 and in Ex. 19. For example, in Ex. 18 the area may be found either by

$$n = (2 \times 4\tfrac{7}{8}) + (2 \times 1\tfrac{7}{8})$$

or

$$n = (2 \times 3) + (4 \times 1\tfrac{7}{8}).$$

• After all pupils have completed the exercises on the page, discuss each type of exercise and have the pupils explain how the exercises should be solved.

Individualizing Instruction

Help *all pupils* to recognize their individual strengths and weaknesses in these areas. Have them restudy appropriate pages and work appropriate extra-practice exercises.

Teaching Page 203

Pupil's Objectives

(a) To have mixed computational practice with decimals; and (b) to review meaning of precision in measurement.

Background

The exercises at the top of the pupil's page provide practice with (a) adding with decimals, (b) subtracting with decimals, (c) renaming from fractions and mixed forms to decimals, (d) solving mathematical sentences requiring adding or subtracting with decimals, and (e) rounding numbers expressed with decimals.

The exercises at the bottom of the page are a review of precision in measurement including work with the unit of precision and the greatest possible error.

Pre-Book Lesson

- Since the pupils have had frequent practice adding and subtracting with decimals, an extensive review of this topic may not be necessary before assigning the written exercises at the top of the page.
- The pupils may have forgotten some of the aspects of precision since the topic was studied in Chapter 4. They may have need for a review of the unit of precision to use and the greatest possible error before doing the written exercises at the bottom of the page. A quick review of pupil's pages 154–157 may help refresh their memories.

Using the Text Page

- Make certain that all pupils understand what they are to do in each of the exercises, particularly the set at the bottom of the pupil's page.
- Pupils should be able to proceed independently with the written work at the top of the page. You may wish to circulate among the pupils while they are working in order to check their work. You may question the pupils who are making errors and guide them to realize what mistakes they are making.

Individualizing Instruction

- A diagnostic analysis of the results of the work on pupil's pages 202 and 203 should provide you with additional information for grouping pupils for the presentation of the new material to be introduced in the remainder of this chapter. It will also help identify skills and concepts which need reviewing or reteaching.
- The *slower learners* will probably have the greatest need for reviewing unit of precision and greatest possible error before beginning the written exercises at the bottom of the page.

NOTES

* Recall that the unit of precision of a measurement is the smallest unit to which the measurement is made and that the greatest possible error is $\frac{1}{2}$ of the unit of precision.

To Keep in Practice

[W]

Ex. 1–5. Copy and add.

1.	2.	3.	4.	5.
33.8	2.683	16.7895	1,603.70	$44.89
17.9	3.772	0.0688	10.08	75.50
51.7	6.455	16.8583	1,613.78	$120.39

Ex. 6–10. Copy and subtract.

6.	7.	8.	9.	10.
7.9	47.8	1.076	35.724	$7,800.91
2.3	9.9	0.034	0.798	109.98
5.6	37.9	1.042	34.926	$7,690.93

Ex. 11–15. Rename with a decimal.

11. $\frac{33}{100}$ 0.33 ^ 12. $2\frac{4}{5}$ 2.8 13. $1\frac{3}{10}$ 1.3 14. $\frac{1,856}{100}$ 18.56 15. $6\frac{7}{25}$ 6.28

Ex. 16–19. Find the number represented by n.

16. $n = 3.4 + 5.8 + 6.6$ 15.8 1.3487 17. $n + 34.772 = 160.881$ 126.109

18. $3.9094 - n = 2.5607$ ^ 19. $n = 0.067 + 0.010 + 0.009$ 0.086

20. Find the perimeter of a triangle whose sides are 14.3 cm., 13.9 cm., and 12.5 cm. 40.7 cm.

21. How much less than the sum of 3.78 and 34.44 is 35.74? 2.48

Ex. 22–26. Round to the nearest hundredth.

22. 4.794 4.79 ^ 23. 80.009 80.01 ^ 24. 1.025 1.03 ^ 25. 0.0825 0.08 ^ 26. 0.0081 0.01

Remembering about Precision

[W]

* Ex. 1–5. Name the unit of precision for the measurement.

1. 17 in. 1 inch ^ 2. $5\frac{0}{4}$ cm. 3. $6\frac{2}{5}$ yd. $\frac{1}{5}$ yard ^ 4. 5 ft. 3 in. 1 inch ^ 5. 2 yd. 0 ft. 1 foot

2. $\frac{1}{4}$ centimeter

6. Copy and complete: A measurement of $4\frac{0}{2}$ in. means that the length is either $3\frac{3}{4}$ in. or is between _?_ $3\frac{3}{4}$ in. and _?_ $4\frac{1}{4}$ in.

7. Name the unit of precision for a measurement if the greatest possible error for the measurement is $\frac{1}{4}$ cm. $\frac{1}{2}$ centimeter

8. Is the greatest possible error *larger* or *smaller* when a smaller unit of measurement is used? smaller

* The answer is not the remainder after all the subtracting, but it is the number of times that $\frac{1}{2}$ was subtracted.

Dividing by a Rational Number

Developing the common-denominator method [O]

Mrs. Davis works at a candy shop. One of her jobs is to cut and package fudge into $\frac{1}{4}$-lb., $\frac{1}{2}$-lb., $\frac{3}{4}$-lb., and 1-lb. packages.

1. How many $\frac{3}{4}$-lb. packages of fudge can Mrs. Davis make from a tray of fudge if the tray contains 3 pounds of fudge?

A

$3 - \frac{3}{4} = 2\frac{1}{4}$

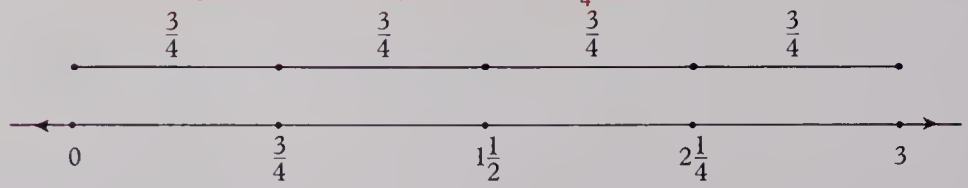
$2\frac{1}{4} - \frac{3}{4} = 1\frac{1}{2}$

$1\frac{1}{2} - \frac{3}{4} = \frac{3}{4}$

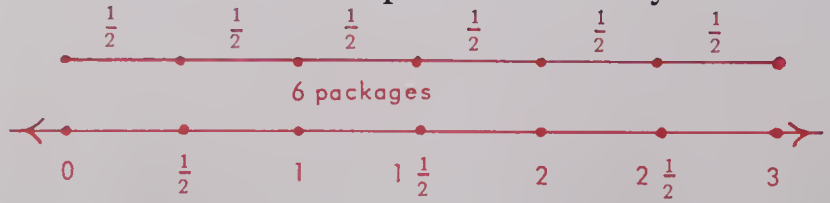
$\frac{3}{4} - \frac{3}{4} = 0$

a. One way to find the answer is to subtract as shown in box A. Explain the work shown. Key idea: $\frac{3}{4}$ is repeatedly subtracted. How many times was $\frac{3}{4}$ subtracted? 4 packages Give the answer for the problem.

b. Is it true that $3 = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$? Yes Explain how the number-line picture below shows this relationship. The line for 3 is shown separated into 4 segments to show $\frac{3}{4}$ each.



* 2. At the board, use subtraction to find how many $\frac{1}{2}$-lb. packages of fudge can be made from 3 pounds of fudge. Draw a number-line picture to show your answer.



$3 - \frac{1}{2} = 2\frac{1}{2}$ $1\frac{1}{2} - \frac{1}{2} = 1$

$2\frac{1}{2} - \frac{1}{2} = 2$ $1 - \frac{1}{2} = \frac{1}{2}$

$2 - \frac{1}{2} = 1\frac{1}{2}$ $\frac{1}{2} - \frac{1}{2} = 0$

Teaching Pages 204 and 205

Pupil's Objective

To learn to divide by a rational number, using the common-denominator method.

Background

There are two widely used methods for dividing with rational numbers. One is the *common-denominator* method and the other is the *reciprocal* method. The two methods differ in the mathematical justifications for the techniques used, but they are similar in that they both involve the use of a substitute technique. The common-denominator method (used on text pages 204–211) makes use of division, and the reciprocal method makes use of multiplication. Although the reciprocal method (introduced on text page 214) is more widely used because it is a very efficient method for performing the computation, the concept of why it results in the correct answer often is not understood by the person using the method. The common-denominator method is not as mechanically efficient as the reciprocal method, but the concept behind it is more easily understood.

When dividing by using the common-denominator method, the first step is to express both numbers with fractions showing a common denominator:

$$\begin{aligned} n &= 11 \div \tfrac{3}{4} \\ &= \tfrac{11}{1} \div \tfrac{3}{4} \\ &= \tfrac{44}{4} \div \tfrac{3}{4} \end{aligned}$$

The next step is to divide the numerators:

$$\begin{aligned} n &= 44 \div 3 \\ &= \tfrac{44}{3} \\ &= 14\tfrac{2}{3} \end{aligned}$$

There are three mathematical justifications for this step:

(1) In the first interpretation, the division is considered to be

$$\begin{aligned} n &= \tfrac{44}{4} \div \tfrac{3}{4} \\ &= 44 \text{ fourths} \div 3 \text{ fourths} \\ &= 44 \div 3 \\ &= 14\tfrac{2}{3} \end{aligned}$$

(2) The second interpretation is based on the idea that, in a division, the product and the known factor may be multiplied by the same number without affecting the unknown factor.

$$\begin{aligned} n &= \tfrac{44}{4} \div \tfrac{3}{4} \\ &= (4 \times \tfrac{44}{4}) \div (4 \times \tfrac{3}{4}) \\ &= (\overset{1}{\cancel{4}} \times \tfrac{44}{\underset{1}{\cancel{4}}}) \div (\overset{1}{\cancel{4}} \times \tfrac{3}{\underset{1}{\cancel{4}}}) \\ &= 44 \div 3 \\ &= 14\tfrac{2}{3} \end{aligned}$$

(3) In the third interpretation, the division is written so that the product and the known factor have a common factor.

$$\begin{aligned} n &= \tfrac{44}{4} \div \tfrac{3}{4} \\ &= (44 \times \tfrac{1}{4}) \div (3 \times \tfrac{1}{4}) \\ &= \frac{44 \times \frac{1}{4}}{3 \times \frac{1}{4}} \\ &= \tfrac{44}{3} \\ &= 14\tfrac{2}{3} \end{aligned}$$

The algebraic proof that if $\frac{a}{b} \div \frac{c}{b} = n$ then $n = a \div c$ is mathematically justified.

The division $\frac{a}{b} \div \frac{c}{b} = n$ may also be expressed by the factors-product relationship $\frac{c}{b} \times n = \frac{a}{b}$. Performing the multiplication indicated on the left side,

$$\frac{c \times n}{b} = \frac{a}{b}.$$

Multiplying each side by b,

$$\overset{1}{\cancel{b}} \times \frac{c \times n}{\underset{1}{\cancel{b}}} = \frac{a}{\underset{1}{\cancel{b}}} \times \overset{1}{\cancel{b}}$$

$$c \times n = a.$$

Dividing each side by c,

$$\frac{\overset{1}{\cancel{c}} \times n}{\underset{1}{\cancel{c}}} = \frac{a}{c}$$

$$n = \frac{a}{c}.$$

Although this proof may be of interest to the teacher, it is not intended to be presented to the pupils.

Pre-Book Lesson

- Review the concept of division (a) by repeated subtraction of the known factor, and (b) by finding the unknown factor.

• Ask, "How many slices $\frac{5}{8}$ inch thick can be cut from a piece of butter 5 inches long?" Discuss different ways of solving the problem. Ask if the problem would be easier to solve if it had been stated, "How many slices $\frac{5}{8}$ inch thick can be cut from a piece of butter $\frac{40}{8}$ inches long?" Lead the pupils to the realization that the problem is easier to solve if the numbers are considered in common-denominator form.

Using the Text Pages

• The purpose of Ex. 1 is to show that the result of dividing a whole number by a number named by a fraction can be found by repeated subtraction as shown in box A. However, some pupils might suggest that the subtraction would be easier if, in the subtraction $3 - \frac{3}{4}$, the 3 were written as $\frac{12}{4}$. The repeated subtraction would then be

$$\begin{aligned} \tfrac{12}{4} - \tfrac{3}{4} &= \tfrac{9}{4} \\ \tfrac{9}{4} - \tfrac{3}{4} &= \tfrac{6}{4} \\ \tfrac{6}{4} - \tfrac{3}{4} &= \tfrac{3}{4} \\ \tfrac{3}{4} - \tfrac{3}{4} &= 0 \end{aligned}$$

If any pupil has difficulty understanding that $3 = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$, he may understand it better written as $\frac{12}{4} = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$. This method of finding the result of dividing a whole number by a number named by a fraction by using repeated subtraction must be restricted to examples where the product is a multiple of the known factor.

• In Ex. 4, the pupils will probably need help in explaining the steps shown in box B. The fact that the answer is correct may be verified by expressing the factors-product relationship, $n = \frac{12}{4} \div \frac{3}{4}$, as $\frac{3}{4} \times n = \frac{12}{4}$. From this, the pupils should be able to see that n must represent 4.

• In Ex. 8–22, have the pupils solve these exercises using the common-denominator method. Then have them try to verify their answers by using some other method of finding the result, such as repeated subtraction or the number-line model. This should demonstrate the advantages of finding the quotient by using the common-denominator method.

Individualizing Instruction

• Have *all pupils* try to explain in their own words how to divide by a whole number using the common-denominator method.

• Ask the *more capable pupils* how the common-denominator method can be extended to solving these divisions:

(a) $\frac{7}{8} \div \frac{3}{4}$; (b) $6 \div 4\frac{3}{4}$; (c) $2\frac{1}{2} \div 1\frac{2}{3}$.

• Provide each of the *slower learners* with four cards. Have them cut each card into three parts of the same size. Ask "How many $\frac{2}{3}$'s are there in 4?" Show these pupils how to use the cards to demonstrate the division

$$\begin{aligned} 4 \div \tfrac{2}{3} &= \tfrac{12}{3} \div \tfrac{2}{3} \\ &= 12 \div 2 \\ &= 6 \end{aligned}$$

Have them use this method to verify the work in the boxes B, C, and D, and to show that $\frac{a}{b} \div \frac{c}{b} = a \div c$.

NOTES

3. Another way to find the answer for Ex. 1 is to think, *how many $\frac{3}{4}$'s are there in 3?* Then to find the answer, we solve the mathematical sentence $n = 3 \div \frac{3}{4}$.

a. By subtracting, we found that $3 = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$, so $3 = \_?\_ \times \frac{3}{4}$. 4

b. By the factors-product relationship, we can express the relationship $3 = 4 \times \frac{3}{4}$ as $4 = 3 \div \frac{3}{4}$. Why? They express the same relationship. Then the number represented by n in $n = 3 \div \frac{3}{4}$ must be $\_?\_$. 4

4. Another way to solve $n = 3 \div \frac{3}{4}$ is to work as in box B. Both 3 and $\frac{3}{4}$ were named with fractions showing a common denominator of $\_?\_$. 4 Explain each step shown. This way is called the *common-denominator method.*

5. Box C. Which steps in box B are done mentally in box C? Third and fourth Is the work in box C shorter than the work in box B? Yes

6. Box D. Explain the work shown for finding the number represented by n in $n = 5 \div \frac{2}{3}$. Do you see that the result of dividing a whole number by a rational number is not always a whole number? Yes

7. For each of Ex. a–c, tell how the product is to be renamed and then, at the board, solve.

a. $n = 6 \div \frac{2}{5}$ 15 $\frac{30}{5}$ **b.** $n = 8 \div \frac{4}{7}$ 14 $\frac{56}{7}$ **c.** $n = 7 \div \frac{3}{5}$ $11\frac{2}{3}$ $\frac{35}{5}$ [W]

B

$n = 3 \div \frac{3}{4}$
$= \frac{12}{4} \div \frac{3}{4}$
$= \frac{12 \div 3}{4 \div 4}$
$= \frac{12 \div 3}{1}$
$= 12 \div 3$
$n = 4$

C

$n = 3 \div \frac{3}{4}$
$= \frac{12}{4} \div \frac{3}{4}$
$= 12 \div 3$
$n = 4$

D

$n = 5 \div \frac{2}{3}$
$= \frac{15}{3} \div \frac{2}{3}$
$= 15 \div 2$
$n = \frac{15}{2}$, or $7\frac{1}{2}$

Ex. 8–22. Solve. Express the answer in simplest form.

8. $n = 6 \div \frac{3}{8}$ 16
9. $n = 7 \div \frac{1}{2}$ 14
10. $x = 16 \div \frac{4}{5}$ 20
11. $x = 12 \div \frac{3}{4}$ 16
12. $3 \div \frac{2}{7} = n$ $10\frac{1}{2}$
13. $5 \div \frac{2}{5} = x$ $12\frac{1}{2}$
14. $n = 15 \div \frac{2}{9}$ $67\frac{1}{2}$
15. $n = 18 \div \frac{5}{7}$ $25\frac{1}{5}$
16. $x = 36 \div \frac{9}{10}$ 40
17. $y = 45 \div \frac{5}{8}$ 72
18. $x = 72 \div \frac{5}{6}$ $86\frac{2}{5}$
19. $n = 100 \div \frac{2}{3}$ 150
20. $2 \div \frac{1}{8} = n$ 16
21. $49 \div \frac{7}{12} = y$ 84
22. $25 \div \frac{4}{9} = x$ $56\frac{1}{4}$

◆ **Extra Examples.** Sets 101–102.

* Since the number-line picture shows the factors-product relationship, the picture for division is the same as the picture for multiplication.

Divisors Greater Than One

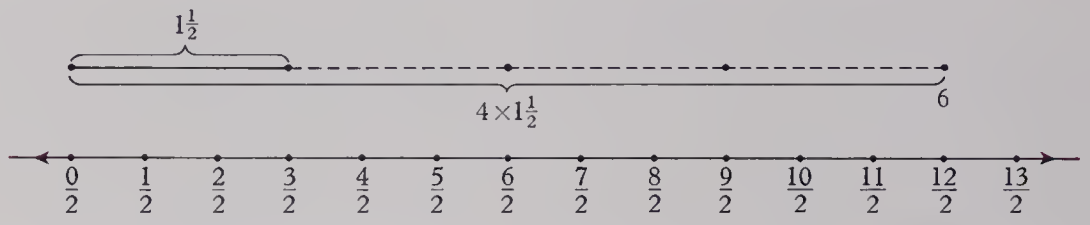
See left. [O]

1. At the board, use subtraction to find $6 \div 1\frac{1}{2}$. How many times did you subtract $1\frac{1}{2}$? 4 $6 \div 1\frac{1}{2} = \_?\_$ 4

$6 - 1\frac{1}{2} = 4\frac{1}{2}$
$4\frac{1}{2} - 1\frac{1}{2} = 3$
$3 - 1\frac{1}{2} = 1\frac{1}{2}$
$1\frac{1}{2} - 1\frac{1}{2} = 0$

2. In the mathematical sentence $6 \div 1\frac{1}{2} = 4$, which numeral names the product? 6 Which numerals name the factors? $1\frac{1}{2}$ and 4 Then does $4 \times 1\frac{1}{2} = 6$ express the same relationship as $6 \div 1\frac{1}{2} = 4$? Yes

* Does the number-line picture below show this relationship? Yes


$1\frac{1}{2}$
$4 \times 1\frac{1}{2}$
6
$\frac{0}{2}$ $\frac{1}{2}$ $\frac{2}{2}$ $\frac{3}{2}$ $\frac{4}{2}$ $\frac{5}{2}$ $\frac{6}{2}$ $\frac{7}{2}$ $\frac{8}{2}$ $\frac{9}{2}$ $\frac{10}{2}$ $\frac{11}{2}$ $\frac{12}{2}$ $\frac{13}{2}$

A

$n = 6 \div 1\frac{1}{2}$
$= \frac{12}{2} \div \frac{3}{2}$
$= 12 \div 3$
$n = 4$

B

$n = 3 \div 2\frac{2}{5}$
$= \frac{15}{5} \div \frac{12}{5}$
$= 15 \div 12$
$n = 1\frac{1}{4}$

C

$n \times 2\frac{1}{3} = 4$
$n = 4 \div 2\frac{1}{3}$
$= \frac{12}{3} \div \frac{7}{3}$
$= 12 \div 7$
$n = 1\frac{5}{7}$

3. Box A. Both the product, 6, and the known factor, $1\frac{1}{2}$, have been renamed with fractions each showing a denominator of $\_?\_$ 2. Explain the work shown.

4. Box B. Explain the work for solving $n = 3 \div 2\frac{2}{5}$. $15 \div 12 = \frac{15}{12} = \frac{?}{4}$ 5 $= \_?\_$ $1\frac{1}{4}$

5. For Ex. a–c, tell how the product and the known factor are to be renamed. Solve as in box A or box B.

a. $n = \underset{\frac{16}{4}}{4} \div 1\frac{1}{4}$ $3\frac{1}{5}$ b. $n = \underset{\frac{25}{5}}{5} \div 1\frac{1}{5}$ $4\frac{1}{6}$ c. $n = \underset{\frac{72}{8}}{9} \div 2\frac{1}{8}$ $4\frac{4}{17}$

6. Box C. In $n \times 2\frac{1}{3} = 4$, does n represent the product or a factor? Explain the work shown.

7. For each of Ex. 8–16, tell if the letter represents the product or a factor and how to solve the sentence. See below.

[W]

Solve each of Ex. 8–16.

8. $n = 5 \div 2\frac{1}{2}$ 2
9. $3 = n \div 1\frac{1}{2}$ $4\frac{1}{2}$
10. $x = 8 \div 4\frac{1}{2}$ $1\frac{7}{9}$
11. $n = 15 \div 7\frac{1}{2}$ 2
12. $n \times 3\frac{1}{8} = 5$ $1\frac{3}{5}$
13. $6 \times 1\frac{1}{3} = x$ 8
14. $x \times 6\frac{2}{3} = 8$ $1\frac{1}{5}$
15. $7 = n \div 2\frac{2}{5}$ $16\frac{4}{5}$
16. $n = 15 \div 4\frac{1}{7}$ $3\frac{18}{29}$

◆ **Extra Examples.** Set 103.

8. factor; divide
9. product; multiply
10. factor; divide
11. factor; divide
12. factor; divide
13. product; multiply
14. factor; divide
15. product; multiply
16. factor; divide

Teaching Page 206

Pupil's Objectives

(a) To learn to divide by a rational number named with a mixed form; and (b) to practice solving mathematical sentences which require dividing by a rational number expressed in mixed form.

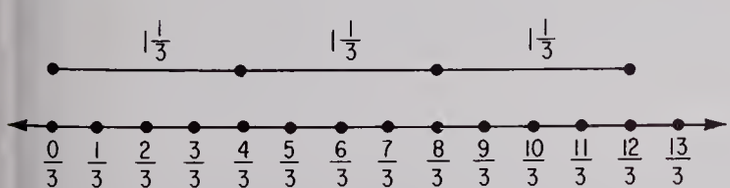
Background

The concept of dividing by a rational number using the common-denominator method is easily extended to dividing a whole number by a rational number expressed in mixed form. The whole number and the rational number expressed in mixed form are simply renamed with fractions showing a common denominator. The division is then performed using the common-denominator method as introduced on the previous text pages.

Pre-Book Lesson

• Review solving mathematical sentences involving the factors-product relationship that require multiplication and that require division. Emphasize (a) how to recognize when the unknown to be found is a factor, and (b) how to determine what operation is required.

• Draw on the board a number-line picture such as that shown below, and ask the pupils to write three mathematical sentences representing the relationship shown by the number-line picture—one sentence containing the symbol for multiplication and two sentences containing the symbol for division:


$1\frac{1}{3}$
$1\frac{1}{3}$
$1\frac{1}{3}$
$\frac{0}{3}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$ $\frac{4}{3}$ $\frac{5}{3}$ $\frac{6}{3}$ $\frac{7}{3}$ $\frac{8}{3}$ $\frac{9}{3}$ $\frac{10}{3}$ $\frac{11}{3}$ $\frac{12}{3}$ $\frac{13}{3}$

(1) $3 \times 1\frac{1}{3} = n$ (2) $n \div 1\frac{1}{3} = 3$ (3) $n \div 3 = 1\frac{1}{3}$

Using the Text Page

• In Ex. 1–5, the common-denominator method of dividing a whole number by a rational number is extended to include dividing a whole number by a rational number expressed in mixed form. The whole number and the rational number in mixed form are renamed with fractions showing a common denominator, and the division is performed by the procedure introduced on text page 205.

• Ex. 6–7 are intended to refresh pupils' memories about determining from the statement of the factors-product relationship whether the unknown in the mathematical sentence represents the product or a factor, and about determining what operation is required to solve for the unknown.

• Ex. 8–16 give the pupils mixed computational practice in multiplying and dividing rational numbers named with mixed forms. Once the pupil recognizes that multiplication is required, he simply multiplies the two numbers shown in the problem. Since multiplication is commutative, the order in which the two factors are shown does not matter.

When the pupil recognizes that division is required, he must decide which number shown is the product and which is the known factor. Since division is not commutative, the order in which the numbers are shown in the mathematical sentence is very important. If pupils fail to get the correct answers for Ex. 12 and 14, check to see if they have shown the numbers in correct order before performing the division—that is, the product divided by the known factor.

Individualizing Instruction

• See if the *more capable pupils* can do any of Ex. 8–16 without pencil and paper.

• Have *all pupils* select some of Ex. 8–16 and try to write a problem for the relationship expressed by the given mathematical sentence.

• Help the *slower learners* analyze their difficulties. They may need a more extensive review than the other pupils in determining whether the unknown in a mathematical sentence is the product or a factor, and in determining what operation is required to solve for the unknown.

Pupil's Objectives

(a) To have practice in differentiating between problems requiring multiplication and those requiring division for their solution; and (b) to have practice in solving problems requiring multiplication of or division by rational numbers expressed in mixed form.

Background

Differentiating between problems requiring multiplication and those requiring division is essentially a task of determining whether the product is named in the problem or is to be found. If the product is to be found, the two factors are given and multiplication is required. If the product is given, one factor also is given and the other factor is to be found. This requires division.

For example, in solving Ex. 3, pupils who do not analyze the problem carefully may simply decide to multiply the two numbers shown. The problem actually is one of determining how many sets of $2\frac{1}{2}$ hours there are in a set of 26 hours. The total number of hours (26) is given and the number of equivalent sets of $2\frac{1}{2}$ is to be determined, so division is required. An analysis of this type in terms of equivalent sets is not applicable to all problem situations, but it may be helpful in many situations.

Pre-Book Lesson

• Ask the pupils for suggestions about how to differentiate between problems requiring multiplication and those requiring division.

• Use problems without numbers as practice exercises, such as indicating what operation would be used to solve problems like the following:

1. The area and the width of a rectangular region are given and the length is to be found.

2. One side of a regular polygon is given and the perimeter is to be found.

Through careful questioning, lead the pupils to the realization that either the product and a factor are given in the problem and the unknown factor is to be found or two factors are given and the product is to be found. When a pupil gives an incorrect answer, it is important that he realize why it is incorrect.

Individualizing Instruction

• The *more capable pupils* may be asked to write original problems requiring multiplication of a whole number and a rational number expressed in mixed form or division of a whole number by a rational number expressed in mixed form. The other pupils then can try to determine what operation is required to solve the problem.

• Emphasize for *all pupils* that the main purpose of these exercises is to learn to differentiate between problems requiring multiplication and those requiring division. Give encouragement to those pupils who analyze the problem correctly and write a correct mathematical sentence even though they may make a computational error and obtain an incorrect answer.

• If *slower learners* have difficulty in determining whether multiplication or division is required, give them additional problems without numbers like those suggested in the Pre-Book Lesson.

Multiply or Divide?

Differentiating M. and D. [O]

For each of Ex. 1–7, tell what operation to use to find the answer for the problem.

1. How many pieces of ribbon each $3\frac{1}{4}$ in. long can be cut from a 26-in. ribbon? Division; $n = 26 \div 3\frac{1}{4}$; 8

2. If gasoline costs $28\frac{1}{2}¢$ per gallon, how much will 16 gallons of gasoline cost? Multiplication; $n = 28\frac{1}{2} \times 16$; 456¢, or \$4.56

3. Mr. James said that it takes one gallon of gasoline to operate his power lawn mower for $2\frac{1}{2}$ hours. How many gallons of gasoline would it take to operate the mower for 26 hours? Division; $n = 26 \div 2\frac{1}{2}$; $10\frac{2}{5}$

4. An angle whose measurement is 33° can be partitioned into how many congruent angles each measuring $8\frac{1}{4}$ in degrees? Division; $n = 33 \div 8\frac{1}{4}$; 4

5. Find the perimeter of a regular octagon if the length of each side is $3\frac{3}{4}$ cm. Multiplication; $n = 8 \times 3\frac{3}{4}$; 30 cm.

6. Find the area of a rectangular region 15 ft. by $4\frac{1}{3}$ ft. Multiplication; $n = 15 \times 4\frac{1}{3}$; 65 sq. ft.

7. Find the width of a rectangular region if the area of the region is 44 sq. in. and the length is $7\frac{3}{8}$ in. Division; $n = 44 \div 7\frac{3}{8}$; $5\frac{57}{59}$ in.

[W]

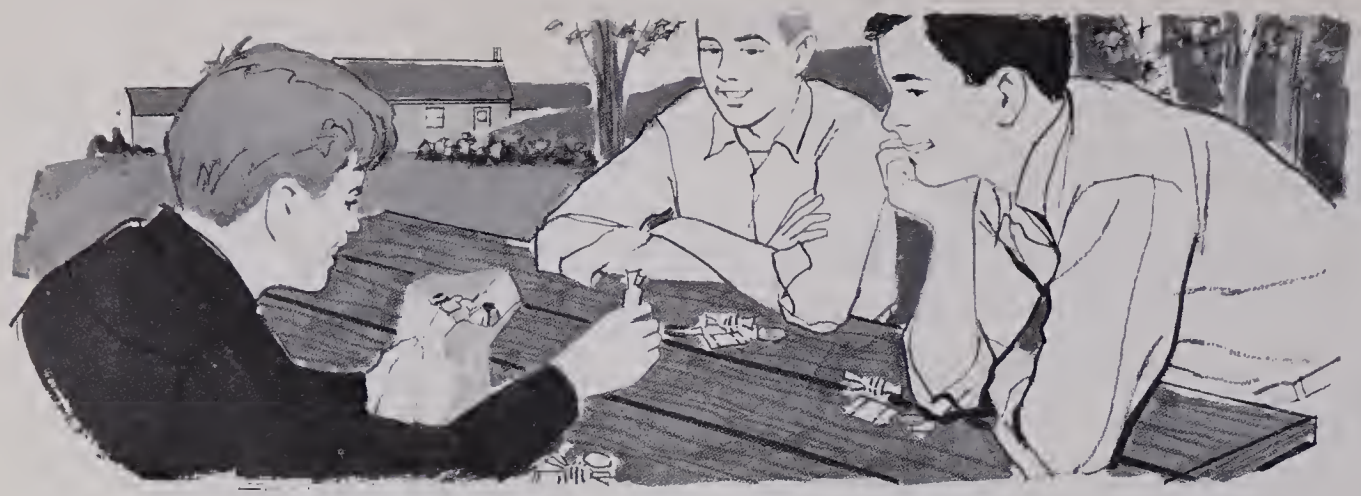
Now write and solve a mathematical sentence to find the answer for each of Ex. 1–7.

Ex. 8–10. Different units are used in stating the problem. Express all measurements in terms of the smaller unit of measurement used and then solve the problem.

8. How many $18\frac{1}{2}$-in. boards can be cut from a board that is 5 feet long? 60 in. $3\frac{9}{37}$

9. If the perimeter of a regular polygon is 30 yd. and the length of each side is $3\frac{1}{3}$ ft., how many sides does the polygon have? 90 ft. 27

10. A segment that is 55 cm. long can be partitioned into how many segments that are $2\frac{1}{5}$ mm. in length? 550 mm. 250



Whole-Number Divisors

Using common denominators [O]

1. Three boys shared equally $\frac{1}{2}$ lb. of candy. How much candy did each boy receive?

a. Did each boy receive *one third* of the $\frac{1}{2}$ lb. of candy? Yes Then each boy received $\frac{1}{3} \times \frac{1}{2}$, or $\frac{1}{6}$, pound of candy.

b. Since each of the three boys received the same amount of candy, another way to find the answer is to divide $\frac{1}{2}$ by 3. Does $n = \frac{1}{2} \div 3$ show the relationship expressed in the problem? Yes

c. Explain the work in box A for solving $n = \frac{1}{2} \div 3$. Key idea: The known factor is renamed with $\frac{6}{2}$ and the work is done.

A

$$n = \frac{1}{2} \div 3$$
$$= \frac{1}{2} \div \frac{6}{2}$$
$$= 1 \div 6$$
$$n = \frac{1}{6}$$

B

$$n = \frac{4}{7} \div 2$$
$$= \frac{4}{7} \div \frac{14}{7}$$
$$= 4 \div 14$$
$$n = \frac{4}{14}, \text{ or } \frac{2}{7}$$

2. For each of Ex. a–c, tell how the known factor is to be renamed and then, at the board, solve as in box A.

a. $n = \frac{1}{5} \div 2$ $\frac{10}{5}$ $\frac{1}{10}$ **b.** $n = \frac{3}{4} \div 5$ $\frac{20}{4}$ $\frac{3}{20}$ **c.** $n = \frac{2}{3} \div 7$ $\frac{21}{3}$ $\frac{2}{21}$

3. Box B. Explain the work shown for solving the mathematical sentence $n = \frac{4}{7} \div 2$. Key idea: The answer is expressed in simplest form. Is the fraction $\frac{4}{14}$ in simplest form? No Is $\frac{2}{7}$? Yes

4. If an answer is expressed with a fraction that is not in simplest form, tell how to find a fraction in simplest form for the answer. Divide both numerator and denominator by their greatest common factor.

5. At the board, solve each of Ex. a–c. Remember, express all answers in simplest form.

a. $n = \frac{2}{3} \div 6$ $\frac{1}{9}$ **b.** $\frac{6}{7} \div 3 = n$ $\frac{2}{7}$ **c.** $x = \frac{15}{16} \div 25$ $\frac{3}{80}$

Teaching Pages 208 and 209

Pupil's Objectives

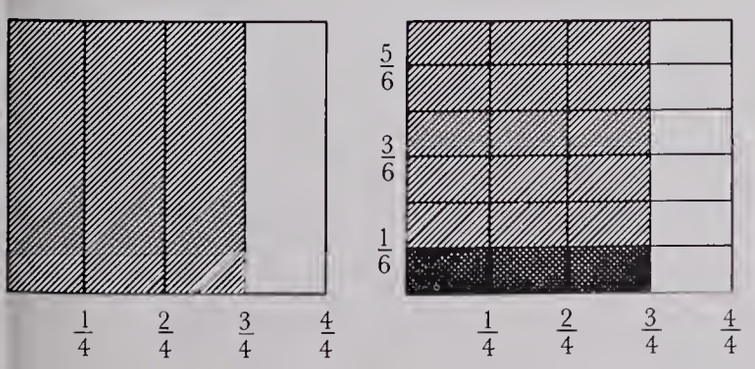
(a) To learn to divide a rational number by a whole number; (b) to have practice solving mathematical sentences involving a factors-product relationship; and (c) to have practice in solving problems requiring division by a whole number of a rational number shown by a fraction.

Background

On these pages, for dividing a rational number named by a fraction by a whole number, the method is introduced as an extension of the common-denominator method of division. The rational number shown by a fraction or a mixed form and the whole number both are renamed with fractions showing a common denominator and then the common-denominator procedure described on previous pages is carried out.

If pupils have difficulty understanding how it is possible to divide a rational number named by a fraction by a whole number, a model of a square-unit region may be used. Have pupils consider the problem, "If $\frac{3}{4}$ lb. of hamburg is to be distributed equally among 6 boys, how much will each boy receive?"

Use the model of a square-unit region to represent 1 lb. of hamburg. Partition (vertically) the unit region shown into 4 congruent smaller regions and shade 3 of them to represent $\frac{3}{4}$ lb. of hamburg. Then partition the entire region horizontally to show 6 congruent regions and shade $\frac{1}{6}$ of the 3 fourths to represent the amount of hamburg one boy will receive.



The square-unit region has now been shown partitioned into 24 smaller regions, each representing $\frac{1}{24}$ lb. of hamburg. 18 of these smaller regions represent the $\frac{3}{4}$ lb. of hamburg, so $18 \div 6$, or 3 of these smaller regions represent the amount of hamburg to be distributed to each boy. These 3 smaller regions represent $\frac{3}{24}$, or $\frac{1}{8}$ lb., and are shown by cross-hatching. Therefore, $\frac{3}{4} \div 6 = \frac{18}{24} \div 6 = \frac{3}{24}$, or $\frac{1}{8}$. This is the same answer that is obtained by using the common-denominator method.

Although the use of the model of a square-unit region may be helpful in giving meaning to dividing a rational number shown by a fraction by a whole number, it is intended that the pupils use the common-denominator method of division in solving the exercises on these pages.

Pre-Book Lesson

Ask for examples of real-life problems requiring division of a rational number (shown by a fraction) by a whole number. Some possible examples would be:

1. If $\frac{2}{3}$ of a ton of topsoil is to be distributed equally among 4 landowners, how much topsoil should each landowner receive?

2. If a board $4\frac{2}{3}$ ft. long is to be cut into 5 shorter boards of the same length, how long will each of the shorter boards be?

When a problem requires dividing a rational number named by a fraction by a whole number, some pupils may suggest that the rational number be multiplied by the reciprocal of the whole number. That is, to divide by 3 you may multiply by $\frac{1}{3}$. This would be mathematically correct. In fact this is called the *reciprocal* method and it is introduced on text page 212. Explain that for now the common-denominator method will be used, and guide them toward using this method in suggesting ways of solving the problems discussed.

Using the Text Pages

• Ex. 1 brings out the fact that multiplying by $\frac{1}{3}$ and dividing by 3 produce the same result. This may be demonstrated by use of a model of a square-unit region or by using a number-line

picture. However, do not encourage the pupils to find the result of division by using multiplication. That concept will be introduced shortly. For now, have pupils perform the division by the common-denominator method.

• In Ex. 6–7 the concept of dividing a rational number named by a fraction by a whole number, using the common-denominator method, is extended to include rational numbers named with mixed forms.

• Check the pupils' work for Ex. 13, 16, 17, 19, and 20. For Ex. 13 and 19 the product is unknown so multiplication is required. In Ex. 16, 17, and 20, a factor is unknown so division is required. Care must be taken that the mathematical sentence for division is first shown correctly as: *known product* ÷ *known factor* = *unknown factor*. Review this idea with those pupils making errors in these examples where division is required.

Individualizing Instruction

• Give the *more capable pupils* more challenging exercises, such as (a) $23\frac{1}{2} \div 3$; (b) $17\frac{2}{5} \div 11$; (c) $9\frac{1}{2} \div 23$.

• Have *all pupils* suggest items for a checklist which would be useful in locating sources of error in working Ex. 9–20. Some suggestions are:

1. Did you use the correct operation?
2. Did you change from the mixed form to a fraction correctly?
3. Did you rename the whole number with a fraction correctly?
4. In exercises requiring division, did you write the correct mathematical sentence before performing the division?
5. Did you write the answer in simplest form?

• You may wish to have the *more capable pupils* help the *slower learners* find sources of their errors in these exercises.

NOTES

* Pupils should include "about" in their answer.

6. If the three boys had shared $1\frac{1}{4}$ lb. of candy, how much candy would each boy have received?

a. Does $n = 1\frac{1}{4} \div 3$ show the relationship expressed in the problem? Yes

b. The work in box C shows one way to solve $n = 1\frac{1}{4} \div 3$. Were both the product and the known factor renamed with fractions that show the same denominator? Yes Explain the work shown. The division was done after factor and product were renamed with a common denominator.

c. Each boy would have received $\frac{5}{12}$ lb. of candy.

C
$$n = 1\tfrac{1}{4} \div 3$$
$$= \tfrac{5}{4} \div \tfrac{12}{4}$$
$$= 5 \div 12$$
$$n = \tfrac{5}{12}$$

7. For each of Ex. a–c, tell how the product and the known factor are to be renamed and then, at the board, solve as in box C.

a. $n = 1\frac{3}{8} \div 2$ $\frac{11}{16}$ b. $n = 2\frac{2}{5} \div 3$ $\frac{4}{5}$ c. $n = 3\frac{3}{4} \div 2$ $1\frac{7}{8}$

8. For each of Ex. 9–20, tell whether the letter represents the product or a factor. When the product and one of two factors are known, we divide to find the other factor.

9. factor; 10. factor; 11. product; 12. factor; 13. product; 14. factor; 15. factor; 16. factor; 17. factor; 18. factor; 19. product; 20. factor [W]

Now solve each of Ex. 9–20.

9. $n = \frac{1}{4} \div 5$ $\frac{1}{20}$
10. $n = \frac{2}{9} \div 7$ $\frac{2}{63}$
11. $x = \frac{4}{5} \times 8$ $6\frac{2}{5}$
12. $x = 12\frac{2}{3} \div 2$ $\frac{6}{23}$
13. $n \div 8 = \frac{1}{4}$ 2
14. $n = 1\frac{3}{4} \div 5$ $\frac{7}{20}$
15. $y = 5\frac{1}{2} \div 3$ $1\frac{5}{6}$
16. $4 \div n = \frac{7}{8}$ $4\frac{4}{7}$
17. $8 \times n = \frac{10}{11}$ $\frac{5}{44}$
18. $y = 6\frac{1}{3} \div 4$ $1\frac{7}{12}$
19. $n \div 5 = 7\frac{2}{3}$ $38\frac{1}{3}$
20. $4 \times x = 2\frac{4}{5}$ $\frac{7}{10}$

Ex. 21–23. Write and solve a mathematical sentence to find the answer for the problem. Show your work.

21. If a $6\frac{1}{2}$-cm. segment is partitioned into 5 congruent segments, what will be the length of each of the smaller segments? $n = 6\frac{1}{2} \div 5$; $1\frac{3}{10}$ cm.

22. Mrs. Conway had $1\frac{1}{4}$ cups of grated walnuts to put on three cakes that she had baked for a picnic. That was how many cups of walnuts for each cake? $n = 1\frac{1}{4} \div 3$; $\frac{5}{12}$

23. If 1 gal. of water weighs about $8\frac{1}{3}$ lb., about how much does 1 qt. of water weigh? $n = 8\frac{1}{3} \div 4$; about $2\frac{1}{12}$ lb. *

♦ **Extra Examples.** Set 104.

More Dividing with Fractions

Product and known factor—rational numbers [O]

1. It takes Mr. Meyer $\frac{3}{5}$ of an hour to spray each of his cherry trees. How many trees can he spray in $4\frac{1}{2}$ hr.?

a. Does $n = 4\frac{1}{2} \div \frac{3}{5}$ show the relationship expressed in the problem? Yes Explain. It expresses the factors-product relationship.

b. The work in box A shows how to solve $n = 4\frac{1}{2} \div \frac{3}{5}$ using the common-denominator method.

A

$$
\begin{aligned}
n &= 4\tfrac{1}{2} \div \tfrac{3}{5} \\
&= \tfrac{9}{2} \div \tfrac{3}{5} && (1) \\
&= \tfrac{45}{10} \div \tfrac{6}{10} && (2) \\
&= 45 \div 6 && (3) \\
&= \tfrac{45}{6} && (4) \\
n &= 7\tfrac{3}{6}, \text{ or } 7\tfrac{1}{2} && (5)
\end{aligned}
$$

Step 1: Rename the product and the known factor with fractions. Do $\frac{9}{2}$ and $\frac{3}{5}$ show a common denominator? No

Step 2: Express $\frac{9}{2}$ and $\frac{3}{5}$ with fractions showing the same denominator. Is 10 the *least common denominator* (L.C.D.) of $\frac{9}{2}$ and $\frac{3}{5}$? Yes For what other operations do we sometimes find the L.C.D.? Addition and subtraction Explain the other steps shown.

c. In $4\frac{1}{2}$ hr., Mr. Meyer can spray _?_ $7\frac{1}{2}$ trees.

2. For each of Ex. a–c, explain how to rename to show a common denominator and then, at the board, solve as in box A.

a. $n = 3\frac{1}{2} \div \frac{1}{5}$ $\frac{35}{10}$ $\frac{2}{10}$ $17\frac{1}{2}$ **b.** $n = 5\frac{2}{3} \div \frac{1}{4}$ $\frac{68}{12}$ $\frac{3}{12}$ $22\frac{2}{3}$ **c.** $n = 4\frac{2}{5} \div \frac{3}{4}$ $\frac{88}{20}$ $\frac{15}{20}$ $5\frac{13}{15}$

Teaching Pages 210 and 211

Pupil's Objectives

(a) To learn to divide with fractions, using the common-denominator method; and (b) to learn to recognize for a division whether the unknown factor is greater than 1 or less than 1.

Background

The concept of dividing rational numbers using the common-denominator method is extended to division involving a common fraction and a mixed form and involving two mixed forms. These rational numbers are first renamed with fractions showing a common denominator, and then the principle that $\frac{a}{b} \div \frac{c}{b} = a \div c$ is applied in accordance with the common-denominator method of division.

Pre-Book Lesson

• The exercises on these pages require that the pupils be familiar with the method of finding the least common denominator (L.C.D.) of two rational numbers. The method of finding the L.C.D. introduced on pupil's page 104 should be reviewed and the pupils given practice exercises in finding the L.C.D.

• The *more capable pupils* may be able to do these exercises without benefit of the development presented. See if they can solve Ex. 7–18 correctly without previous development.

Using the Text Pages

• In Ex. 1, step 2 requires the pupil to express the numbers with fractions showing the same denominator. It is not necessary that the denominator be the least common denominator of the two rational numbers, but, if it is not, the answer will be a fraction that is not in simplest form. The answer will then have to be expressed in simplest form. You may wish to have the pupils explain what would happen if the common denominator selected were not the least common denominator.

• If pupils have difficulty with Ex. 5–6, the following relationships which were introduced on pupil's page 198 may be summarized: For two factors that are greater than zero

if one factor is	the product is
less than 1,	less than the other factor.
greater than 1,	greater than the other factor.
equal to 1,	equal to the other factor.

From this table, we can see that if

the product is	the unknown factor is
less than the known factor,	less than 1.
greater than the known factor,	greater than 1.
equal to the known factor,	equal to 1.

This concept is useful when estimating answers for division exercises and determining the reasonableness of answers obtained.

• If the number of pupils making errors in Ex. 7–18 is great, attempt to determine the main cause of the errors and explain to the class the reason for these errors. Give more exercises until the pupils have mastered the common-denominator method.

Individualizing Instruction

• The *more capable pupils* may be able to solve without pencil and paper exercises of the type found on pupil's page 211. They may be given additional exercises to do mentally, but the exercises on this page should be written so that if wrong answers are obtained, the cause of the errors may be determined.

• *All pupils* may be asked to summarize the common-denominator method of dividing rational numbers expressed by fractions or by mixed forms.

• The *slower learners* may be given additional exercises as needed in (a) changing mixed forms to fractions, (b) determining least common denominator of two rational numbers, and (c) renaming two rational numbers so they have a common denominator, preferably the least common denominator. Mastery of these techniques is necessary in dividing using the common-denominator method.

NOTES

* See page 198.

3. If it took Mr. Meyer $1\frac{1}{4}$ hr. to spray each tree instead of $\frac{3}{5}$ of an hour, how many trees would he be able to spray in $4\frac{1}{2}$ hr.?

a. Is $1\frac{1}{4}$ greater than $\frac{3}{5}$? Yes Then would he be able to spray more trees or fewer trees? Fewer Explain. Key idea: More time would be spent on each tree.

b. Explain how $n = 4\frac{1}{2} \div 1\frac{1}{4}$ shows the relationship expressed in the problem. It expresses the factors-product relationship.

c. Explain the work in box B for solving $n = 4\frac{1}{2} \div 1\frac{1}{4}$. Key idea: The common denominator method is used.

d. Give the answer for the problem. $3\frac{3}{5}$

B

$$n = 4\tfrac{1}{2} \div 1\tfrac{1}{4}$$
$$= \tfrac{9}{2} \div \tfrac{5}{4}$$
$$= \tfrac{18}{4} \div \tfrac{5}{4}$$
$$= 18 \div 5$$
$$n = \tfrac{18}{5}, \text{ or } 3\tfrac{3}{5}$$

4. For each of Ex. a–c, explain how to rename to show a common denominator and then, at the board, solve as in box B.

a. $n = 3\frac{1}{5} \div 1\frac{1}{3}$ $\left(\frac{48}{15} \div \frac{20}{15}\right)$ $2\frac{2}{5}$ **b.** $n = 5\frac{2}{3} \div 2\frac{1}{2}$ $\left(\frac{34}{6} \div \frac{15}{6}\right)$ $2\frac{4}{15}$ **c.** $n = 2\frac{4}{5} \div 5\frac{1}{3}$ $\left(\frac{42}{15} \div \frac{80}{15}\right)$ $\frac{21}{40}$

* 5. Suppose for a division that both the product and the known factor are greater than 0. Then, if the product is less than the known factor, the unknown factor is _?_ less than 1. If the product is greater than the known factor, the unknown factor is _?_ greater than 1.

6. For each of Ex. 7–18, tell whether the number represented by the letter *is less than* or *is greater than* 1.

7. greater; 8. less; 9. greater; 10. greater; 11. greater; 12. less; 13. greater; 14. less; 15. greater; 16. greater; 17. less; 18. greater [W]

Ex. 7–18. Solve. Use the common-denominator method.

7. $n = \frac{2}{3} \div \frac{1}{6}$ 4

8. $n = \frac{7}{8} \div 1\frac{1}{4}$ $\frac{7}{10}$

9. $n = 3\frac{1}{5} \div 3$ $1\frac{1}{15}$

10. $y = 4 \div 2\frac{4}{5}$ $1\frac{3}{7}$

11. $x = 2\frac{1}{3} \div \frac{3}{5}$ $3\frac{8}{9}$

12. $n = 5\frac{1}{5} \div 6\frac{1}{2}$ $\frac{4}{5}$

13. $n = 8\frac{1}{3} \div 1\frac{1}{5}$ $6\frac{17}{18}$

14. $n = \frac{14}{5} \div \frac{9}{3}$ $\frac{14}{15}$

15. $n = \frac{20}{3} \div 3\frac{2}{3}$ $1\frac{9}{11}$

16. $n = 5\frac{4}{7} \div 2\frac{2}{3}$ $2\frac{5}{56}$

17. $n = \frac{7}{8} \div 3\frac{5}{6}$ $\frac{21}{92}$

18. $x = 12\frac{1}{2} \div 4\frac{1}{3}$ $2\frac{23}{26}$

Ex. 19–24. Copy and complete by writing $<$ or $>$.

19. $3 \div \frac{3}{7}$ _?_ ≥ 6

20. $4\frac{1}{2} \div 1\frac{1}{2}$ _?_ $> 2\frac{2}{3} \div 2$

21. $5 \div \frac{2}{3}$ _?_ $\geq 6\frac{1}{2}$

22. $\frac{3}{7} \div \frac{2}{5}$ _?_ ≥ 1

23. $\frac{4}{5} + 1\frac{3}{4}$ _?_ $< 3\frac{1}{3}$

24. $5\frac{7}{8}$ _?_ $> 11\frac{1}{2} \div 2$

25. Using $3\frac{5}{7}$ and $14\frac{2}{3}$ for the product and the known factor and n to represent the unknown factor, write four mathematical sentences which illustrate the factors-product relationship.

$14\frac{2}{3} \times n = 3\frac{5}{7}$; $n = 3\frac{5}{7} \div 14\frac{2}{3}$; $3\frac{5}{7} \div n = 14\frac{2}{3}$; $n \times 14\frac{2}{3} = 3\frac{5}{7}$

*Point out that the reciprocal for a number named with a mixed form is easier to find if the number is first renamed with a fraction.

The Reciprocal of a Number

Meaning [O]

1. Is $2 \times \frac{1}{2} = 1$ a true mathematical sentence? Yes Because the product of 2 and $\frac{1}{2}$ is 1, 2 is called the *reciprocal* of $\frac{1}{2}$, and $\frac{1}{2}$ is called the reciprocal of 2.

If the product of two numbers is 1, then either of the two numbers is called the reciprocal of the other.

2. Explain why $\frac{1}{3}$ is the reciprocal of 3. Their product is 1.

3. For each of Ex. a–e, name the reciprocal of the number.

a. 7 $\frac{1}{7}$ **b.** 10 $\frac{1}{10}$ **c.** $\frac{1}{9}$ 9 **d.** 23 $\frac{1}{23}$ **e.** $\frac{1}{15}$ 15

4. $\frac{2}{3} \times \frac{3}{2} = \frac{?}{6} = \_?\_1$ Is $\frac{2}{3}$ the reciprocal of $\frac{3}{2}$? Yes Is $\frac{3}{2}$ the reciprocal of $\frac{2}{3}$? Yes

$\frac{3}{5} \times n = 1$
$n = 1 \div \frac{3}{5}$
$= \frac{5}{5} \div \frac{3}{5}$
$= 5 \div 3$
$n = \frac{5}{3}$

5. For $\frac{3}{5} \times n = 1$, is the number represented by n the reciprocal of $\frac{3}{5}$? Yes Why? The product is 1. Explain the work in the box. The reciprocal of $\frac{3}{5}$ is $\_?\frac{5}{3}$. Tell a short way to find the reciprocal of a number that is named with a fraction. By showing the fraction with numerator and denominator interchanged.

6. For each of Ex. a–c, name the number represented by n by thinking about reciprocals.

a. $\frac{4}{5} \times n = 1$ $\frac{5}{4}$ **b.** $n \times \frac{7}{8} = 1$ $\frac{8}{7}$ **c.** $1 = n \times \frac{15}{7}$ $\frac{7}{15}$

7. At the board, find $2\frac{1}{2} \times \frac{2}{5}$. 1 Is $\frac{2}{5}$ the reciprocal of $2\frac{1}{2}$? Yes

8. For each of Ex. a–e, rename with a fraction and then name the reciprocal.

a. $1\frac{1}{2}$ $\frac{3}{2}$; $\frac{2}{3}$ **b.** $4\frac{1}{3}$ $\frac{13}{3}$; $\frac{3}{13}$ **c.** $3\frac{4}{5}$ $\frac{19}{5}$; $\frac{5}{19}$ **d.** $5\frac{3}{8}$ $\frac{43}{8}$; $\frac{8}{43}$ **e.** $10\frac{3}{10}$ $\frac{103}{10}$; $\frac{10}{103}$

[W]

* Ex. 9–20. Name the reciprocal of the number.

9. 17 $\frac{1}{17}$ 10. $\frac{34}{43}$, or $1\frac{1}{3}$ 11. $\frac{1}{100}$ 100 12. $25\frac{3}{4}$ $\frac{4}{103}$ 13. $7\frac{1}{9}$ $\frac{9}{64}$ 14. $5\frac{2}{7}$ $\frac{7}{37}$
15. $\frac{5}{3}$ $\frac{3}{5}$ 16. $\frac{9}{10}$ $\frac{10}{9}$, or $1\frac{1}{9}$ 17. 6.1 $\frac{1}{6.1}$, or $\frac{10}{61}$ 18. $1\frac{4}{5}$ $\frac{5}{9}$ 19. $\frac{78}{87}$, or $1\frac{1}{7}$ 20. 1 1

21. List the members of $\{0, \frac{3}{4}, \frac{7}{8}, 1\frac{1}{3}, 1\frac{1}{2}\}$ that are

a. less than the reciprocal of $1\frac{1}{5}$. 0, $\frac{3}{4}$

b. greater than the reciprocal of $\frac{4}{5}$. $1\frac{1}{3}$, $1\frac{1}{2}$

22. Which counting number is the reciprocal of itself? 1

Teaching Page 212

Pupil's Objective

To learn the meaning of the reciprocal of a number.

Background

The exercises on this page deal with the concept of the reciprocal of a number, as preparation for learning how to divide rational numbers using the reciprocal method. The reciprocal method is presented on pupil's page 214.

If the product of two numbers is 1, then either of the two numbers is called the *reciprocal* of the other. Another mathematical expression for *reciprocal* is *multiplicative inverse*. Some examples of using reciprocals are:

a. $\frac{5}{7} \times \frac{7}{5} = 1$ b. $\frac{9}{2} \times \frac{2}{9} = 1$

c. $1 \times 1 = 1$ d. $\frac{5}{5} \times \frac{5}{5} = 1$

From these examples, we can state that $\frac{5}{7}$ is the reciprocal of $\frac{7}{5}$ and $\frac{7}{5}$ is the reciprocal of $\frac{5}{7}$; $\frac{9}{2}$ is the reciprocal of $\frac{2}{9}$ and $\frac{2}{9}$ is the reciprocal of $\frac{9}{2}$; 1 is the reciprocal of itself.

Zero, as shown by any fraction that names zero such as $\frac{0}{2}$, has no reciprocal because there is no number by which we can multiply zero to obtain a product of 1.

If $\frac{a}{b}$ represents a rational number in which $a \neq 0$, its reciprocal can be shown to be $\frac{b}{a}$ because

$$\frac{a}{b} \times \frac{b}{a} = \frac{\overset{1}{\cancel{a}} \times \overset{1}{\cancel{b}}}{\underset{1}{\cancel{b}} \times \underset{1}{\cancel{a}}} = \frac{1}{1}, \text{ or } 1.$$

Pre-Book Lesson

• Write on the board the mathematical sentence $\frac{2}{3} \times n = 1$ and ask, "What is unknown, the product or a factor? What operation is required to find the unknown? Express the relationship in a mathematical sentence involving division. Solve, using the common-denominator method."

$$n = \frac{3}{3} \div \frac{2}{3} = 3 \div 2 = \frac{3}{2}, \text{ or } 1\frac{1}{2}.$$

• Repeat this procedure for

(a) $3 \times n = 1$; (b) $\frac{7}{3} \times n = 1$; (c) $\frac{1}{5} \times n = 1$.

• Ask if there is a number which, when substituted for n, will make $0 \times n = 1$ a true statement. Summarize the results of these exercises when the pupils realize that for any number except zero there is a number such that their product is 1.

Using the Text Page

• After Ex. 1, ask "What is the reciprocal of 1?" (1) "What number has no reciprocal?" (0) "How do you know that zero has no reciprocal?" (The product of zero and any number is always zero, so the product can never be 1.)

• In Ex. 5, the reciprocal of $\frac{a}{b} (a \neq 0)$ is $\frac{b}{a}$. You may find the pupils using such incorrect expressions as "turn the known factor upside down," "flip the known factor," and "invert the known factor." Pupils often find it difficult to verbalize the idea in a manner which is mathematically correct. Such expressions as *interchange numerator and denominator* and *exchange numerator and denominator* are more correct.

• In Ex. 7 have the pupils explain how to find the reciprocal of a rational number shown in mixed form.

• If the answers for Ex. 3 and 8 are shown on the board, the pupils can be asked, "The reciprocal of a number greater than 1 is always _ ? _ than 1. The reciprocal of a number less than 1 is always _ ? _ than 1." This can be used as a check on the reasonableness of an answer given in finding the reciprocal of a number.

Individualizing Instruction

• The *more capable pupils* may be given exercises of increasing difficulty to find the reciprocal of a number without using pencil and paper.

• Be sure *all pupils* can explain accurately in their own words the meaning of the reciprocal of a number. Be sure they know which number is its own reciprocal and which number has no reciprocal.

• Check the *slower learners* to be certain they understand how to find the reciprocal of a number shown in mixed form.

Pupil's Objectives

(a) To check on some of the major understandings taught in this and previous chapters; and (b) to relate some of the concepts learned in Chapter 5 to ideas presented in previous chapters.

Background

The exercises at the top of the page review concepts such as the reciprocal of a number, multiplying rational numbers and dividing rational numbers, the concepts of set, union of two sets, intersection of two sets, equivalent sets, and adding rational numbers. These exercises help reinforce the learning in this chapter and help maintain ideas presented in previous chapters.

The exercises at the bottom of the page involve mixed computational practice in which words are used instead of the usual symbols, $+$, $-$, $\times$ and $\div$. These exercises also involve the concepts of reciprocal, prime number, and mean average.

Pre-Book Lesson

You may wish to review quickly the meaning of set, union of sets, intersection of sets, equivalent sets, prime number, and mean average before beginning the work on the text page.

Using the Text Page

• The exercises at the top of the page are intended to be oral excrcises, and most of the pupils should be able to do them without using pencil and paper. If some of thc *slower learners* are simply unable to do the exercises without pencil and paper, they should not be required to do so.

• For Ex. 12, make sure that pupils realize that since $1\frac{1}{3}$ and $\frac{4}{3}$ are both names for the same number, the intersection of sets R and S is not empty.

• In Ex. 13, the pupils should know that the set of counting numbers is $\{1, 2, 3, \cdots\}$, and that the set of whole numbers is $\{0, 1, 2, \cdots\}$. They may not be certain as to what is included in the set of rational numbers. The reason for this is that the complete set of rational numbers is not considered at this grade level. A rational number is any number which can be named by $\frac{a}{b}$ $(b \neq 0)$ in which a and b are integers. Since integers are non-negative, negative, or zero, rational numbers are non-negative, negative, or zero. At this grade level, only the non-negative rational numbers and zero are considered. One way of describing a rational number for these pupils is "Any number that can be expressed in the form $\frac{a}{b}(b \neq 0)$ is a rational number." Since $1 = \frac{1}{1}$ and $4 = \frac{4}{1}$, and since $\frac{1}{1}$ and $\frac{4}{1}$ show rational numbers, we know that 1 and 4 are members of the set of rational numbers.

Individualizing Instruction

• *All pupils* may be given mathematical sentences involving the use of the symbols $+$, $-$, $\times$, and $\div$ and be asked to rewrite them in sentences using words instead of the mathematical symbols.

• Help *slower learners* to recognize areas of weakness revealed by these exercises, and have them work additional exercises to strengthen the weak areas.

Without Paper and Pencil

[o]

Ex. 1–5. Name the reciprocal of the number.

1. 15 $\frac{1}{15}$ 2. $\frac{7}{8}$ $\frac{8}{7}$, or $1\frac{1}{7}$ 3. $\frac{1}{3}$ 3 4. $\frac{35}{9}$ $\frac{9}{35}$ 5. $3\frac{1}{2}$ $\frac{2}{7}$

Ex. 6–11. Name the number represented by n.

6. $n = \frac{5}{7} \times \frac{2}{3}$ $\frac{10}{21}$ 7. $n = \frac{8}{9} \times \frac{9}{8}$ 1 8. $n = 6\frac{1}{2} \div (\frac{3}{4} + \frac{1}{4})$ $6\frac{1}{2}$

9. $n = \frac{4}{5} \div \frac{2}{5}$ 2 10. $n \times \frac{3}{10} = 1$ 11. $n = (7\frac{2}{3} + 8\frac{1}{3}) \div 4$ 4

10. $\frac{10}{3}$, or $3\frac{1}{3}$

Ex. 12–15. Refer to the sets of numbers listed at the right.

$R = \{0, \frac{5}{6}, 1\frac{1}{3}, 2\}$
$S = \{\frac{3}{5}, \frac{4}{3}, 1, 4\}$

12. Is it true that $R \cap S = \{\ \}$? No Explain. Since $1\frac{1}{3} = \frac{4}{3}$, $R \cap S = \{1\frac{1}{3}\}$ or $\{\frac{4}{3}\}$

13. Which members of $R \cup S$ are counting numbers? 1, 2, 4 whole numbers? 0, 1, 2, 4 rational numbers? $0, \frac{5}{6}, 1\frac{1}{3}, 2, \frac{3}{5}, 1, 4$

14. Name the numbers obtained when each member of S is multiplied by $\frac{3}{4}$. $\frac{9}{20}, 1, \frac{3}{4}, 3$

15. Explain why sets R and S are equivalent sets. Key idea: They have the same number of members.

Can You Follow Directions?

[w]

1. Multiply $5\frac{3}{4}$ by the reciprocal of $\frac{2}{5}$. $14\frac{3}{8}$
2. Divide $4\frac{1}{2}$ by the sum of $\frac{4}{5}$ and $3\frac{2}{3}$. $1\frac{1}{134}$
3. Divide the product of 24 and $3\frac{1}{8}$ by $4\frac{1}{3}$. $17\frac{4}{13}$
4. Divide 12 by $\frac{2}{3}$ and then subtract $15\frac{4}{7}$ from the result. $2\frac{3}{7}$
5. Divide $33\frac{1}{2}$ by the prime number that is less than 3. $16\frac{3}{4}$
6. Multiply 33 by the reciprocal of the result of adding $2\frac{3}{4}$ and $5\frac{1}{3}$. $4\frac{8}{97}$
7. Find the product of 3, $4\frac{1}{5}$, and $15\frac{1}{3}$. $193\frac{1}{5}$
8. Find the mean average for $14\frac{3}{4}$, 15, 15, $15\frac{1}{3}$, and $15\frac{3}{4}$. $15\frac{1}{6}$
9. Find the sum of the reciprocal of 5 and the reciprocal of $6\frac{1}{3}$. $\frac{34}{95}$

* Continue to have pupils explain the reasoning behind this algorithm even after they become proficient in its use.

Using Reciprocals in Division

The reciprocal method [O]

A

$$n = 5 \div \tfrac{2}{3}$$
$$= \tfrac{15}{3} \div \tfrac{2}{3}$$
$$= 15 \div 2$$
$$n = \tfrac{15}{2}, \text{ or } 7\tfrac{1}{2}$$

B

$$n = 5 \div \tfrac{2}{3}$$
$$= (5 \times \tfrac{3}{2}) \div (\tfrac{2}{3} \times \tfrac{3}{2}) \quad (1)$$
$$= (5 \times \tfrac{3}{2}) \div 1 \quad (2)$$
$$= 5 \times \tfrac{3}{2} \quad (3)$$
$$n = \tfrac{15}{2}, \text{ or } 7\tfrac{1}{2}$$

C

$$n = 5 \div \tfrac{2}{3}$$
$$= 5 \times \tfrac{3}{2}$$
$$= \tfrac{15}{2}, \text{ or } 7\tfrac{1}{2}$$

1. The work in box A shows how to solve $n = 5 \div \frac{2}{3}$ by the common-denominator method. Explain the work shown. Key idea: After the numbers are named with common denominators the division is done.

2. The work in box B shows how to solve $n = 5 \div \frac{2}{3}$ by the *reciprocal method.*

Step 1: Multiply both the product and the known factor by $\frac{3}{2}$, the reciprocal of the known factor. Explain why this does not change the number represented by n. Key idea: Multiplying a product and known factor by the same number does not change the unknown factor.

Step 2: Write *1* for $(\frac{2}{3} \times \frac{3}{2})$. Why may we do this? $\frac{2}{3} \times \frac{3}{2} = 1$

Step 3: Explain why we just write $5 \times \frac{3}{2}$. Key idea: 1 is the identity element for division. Explain the other work shown.

3. Which steps in box B are done mentally in box C? (1) and (2) Is the work in box C shorter than the work in box B? Yes box A? Yes

*** When the product and one of two factors are known, the unknown factor can be found by multiplying the product by the reciprocal of the known factor.**

4. At the board, solve $n = 7 \div \frac{4}{5}$ first as in box A, then as in box B, and then as in box C. Explain your work. See left.

4. $n = 7 \div \frac{4}{5}$
$= \frac{35}{5} \div \frac{4}{5}$
$= 35 \div 4$
$n = \frac{35}{4}$, or $8\frac{3}{4}$

$n = 7 \div \frac{4}{5}$
$= (7 \times \frac{5}{4}) \div (\frac{4}{5} \times \frac{5}{4})$
$= (7 \times \frac{5}{4}) \div 1$
$= 7 \times \frac{5}{4}$
$n = \frac{35}{4}$, or $8\frac{3}{4}$

$n = 7 \div \frac{4}{5}$
$= 7 \times \frac{5}{4}$
$= \frac{35}{4}$, or $8\frac{3}{4}$

5. At the board, solve Ex. a–f as in box B and then as in box C.

a. $n = 3 \div \frac{7}{8}$ $3\frac{3}{7}$ **b.** $n = 4 \div \frac{3}{5}$ $6\frac{2}{3}$ **c.** $n = 13 \div \frac{7}{9}$ $16\frac{5}{7}$
d. $x = 6 \div \frac{5}{7}$ $8\frac{2}{5}$ **e.** $n = 10 \div \frac{7}{8}$ $11\frac{3}{7}$ **f.** $y = 1 \div \frac{5}{3}$ $\frac{3}{5}$

6. Explain the work in boxes D and E below. Key idea: The reciprocal method and division by common factors is used.

D

$$\tfrac{3}{7} \div \tfrac{4}{5} = \tfrac{3}{7} \times \tfrac{5}{4} = \tfrac{15}{28}$$

E

$$\tfrac{2}{9} \div \tfrac{4}{3} = \tfrac{\overset{1}{\cancel{2}}}{\underset{3}{\cancel{9}}} \times \tfrac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}} = \tfrac{1}{6}$$

Teaching Pages 214 and 215

Pupil's Objective

To learn to use the reciprocal method when dividing rational numbers.

Background

If the product and known factor are both multiplied by the same number, the unknown factor is not changed, as illustrated below.

a. $n = 12 \div 4$
$= \frac{12}{4}$, or 3

b. $n = 12 \div 4$
$= (3 \times 12) \div (3 \times 4)$
$= 36 \div 12 = \frac{36}{12}$, or 3

The reciprocal method of dividing rational numbers is based on this concept. This principle also applies to rational numbers expressed by fractions. For $n = \frac{2}{3} \div \frac{3}{4}$, the product $\frac{2}{3}$ and the known factor $\frac{3}{4}$ may be multiplied by any number other than zero without changing the unknown factor. If each of the numbers and the reciprocal of the known factor are multiplied, the known factor becomes 1.

$$n = (\tfrac{2}{3} \times \tfrac{4}{3}) \div (\tfrac{3}{4} \times \tfrac{4}{3})$$
$$= (\tfrac{2}{3} \times \tfrac{4}{3}) \div 1 = \tfrac{2}{3} \times \tfrac{4}{3}$$

The result is a multiplication in which the factors are (a) the product shown in the original mathematical sentence and (b) the reciprocal of the known factor. The fact that this is always true may be proved as follows:

$$\tfrac{a}{b} \div \tfrac{c}{d} = (\tfrac{a}{b} \times \tfrac{d}{c}) \div (\tfrac{c}{d} \times \tfrac{d}{c})$$
$$= (\tfrac{a}{b} \times \tfrac{d}{c}) \div 1$$
$$= \tfrac{a}{b} \times \tfrac{d}{c}$$

This method of dividing rational numbers is called the *reciprocal method* and is the most efficient method. Its major drawback is that pupils soon forget the reasoning—that is, why the method results in the correct quotient. It is important that pupils remember that the product and known factor are both multiplied by the reciprocal of the known factor so that the known factor becomes 1.

Pre-Book Lesson

• Ex. 1 on pupil's page 208 demonstrates that multiplying by $\frac{1}{3}$ and dividing by 3 produce the same result. Refer to this exercise and have the pupils suggest other problems which can be solved by performing a division or a multiplication.

• Have the pupils explain how to find the answers for Ex. 9, 10, 12, 14, 15, and 18 on pupil's page 209, by performing multiplications.

Using the Text Pages

• In connection with Ex. 2, it may be necessary to do more examples of this type on the board for all pupils to understand the three steps in the procedure for the development of the reciprocal method of dividing rational numbers. The pupils must understand that (a) multiplying the product and the known factor by the same number other than zero does not change the value of the unknown factor, n; (b) the product and the known factor are multiplied by the reciprocal of the known factor because this will change the known factor to 1; and (c) the result is a multiplication involving the product and the reciprocal of the known factor.

• Enough examples should be given so that the pupils will be convinced that the result always is a multiplication involving the product and the reciprocal of the known factor. Ex. 4–10 may be sufficient, but pupils may also be encouraged to test this eoncept with any examples of which they can think.

• Sometimes pupils forget which reciprocal they are supposed to multiply by, particularly when the examples are presented in different ways as in Ex. 12–26. Point out that each of the following is solved by a division exercise and the reciprocal method can be used. In each of these examples, $3\frac{1}{3}$ is the product and $4\frac{3}{4}$ is the known factor:

a. $n = 3\frac{1}{3} \div 4\frac{3}{4}$
b. $n \times 4\frac{3}{4} = 3\frac{1}{3}$
c. $4\frac{3}{4} \times n = 3\frac{1}{3}$
d. $3\frac{1}{3} \div 4\frac{3}{4} = n$

Emphasize that the reciprocal method involves multiplying the *product* by the *reciprocal of the known factor*.

• For Ex. 27–34, pupils must perform the operation indicated in the parentheses first.

Individualizing Instruction

• Division of rational numbers may be performed using either the common-denominator method or the reciprocal method. Since the reciprocal method is so efficient, every effort should be made to have *all pupils* master this method. It would be desirable to have as many pupils as possible be able to explain the rationale of this method, but it is understandable if the *slower learners* are not able to explain why the technique results in the correct answer.

• You may wish to have the *more capable pupils* argue that $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$. The proof is contained in the Background section.

• Some of the *more capable pupils* may feel that it should be possible to apply the distributive property in some of Ex. 27–34. Multiplication is not distributive with respect to division, nor is division distributive with respect to multiplication. However, as pupils know, multiplication is distributive with respect to addition. It also distributes over subtraction:

$$a \times (b - c) = (a \times b) - (a \times c)$$
$$\text{and } (b - c) \times a = (b \times a) - (c \times a)$$

Therefore, in Ex. 29,

$$(4\tfrac{1}{2} - 2\tfrac{7}{8}) \times 3\tfrac{1}{3} = (4\tfrac{1}{2} \times 3\tfrac{1}{3}) - (2\tfrac{7}{8} \times 3\tfrac{1}{3})$$

Division distributes from right to left over addition and also over subtraction:

$$(a + b) \div c = (a \div c) + (b \div c)$$
$$(a - b) \div c = (a \div c) - (b \div c)$$

Therefore, in Ex. 33,

$$(15 - 12\tfrac{3}{5}) \div 5 = (15 \div 5) - (12\tfrac{3}{5} \div 5)$$

The question of use of the distributive property probably will not arise since the pupils have been taught to perform the operation indicated within the parentheses first. However, if the question should arise the information presented above can be used in answering any such question.

NOTES

* These problems may be worked using either the common-denominator or the reciprocal method. However, if pupils become confused about the two methods, it is advisable for them to use only one method.

7. At the board, work Ex. a–d as in boxes D and E.

a. $\frac{5}{12} \div \frac{3}{7}$ $\frac{35}{36}$ **b.** $\frac{2}{5} \div \frac{4}{7}$ $\frac{7}{10}$ **c.** $\frac{3}{4} \div \frac{5}{8}$ $1\frac{1}{5}$ **d.** $\frac{10}{3} \div \frac{20}{9}$ $1\frac{1}{2}$

8. Box F. Explain the work shown for solving $n = 3\frac{1}{5} \div 6\frac{2}{3}$ by the common-denominator method.

9. Box G. Explain the work shown for solving $n = 3\frac{1}{5} \div 6\frac{2}{3}$ by the reciprocal method. Does the reciprocal method seem easier to use than the common-denominator method? Yes You may use the method you prefer unless directed otherwise.

F	G
$n = 3\frac{1}{5} \div 6\frac{2}{3}$	$n = 3\frac{1}{5} \div 6\frac{2}{3}$
$= \frac{16}{5} \div \frac{20}{3}$	$= \frac{16}{5} \div \frac{20}{3}$
$= \frac{48}{15} \div \frac{100}{15}$	$= \frac{\overset{4}{\cancel{16}}}{5} \times \frac{3}{\cancel{20}_{5}}$
$= 48 \div 100$	$n = \frac{12}{25}$
$n = \frac{48}{100}$, or $\frac{12}{25}$	

10. At the board, solve each of Ex. a–c.

a. $n = 2\frac{3}{4} \div 5\frac{1}{5}$ $\frac{55}{104}$ **b.** $n = 1\frac{1}{3} \div \frac{4}{9}$ 3 **c.** $n \times 4\frac{3}{5} = 1\frac{3}{5}$ $\frac{8}{23}$

11. For each of Ex. 12–26, tell whether the number represented by the letter *is less than* or *is greater than* 1.

12. greater; 13. less; 14. greater; 15. greater; 16. greater; 17. less; 18. greater; 19. less; 20. less; 21. less; 22-26. greater. [W]

Ex. 12–26. Solve. Use the reciprocal method. Remember, express all answers in simplest form.

12. $n = 6 \div \frac{2}{5}$ 15
13. $n = \frac{3}{4} \div \frac{7}{8}$ $\frac{6}{7}$
14. $n = 5\frac{1}{3} \div \frac{5}{6}$ $6\frac{2}{5}$
15. $n = 17 \div \frac{8}{3}$ $6\frac{3}{8}$
16. $n \times \frac{6}{7} = 5\frac{2}{5}$ $6\frac{3}{10}$
17. $n = \frac{7}{3} \div 8\frac{5}{6}$ $\frac{14}{53}$
18. $x \times \frac{7}{4} = 2\frac{1}{2}$ $1\frac{3}{7}$
19. $\frac{3}{8} \div n = 15$ $\frac{1}{40}$
20. $x = 3\frac{3}{4} \div 6\frac{1}{2}$ $\frac{15}{26}$
21. $n \times 5\frac{1}{10} = \frac{2}{5}$ $\frac{4}{51}$
22. $\frac{6}{7} \div \frac{8}{21} = x$ $2\frac{1}{4}$
23. $6\frac{2}{9} = n \times 3\frac{2}{3}$ $1\frac{23}{33}$
24. $50\frac{1}{2} \div \frac{3}{4} = n$ $67\frac{1}{3}$
25. $n = 100 \div 6\frac{1}{4}$ 16
26. $x = 3\frac{15}{16} \div \frac{5}{8}$ $6\frac{3}{10}$

*Ex. 27–34. Solve.

27. $n = \frac{5}{7} \div (2\frac{1}{2} + \frac{5}{8})$ $\frac{8}{35}$
28. $n = 3\frac{1}{2} \times (\frac{5}{6} \div \frac{8}{3})$ $1\frac{3}{32}$
29. $y = (4\frac{1}{2} - 2\frac{7}{8}) \times 3\frac{1}{3}$ $5\frac{5}{12}$
30. $n = \frac{4}{3} \times (1\frac{7}{8} \div 2\frac{1}{2})$ 1
31. $n = 6\frac{3}{4} \div (\frac{4}{5} \times \frac{2}{3} \times \frac{3}{8})$ $33\frac{3}{4}$
32. $n = (4\frac{2}{5} \times 1\frac{1}{8}) \div 3\frac{5}{6}$ $1\frac{67}{230}$
33. $(15 - 12\frac{3}{5}) \div 5 = x$ $\frac{12}{25}$
34. $n = (\frac{9}{10} \div \frac{4}{15}) \div 1\frac{1}{2}$ $2\frac{1}{4}$

♦ **Extra Examples.** Sets 105–107.

Mixed Practice

[W]

Ex. 1–12. Solve. Use the common-denominator method.

1. $n = \frac{3}{5} \div \frac{1}{5}$ 3
2. $n = \frac{7}{3} \div \frac{5}{3}$ $1\frac{2}{5}$
3. $n = 8 \div \frac{5}{3}$ $4\frac{4}{5}$
4. $x = 3\frac{1}{3} \div \frac{5}{6}$ 4
5. $x = 1\frac{2}{5} \div \frac{3}{10}$ $4\frac{2}{3}$
6. $n = 5\frac{1}{3} \div 2\frac{5}{6}$ $1\frac{15}{17}$
7. $n \times 7 = \frac{4}{5}$ $\frac{4}{35}$
8. $x \times 2\frac{1}{2} = 30$ 12
9. $n = 5\frac{3}{4} \div 6\frac{1}{8}$ $\frac{46}{49}$
10. $n = 7 \div 4\frac{2}{3}$ $1\frac{1}{2}$
11. $x = 5 \div 7\frac{3}{10}$ $\frac{50}{73}$
12. $32\frac{1}{2} \times x = \frac{7}{8}$ $\frac{7}{260}$

Ex. 13–24. Solve. Use the reciprocal method.

13. $n = 14 \div \frac{7}{2}$ 4
14. $n = 420 \div \frac{6}{7}$ 490
15. $x \times 2\frac{2}{7} = 32$ 14
16. $n = \frac{3}{10} \div \frac{3}{7}$ $\frac{7}{10}$
17. $x \times \frac{8}{21} = 5\frac{1}{3}$ 14
18. $x = 13\frac{1}{2} \div 3$ $4\frac{1}{2}$
19. $n = \frac{7}{8} \div \frac{3}{4}$ $1\frac{1}{6}$
20. $\frac{3}{10} \times n = \frac{4}{7}$ $1\frac{19}{21}$
21. $3\frac{1}{4} = 5\frac{2}{5} \times n$ $\frac{65}{108}$
22. $x = 6\frac{2}{11} \div \frac{5}{22}$ $27\frac{1}{5}$
23. $\frac{1}{9} \times n = 17\frac{2}{3}$ 159
24. $n = \frac{2}{9} \div 5\frac{2}{3}$ $\frac{2}{51}$

Ex. 25–29. Copy and multiply.

25. $44 \times 2\frac{1}{2}$ = 110
26. $873 \times 15\frac{2}{3}$ = 13,677
27. $105 \times 26\frac{3}{5}$ = 2,793
28. $36 \times 9\frac{5}{12}$ = 339
29. $1{,}320 \times 12\frac{3}{8}$ = 16,335

Ex. 30–38. Solve.

30. $n = 4 \times 5\frac{1}{3}$ $21\frac{1}{3}$
31. $260 = n \times 3\frac{1}{2}$ $74\frac{2}{7}$
32. $x \div \frac{3}{2} = 5\frac{1}{2}$ $8\frac{1}{4}$
33. $x \div 4\frac{2}{3} = \frac{5}{9}$ $2\frac{16}{27}$
34. $46 \times n = 4\frac{3}{5}$ $\frac{1}{10}$
35. $\frac{7}{8} \div x = 16\frac{2}{3}$ $\frac{21}{400}$
36. $x = 438 \times 5\frac{1}{6}$ 2,263
37. $x \div 4\frac{4}{7} = \frac{2}{5}$ $1\frac{29}{35}$
38. $6\frac{1}{8} = x \times \frac{5}{12}$ $14\frac{7}{10}$

39. Multiply $4\frac{2}{3}$ by the sum of $6\frac{1}{5}$ and $3\frac{2}{3}$. $46\frac{2}{25}$

40. From the product of 440 and $6\frac{3}{4}$, subtract the sum of $100\frac{3}{4}$ and $278\frac{1}{3}$. $2{,}590\frac{11}{12}$

Ex. 41–46. Solve.

41. $n = 30\frac{1}{2} \div (3\frac{1}{3} + 4\frac{1}{4} + 5\frac{1}{5})$ $2\frac{296}{767}$
42. $n \times 1\frac{3}{10} = 6\frac{3}{4} + 1\frac{1}{2}$ $6\frac{9}{26}$
43. $x = 3\frac{1}{5} \times (1\frac{1}{3} + 2\frac{3}{4} + 4\frac{5}{6})$ $28\frac{8}{15}$
44. $x = (14 - 3\frac{3}{8}) \div \frac{3}{4}$ $14\frac{1}{6}$
45. $4\frac{2}{3} \div 1\frac{1}{4} = y + \frac{7}{8}$ $2\frac{103}{120}$
46. $n \div \frac{3}{4} = 1\frac{1}{2} + \frac{3}{10}$ $1\frac{7}{20}$

Teaching Page 216

Pupil's Objective

To have mixed computational practice with rational numbers named by fractions and mixed forms.

Background

These exercises give the pupils an opportunity to gain experience in dividing rational numbers using the common-denominator method, dividing rational numbers using the reciprocal method, multiplying rational numbers using the vertical form, solving mathematical sentences requiring multiplication or division of rational numbers, and experience in following directions.

These exercises may be of help to the teacher in evaluating the learning that has taken place so far in this chapter, and in determining areas where remedial work is necessary.

Pre-Book Lesson

As a brief review before assigning the written work, put on the board examples of the types found in the five sets of exercises on this page. Have pupils explain (a) how to divide rational numbers using the common-denominator method, (b) how to divide using the reciprocal method, (c) how to multiply rational numbers using the vertical form, and (d) how to determine whether multiplication or division is required in solving a mathematical sentence.

Using the Text Page

- There are enough exercises on this text page so you may have a great deal of flexibility in using them for review, practice, or testing. If you wish, you may use some of the exercises for review, some for practice, and some for testing purposes.
- Concepts in which the pupils show weakness should be retaught before going on to the new work in the chapter. Do remedial work with groups or individuals who have difficulty with any of the exercises.
- The exercises may be performed over a period of more than one day, as the number of exercises is probably excessive for use in one day.

Individualizing Instruction

Individual help can be provided for *all pupils*. Pupils with the same weakness may be grouped and given extra practice sets provided at the back of the text.

NOTES

Pupil's Objective

To solve problems requiring multiplication and division of rational numbers.

Background

These problems give the pupils practice in (a) differentiating between problems requiring multiplication and those requiring division, and (b) computing with rational numbers.

Having the pupil learn to compute with rational numbers is of little practical value unless he also learns how to apply his computational skill in solving problems. For this reason, the problem-solving aspects of the mathematics program are extremely important. The pupil must have frequent practice in solving problems so as to maintain and strengthen his ability to determine the operation required for solving the problem and his ability to express the relationship in the problem in a correct mathematical sentence.

Problems relating to real-life situations of the pupils in home and at school usually create the greatest interest among the pupils. Problems of this type should be used freely.

Pre-Book Lesson

- Review with pupils circumstances in which multiplication and division are used. Have pupils suggest problems requiring each of these operations.
- Review problem-solving procedures.

Using the Text Page

- Discuss each problem so that pupils will understand the situation being described in the problem. Make sure pupils understand what question is being asked in the problem.
- In Ex. 4, the expression "what part of" in the question "The expenses were what part of the money that he received for the rabbits?" may cause some confusion. The pupils are being asked what fractional part of the set of 25 dollars is $18\frac{3}{4}$ dollars, for which the answer may be found by the division, $18\frac{3}{4} \div 25$. The pupils probably will be better equipped to answer questions of this type after studying the topic of *ratio* later in the chapter. This question may serve to show the need for having a way of expressing a comparison between two numbers.

Individualizing Instruction

For *all pupils* who obtained incorrect answers for any of the written exercises, determine whether the errors were in analyzing the problem or were of a computational nature. Assign appropriate remedial work as necessary.

NOTES

Problem-Solving

[O]

For each of Ex. 1–5, tell what operation or operations to use to find the answer for the problem.

1. Each day, Bill feeds his rabbits $4\frac{1}{2}$ lb. of feed. A 100-lb. sack of feed is enough feed for how many days? Division; $n = 100 \div 4\frac{1}{2}$; $22\frac{2}{9}$

2. The feed costs Bill $5\frac{3}{4}¢$ per pound. What is the cost of a day's supply of feed ($4\frac{1}{2}$ lb.)? Multiplication; $n = 5\frac{3}{4} \times 4\frac{1}{2}$; $25\frac{7}{8}¢$

3. One day, Bill sold four rabbits that weighed $5\frac{1}{8}$ lb., $3\frac{5}{8}$ lb., $5\frac{1}{4}$ lb., and $5\frac{1}{2}$ lb. Find the mean average for these weights. Addition and division; $n = (5\frac{1}{8} + 3\frac{5}{8} + 5\frac{1}{4} + 5\frac{1}{2}) \div 4$; $4\frac{7}{8}$ lb.

4. Last July, Bill received 25 dollars for 12 rabbits. His expenses for raising those 12 rabbits amounted to $18\frac{3}{4}$ dollars. The expenses were what part of the money that he received for the rabbits? Division; $n = 18\frac{3}{4} \div 25$; $\frac{3}{4}$

5. Bill mixes $\frac{3}{8}$ lb. of insect powder with 6 gal. of water. That is how many pounds of insect powder per gallon?
Division; $n = \frac{3}{8} \div 6$; $\frac{1}{16}$

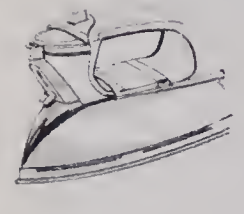
[W]

Now write and solve a mathematical sentence to find the answer for each of Ex. 1–5.

◆ **Extra Problems.** Set 146.

Using Mathematics

[O]



A *watt-hour* is a unit for measuring electrical energy. To find how many watt-hours of electrical energy are used by an appliance, we multiply the watt rating of the appliance by the number of hours it is used.

1. A 1,200-watt electric steam iron operating for 2 hours uses $2 \times 1{,}200$, or 2,400 ? watt-hours of electrical energy.

2. How much electrical energy is used by a 25-watt light bulb that is used for **a.** 4 hours? 100 watt-hours **b.** $6\frac{1}{5}$ hours? 155 watt-hours

3. At the board, find the watt rating of an appliance which uses 1,750 watt-hours of electrical energy in $3\frac{1}{2}$ hours.

$n = 1{,}750 \div 3\frac{1}{2}$

$= 500$

[W]

4. An electric 500-watt toaster uses $787\frac{1}{2}$ watt-hours of electrical energy in $\frac{3}{4}$ hr. What is the watt rating of the toaster? 1,050 watt

5. Power companies measure electrical energy in *kilowatt-hours*. 1 kilowatt-hour = 1,000 watt-hours Use mixed forms to express each of a–c in terms of kilowatt-hours.

a. 3,000 watt-hours 3 kilowatt-hours **b.** 4,500 watt-hours $4\frac{1}{2}$ kilowatt-hours **c.** 15,250 watt-hours $15\frac{1}{4}$ kilowatt-hours

6. At $3\frac{1}{2}¢$ per kilowatt-hour, what would be the cost, to the nearest cent, for $67\frac{9}{10}$ kilowatt-hours of electrical energy? 238¢, or $2.38

7. $5\frac{1}{2}$ kilowatt-hours of electrical energy will operate a 750-watt appliance for how many hours? (Hint: $5\frac{1}{2}$ kilowatt-hours is how many *watt-hours*?) $7\frac{1}{3}$

● **Extra Activity.** Set 169.

Teaching Page 218

Pupil's Objectives

(a) To have practice in solving problems requiring multiplication and division of rational numbers; and (b) to learn the meaning of watt-hour and kilowatt-hour.

Background

The exercises on this page give the pupil practice in solving problems requiring multiplication and division with fractions and mixed forms. The exercises also introduce the *watt-hour* and *kilowatt-hour* as units for measuring electrical energy.

This work may be considered as a scientific application of mathematics. It is not intended that these problems be used as an introduction to a study of electricity, but you will want to be aware of a few fundamental concepts pertaining to electricity and the measuring of electrical energy so as to be able to answer questions that may arise.

All atoms contain charged particles called *electrons* which revolve around the center or nucleus of the atom. In eertain metals, such as copper, silver, and aluminum, some of these electrons can be attracted away from the atom and made to flow through the metal. This movement of electrons is called an *electric current*. The electric current has volume and pressure, much as water in a pipe has volume and pressure.

The unit for measuring the volume of electric current is the *ampere*, while the unit of pressure is the *volt*. The total amount of electrical energy in the current depends on both the amperage and voltage. One ampere of current flowing under a pressure of one volt is a unit of electrical energy called the *watt*.

number of watts = number of volts × number of amperes

The total amount of electrical energy used by an appliance depends on the amount of electrical energy used by the device, measured in watts, and the time for which the appliance is operated. A convenient unit for measuring the total amount of electrical energy used is the *watt-hour*. An appliance needing one watt of energy will use, in a period of one hour, one watt-hour of energy.

$$\text{Number of watt-hours} = \begin{matrix}\text{number of}\\ \text{hours}\end{matrix} \times \text{number of watts}$$

An even more convenient unit is the *kilowatt-hour* (the prefix *kilo* meaning *one thousand*).

The number of kilowatt-hours of electrical energy used by an appliance can be found by applying the relationship

$$\left(\begin{matrix}\text{number of}\\ \text{kilowatt-hours}\end{matrix}\right) = \left(\begin{matrix}\text{number of}\\ \text{hours}\end{matrix} \times \begin{matrix}\text{number of}\\ \text{watts}\end{matrix}\right) \div 1{,}000$$

Pre-Book Lesson

Refer to the enrichment exercises at the bottom of text page 158. Then have pupils suggest a name for the unit of electrical energy used when one watt is used for a period of one hour. This can lead to the introduction of the terms *watt-hour* and *kilowatt-hour*.

Using the Text Page

The oral exercises stress the relationship

$$\begin{matrix}\text{number of}\\ \text{watt-hours}\end{matrix} = \begin{matrix}\text{number}\\ \text{of hours}\end{matrix} \times \begin{matrix}\text{number}\\ \text{of watts}\end{matrix}$$

From this relationship we see that

$$\begin{matrix}\text{number}\\ \text{of watts}\end{matrix} = \begin{matrix}\text{number of}\\ \text{watt-hours}\end{matrix} \div \begin{matrix}\text{number}\\ \text{of hours}\end{matrix}$$

$$\begin{matrix}\text{number}\\ \text{of hours}\end{matrix} = \begin{matrix}\text{number of}\\ \text{watt-hours}\end{matrix} \div \begin{matrix}\text{number}\\ \text{of watts}\end{matrix}$$

In the written exercises, the term *kilowatt-hour* is introduced, and the relationship becomes

$$\left(\begin{matrix}\text{number of}\\ \text{kilowatt-hours}\end{matrix}\right) = \left(\begin{matrix}\text{number of}\\ \text{hours}\end{matrix} \times \begin{matrix}\text{number of}\\ \text{watts}\end{matrix}\right) \div 1{,}000$$

Individualizing Instruction

• Suggest to *all pupils* that they try to read the electric meter at home and then read it again after a few days. If they are able to determine the number of kilowatt-hours of energy used in the interval have them determine the cost of this energy if one kilowatt-hour of energy costs $3\frac{1}{2}¢$.

• You may wish to point out to the *more capable pupils* some of the facts stated in the Background section, making it clear that this is somewhat of an oversimplification of the concept of electricity and electric current.

Teaching Page 219

Pupil's Objective

To review meaning and use of exponent forms.

Background

The meaning and use of exponent forms were first introduced on pupil's page 19. Using exponent forms in expressing a number as a product of primes was introduced on pupil's page 69. Since then, exponent forms have been used or reviewed in each chapter. However, this is the first comprehensive review of the meaning and use of exponent forms since it was introduced in Chapter 1.

Pre-Book Lesson

Briefly review the meaning and use of exponent forms before assigning the written work. Write on the board one or more exercises of each type contained in the exercises on the text page, and have the pupils work the exercises and explain their answers. Encourage pupils to ask questions on any aspect of the topic which they feel they do not completely understand.

Using the Text Page

• After an adequate review of the meaning and use of exponent forms, *all pupils* should be able to complete the review exercises on this page.

• Enrichment exercises, such as those found at the bottom of the page, may be handled in any one of a variety of ways. You may want to have the pupils who finish the review exercises early go on and try the enrichment exercises on their own. Perhaps you would prefer to develop the enrichment exercises orally with the *more capable pupils* only and then ask them to write answers to the questions. Or, you may want to develop the material with *all pupils*, and then assign it as written work for the *more capable pupils* only. You may even find that the material is suitable for *all pupils* as a written assignment. Feel free to use the enrichment exercises in the way that best meets the needs and the abilities of your pupils.

Individualizing Instruction

• All pupils who have difficulty with the review exercises should restudy appropriate text pages and be assigned practice exercises to correct any weaknesses revealed by these review exercises.

• For the *more capable pupils*, the enrichment exercises may be extended to exercises in writing fractions for each of the following types:

a. $(3\frac{1}{3})^3$ b. $(\frac{a}{b})^x$ c. $(\frac{2}{3})^0$

A factor (except 0) raised to the zero power is equal to 1.

NOTES

* Review the terms "power" and "multiple" so that pupils do not confuse their meanings.

Working with Exponent Forms

Resurvey [W]

1. Copy and complete: In the exponent form 3^4, the 4 names the exponent and indicates how many times the number 3 is used as a factor. $3^4 = 3 \times 3 \times 3 \times 3 = 81$

Ex. 2–6. Write an exponent form for the number.

2. 5×5 5^2 3. $7 \times 7 \times 7$ 7^3 4. $3 \times 3 \times 3 \times 3 \times 3$ 3^5

5. Two to the third power 2^3 6. Six squared 6^2

7. List within braces the members of $\{3^5, 2^3, 5^6, 10^5, 3, 10\}$ which are powers of a. 2 $\{2^3\}$ b. 3 $\{3^5, 3\}$ c. 5 $\{5^6\}$ d. 10 $\{10^5, 10\}$

Ex. 8–12. Write the standard numeral for the number.

8. 3^2 9 9. 4^3 64 10. 2^5 32 11. 10^3 1,000 12. 5^3 125

Ex. 13–18. Find the number represented by the letter.

13. $n = 3^3 \times 5$ 135 14. $n = 2^4 \times 10$ 160 15. $x = 5^2 \times 3^4$ 2,025

16. $x = 10^2 \times 40$ 4,000 17. $x = 8^2 \times 10^2$ 6,400 18. $n = 10^4 \times 32$ 320,000

*19. Express 16 first as a power of 4 and then as a power of 2. 4^2; 2^4
Is 16 a power of 8? No

Try These Exercises

Enrichment [W]

Study the work in box A for finding a fraction for the number named by the exponent form $(\frac{2}{3})^3$. Then, for each of Ex. 1–5, find a fraction for the number. Work as in box A.

A
$$(\tfrac{2}{3})^3 = \tfrac{2}{3} \times \tfrac{2}{3} \times \tfrac{2}{3}$$
$$= \tfrac{2 \times 2 \times 2}{3 \times 3 \times 3}$$
$$= \tfrac{8}{27}$$

1. $(\frac{1}{2})^3$ $\frac{1}{8}$ 2. $(\frac{2}{5})^3$ $\frac{8}{125}$ 3. $(\frac{2}{3})^2$ $\frac{4}{9}$ 4. $(\frac{5}{7})^2$ $\frac{25}{49}$ 5. $(\frac{3}{10})^3$ $\frac{27}{1,000}$

Box B shows a shorter way to find a fraction for the number named by $(\frac{2}{3})^3$. Work each of Ex. 6–10 as in box B.

B
$$(\tfrac{2}{3})^3 = \tfrac{2^3}{3^3}$$
$$= \tfrac{8}{27}$$

6. $(\frac{4}{5})^2$ $\frac{16}{25}$ 7. $(\frac{1}{10})^4$ $\frac{1}{10,000}$ 8. $(\frac{3}{8})^2$ $\frac{9}{64}$ 9. $(\frac{7}{10})^3$ $\frac{343}{1,000}$ 10. $(\frac{5}{12})^2$ $\frac{25}{144}$

Ex. 11–15. Work either as in box A or box B.

11. $(\frac{1}{3})^3$ $\frac{1}{27}$ 12. $(\frac{5}{3})^2$ $\frac{25}{9}$ 13. $(\frac{5}{8})^3$ $\frac{125}{512}$ 14. $(\frac{1}{10})^5$ $\frac{1}{100,000}$ 15. $(\frac{7}{8})^2$ $\frac{49}{64}$

Scale Drawings

[o]

1. The drawing above shows the front of a building which is to be built in Elmville. The drawing is the same shape as the proposed shape of the front of the building. To make this drawing, a draftsman used a *scale*. He let 1 inch on the drawing represent 10 feet of the building. The front of the building is to be 45 ft. long and 20 ft. high, so the drawing is _? $4\frac{1}{2}$ in. by _? $\frac{2}{\ }$ in. This is a *scale drawing* with a scale of *1 in. to 10 ft.*

2. Suppose the draftsman used a scale of *1 in. to 5 ft.* Then, since 1 inch on the drawing would represent 5 feet of the building, to find how many inches long the drawing would be, we divide 45 by _? $\frac{5}{\ }$. The drawing would be _? $\frac{9}{\ }$ in. long.

3. At the board, find what the dimensions of the drawing would be if the draftsman used a scale of *1 in. to 4 ft.* $11\frac{1}{4}$ in. by 5 in.

4. What scale is indicated on the floor plans at the left? 1 in. to 24 ft. Explain what the scale means. 1 inch on the drawing represents 24 feet of the room.

5. Measure the floor plan of the living room with your ruler. The floor plan is $1\frac{1}{2}$ in. long and _? $\frac{3}{4}$ in. wide. At the board, find the real dimensions of the living room. 36 ft. long and 18 ft. wide

6. Find the real dimensions of the bedroom and the kitchen. 18 ft. long and 15 ft. wide; 18 ft. long and 12 ft. wide

♦ **Extra Problems.** Set 147.

Teaching Page 220

Pupil's Objectives

(a) To learn the meaning of *scale*; and (b) to learn to read a scale drawing.

Background

This is the first of several lessons pertaining to the meaning and interpretation of scale drawings. If a pupil is given a scale drawing and is told the scale, he should be able to convert any measurement on the drawing to the actual dimension of the object represented by the drawing.

If a drawing is made to a scale of *1 in. to f ft.*, then a inches on the drawing represents an actual distance of $(a \times f)$ ft. An actual dimension of c ft. of the object being represented will be represented on the drawing by a distance of $(c \div f)$ in.

Pre-Book Lesson

Have the pupils determine to the nearest foot the actual length and width of the classroom floor. Discuss the problem of how to make an accurate drawing of the floor on a piece of paper or on the board. This should lead to the concept of *scale*, and pupils may be asked to suggest an appropriate scale. Then let them attempt to make a drawing to the scale suggested.

Using the Text Page

- Ex. 1–4 should lead to the understanding that if the scale of a drawing is *1 in. to f ft.*, then an actual measurement of a ft. will be represented by $\frac{a}{f}$ in. on the drawing.
- Ex. 5–6 should lead to the understanding that if the scale of a drawing is *1 in. to f ft.*, then a measurement of m inches on the drawing represents an actual distance of $(m \times f)$ ft.

Individualizing Instruction

- The *more capable pupils* may attempt to make a scale drawing of one floor of their house.
- The *slower learners* may be asked to make a scale drawing of their own room or of a room at school.
- *All pupils* may be asked questions pertaining to scale drawings such as

(1) If a drawing is made to a scale of *1 in. to k ft.*, a measurement of d inches on the drawing represents an actual distance of _ ? _ ft. $(k \times d)$

(2) If a drawing is made to a scale of *1 in. to p mi.*, an actual distance of m miles will be represented by _ ? _ in. on the drawing. $(m \div p)$

(3) If an actual distance of a ft. is represented by b in. on the drawing, the scale for the drawing is 1 in. to _ ? _ ft. $(a \div b)$

NOTES

Teaching Page 221

Pupil's Objective

To learn to find actual sizes of things from scale drawings.

Background

The work on this page is an extension of the concepts of scale drawing introduced on pupil's page 220. The exercises on page 220 gave the pupils an opportunity to determine actual measurements of a room from a scale drawing of the room, while the exercises on this page give the pupils an opportunity to determine the measurements of various animals from scale drawings of those animals.

Pre-Book Lesson

Ask for examples of drawings or models of people and animals, such as statues that are smaller or larger than the thing they represent. Try to discover such examples in the classroom or use reference books such as dictionaries and encyclopedias.

Using the Text Page

Provide a thorough discussion of Ex. 1–3 to make sure all pupils understand the meaning of the scales *1 in. to 60 in.* and *1 in. to 72 in.* Make sure they understand how to convert measurements on the drawings to actual measurements of the thing being represented.

Individualizing Instruction

- You may wish to have the *more capable pupils* make scale models in clay or paper maché.

All the scale drawings studied so far are drawings in which the object being represented is much larger than the drawing. Have the *more capable pupils* consider examples where the drawing is larger than the object being represented, such as a scale drawing of a snowflake, in which a snowflake $\frac{1}{4}$ inch across is represented by a scale drawing 4 inches across. Ask the pupils to determine the scale for such a drawing.

- Give *all pupils* examples of scale drawings involving metric units of measurement.
- Have *all pupils* describe how to convert scale-drawing measurements to measurements of the object being represented.
- Have *all pupils* make scale drawings of an object in the classroom or an object of their choice, using quarter-inch graph paper.
- Put on the board pictures of geometric figures such as rectangles, squares, and parallelograms, and have the *slower learners* make scale drawings of these figures. You may want to suggest the scale to be used.

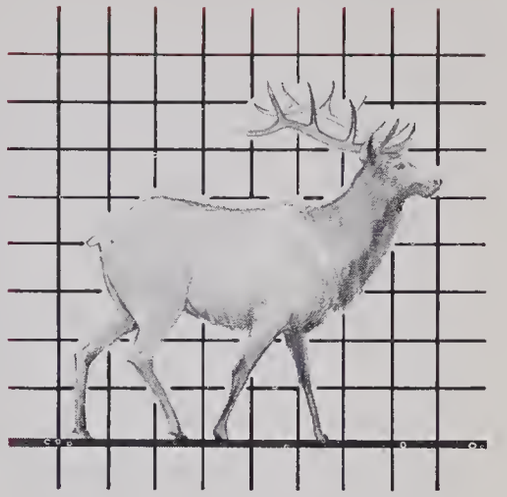
NOTES

* A scale may be named with a fraction, but before expressing with a fraction, be sure the same unit is used for both measurements.

Scale Drawings in Science

[O]

* Miss Pike's class looked in an encyclopedia for some scale drawings of well-known animals. They found this picture of an elk drawn to a scale of *1 in. to 60 in.*



1. On the drawing, the elk's shoulder height measures 1 in inches. The real shoulder height is _?_ 60 in., or _?_ 5 ft.

2. The class found other scale drawings of members of the deer family with a scale of *1 in. to 72 in.* Explain this scale. 1 inch on the drawing represents 72 inches on the deer.

3. If a scale drawing with a scale of *1 in. to 72 in.* of a white-tailed deer has a shoulder height of $\frac{1}{2}$ in., what is the real shoulder height? 36 in.

[W]

Find the real heights and lengths if scale drawings with a scale of *1 in. to 72 in.* have these measurements. (Give your answers in terms of feet or in terms of feet and inches.)

	Height		Length	
4. Black-tailed deer	$\frac{1}{2}$ in.	3 ft.	1 in.	6 ft.
5. Moose	1 in.	6 ft.	$1\frac{7}{16}$ in.	8 ft. $7\frac{1}{2}$ in.
6. Woodland caribou	$\frac{3}{4}$ in.	4 ft. 6 in.	1 in.	6 ft.

7. If the scale for a scale drawing of a whale is *1 in. to 64 in.*, find the real length of

   a. a right whale, if the scale drawing is $7\frac{1}{2}$ in. long. 480 in., or 40 ft.

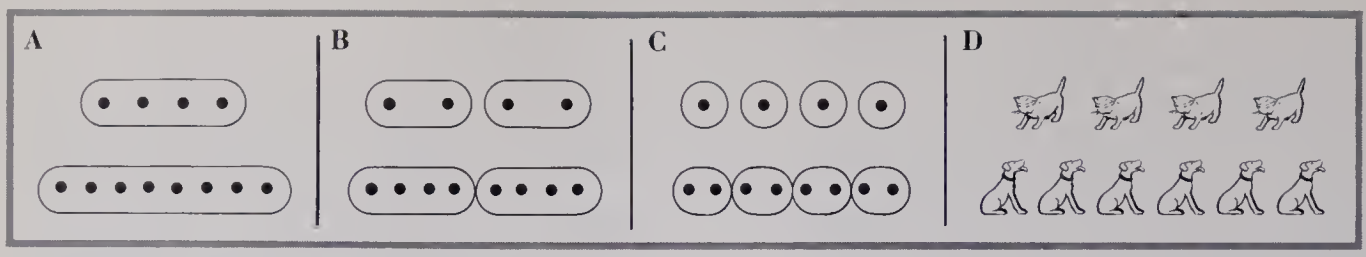
   b. a sperm whale, if the scale drawing is $10\frac{1}{2}$ in. long. 672 in., or 56 ft.

8. Show a segment for 60 ft. Scale: $\frac{1}{8}$ in. to 10 ft. $\frac{3}{4}$ in.

9. Show a segment for 850 mi. Scale $\frac{1}{4}$ in. to 50 mi. $4\frac{1}{4}$ in.

10. Make a scale drawing of a square which is 10 ft. on a side. Use a scale of $\frac{1}{4}$ in. to 1 ft. Picture a square with each side $2\frac{1}{2}$ in. in length.

● **Extra Activity.** Set 170.



Ratio

Resurvey; meaning [O]

1. Box A. How many black dots are shown? 4 How many blue dots are shown? 8 We may say that the *ratio* of the number of black dots to the number of blue dots is 4 to 8. To express this ratio, we may write *4 to 8* or *4:8* or $\frac{4}{8}$. At the board, express the ratio of the number of blue dots to the number of black dots in these three ways. 8 to 4; 8:4; $\frac{8}{4}$

2. Box B. Both the set of black dots and the set of blue dots are shown partitioned into two equivalent subsets. Explain what *equivalent subsets* means. Each subset has the same number of members. For every 2 black dots, there are _? 4 blue dots. This is a ratio of 2 to _? 4. We may write 2 to _? 4 to express the ratio of the number of black dots in box A to the number of blue dots in box A.

3. Explain how box C shows that the ratio of the number of black dots to the number of blue dots may also be expressed as 1 to 2. Both sets are shown partitioned into 4 equivalent subsets. Can 1:2 be used to express this ratio? Yes Can $\frac{1}{2}$? Yes

4. Box D. How many cats are shown? 4 How many dogs are shown? 6 To express the ratio of the number of cats to the number of dogs, we may write $\frac{4}{?}$. Can the fraction $\frac{2}{3}$ be used to express this ratio? Yes Explain. $\frac{4}{6}$ and $\frac{2}{3}$ name the same number

> **When using a fraction to express a ratio, we usually use a fraction in simplest form.**

5. At the board, express the following ratios with fractions in simplest form.

a. 5 to 9 $\frac{5}{9}$ b. 4:10 $\frac{2}{5}$ c. 7 to 21 $\frac{1}{3}$ d. 10 to 18 $\frac{5}{9}$ e. 32:48 $\frac{2}{3}$

f. $\frac{10}{12}$ $\frac{5}{6}$ g. 75 to 100 $\frac{3}{4}$ h. 33 to 99 $\frac{1}{3}$ i. 55 to 75 $\frac{11}{15}$ j. 250:1500 $\frac{1}{6}$

Teaching Pages 222 and 223

Pupil's Objectives

(a) To learn to use a ratio to express the relationship between two sets or between two numbers; and (b) to learn three ways of expressing a ratio.

Background

A ratio may be thought of as comparing the number property of one set with that of another set. The number for one set serves as a base for the comparison.

The ratio of the number of pupils to the number of teachers in your room is entirely different from the ratio of the number of teachers to the number of pupils. Therefore, the order of the numerals showing parts of the ratio is very important.

If a school has 308 pupils and 14 teachers, the ratio of the number of teachers to the number of pupils may be shown in any of the following ways:

(a) 14 to 308 (b) 14:308 (c) $\frac{14}{308}$

The ratio of the number of pupils to the number of teachers is expressed in any of these ways:

(a) 308 to 14 (b) 308:14 (c) $\frac{308}{14}$

Of these three methods of expressing a ratio, the fraction method is widely used because most ratio problems can be solved by expressing the ratio by a fraction and using the arithmetic of rational numbers. The ratio, a to b, is shown in fraction form as $\frac{a}{b}$.

One of the basic principles of rational numbers is that when the numerator and the denominator are both multiplied or both divided by the same number, other than zero, the result is unchanged. This same concept applies to two numbers forming a ratio. If both numbers forming the ratio are multiplied or both divided by the same number, other than zero, the resulting ratio is equal to the original ratio. One application of this principle is that when a fraction is used to express a ratio, the fraction may be changed to simplest form.

The ratio of 14 teachers to 308 pupils may be expressed in fraction form as $\frac{14}{308}$ or $\frac{7}{154}$ or $\frac{1}{22}$, the last being the simplest form for expressing the ratio. The fraction $\frac{1}{22}$ does not tell us the total number of teachers nor the total number of pupils, but simply expresses the relationship between the number property of the set of teachers and the number property of the set of pupils. It states that for every teacher there are 22 pupils.

Much of the arithmetic of rational numbers can be applied to ratios. To determine which of two ratios is greater, the procedure is the same as that for determining which of two rational numbers is greater. The procedure for determining whether two ratios are equal is the same as that for determining whether two rational numbers are equal.

When ratios are used to compare quantities involving units of measurement, both quantities must be named by the same unit of measurement. The ratio of a distance of 3 inches to a distance of 7 feet is *not* $\frac{3}{7}$. The ratio of 3 inches to 7 feet is the ratio of 3 inches to 84 inches, which is $\frac{3}{84}$, or $\frac{1}{28}$. On pupil's page 220, the scale of the drawing at the bottom of the page is *1 in. to 24 ft.* The ratio of the length of the segment in the drawing *representing* the length of the living room to the actual length of the living room is *not* 1:24. The scale of *1 in. to 24 ft.* is *1 in. to 288 in.* Therefore, the ratio is 1:288, or $\frac{1}{288}$.

Pre-Book Lesson

Provide pupils A, B, and C with 3 marbles, 9 marbles, and 15 marbles (or some other object) respectively. Provide pupils D, E, and F with 1 marble, 3 marbles, and 5 marbles respectively. Ask pupils to compare (a) the number of marbles A has with the number of marbles D has; (b) the number B has with the number E has; and (c) the number C has with the number F has.

Each of these comparisons should be expressed in four different ways, such as:

(1) A has 2 more than D.
(2) A has 3 times as many as D.
(3) D has 2 fewer than A.
(4) D has $\frac{1}{3}$ as many as A.

Ask in what way the comparisons in the ratio forms (2) and (4) show a common property, leading pupils to the realization that the comparisons of 3 to 1, 9 to 3, and 15 to 5 all have the common

property of the first number being three times as great as the second; and the comparisons of 1 to 3, 3 to 9, and 5 to 15 all have the common property of the first number being $\frac{1}{3}$ the second. Explain to the pupils that on these pages they should learn several ways of expressing ratios.

Using the Text Pages

• In Ex. 1 the pupils are introduced to the meaning of ratio and three ways of expressing a ratio, including the fraction form.

• Ex. 2–4 show, by use of sets, that a ratio has the property of a rational number in that both numbers forming the ratio may be divided by the same number to express it in simpler form. This is similar to finding simplest form for rational numbers.

• Ex. 6–8 show, by use of sets, that a ratio has another property of rational numbers in that if both numbers forming the ratio are multiplied by the same number, the result is unchanged. If any pupils have difficulty understanding the steps shown in box E, the work may be written on the board as:

$$\frac{2}{3} = \frac{16}{n}$$
$$\frac{? \times 2}{? \times 3} = \frac{16}{n}$$

The pupil should be able to see that the numerator of the ratio $\frac{2}{3}$ must be multiplied by 8 to obtain 16, and so the denominator must also be multiplied by 8, resulting in 24.

Individualizing Instruction

• Have *more capable pupils* determine whether the first ratio is greater than or less than or equal to the second ratio in exercises such as:

a. 6:14 and 9:24 b. 8:9 and $\frac{40}{36}$

c. $\frac{11}{12}$ and 5 to 6

• Have *all pupils* suggest problem situations involving ratios whose solution requires work such as that shown in box E. For example,

"The ratio of the number of pupils to the number of teachers at Henderson Elementary School is 45 to 2. If Henderson School has 270 pupils, how many teachers does it have?"

• Give *all pupils* exercises expressing ratios of quantities involving units of measurement. Point out instances where it is best to rename the quantities using the same unit of measurement before comparing them by means of a ratio. Some possible problems are:

1. If eggs are packaged 24 dozen to the crate, what is the ratio of eggs to crates in a shipment of 10 crates?

2. The drawing on page 220 is made to a scale of *1 in. to 24 ft.* What is the ratio of the length of the living room to the length shown in the drawing?

3. A map is made to the scale *1 in. to 1 mi.* What is the ratio of the actual distance between cities and the length of the distance on the map?

• Emphasize for *slower learners* the similarity of ratios and rational numbers. Point out that the arithmetic of ratios is very much like that of rational numbers, so they should express ratios in fraction form whenever convenient. Urge these pupils to be careful to have the numbers in correct order when comparing them by use of ratios.

6. One afternoon while at camp, Bob and Jim decided to go fishing. The ratio of the number of fish that Bob caught to the number of fish that Jim caught was 3 to 5. Does this mean that Bob caught 3 fish and Jim caught 5 fish? Not necessarily Explain your answer. It only means that for every 8 fish Bob caught, Jim caught 5 fish.

7. The ratio of the number of worms that Bob used to the number of worms that Jim used was 2 to 3. If Bob used 16 worms, how many did Jim use? 24

a. Below are pictured 16 black dots. The dots are shown partitioned into subsets of 2 dots each. The ratio is 2 to 3 so for each subset of 2 dots _?3_ blue dots are shown. Count the blue dots. 24

b. Does 2 to 3 name the same ratio as 16 to 24? Yes

8. Box E shows another way to find the answer for Ex. 7. The ratio 2 to 3 may be expressed with the fraction $\frac{2}{3}$, so to find the answer we need to find a fraction showing numerator 16 which expresses the same ratio as 2 to 3. Explain the work shown.

E

$$\frac{2}{3} = \frac{16}{n}$$
$$\frac{2}{3} = \frac{2 \times 8}{3 \times ?}$$
$$\frac{2}{3} = \frac{16}{24}$$

[W]

Ex. 9–12. Express the ratio with a fraction in simplest form.

9. 25 to 125 $\frac{1}{5}$ **10.** 22:330 $\frac{1}{15}$ **11.** 17:51 $\frac{1}{3}$ **12.** 6 to 44 $\frac{3}{22}$

Ex. 13–16. Find the number represented by n so that the two fractions express the same ratio. Work as in box E.

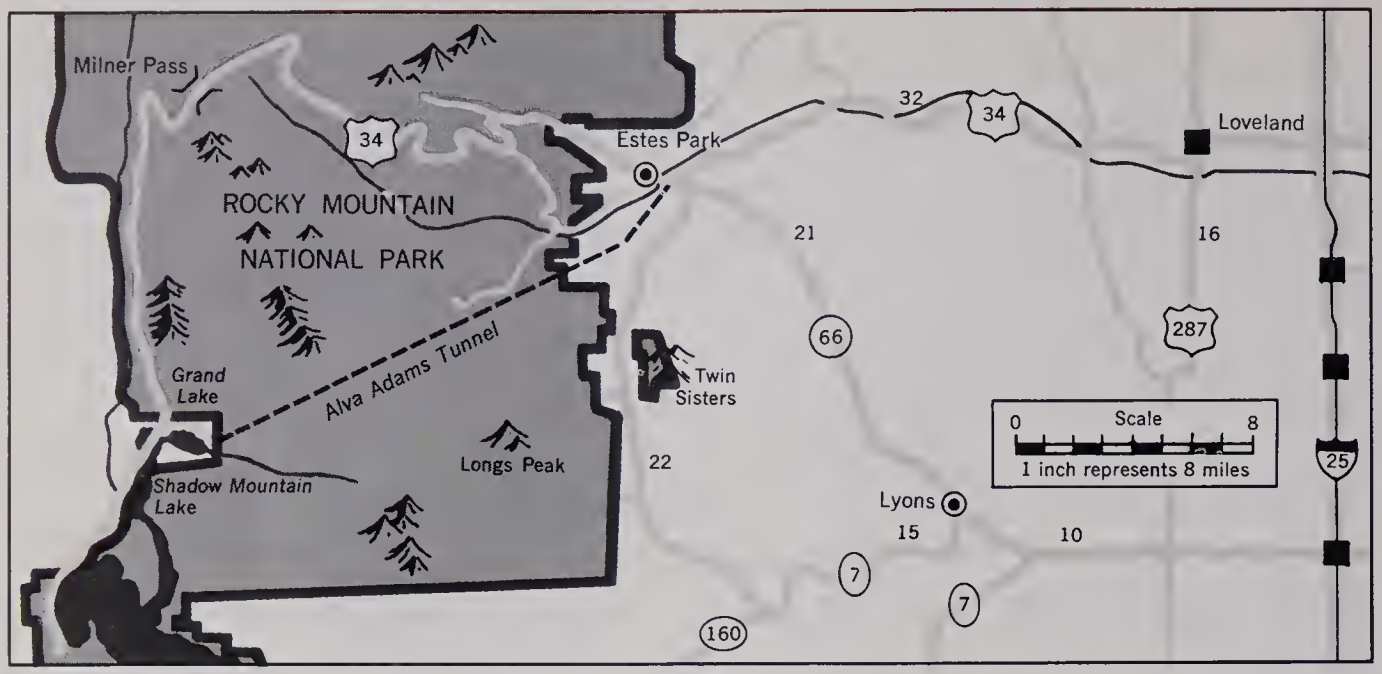
13. $\frac{5}{7}, \frac{25}{n}$ 35 **14.** $\frac{7}{3}, \frac{28}{n}$ 12 **15.** $\frac{n}{6}, \frac{5}{3}$ 10 **16.** $\frac{14}{8}, \frac{n}{56}$ 98

17. If an ice-cream cone costs 15¢ and a sundae costs 35¢, find the ratio of the cost of a cone to the cost of a sundae. $\frac{3}{7}$

18. Find the ratio of the number of prime numbers between 1 and 10 to the number of primes between 10 and 30. 2 to 3

♦ **Extra Examples.** Sets 108–109. ♦ **Extra Problems.** Set 148.

* The distances given are road distances, so they will not agree with distances found by measuring on the map.



Reading a Road Map

[W]

* For Ex. 1–5, use road distances when given and measure and compute other distances, using a scale of *1 in. to 8 mi.*

1. In going from Loveland to Estes Park by Route 34 and then returning by Routes 66 and 287, about how far would you travel? 79 mi. ✓

2. Alva Adams Tunnel carries water from Grand Lake to a point near Estes Park. Measure and compute to find about how many miles long the tunnel is. 18 mi.

3. The distance from Milner Pass to Long's Peak measures about $2\frac{0}{8}$ in inches on the map. About how many miles does that represent? 16 mi.

4. Find the ratio of the road distance between Lyons and Estes Park to the road distance between Lyons and the intersection of Routes 160 and 7. Express the ratio with a fraction. $\frac{7}{5}$

5. The distance from Loveland to Lyons via Routes 34 and 66 is about how many times the distance from Loveland to Lyons via Routes 287 and 66? 2 times

● **Extra Activity.** Set 171.

Teaching Page 224

Pupil's Objective

To learn to find distances on a map, using the map scale.

Background

Map reading is a topic more thoroughly covered in courses other than mathematics. The work on this page is intended to be limited to the arithmetic involved in finding distances on a map by using the map scale.

Pre-Book Lesson

Have on display two commercial maps of your state, each made to a different scale. Road maps produced by two different companies will probably suffice. Lead the pupils to the realization that, to find distances on a map, we need to know the map scale. Have them find on the maps the scales indicated.

The most accurate map of a region is a topographic map published by the United States Geological Survey. Obtain one of these maps, if possible, and have it on display. Explain to the pupils that this map may be too difficult for them to read but there is a simpler map printed in their books.

Using the Text Page

• Make sure all pupils understand that a map scale of 1 in. to 8 mi. means that the number of inches measured on the map must be multiplied by 8 to convert the measure in inches to the number of miles represented.

• The small numerals tell the road mileage between major locations or road intersections shown on the map. The fact that the map is really made to scale can be proved by measuring the map distance between two towns, multiplying the number of inches by 8, and comparing this with the number of miles indicated.

Individualizing Instruction

• A real topographical map of the United States Geological Survey can afford many interesting challenges for the *more capable pupils*. The map scale is indicated in a true ratio. Instead of the map scale being indicated as *1 in. to 1 mi.*, it is indicated as 1:63,360 (one mile to 63,360 inches). Actually, most United States Geological Survey maps are drawn to a scale of 1:62,250 which may be described as *1 in. represents approximately 1 mile.*

• *All pupils* may be given exercises such as the following:

1. What is the scale of this map? (1:62,250)
2. Express the map scale of *1 in. to 1 mi.* as a ratio. (1:63,360)
3. If a map of your state and a map of the United States are approximately the same size, which map is made to the greater scale? (the map of your state)
4. Select two points on the map and determine the approximate distance between these two points on the surface of the earth, using the map scale.

• The *slower learners* may be asked to make a scale map of some area, with dimensions being furnished by the teacher.

Teaching Pages 225, 226, and 227

Pupil's Objective

To evaluate the learning of the chapter content by taking the four end-of-chapter tests: (a) Test of Information and Meaning 5, (b) Diagnostic Test 5, (c) Problem Test 5, and (d) Computation Test 5.

Background

These tests not only measure how well the ideas, skills, and concepts presented in Chapter 5 have been learned, but also measure how well ideas, skills, and concepts presented in earlier chapters have been retained.

These tests should not only be used for marking purposes but also for diagnosing weaknesses. Remedial work may be assigned as necessary to correct weaknesses in concepts presented in this and previous chapters.

Pre-Book Lesson

- Before giving these tests, you may wish to review topics in which the pupils previously have shown weaknesses.
- Before assigning Problem Test 5, you may wish to review with the pupils how to differentiate among problems requiring addition, subtraction, multiplication, and division, and to review problem-solving procedures.

Using the Text Pages

- Space the tests so as to minimize fatigue, giving no more than two tests a day.
- Have the pupils scan each test to be sure they understand the directions.
- After each test have the pupils explain the reasons for their responses. For the problems in Problem Test 5, have the pupils explain their choice of operation used in solving each problem.

Individualizing Instruction

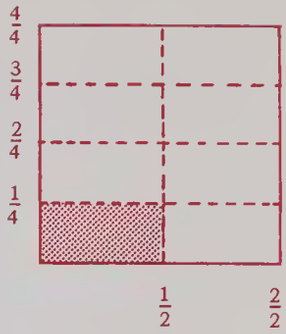
- After the tests have each been scored (see below for Table of Per Cents for Chapter 5 Scores) and returned, have *all pupils* correct their mistakes and work appropriate practice sets in order to strengthen weaknesses.
- If a sizable number of pupils show weakness in any one area, the concept should be retaught and the pupils retested in order to be sure that they understand the material.
- *More capable pupils* will probably not need to have reteaching experiences and may work the Supplementary Activities which follow.

Table of Per Cents for Chapter 5 Scores

Problem Test		Computation Test			
Score	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*
1	11	1	5	12	54
2	22	2	9	13	59
3	33	3	14	14	63
4	44	4	18	15	68
5	56	5	23	16	72
6	67	6	27	17	77
7	78	7	32	18	81
8	89	8	36	19	86
9	100	9	41	20	90
		10	45	21	95
		11	50	22	100

Do You Understand?

Test of Information and Meaning 5



Ex. 1–3. Copy and complete by writing $=$ or $<$ or $>$.

1. $\frac{1}{2} \times \frac{1}{3}$ _?_ $\frac{1}{5}$ $<$ 2. $4 \times \frac{2}{3}$ _?_ $2\frac{2}{3}$ $=$ 3. $\frac{1}{2} \div \frac{1}{3}$ _?_ 1 $>$

4. Represent $\frac{1}{2} \times \frac{1}{4}$ by shading part of a model of a square unit region.

5. Find the area of a rectangular region $3\frac{1}{2}$ ft. by $2\frac{5}{8}$ ft. $9\frac{3}{16}$ sq. ft.

Ex. 6–10. Write a fraction in simplest form for the reciprocal of the number.

6. $\frac{5}{7}$ $\frac{7}{5}$ 7. $\frac{100}{23}$ $\frac{23}{100}$ 8. $3\frac{2}{3}$ $\frac{3}{11}$ 9. 1.4 $\frac{5}{7}$ 10. $15\frac{3}{10}$ $\frac{10}{153}$

Ex. 11–13. Tell whether n represents the product or a factor and then solve.

11. $n \times 3\frac{1}{3} = 15$ factor; $4\frac{1}{2}$ 12. $n \div \frac{4}{5} = \frac{7}{8}$ product; $\frac{7}{10}$ 13. $6\frac{2}{5} \div n = \frac{3}{7}$ factor; $14\frac{14}{15}$

14. Using 5 and $4\frac{2}{3}$ for the factors and n to represent the product, write four mathematical sentences which illustrate the factors-product relationship. $n = 5 \times 4\frac{2}{3}$; $n \div 5 = 4\frac{2}{3}$; $n \div 4\frac{2}{3} = 5$; $n = 4\frac{2}{3} \times 5$

Ex. 15–16. Copy and complete.

15. $3\frac{2}{5}$ ft. $=$ _?_ $1\frac{2}{15}$ yd. 16. $20\frac{2}{3}$ sq. ft. $=$ _?_ $2\frac{8}{27}$ sq. yd.

17. If the two factors of a product are greater than 0 and one of the factors is less than 1, is the product *less than* or *greater than* the other factor? less than

18. Copy and complete: The product of a number and its reciprocal is equal to _?_ 1.

Ex. 19–23. Express the ratio with a fraction in simplest form.

19. 3 to 7 $\frac{3}{7}$ 20. 12 to 8 $\frac{3}{2}$ 21. 45:75 $\frac{3}{5}$ 22. 2:100 $\frac{1}{50}$ 23. 81:63 $\frac{9}{7}$

24. Make a scale drawing of a rectangle which is 40 in. long and 25 in. wide. Use a scale of $\frac{1}{4}$ in. to 5 in. Show a rectangle with length 2 in. and width $1\frac{1}{4}$ in.

25. Find the length of one side of a regular hexagon if the perimeter is $21\frac{1}{2}$ cm. $3\frac{7}{12}$ cm.

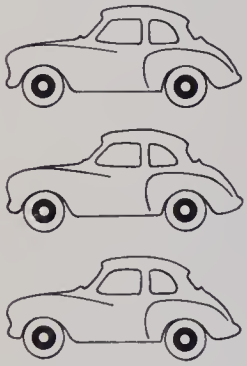
Do You Make Mistakes?

Diagnostic Test 5

	a	b	c	d	Study Pages	Practice: Use Sets
1. Multiply	$15 \times \frac{4}{5}$ 12	$\frac{2}{3} \times \frac{3}{4}$ $\frac{1}{2}$	$\frac{5}{9} \times 3\frac{1}{2}$ $1\frac{17}{18}$	$\frac{6}{11} \times 4\frac{2}{5} \times 2\frac{1}{3}$ $5\frac{3}{5}$	184–189, 192–197	92, 96, 99
2. Multiply	$2\frac{1}{2}$ 4 10	$14\frac{2}{3}$ 33 484	15 $2\frac{3}{5}$ 39	68 $32\frac{3}{4}$ 2,227	189	93–94
3. Divide	Use the common-denominator method: $3 \div \frac{4}{5}$ $3\frac{3}{4}$	$\frac{2}{3} \div 7$ $\frac{2}{21}$	$\frac{3}{5} \div \frac{2}{7}$ $2\frac{1}{10}$	$8\frac{1}{3} \div \frac{5}{6}$ 10	204–206, 208–211	101
4. Divide	Use the reciprocal method: $\frac{6}{7} \div \frac{2}{3}$ $1\frac{2}{7}$	$\frac{4}{5} \div 8$ $\frac{1}{10}$	$5\frac{1}{5} \div 8\frac{3}{5}$ $\frac{26}{43}$	$\frac{7}{8} \div 1\frac{1}{10}$ $\frac{35}{44}$	212, 214–215	105–106

Can You Solve Problems?

Problem Test 5



Let n represent the number which will answer the question in the problem. Write a mathematical sentence for the problem and then solve to find the answer for the problem.

1. If a machine can produce small plastic cars at a rate of 1 car every $\frac{4}{5}$ sec., how many cars can it produce in 1 min.? $n = 70 \div \frac{4}{5}$; 75

2. A certain telephone crew can lay $1\frac{1}{2}$ mi. of cable per day. How many days will it take this crew to lay 20 mi. of cable? $n = 20 \div 1\frac{1}{2}$; $13\frac{1}{3}$

3. How many $1\frac{1}{4}$-lb. packages of hamburger can be made from 17 lb. of hamburger? $n = 17 \div 1\frac{1}{4}$; $13\frac{3}{5}$

4. Bob has read $\frac{4}{7}$ of a 357-page book. How many more pages does he have to read to finish the book? $n = 357 - (\frac{4}{7} \times 357)$; 153

Supplementary Activities

• Explain errors in any of the following examples:

a. $\frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{5} = \frac{4}{5}$

b. $\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{4}}} \div \frac{\overset{3}{\cancel{12}}}{\underset{5}{\cancel{15}}} = \frac{3}{5}$

c. $\frac{5 + \overset{1}{\cancel{6}}}{7 + \underset{1}{\cancel{6}}} = \frac{5}{7}$

d. $\frac{\overset{2}{\cancel{2}}}{\cancel{3}} \times \frac{\cancel{3}}{\underset{2}{\cancel{4}}} = \frac{0}{2}$, or 0

e.
$$\begin{array}{r} 126 \\ \times\ 18\frac{2}{3} \\ \hline 84 \\ 10\,08 \\ 12\,6 \\ \hline 12{,}764 \end{array}$$

f. $\frac{2}{3} \div \frac{4}{3} = \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{3}}}$

$= 2$

g. $\frac{1}{2} + (\frac{1}{2} \times \frac{3}{2}) = (\frac{1}{2} + \frac{1}{2}) \times (\frac{1}{2} + \frac{3}{2})$

$= 1 \times 2$

$= 2$

h. $\frac{3}{4} \times (\frac{4}{5} + \frac{8}{5}) = \frac{3}{4} \times \frac{4}{5} + \frac{8}{5}$

$= \frac{3 \times \overset{1}{\cancel{4}}}{\underset{1}{\cancel{4}} \times 5} + \frac{8}{5}$

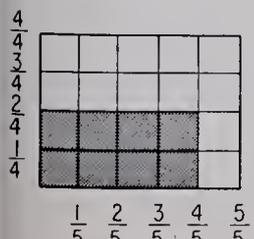
$= \frac{11}{10}$

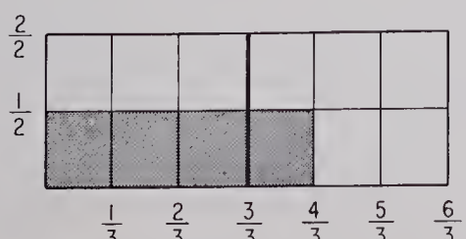
i. $\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b + b}$

j. $\frac{a}{b} \times \frac{c}{d} = \frac{a + c}{b \times d}$

(Answers: a. Divided 4 twice in numerators and only once in denominators. b. Did not multiply by reciprocal of known factor. c. Can't divide out factor of a single addend. d. Didn't show 1's; numerator is 1. e. $\frac{2}{3} \times 126$ and 8×126 are numbers of ones, so product numerals should align. f. Multiplied using reciprocal of product instead of known factor. g. Addition does not distribute over multiplication. h. Did not multiply addend $\frac{8}{5}$ by $\frac{3}{4}$. i. Added denominators. j. Added numerators instead of multiplying.)

• For each of the following diagrams, write one mathematical sentence containing the symbol for multiplication and two mathematical sentences each containing the symbol for division which describe the relationship for which the diagram is a model:

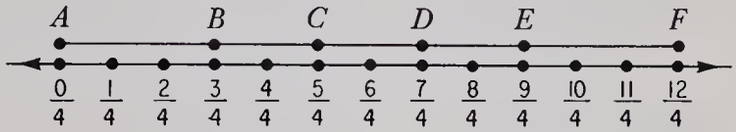




($\frac{2}{4} \times \frac{4}{5} = \frac{2}{5}$; $\frac{2}{5} \div \frac{2}{4} = \frac{4}{5}$; $\frac{2}{5} \div \frac{4}{5} = \frac{2}{4}$.) ($\frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$; $\frac{2}{3} \div \frac{1}{2} = \frac{4}{3}$; $\frac{2}{3} \div \frac{4}{3} = \frac{1}{2}$.)

• Name the segment shown below whose measure is

a. $\frac{1}{3} \times \frac{15}{4}$ $(\overline{AC})$; b. $\frac{1}{2} \times \frac{7}{2}$ $(\overline{AD})$; c. $\frac{3}{2} \times 2$ $(\overline{AF})$; d. $\frac{1}{3} \times \frac{9}{4}$ $(\overline{AB})$; e. $\frac{3}{2} \times 1\frac{1}{2}$ $(\overline{AE})$.



• Work the following without pencil and paper, applying the principle $\frac{a}{b} \div \frac{c}{d} = \frac{a \times d}{b \times c}$.

(a) $\frac{2}{3} \div \frac{5}{7} = n$ (b) $\frac{11}{2} \div \frac{1}{5} = n$ (c) $\frac{7}{3} \div \frac{2}{5} = n$
(d) $\frac{11}{13} \div \frac{1}{2} = n$ (e) $\frac{2}{7} \div \frac{3}{5} = n$ (f) $\frac{3}{11} \div \frac{7}{5} = n$
(g) $\frac{17}{13} \div \frac{3}{2} = n$ (h) $\frac{5}{7} \div \frac{5}{5} = n$ (i) $\frac{3}{5} \div \frac{3}{5} = n$
(a. $\frac{14}{15}$; b. $27\frac{1}{2}$; c. $5\frac{5}{6}$; d. $1\frac{9}{13}$; e. $\frac{10}{21}$; f. $\frac{15}{77}$; g. $\frac{34}{39}$; h. $\frac{25}{49}$; i. 1.)

• Without using pencil and paper, determine which of the three choices is the correct answer.

1. $(4\frac{1}{2} + 1\frac{1}{2}) \times 3 = n$	(a) 18	(b) 6	(c) 24
2. $\frac{2}{3} \times 17\frac{1}{2} = n$	(a) 21	(b) 8	(c) $11\frac{2}{3}$
3. $\frac{7}{3} \div \frac{7}{3} = n$	(a) 1	(b) $\frac{49}{9}$	(c) $\frac{9}{49}$
4. $5 \div \frac{1}{5} = n$	(a) 1	(b) 5	(c) 25
5. $5 \times \frac{1}{5} = n$	(a) 1	(b) 5	(c) 25

• The following is a multiplication chart for a system of numeration which uses the letters *a*, *c*, *e*, and *g* for numerals. The questions are based on this chart.

×	*a*	*c*	*e*	*g*
a	*a*	*a*	*a*	*a*
c	*a*	*c*	*e*	*g*
e	*a*	*e*	*a*	*e*
g	*a*	*g*	*e*	*c*

1. In this number system, is there an identity element for multiplication? (Yes, *c*.)

2. If the product of two numbers is the identity

Teacher's Page 226

element for multiplication, the two numbers are said to be reciprocals of each other. What is the reciprocal of g? ($g \times g = c$, so g is its own reciprocal.)

3. What numbers have no reciprocals? (a and e)

• Tell which of the given answers says the same thing.

1. Divide the sum of a and b by c.

(a) $(a \div b) + c$ (b) $\underline{(a + b) \div c}$
(c) $c \div (a + b)$

2. Subtract c from the product of k and m.

(a) $c - (k \times m)$ (b) $\underline{(m \times k) - c}$
(c) $k \times (m \div c)$

3. Rename $h\frac{t}{r}$ with a fraction.

(a) $\underline{\frac{(r \times h) + t}{r}}$ (b) $\frac{r + (h \times t)}{r}$ (c) $\frac{(r + h) + t}{r}$

4. Divide $\frac{b}{c}$ by $\frac{f}{g}$.

(a) $(b \times f) \div (c \times g)$ (b) $\underline{(b \times g) \div (c \times f)}$
(c) $(b \div g) \times (c \div f)$

5. To the sum of $\frac{h}{e}$ and $\frac{d}{e}$ add the product of $\frac{h}{e}$ and $\frac{e}{h}$.

(a) $\frac{1 + h + d}{e}$ (b) $\frac{h \times d}{e} + 1$
(c) $\underline{\frac{h + d}{e} + 1}$

• First rename with a base-three numeral, and then express each as a sum using powers of 3.

[a. 3 b. 36 c. 13 d. 82 e. 37 f. 40
a. $3 = 10_{\text{three}} = (1 \times 3) + 0$; b. $36 = 1100_{\text{three}} = (1 \times 3^3) + (1 \times 3^2) + (0 \times 3^1) + 0$; c. $13 = 111_{\text{three}} = (1 \times 3^2) + (1 \times 3^1) + 1$; d. $82 = 10001_{\text{three}} = (1 \times 3^4) + (0 \times 3^3) + (0 \times 3^2) + (0 \times 3^1) + 1$; e. $37 = 1101_{\text{three}} = (1 \times 3^3) + (1 \times 3^2) + (0 \times 3^1) + 1$; f. $40 = 1111_{\text{three}} = (1 \times 3^3) + (1 \times 3^2) + (1 \times 3^1) + 1$]

• What numbers can be substituted for a and b to make each sentence true?

1. $(\frac{a}{b})^2 = \frac{25}{36}$ 2. $(\frac{a}{b})^2 = \frac{49}{81}$ 3. $(\frac{a}{b})^3 = \frac{8}{27}$
(1. $a = 5, b = 6$; 2. $a = 7, b = 9$; 3. $a = 2, b = 3$)

• If the area of a square is $\frac{25}{49}$ square inches, what is the length of one side? ($\frac{5}{7}$ in.) What is the measure of the perimeter? ($\frac{20}{7}$ in. or $2\frac{6}{7}$ in.)

• For each of the following, write *always* if the statement is true for *any* whole number, *sometimes* if it is true for only *some* whole numbers, or *never* if it is true for *no* whole numbers. When dividing by n, $n \neq 0$.

1. $n = n \times n$
2. $n = n + n$
3. $n + n = 2 \times n$
4. $n \div n = 1$
5. $n \times n = n^2$
6. $n + n = n \times n$
7. $n \times \frac{n}{n} = n$
8. $n \times \frac{1}{n} = 2$

(Answers: 1. Sometimes; 0 and 1. 2. Sometimes; 0. 3. Always. 4. Always. 5. Always. 6. Sometimes; 0 and 2. 7. Always. 8. Never.)

• What is the reciprocal of

$4\frac{1}{2}$? $(\frac{2}{9})$ $\frac{a}{b}(a \neq 0)$? $(\frac{b}{a})$ $(a + b)$? $(\frac{1}{a + b})$
$\frac{1}{k}$? (k) $\frac{1}{(a + b)}$? $(a + b)$ $\frac{3}{5 \times a}$? $(\frac{5 \times a}{3})$

• Find the area of each of the rooms by measuring the plans below. The scale is *1 in. to 8 ft.*

• Determine the pattern in each of the sequences and show the next number in the sequence.

1. $\frac{1}{7}, \frac{3}{7}, \frac{5}{7}, \frac{7}{7}, \cdots$ ($\frac{9}{7}$; add 2 to the numerator.)
2. $\frac{4}{7}, 2\frac{2}{7}, 9\frac{1}{7}, 36\frac{4}{7}, \cdots$ ($146\frac{2}{7}$; multiply by 4.)
3. $\frac{2}{3}, \frac{1}{3}, \frac{1}{9}, \frac{1}{18}, \cdots$ ($\frac{1}{54}$; multiply by $\frac{1}{2}$ and then by $\frac{1}{3}$)
4. $\frac{3}{4}, 2\frac{1}{2}, 4\frac{1}{4}, 6, \cdots$ ($7\frac{3}{4}$; add $1\frac{3}{4}$)
5. $14, 7, 3\frac{1}{2}, 1\frac{3}{4}, \cdots$ ($\frac{7}{8}$; multiply by $\frac{1}{2}$)

5. At a park, a $\frac{3}{4}$-acre rectangular plot was marked off to form 6 campsites, each of the same size. What was the acreage of each campsite? $n = \frac{3}{4} \div 6$; $\frac{1}{8}$ acre

6. The ranger at the park told Jerry that the ratio of the number of birch trees to the number of pine trees in the park is about 3 to 4. Jerry counted 42 birch trees. About how many pine trees are there in the park? $\frac{42}{n} = \frac{3}{4}$; 56

7. Find the perimeter of a regular octagon if each side measures $17\frac{3}{5}$ in inches. $n = 8 \times 17\frac{3}{5}$; $140\frac{4}{5}$ in.

8. How much greater is $35\frac{3}{7}$ than the product of $\frac{3}{4}$ and $41\frac{2}{3}$? $n = 35\frac{3}{7} - (\frac{3}{4} \times 41\frac{2}{3})$; $4\frac{5}{28}$

9. If a $57\frac{1}{2}°$ angle is partitioned into 9 congruent angles, what will be the measure in degrees of each of the 9 angles? $n = 57\frac{1}{2} \div 9$; $6\frac{7}{18}$

How Well Can You Compute?

Computation Test 5

Ex. 1–5. Multiply the number by $3\frac{1}{4}$.

1. 24 78
2. 37 $120\frac{1}{4}$
3. $\frac{5}{8}$ $2\frac{1}{32}$
4. $\frac{15}{16}$ $3\frac{3}{64}$
5. $8\frac{3}{5}$ $27\frac{19}{20}$

Ex. 6–10. Divide the number by $2\frac{1}{3}$.

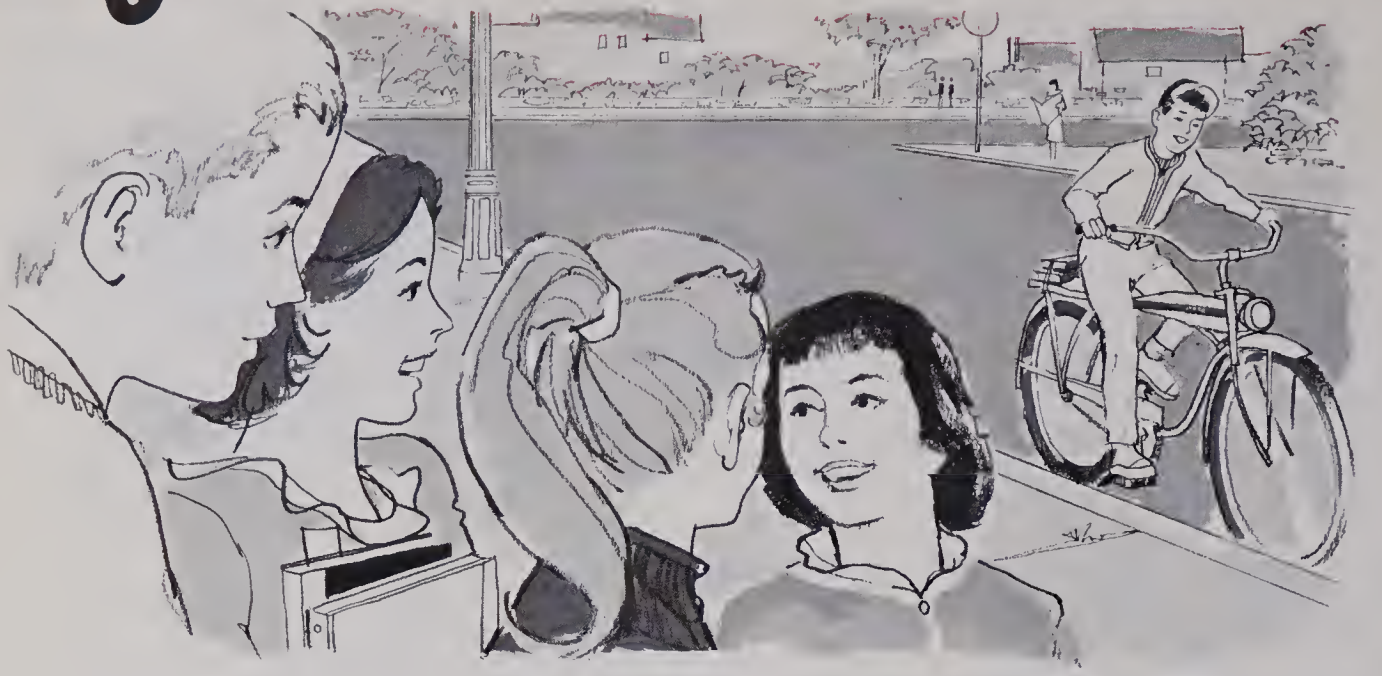
6. 35 15
7. $3\frac{1}{5}$ $1\frac{13}{35}$
8. $25\frac{1}{2}$ $10\frac{13}{14}$
9. 5.2 $2\frac{8}{35}$
10. $\frac{3}{10}$ $\frac{9}{70}$

Ex. 11–22. Find the number represented by the letter.

11. $n = 4\frac{1}{3} \times \frac{3}{4}$ $3\frac{1}{4}$
12. $12 \times x = 3\frac{1}{5}$ $\frac{4}{15}$
13. $\frac{3}{4} \div \frac{5}{6} = x$ $\frac{9}{10}$
14. $n = \frac{2}{3} \times 6 \times 3\frac{1}{2}$ 14
15. $n = 33 + (\frac{1}{4} \times \frac{2}{3})$ $33\frac{1}{6}$
16. $5\frac{3}{4} \times x = 46$ 8
17. $n = 7\frac{1}{3} \times (3 \div \frac{4}{5})$ $27\frac{1}{2}$
18. $n = (\frac{3}{8} \times 2\frac{1}{2}) + (23 \div \frac{3}{7})$ $54\frac{29}{48}$
19. $n = (5\frac{6}{7} + \frac{3}{7} + 4\frac{2}{7}) \div 3$ $3\frac{11}{21}$
20. $x - 3\frac{5}{8} = 36\frac{1}{2}$ $40\frac{1}{8}$
21. $15\frac{2}{3} \div x = 55\frac{5}{6}$ $\frac{94}{335}$
22. $n = 1\frac{1}{5} \times (14\frac{1}{2} \div 5)$ $3\frac{12}{25}$

6

* In discussing the number-line picture, point out how each factor and the product are shown.



Multiplying with Decimals

Multiplying tenths [O]

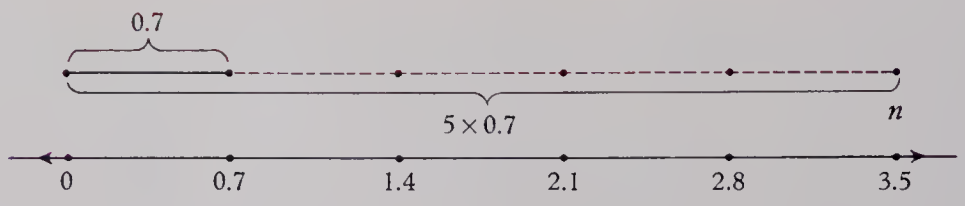
1. Each day, Bob rides his bicycle 0.7 miles in making the round trip to and from school. How many miles does Bob ride in making 5 such trips?

a. Box A. What operation was used to find the answer for the problem? Addition Give the answer for the problem. 3.5 miles

b. You know that $n = 0.7 + 0.7 + 0.7 + 0.7 + 0.7$ shows the relationship expressed in the problem. Since the addends are equal, can we use multiplication to find the answer? Yes $n = \_? 5 \times 0.7$

* **c.** Explain how the number-line picture below shows the relationship expressed by $n = 5 \times 0.7$. $n = \_? 3.5$

The segment above the number-line picture shows 0.7 five times.



d. Box B shows how to solve $n = 5 \times 0.7$. Does the numeral for the product show tenths? Yes Explain. 3.5 means three and five tenths.

A

Add

0.7
0.7
0.7
0.7
0.7
3.5

B

$n = 5 \times 0.7$

Multiply

0.7
5
3.5

$n = 3.5$

Overview -- Chapter 6

• The developmental work in Chapter 6 concerns multiplying and dividing rational numbers including rational numbers named with decimals; working with measures of parts of circular regions; naming rational numbers and operating with per cent forms. Other topics include the commutative property used with decimals; factors which are powers of 10; multiplication and division using the vertical form; rounding quotients; use and meaning of the symbols $\approx$ and π; forumulas for parts of circular regions; and ratio and base factors.

• **Problem-Solving.** Problem-solving skills are further developed by sets of exercises related to multiplication and division with decimals and a variety of significant scientific and social situations involving per cent, ratio factors, and base factors; mathematical sentences with metric units of measurement; measure in the English and metric systems expressed with decimals; "money numbers"; astronomy and satellites; and per cent in business. Extra Problem Sets are provided as indicated on pages 229, 245, and 250.

• **Maintenance.** Extra sets of exercises appear on pages 235, 247, 250, 263, and 270. Newly acquired skills immediately reinforced are related to such topics as multiplieation and division of rational numbers in fraction, decimal, and mixed form; measurement of area, angles, radii, diameters, circumferences, length, weight, perimeter in English and metric units; factoring; multiplication with reciprocals; rounding numbers expressed in decimal form; factors which are powers of ten; arithmetic mean; exponent form; addition and subtraction with per cent forms; and set terminology in geometry.

• **Enrichment.** Extensive opportunities for inriching the basic textbook program are provided within the lesson plans of the Teachers' Edition. Further enrichment within the pupil's textbook is provided through problem-solving with scientific applications. A section entitled Supplementary Activities can bc found on the teacher's pages at the end of this chapter.

• **Tests.** In addition to the regular end-of-chapter tests, you may wish to use summary or practice exercises for testing purposes. Topics covered are inequalities; changing from decimals and fractions to per cent forms; the factors-product relationship and ratio; geometric concepts including circles, radii, diameters, circumferences and their measures; multiplication and division with decimals and rounding in division; problems involving social situations, geometry, the arithmetic mean, and per cent.

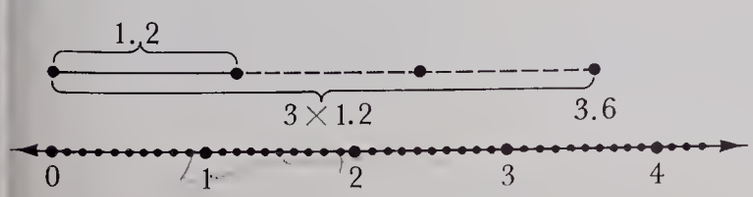
Teaching Pages 228 and 229

Pupil's Objective

To learn to multiply a whole number by a rational number expressed with a decimal.

Background

The number-line picture illustrates a multiplication when one factor is between counting numbers, such as $3 \times 1.2 = 3.6$.

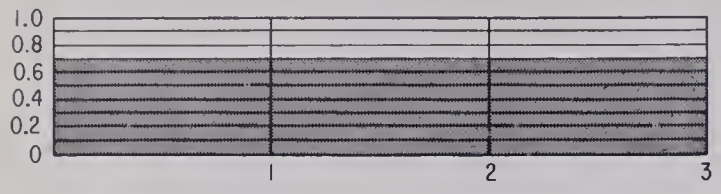


When the number of tenths in a product is greater than 9, it is necessary to rename it as a number of ones and tenths. In the multiplication 5×0.7, the product can be thought of as 35 tenths = 30 tenths + 5 tenths. But 30 tenths can be named $\frac{30}{10}$ or 3, so the final product is 3 + 5 tenths or 3.5.

Another way to show this is:

$$\begin{aligned} 3 \times 0.7 &= 3 \times 7 \text{ tenths} \\ &= 3 \times (7 \times 1 \text{ tenth}) \\ &= (3 \times 7) \times 1 \text{ tenth} \\ &= 21 \times 1 \text{ tenth} \\ &= 21 \text{ tenths, or } 2.1 \end{aligned}$$

In the diagram, a whole has been marked off into tenths. 7 of the 10 tenths are being considered. If we consider the 7 tenths three times, there is a total of 21 tenths.



Pre-Book Lesson

• Review multiplication interpreted as the addition of equal addends as it applies to counting numbers. On the board, write $7 + 7 + 7 + 7 + 7 = 5 \times 7 = 35$. Draw a number-line picture resembling the one from the Background to illustrate this.

• Ask the class to tell what they know about the decimal form for a rational number. (It suggests a denominator which is a power of ten by use of a decimal point.) Review the powers of ten. $10^0 = 1$, $10^1 = 10$, $10^2 = 100$, $10^3 = 1{,}000$, and so on.

• Review the factors-product relationship and vocabulary indicating that it can be applied to decimals.

Using the Text Pages

• Ex. 1–2. Pupils should notice the two sets of numbers involved, namely whole numbers and numbers between 0 and 1. To indicate that the numeral for the product shows tenths,

write $\begin{array}{r} 7 \text{ tenths} \\ \times 5 \\ \hline 35 \text{ tenths} \end{array} = 30 \text{ tenths} + 5 \text{ tenths}$

$= \frac{30}{10} + 5 \text{ tenths}$

$= 3 + 5 \text{ tenths, or } 3.5$

You may work through a similar example at the board, using the process from Ex. 1 and 3 and the material and diagrams from the Background. Practice expressing 10 or more tenths as a whole number and a number of tenths.

• Ex. 3. Have pupils practice expressing numerals such as 2.3 in the form 20 tenths + 3 tenths. Use the method for multiplying 3×14:

$3 \times 14 = 3 \times (10 + 4)$

$= (3 \times 10) + (3 \times 4)$

$= 30 + 12$

$= 30 + 10 + 2, \text{ or } 42$

Work through box C as follows:

$1.4 = 1 + 0.4 = 10 \text{ tenths} + 4 \text{ tenths}$

$= 1 + 4 \text{ tenths.}$

Then: $\begin{array}{r} 1 + \ 4 \text{ tenths} \\ \times 3 \\ \hline 3 + 12 \text{ tenths} \end{array}$ or $\begin{array}{r} 10 \text{ tenths} + \ 4 \text{ tenths} \\ \times 3 \\ \hline 30 \text{ tenths} + 12 \text{ tenths} \end{array}$

From their previous experience, they should see that this is 42 tenths or 4.2. You may also wish to work Ex. 4a by this method.

• Ex. 15–20. Select one example from Ex. 15–20 and work it through using a horizontal form as follows:

$n = 4 \times 6.7$

$= 4 \times (6 + 0.7)$

$= (4 \times 6) + (4 \times 0.7)$

$= 24 + 28 \text{ tenths}$

$= 24 + 20 \text{ tenths} + 8 \text{ tenths}$

$= 24 + 2 + 8 \text{ tenths}$

$= 26 + 8 \text{ tenths}$

$n = 26.8$

• Ex. 21. Review the meaning of perimeter of a polygon and the characteristics of an equilateral triangle. Ask for the answer in terms of millimeters, decimeters, or meters. Require pupils to make and use a carefully labeled sketch.

• Ex. 22. Illustrate the distributive property: $4 \times (2.1 + 4.3) = (4 \times 2.1) + (4 \times 4.3)$

Individualizing Instruction

• *More capable pupils* might consider how the rule for multiplication of rational numbers expressed in fraction form might be applied to several of the exercises. They might illustrate some examples with both rectangular arrays and number-line pictures as shown in the Background.

• *Slower learners* should not be expected to master the horizontal process for multiplication. They could construct simple multiplication examples of their own and submit step-by-step solutions.

*Have pupils place the decimal point by "understanding," not by "rule."

2. At the board, solve each of Ex. a–c as in box B.

a. $n = 3 \times 0.7$ 2.1 **b.** $n = 4 \times 0.6$ 2.4 **c.** $n = 6 \times 0.9$ 5.4

3. Over a 3-day period, Bob rode his bicycle a mean average of 1.4 miles per day. That was how many miles in all?

a. At the board, use addition to find the answer. 1.4 + 1.4 + 1.4 = 4.2

***b.** Box C shows how to find the answer by multiplying in vertical form. Think of 1.4 as 1 one + 4 tenths.

Multiply tenths: $3 \times 4 =$ _?12 Think of 12 tenths as 10 tenths + 2 tenths or 1 one + 2 tenths. Write a 2 in tenth's place in the numeral for the product. Remember 1 one.

Multiply ones: $3 \times 1 =$ _?3 3 ones + 1 one = _?4 ones. Write a 4 in one's place in the numeral for the product.

c. Bob rode _?4.2 miles in all over that 3-day period.

C Multiply

$$\begin{array}{r} 1.4 \\ \times\ 3 \\ \hline 4.2 \end{array}$$

4. On the board, work each of Ex. a–d as in box C.

a. 4×1.1 4.4 **b.** 5×1.7 8.5 **c.** 3×2.6 7.8 **d.** 7×4.7 32.9

[W]

Ex. 5–14. Copy and multiply.

5. $\begin{array}{r} 0.3 \\ \times\ 2 \\ \hline 0.6 \end{array}$	6. $\begin{array}{r} 0.8 \\ \times\ 4 \\ \hline 3.2 \end{array}$	7. $\begin{array}{r} 0.9 \\ \times\ 8 \\ \hline 7.2 \end{array}$	8. $\begin{array}{r} 1.2 \\ \times\ 3 \\ \hline 3.6 \end{array}$	9. $\begin{array}{r} 2.1 \\ \times\ 7 \\ \hline 14.7 \end{array}$
10. $\begin{array}{r} 2.7 \\ \times\ 2 \\ \hline 5.4 \end{array}$	11. $\begin{array}{r} 4.5 \\ \times\ 3 \\ \hline 13.5 \end{array}$	12. $\begin{array}{r} 6.2 \\ \times\ 3 \\ \hline 18.6 \end{array}$	13. $\begin{array}{r} 5.2 \\ \times\ 9 \\ \hline 46.8 \end{array}$	14. $\begin{array}{r} 8.9 \\ \times\ 7 \\ \hline 62.3 \end{array}$

Ex. 15–20. Find the number represented by the letter.

15. $n = 4 \times 6.7$ 26.8 **16.** $n = 5 \times 3.3$ 16.5 **17.** $x = 3 \times 7.2$ 21.6

18. $7 \times 3.1 = x$ 21.7 **19.** $7 \times 8.8 = n$ 61.6 **20.** $n = 2 \times 0.4$ 0.8

Ex. 21–23. Write a mathematical sentence and then solve.

21. Find the perimeter of an equilateral triangle if the length of each side is 4.9 cm. $n = 3 \times 4.9$; 14.7 cm.

22. Find the product of 4 and the sum of 2.1 and 4.3. $n = 4 \times (2.1 + 4.3)$; 25.6

23. What is the total weight of 6 small springs if each spring weighs 7.3 grams? $n = 6 \times 7.3$; 43.8 grams

♦ **Extra Examples.** Set 110. ♦ **Extra Problems.** Set 149.

Multiplying Hundredths

[o]

1. Find the perimeter of a regular hexagon if the length of each side is 3.12 cm.

a. A regular hexagon has _?_ 6 sides of the same length, so one way to find the answer is to multiply 3.12 by _?_ 6.

b. Box A shows one way to find 6×3.12. Explain the renaming and the work shown. See below.

c. The perimeter of the hexagon is _?_ 18.72 cm.

A

$$6 \times 3.12 = 6 \times \frac{312}{100}$$
$$= \frac{1,872}{100}$$
$$= 18.72$$

2. Box B shows how to find 6×3.12 by multiplying in vertical form without renaming either factor. The decimal 3.12 shows two *decimal places* (places to the right of the decimal point) or hundredths. The decimal for the product shows _?_ 2 decimal places or _?_ hundredths

B

Multiply

$$\begin{array}{r} 3.12 \\ \times\quad 6 \\ \hline 18.72 \end{array}$$

3. At the board, work each of Ex. a–d as in box B.

a. 3×0.23 0.69 b. 5×1.31 6.55 c. 7×2.76 19.32 d. 4×0.07 0.28

4. Box C. Explain the work shown for finding 62×5.37.

C

Multiply

$$\begin{array}{r} 5.37 \\ \times\quad 62 \\ \hline 10\,74 \\ 322\,2 \\ \hline 332.94 \end{array}$$

5. At the board, find 38×2.84 first as in box A and then as in box C. See left.

5.

$$38 \times 2.84 = 38 \times \frac{284}{100}$$
$$= \frac{10,792}{100}$$
$$= 107.92$$

$$\begin{array}{r} 2.84 \\ \times\quad 38 \\ \hline 2272 \\ 852 \\ \hline 107.92 \end{array}$$

6. For each of Ex. 7–16, tell whether the decimal for the product will show tenths or hundredths. 7. tenths; 8. hundredths; 9. hundredths; 10. tenths; 11. hundredths; 12. hundredths; 13. tenths; 14. hundredths; 15. hundredths; 16. tenths [w]

Ex. 7–16. Copy and multiply.

7. $\begin{array}{r} 9.6 \\ \times\quad 4 \\ \hline 38.4 \end{array}$	8. $\begin{array}{r} 3.72 \\ \times\quad 3 \\ \hline 11.16 \end{array}$	9. $\begin{array}{r} 0.63 \\ \times\quad 15 \\ \hline 9.45 \end{array}$	10. $\begin{array}{r} 0.7 \\ \times\quad 8 \\ \hline 5.6 \end{array}$	11. $\begin{array}{r} 23.84 \\ \times\quad 68 \\ \hline 1,621.12 \end{array}$
12. $\begin{array}{r} 0.21 \\ \times\quad 9 \\ \hline 1.89 \end{array}$	13. $\begin{array}{r} 23.4 \\ \times\quad 6 \\ \hline 140.4 \end{array}$	14. $\begin{array}{r} 0.08 \\ \times\quad 2 \\ \hline 0.16 \end{array}$	15. $\begin{array}{r} 0.04 \\ \times\quad 2 \\ \hline 0.08 \end{array}$	16. $\begin{array}{r} 243.8 \\ \times\quad 47 \\ \hline 11,458.6 \end{array}$

Ex. 17–22. Solve.

17. $n = 7 \times 6.93$ 48.51 18. $14 \times 7.23 = n$ 101.22 19. $n = 17 \times 0.28$ 4.76

20. $9 \times 16.4 = n$ 147.6 21. $x = 45 \times 3.24$ 145.80 22. $y = 84 \times 17.1$ 1,436.4

◆ **Extra Examples.** Set 111.

1. b. The decimal was renamed with a fraction, the multiplication done, and the fraction renamed with a decimal.

Teaching Page 230

Pupil's Objectives

(a) To learn to multiply a counting number by a rational number expressed with a decimal showing hundredths; and (b) to improve and extend problem-solving skills.

Background

Students should be prepared to apply the rule for multiplication with fractions to verify the method involved with decimal fractions. This will require being able to rename from decimal form to fraction form. Pupils must also have a clear concept of multiplying a whole number by a rational number expressed with a decimal showing tenths. Material from the previous lesson may be reviewed.

Pre-Book Lesson

Review with pupils the concept that multiplication may be interpreted as addition of equal addends. Have pupils suggest that since a decimal may be thought of in terms of a numerator and denominator, the rules for fractions apply.*

Using the Text Page

• Ex. 1. Pupils should know that a regular hexagon has 6 sides of the same length and that perimeter is the distance around the hexagon. Make a sketch and work through box A with the pupils.

• Ex. 2–3. Practice changing expressions such as 4.12 to 412 hundredths, 41 tenths + 2 hundredths, 400 hundredths + 10 hundredths + 2 hundredths, and so on. Show how to complete the example in box B, using an expanded form and the distributive property. Then indicate how these steps can be done mentally, listing only the summary as in box B. Try to elicit from pupils the fact that this is analogous to the problem of multiplying 312 by 6. They might not reach this conclusion until after completing part of Ex. 3.

• Ex. 4. If pupils have grasped the analogy in Ex. 2–3, this should present no problem. You might consider working one example using an expanded form, but pupils should now realize that 5.37 is 537 hundredths, and 537 hundredths considered 62 times is 33,294 hundredths. If some pupils are still having difficulty, you may wish to present other illustrations of expanded form.

• Ex. 6. Do this entire example before assigning Ex. 7–16. Pupils' responses will tell you if they really understand. Pupils should be able to work these as in boxes B or C.

• Ex. 17–22. Pupils should express the multiplication in vertical form.

Individualizing Instruction

• At an early stage, *more capable pupils* might realize how to consider the problem as the product of two counting numbers which could be labeled hundredths. They should, however, work through each of Ex. 1–6 so they may understand why.

They should be encouraged to try to express Ex. 17–22 with a horizontal form, using an expanded-form technique.

• *Slower learners* might be encouraged to write out examples such as Ex. 10 as 7 tenths multiplied by 8 yields _ ? _ tenths. The vertical form should be used by these pupils with a step-by-step solution shown at the side.

* See item 6, page xix. You may wish to use the number line to show similarity between work with decimals and fractions.

Teaching Page 231

Pupil's Objectives

(a) To review multiplication of numbers of tenths and hundredths by a counting number; and (b) to learn to multiply a counting number by a rational number in decimal form showing thousandths.

Background

Pupils should know the names of decimal places through thousandths and be prepared to change expressions such as 2.374 to 23 tenths + 74 thousandths, or 237 hundredths + 4 thousandths, and so on. They should realize that when one factor is a counting number, the other factor determines the label for the product. That is, the product of a counting number and a number of tenths is a number of tenths, the product of a counting number and a number of hundredths is a number of hundredths, and so on.

Pre-Book Lesson

- Review with pupils multiplication of a number of tenths and hundredths by a counting number. Use easy examples working mentally with counting numbers such as 3×12, 4×21.
- Review the use of the distributive property as applied in multiplying two counting numbers.
- Review the place values for decimals through ten-thousandth's place.

Using the Text Page

- Ex. 1. Practice reading and writing a few decimals involving tenths, hundredths, and thousandths as well as combinations with whole numbers.
- Ex. 2–4. Pupils might be told to express numbers in expanded form to help them visualize the naming of the product. You may wish to work one example of this type at the board using fractions and decimals. Express the product of 3 and 0.004 as 12 thousandths, or 1 hundredth + 2 thousandths, and so on.

In box C, be sure pupils understand why the 2 is placed in the second column from the right. It is the product of 30 and 4 thousandths which is 120 thousandths or 12 hundredths. Pupils must be reminded that these partial products are to be added, and thus must be shown accordingly.

- Ex. 5. Pupil responses should enable the teacher to evaluate the need for more practice in application of the conclusions from Ex. 3 that when a counting number is multiplied by tenths, the product shows tenths; when a counting number is multiplied by hundredths, the product shows hundredths, and so on.
- Ex. 7. Pupils should answer this question for each of Ex. 8–17 before working the examples.
- Ex. 8–17. If pupils wrote out the answers to Ex. 7, they should compare their answers here with their previous decisions.
- Ex. 18–21. Pupils should be required to work these examples in two ways, vertical form and expanded form, verifying the distributive property for decimals.
- Ex. 21. Notice and compare this with Ex. 18–20 to remind pupils of the meaning of parentheses.

Individualizing Instruction

- *More capable pupils* might be asked to make up problems which require the number sentences in Ex. 18–21 for solution. They might also be encouraged to try explaining the step-by-step analysis of a product of thousandths by a counting number greater than 100.
- *Slower learners* might be asked to work the examples expressing both factors as counting numbers.

Multiplying Thousandths

[O]

1. In a decimal, the third place shown to the right of the decimal point is called thousandth's ? place. The decimal in box A is read, *two and* three ? *hundred fifty-four* thousandths ?.

2. Box B. Explain the work shown for finding 3×2.354. Does 2.354 show thousandths? Yes Does 7.062? Yes

3. Complete:

a. When tenths are multiplied by a counting number, the decimal for the product will show tenths ?.

b. When hundredths are multiplied by a counting number, the decimal for the product will show hundredths ?.

c. When thousandths are multiplied by a counting number, the decimal for the product will show thousandths ?.

4. Box C. Explain the work shown for finding 32×2.064.

5. At the board, find 41×4.227 as in box C. 173.307

A	
	2.354
B	Multiply
	2.354
	3
	7.062
C	Multiply
	2.064
	32
	4 128
	61 92
	66.048

6. Without using paper and pencil, give the number represented by n for each of Ex. a–c.

a. $n = 4 \times 0.03$ 0.12 **b.** $n = 7 \times 0.008$ 0.056 **c.** $n = 3 \times 1.112$ 3.336

7. For each of Ex. 8–17, tell whether the decimal for the product will show tenths, hundredths, or thousandths.

8. thousandths; 9. thousandths; 10. thousandths; 11. hundredths; 12. thousandths; 13. tenths; 14. thousandths; 15. thousandths; 16. tenths; 17. thousandths [W]

Ex. 8–17. Copy and multiply.

8. 1.241	**9.** 5.672	**10.** 1.372	**11.** 53.89	**12.** 0.067
3	4	26	31	72
3.723	22.688	35.672	1,670.59	4.824
13. 258.4	**14.** 0.663	**15.** 5.881	**16.** 736.8	**17.** 3.992
33	12	91	9	77
8,527.2	7.956	535.171	6,631.2	307.384

Ex. 18–21. Solve.

18. $n = 34 \times (4.726 + 1.029)$ 195.670 **19.** $n = 4 \times (2.734 + 5.552)$ 33.144

20. $n = 12 \times (16.720 - 3.586)$ 157.608 **21.** $n = (3 \times 0.604) - 0.025$ 1.787

◆ **Extra Examples.** Set 112.

* Establish that the commutative property holds, so that pupils can justify why they may multiply by either factor.

Using the Commutative Property of Multiplication

[o]

A Multiply

$$\begin{array}{r} 23.26 \\ \times\ \ 21 \\ \hline 23\ 26 \\ 465\ 2 \\ \hline 488.46 \end{array}$$

B Multiply

$$\begin{array}{r} 21 \\ \times\ 23.26 \\ \hline 1\ 2?6 \\ 4\ 2 \\ 6?3 \\ 42 \\ \hline 4?8.4? \\ 8\quad 6 \end{array}$$

C Multiply

$$\begin{array}{r} 0.07 \\ \times\ \ 325 \\ \hline \end{array}$$

D Multiply

$$\begin{array}{r} 325 \\ \times\ 0.07 \\ \hline \end{array}$$

1. To find the product of 21 and 23.26, we may work as in box A or as in box B. Hundredths are multiplied by a counting number.

   a. Explain the work in box A. $21 \times 23.26 = \_?\_$ 488.46

   b. Box B. Give the missing numerals. $23.26 \times 21 = \_?\_$ 488.46

* 2. Is $21 \times 23.26 = 23.26 \times 21$ a true mathematical sentence? Yes This is an illustration of the Commutative Property of Multiplication.

3. At the board, find the product of 13 and 5.28 first by multiplying 5.28 by 13 as in box A and then by multiplying 13 by 5.28 as in box B. See left below. Did you get the same result with each arrangement? Yes Which arrangement involved less work? Box A

4. At the board, find the product of 325 and 0.07 first by using the arrangement in box C and then by the arrangement in box D. Which arrangement involved less work? Box D

5. For each of Ex. a–b, give the number represented by n by thinking about the Commutative Property of Multiplication. 22.75

   a. $n \times 3.647 = 3.647 \times 28$ 28 b. $0.06 \times 3{,}567 = n \times 0.06$ 3,567

6. When a counting number is multiplied by tenths, the decimal for the product will show tenths. Make a similar statement for multiplying by hundredths; by thousandths. See below.

7. For each of Ex. 8–19, give the factors and tell whether the arrangement as in box C (decimal above) or the arrangement as in box D (decimal below) would be easier to use to find the product. 8. 34; 5.037; C; 9. 1.78; 63; C; 10. 728; 0.6; D; 11. 46; 23.8; C; 12. 0.116; 9; C; 13. 45.026; 8; C; 14. 19; 3.782; C; 15. 2,627; 1.4; D; 16. 378; 5.3; D; 17. 18; 0.038; C; 18. 3.006; 7; C; 19. 5.598; 48; C

[w]

Find the number represented by the letter.

8. $n = 34 \times 5.037$ 171.258
9. $1.78 \times 63 = n$ 112.14
10. $x = 728 \times 0.6$ 436.8
11. $x = 46 \times 23.8$ 1,094.8
12. $n \div 0.116 = 9$ 1.044
13. $n = 45.026 \times 8$ 360.208
14. $x = 19 \times 3.782$ 71.858
15. $x = 2{,}627 \times 1.4$ 3,677.8
16. $378 \times 5.3 = n$ 2,003.4
17. $18 \times 0.038 = n$ 0.684
18. $x = 3.006 \times 7$ 21.042
19. $x \div 48 = 5.598$ 268.704

3.

$$\begin{array}{r} 5.28 \\ \times\ \ 13 \\ \hline 1584 \\ 528 \\ \hline 68.64 \end{array} \qquad \begin{array}{r} 13 \\ \times\ 5.28 \\ \hline 104 \\ 26 \\ 65 \\ \hline 68.64 \end{array}$$

♦ Extra Examples. Set 113.

6. When a counting number is multiplied by {hundredths / thousandths}, the decimal for the product will show {hundredths / thousandths}.

Teaching Page 232

Pupil's Objectives

(a) To practice and reinforce multiplication of numbers of tenths, hundredths, and thousandths by counting numbers; (b) to verify the Commutative Property of Multiplication as it applies to decimals; and (c) to appreciate the convenience of being able to select the order of the factors.

Background

The reasoning underlying use of the commutative property with whole numbers applies to rational numbers expressed in fraction or decimal form. You may refer to previous teacher's pages dealing with this topic.

Pupils are familiar with the use of the commutative property with whole numbers and rational numbers. Previous class discussions concerning commutativity with counting-number factors apply here. These discussions appear on pages 40, 54, 107, and 187.

Pre-Book Lesson

Review the use of the commutative property showing how 235×20 can be worked using the easier 20×235. This should be carried out in both vertical and horizontal form. Verify the fact that the product of 0.34×32 is the same as the product of 32×0.34 or $\frac{2}{3} \times 5$ is the same as $5 \times \frac{2}{3}$. Perhaps express the decimal form with fractions to show the reasoning behind labeling the product as tenths or hundredths.

Using the Text Page

• Ex. 1. Work through the examples in boxes A and B. You may wish to refer pupils to previous pages to review explanations there, such as in box B where location of the numerals for the partial products is especially important. The work may be shown in expanded form so the product of hundredths, tenths, ones, and tens is clearly visualized.

• Ex. 2. If pupils do not immediately recognize the commutative property, write the corresponding example without decimals.

• Ex. 3–4. Have pupils work these exercises on their own. Then discuss their results before proceeding further.

• Ex. 5. Pupils should recognize the numbers for n at once. If not, try writing the corresponding statements without decimals, and perhaps with only one-place numerals.

• Ex. 6. Pupils who do not recall the generalizations for multiplying counting numbers by rational numbers expressed in decimal form, may need additional maintenance or explanatory exercises.

• Ex. 8–19. Pupils should be encouraged to consider the use of the Commutative Property of Multiplication to ease their solutions for examples such as $728 \times 0.6 = 0.6 \times 728$.

Individualizing Instruction

• *More capable pupils* can be asked to make up examples where using the commutative property makes working the example easier.

• Have *slower learners* express the examples in the written section in reverse form by using the commutative property. They may make and manipulate arrays of X's or dots to illustrate the commutativity. You may need to help them perform the computations.

Teaching Page 233

Pupil's Objectives

(a) To review renaming products when expressed with decimals; and (b) to review the concept of precision in measurement.

Background

You may wish to refer to the Background on Teacher's Pages 154–157 to review the concept of precision. A measurement is recorded in such a way as to indicate the unit of precision involved. Precision is dependent upon the unit used in making the measurement, the measurement made with the smaller unit being the more precise.

The work concerning the renaming of products involves the same principles as in work with whole numbers. Concepts pertaining to our place-value system are important here. For example, the fact that each place has a value which is a power of ten is very useful when renaming products to show tenths instead of hundredths.

Pre-Book Lesson

• Review expressing decimals with fractions as well as using the identity element for multiplication as it applies to expressing a fraction in simplest form.

• Mention that a measurement made with a ruler marked off into fourths of an inch is used to measure only to the nearest fourth of an inch. Discuss what this means in terms of the greatest possible error. The smaller the unit of precision, the lesser the greatest possible error.

Using the Text Page

• Ex. 1–5. Review the idea of naming a rational number many ways in fraction and decimal form. Work several examples at the board. By having pupils work through the example in box A, using the ideas from the review, they should see that renaming the product presents nothing really new.

• Ex. 6. Pupils must realize that there is more involved in a given example than the mere manipulation of digits. In this case, the pupil is really asked for the principles involved in finding the precision for a measurement expressed with decimals.

• Ex. 7–12. The idea of renaming rational numbers expressed in decimal form involves the same principles as expressing fractions in simplest form. Pupils should work these exercises and then be asked if the products would be different if the numbers were measures. The answer is yes since there are many rules for multiplying with measures too complex for this grade level. You may wish to refer to *Mathematics We Need, Level J-2.*

• Ex. 13. Other examples and illustrations such as those on Teacher's Page 154 could be used for reinforcement. Refer to Teacher's Pages 154–156 for a review of the proper terminology associated with precision. You may wish to use number-line pictures with these examples.

Individualizing Instruction

• *More capable pupils* may construct some problems using numbers they obtained by making measurements in the classroom. They should make measurements of the same item, using different units of precision, and compare answers.

• As a break from the work with decimals, have *all pupils* work some Supplementary Activities.

Renaming Products

[O]

1. Box A. Study the work shown for solving the mathematical sentence $n = 5 \times 2.34$.

a. Does 2.34 show two decimal places? Yes Does 11.70 show two decimal places? Yes

b. $11.70 = 11\frac{?\ 70}{100} = 11\frac{?\ 7}{10}$ Do you understand why 11.70 may be renamed 11.7? Yes

2. Another way to understand why 11.70 may be renamed 11.7 is to think, $11.70 = \frac{1,170}{100} = \frac{?}{10} = 11.7$. 117

3. Explain the work in box B for solving the mathematical sentence $n = 25 \times 0.912$. Explain why the product, 22.800, may be renamed 22.8. Key idea: $\frac{800}{1,000} = \frac{8}{10}$

4. At the board, solve $n = 4 \times 3.5$. Yes Did you find: $n = 14.0$? 14.0 The product 14.0 may be renamed with the standard numeral 14.

A

$n = 5 \times 2.34$

Multiply

$$\begin{array}{r} 2.34 \\ \times\ \ 5 \\ \hline 11.70 \end{array}$$

$n = 11.70$, or 11.7

B

$n = 25 \times 0.912$

Multiply

$$\begin{array}{r} 0.912 \\ \times\ \ 25 \\ \hline 4\,560 \\ 18\,24 \\ \hline 22.800 \end{array}$$

$n = 22.800$, or 22.8

5. Ex. a–f. Rename as in Ex. 3 and 4.

a. 23.860 23.86 **b.** 0.50 0.5 **c.** 837.00 837

d. 1.030 1.03 **e.** 0.010 0.01 **f.** 260.00 260

6. Sometimes 0's in decimals must be kept since they indicate the unit of precision. Mr. Harvey measured the length and width of a picture that he wanted to frame and recorded the width as 8.0 in. A measurement of 8.0 in. does not mean the same as a measurement of 8 in. Why not? 8.0 in. means the unit of precision is $\frac{1}{10}$ inch; 8 in. means the unit of precision is 1 inch.

[W]

Ex. 7–12. Solve.

7. $n = 8 \times 4.5$ 36

8. $n = 7.545 \times 14$ 105.63

9. $n = 22 \times 0.075$ 1.65

10. $n = 26 \times (3.17 + 1.14 + 12.64)$ 440.7

11. $x = 15 \times (0.038 + 0.004)$ 0.63

12. $x = 24 \times (2 - 0.25)$ 42

13. Copy and complete. A measurement of $3\frac{0}{8}$ in. means that the unit of precision is $\frac{1}{8}$ inch and that the length is either $2\frac{15}{16}$ in. or is between $2\frac{15}{16}$ in. and $3\frac{1}{16}$ in.

*Do not be too quick to state the generalization. Give pupils many examples and see if they can express, in their own words, a generalization before reading Ex. 4.

A	B	C
$10 \times 3.82 = 10 \times \frac{382}{100}$ $= \frac{3,820}{100}$, or 38.2	$100 \times 3.82 = 100 \times \frac{382}{100}$ $= 382$	$1,000 \times 3.82 = 1,000 \times \frac{382}{100}$ $= 3,820$

When One Factor is a Power of 10

[O]

1. For each of $\underline{10} \times 3.82$, $\underline{100} \times 3.82$, and $\underline{1,000} \times 3.82$, which factor is a power of 10? Name three other numbers which are powers of 10. Sample answers: 10,000; 100,000; 1,000,000

2. Explain the work in boxes A–C for finding 10×3.82, 100×3.82, and $1,000 \times 3.82$. Key idea: The decimal is renamed with a fraction, the multiplication is done, and the fraction is renamed with a decimal.

D
$10 \times 7 = \_?\_^{70}$
$100 \times 7 = \_?\_^{700}$
$1,000 \times 7 = \_?\_^{7,000}$
$10,000 \times 7 = \_?\_^{70,000}$

3. Box D. Complete the sentences shown. Tell a quick way to find the product of two counting numbers when one of the numbers is 10; 100; 1,000; 10,000. Key idea: For each 0 in the numeral for the power of ten, the digits for the other factor are shown one place farther to the left.

* 4. Instead of working as in box B to find 100×3.82, we can use the following procedure. This procedure can be used to find the product of two numbers when one factor is named by a decimal and the other is a power of 10.

Step 1: Note the number of 0's in the standard numeral for the power of 10. For 100, the number of 0's is $\_?\_^{2}$.

Step 2: Write the numeral for the product using the same digits as in 3.82, only write each digit two places farther to the left. The 3 will be in $\_?\_^{\text{hundred's}}$ place, the 8 will be in $\_?\_^{\text{ten's}}$ place, and the 2 will be in $\_?\_^{\text{one's}}$ place. $3.82 \times 100 = \_?\_^{382}$

5. If the power of 10 in Ex. 4 had been 1,000, each digit for 3.82 would have been written $\_?\_^{3}$ places farther to the left.

6. Explain using the above procedure to find $1,000 \times 0.534$. Key idea: Each digit for 0.534 is written 3 places farther to the left. $1,000 \times 0.534 = 534$

[W]

Ex. 7–12. Find the number represented by n.

7. $n = 10 \times 0.78$ 7.8
8. $n = 1,000 \times 3.708$ 3,708
9. $n = 100 \times 26$ 2,600
10. $n = 10,000 \times 47$ 470,000
11. $n = 100,000 \times 13.628$ 1,362,800
12. $n = 1,000 \times 0.003$ 3

♦ **Extra Examples.** Set 114.

Teaching Page 234

Pupil's Objective

To increase skill and speed in multiplying when one factor is named by a decimal and the other factor is a power of ten.

Background

The value of each place in our place-value system is ten times as great as we move consecutively to the left. Thinking of these facts in relation to one another can lead to a generalization or "shortcut" method for multiplying one factor by a power of ten.

$$\begin{aligned}\text{(a) } 10 \times 4.35 &= 10 \times \tfrac{435}{100} \\ &= \tfrac{4,350}{100} \\ &= \tfrac{435}{10} \\ &= 43 \text{ and } \tfrac{5}{10}, \text{ or } 43.5\end{aligned}$$

$$\begin{aligned}\text{(b) } 100 \times 9 &= 1 \text{ hundred} \times 9 \\ &= 9 \text{ hundreds, or } 900\end{aligned}$$

Consider the number of zeros in the numeral for the factor which is a power of ten. To show the product, simply annex this number of zeros to the numeral for the other factor or show the digits in the numeral that many places to the left.

Pre-Book Lesson

- Review briefly multiplying a number of ones by a number of tenths or hundredths. Make sure pupils have mentally established the procedure for placing the decimal point.
- Turn back to pupil's page 60 and review multiplying counting numbers which are multiples or powers of ten.

Using the Text Page

- Ex. 1–2. List the given factors and the resulting products on the board using separate columns so pupils can see the change in position for each digit. Use examples (a) and (b) from the Background to help explain the work in the boxes. Pupils may benefit by working through each step in the multiplication process with the teacher.
- Ex. 3–6. Pupils should deduce from Ex. 1–2 that each digit in the decimal is moved one place to the left for each zero in the numeral for the power of ten.
- Ex. 7–12. Pupils should work only one or two of these examples as in the boxes at the top of the page. They should be encouraged to use the generalization for annexing zeros.

Individualizing Instruction

- *More capable pupils* may be able to complete this page without much assistance. In such an event, if the situation permits, they may assist other pupils in working the examples. These pupils should be asked to consider the possibility of a corresponding rule for division in anticipation of the presentation on page 246.
- *Slower learners* may need to set up a chart whereby multiplication of a few numbers by consecutive powers of ten is shown. This visual aid may help them apply generalizations more directly.

×	10	100	1,000
31	310	3,100	31,000
20	200	2,000	20,000
7	70	700	7,000
4.9	49	490	4,900
0.04	0.4	4	40
20.11	201.1	2,011	20,110
5.327	53.27	532.7	5,327
0.0001	0.001	0.01	0.1

These pupils may also need to refer to page 60 where multiplication by tens is first presented.

Teaching Page 235

Pupil's Objectives

(a) To review some of the concepts and skills presented in Chapters 1–5; and (b) to discover where weaknesses exist so appropriate review may be provided.

Background

Remember that in representing an unknown for a rational number in a mathematical sentence, only a single letter is necessary. As a review, you may wish to refer to previous pages dealing with the topics reinforced on this page such as rounding numbers, numbers expressed in other bases, and operations with fraction numbers.

Pre-Book Lesson

- If this page is to be used as a test, you may wish to have pupils work related examples at the board as a review. Provide those who finish their work early with one or two of the Supplementary Activities at the end of the chapter.
- When using this page as a review, you may want pupils to do all the work on their own; or, you may wish to work some examples at the board as review and then assign remaining examples for written work.

Using the Text Page

- Ex. 1–29. The need for individual help may be determined from board work, and you may wish to work with pupils individually as a type of reteaching.
- Ex. 1–8. Vocabulary difficulties may be encountered in exercises at the bottom of the page. Pupils should try to use the index and refer to the page where the meaning of the term is presented. They may refer to Terms and Illustrations on pages 372–375.

Individualizing Instruction

- *All pupils* should be grouped for reteaching and remedial work according to common errors. If the situation permits, *more capable pupils* may assist with the remedial work by helping individual pupils at their seats. The following list may be referred to for review pages:

For Errors in Ex.	*Review*
(top of page 235)	
1–10	page 123
11–15	page 23
16–20	page 22
21–27	pages 112–114
28	pages 184–187
(bottom of page 235)	
1	page 212
2 and 5	page 166
3 and 6	page 67
4	page 68
7	page 28
8	pages 8, 9

- *Slower learners* may be confused by the appearance of 4.73 and $\frac{1}{4}$ in the same exercise in Ex. 1. They should remember that they can rename from a fraction to a decimal and when the reciprocal of $\frac{1}{4}$ is considered, it can be thought of as 4.0 or 4.00 as well as 4.

To Keep in Practice

[W]

Ex. 1–5. Round to the nearest hundredth.

1. 4.783 4.78 2. 13.955 13.96 3. 0.782 0.78 4. 0.006 0.01 5. 1.0509 1.05

Ex. 6–10. Round to the nearest thousandth.

6. 5.5582 5.558 7. 794.0058 794.006 8. 0.0309 0.031 9. 0.1005 0.101 10. 40.4859 40.486

Ex. 11–15. Rename with a base-five numeral.

11. 20 40_{five} 12. 33 113_{five} 13. 50 200_{five} 14. 76 301_{five} 15. 133 1013_{five}

Ex. 16–20. Rename with a base-ten numeral.

16. 33_{four} 15 17. 41_{five} 21 18. 203_{seven} 101 19. 71_{eight} 57 20. 312_{four} 54

Ex. 21–29. Find the number represented by the letter.

21. $n = \frac{2}{3} + \frac{4}{5}$ $1\frac{7}{15}$ 24. $x - \frac{5}{6} = 5\frac{1}{2}$ $6\frac{1}{3}$ 27. $n = \frac{4}{9} + 3\frac{1}{3} + \frac{5}{9}$ $4\frac{1}{3}$

22. $n = 3\frac{7}{8} + \frac{5}{8}$ $4\frac{1}{2}$ 25. $x = 3\frac{2}{3} - 2\frac{1}{2}$ $1\frac{1}{6}$ 28. $x = \frac{3}{5} \times 4\frac{2}{3}$ $2\frac{4}{5}$

23. $n = 4\frac{2}{7} + \frac{2}{3}$ $4\frac{20}{21}$ 26. $n + 4\frac{4}{5} = 7\frac{3}{5}$ $2\frac{4}{5}$ 29. $x = 4 + (5 \times 2\frac{1}{2})$ $16\frac{1}{2}$

Can You Follow Directions?

[W]

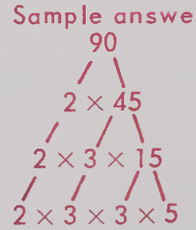
1. Multiply 4.73 by the reciprocal of $\frac{1}{4}$. 18.92

2. Subtract 35.8 from 51.4 and then multiply the result by 8. 124.8

3. Multiply 0.793 by the prime number that is less than 3. 1.586

4. Make a factor tree for 90 and then express 90 as a product of primes. $2 \times 3 \times 3 \times 5$

Sample answer



5. Multiply 3.78 by the sum of 14.7 and 13.3. 105.84

6. List within braces the members of {0, 2, 4, 5, 9} which are
a. counting numbers. {2, 4, 5, 9} b. primes. {2, 5}

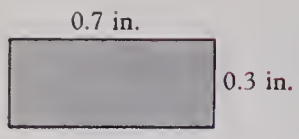
7. Write a mathematical sentence which illustrates the Associative Property of Addition. Sample answer: (2 + 3) + 5 = 2 + (3 + 5)

8. List with braces two 3-element sets whose intersection is a 2-element set. Sample answer: {1, 5, 6} {1, 6, 8}

* Notice that a product is first found by multiplying with fractions and then the same product is found by multiplying with decimals. The decimal point is placed by "understanding," not by "rule."

Both Factors Expressed with Decimals

[O]



1. Give the dimensions of the rectangular region shown at the left. 0.7 inches by 0.3 inches

a. Explain why we can use $n = 0.7 \times 0.3$ to find the area of the region in terms of square inches. Key idea: A = l × w where l = measure of length and w = measure of width.

b. Box A shows how to solve $n = 0.7 \times 0.3$ by renaming the factors with fractions ($0.7 = \frac{?}{10}^{7}$ and $0.3 = \frac{?}{10}^{3}$). Explain the work shown. Do the numerals $\frac{21}{100}$ and 0.21 both name the same number? Key idea: The decimals are renamed with fractions, the work done, and the product shown with a decimal. Yes

c. The area of the region is $\_?\_^{0.21}$ sq. in.

A

$n = 0.7 \times 0.3$

$= \frac{7}{10} \times \frac{3}{10}$

$= \frac{21}{100}$, or 0.21

B

Multiply

$$\begin{array}{r} 0.3 \\ \underline{0.7} \\ 0.21 \end{array}$$

C

$n = 0.5 \times 2.8$

Multiply

$$\begin{array}{r} 2.8 \\ \underline{0.5} \\ 1.40 \end{array}$$

$n = 1.40$, or 1.4

* 2. Box B shows how to find 0.7×0.3 by multiplying in vertical form with decimals. Think, $7 \times 3 = \_?\_^{21}$. To place the decimal point properly in the decimal for the product think, $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$, *so the decimal for the product will show* $\_?\_^{\text{hundredths}}$. $0.7 \times 0.3 = \_?\_^{0.21}$

3. At the board, work each of Ex. a–c as in box B.

a. 0.2×0.4 0.08 b. 0.8×1.6 1.28 c. 0.9×23.4 21.06

4. Explain the work shown in box C for solving $n = 0.5 \times 2.8$. Why was the product renamed 1.4? Key idea: To show the product in simplest form

[W]

Ex. 5–9. Copy and multiply.

5.	6.	7.	8.	9.
0.8	0.3	7.1	25.5	9.6
0.7	0.3	0.8	4.3	8.8
0.56	0.09	5.68	109.65	84.48

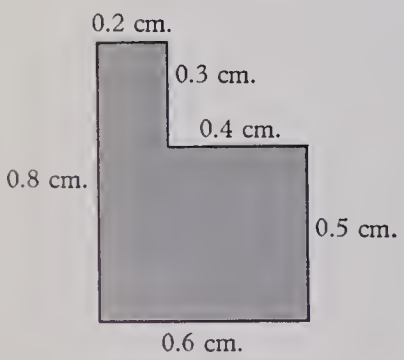
Ex. 10–13. Solve.

10. $n = 23.7 \times 3.4$ 80.58

11. $n = 5.2 \times 6.5$ 33.8

12. $0.4 \times 2.5 = n$ 1

13. $x = 6.7 \times 10.1$ 67.67



14. Find the area of the region shown at the left. 0.36 sq. cm.

15. Subtract the sum of 2.7 and 9.8 from the product of 3.2 and 15.6. 37.42

◆ **Extra Examples.** Set 115.

Teaching Page 236

Pupil's Objective

To find a product when each factor is represented by a decimal showing tenths.

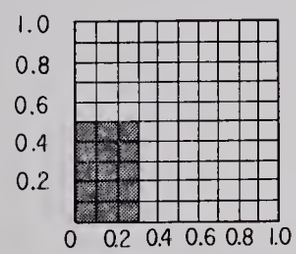
Background

When both factors are named by decimals, the numeral for the product has as many places to the right of the point as the total number of places in the numerals for both factors. The reasonableness of this rule can be illustrated by the following examples:

(a) $0.4 \times 0.6 = \frac{4}{10} \times \frac{6}{10}$
$= \frac{4 \times 6}{10 \times 10}$
$= \frac{24}{100}$, or 0.24

(b) $2.14 \times 0.2 = 2\frac{14}{100} \times \frac{2}{10}$
$= \frac{214}{100} \times \frac{2}{10}$
$= \frac{214 \times 2}{100 \times 10}$
$= \frac{428}{1,000}$, or 0.428

The denominators shown by the fractions are powers of ten and can be expressed in decimal form. In (b), multiplication of a rational number greater than one by a rational number less than one yields a product which is less than one. Pupils should not jump to conclusions in thinking that since one factor is greater than one, the product must necessarily be greater than one. 0.5×0.3 can be illustrated with a diagram. Mark off a model of a square-unit region into ten parts of the same size along each of two adjacent sides. Consider 5 of the ten parts along one side of the model, then consider 3 of the ten parts along the other. In all, you have considered 15 of the 100 square regions representing the whole, so $0.5 \times 0.3 = 0.15$.



Pre-Book Lesson

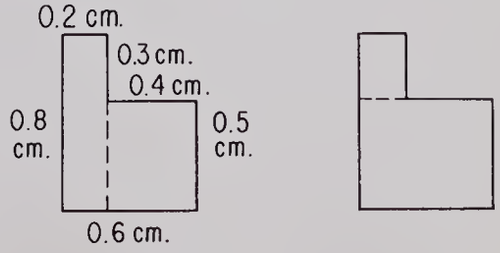
- Review multiplication with fractions and mixed forms. Use several denominators which are powers of ten.
- Review expressing decimals and fractions in simplest form and in expanded form.

Using the Text Page

- Ex. 1. Have pupils draw, at the board, diagrams similar to the one appearing in the Background section. They may use them in conjunction with simple examples to illustrate the multiplications.
- Ex. 2. Emphasize the relation between the number of places to the right of the point in the decimal and the value of the denominator in terms of powers of ten. If the denominators of the factors are 10 and 100, the denominator of the product will be 1,000 and will show 3 places for the decimal form.
- Ex. 5–13. Pupils may need to refer to boxes A–C before they can do the multiplication.

Individualizing Instruction

- *More capable pupils* may work out and explain the basis for a shortcut method of finding the correct number of places in a decimal naming a product. It involves counting the total number of places to the right of the point in decimals naming both factors and marking off this total in the decimal naming the product. The basis was brought out in the Pre-Book Lesson and in boxes A–C on the text page.
- *All pupils* may need help with Ex. 15 since it involves more than one step and requires the use of parentheses if the work is to be expressed with a mathematieal sentence.
- *Slower learners* may need a hint for Ex. 14. If they partition the region either way as shown below, they need find only the sum of the measures of both areas.



Teaching Page 237

Pupil's Objective

To review multiplying in vertical form factors expressed with decimals to hundredth's, thousandth's, and ten-thousandth's places.

Background

Every multiplication having factors expressed with decimals can be worked by changing the decimals to fraction form. For example:

$$\begin{array}{r} 36.42 \\ \times\ 0.7 \\ \hline 25.494 \end{array}$$

(a) multiply with the decimals as if they were whole numbers

(b) *think* of the factors as fractions (7 tenths and 3,642 hundredths)

(c) *think* of the product of the denominators as a power of ten (tenths $\times$ hundredths = thousandths)

(d) using this power of ten, place the decimal point in the numeral for the product (25,494 thousandths)

It is important that pupils establish this reasoning process before using the following general procedure: "When multiplying with decimals, there are as many decimal places in the numeral for the product as there are in the numerals for both the factors."

Pre-Book Lesson

- Briefly review multiplying when both factors are powers of ten.
- At the board, work several multiplication examples with the factors in fraction form showing denominators which are multiples of ten.
- Review material from page 236 to establish the procedure for multiplying when both factors are expressed with decimals.

Using the Text Page

- Ex. 1–2. Pupils may work several extra examples writing out the step-by-step procedure. Then, in Ex. 2, have them "think" the multiplication for the denominators mentally. You may wish to use the procedure as discussed in the Background.
- Ex. 3. You may wish to bring out the possible use of the commutative property with these examples and have pupils compare answers when the order of the factors is changed.
- Ex. 5. Pupils should now be ready to use the generalization quoted in the Background. Have them verify answers obtained in this manner by using the method in box A.
- Ex. 6. Be sure pupils are "thinking" the products and the location of the decimal point.
- Ex. 17–22. Have pupils work these examples in vertical form using the commutative property to reverse the order of the factors.

Individualizing Instruction

- Have *all pupils* estimate the answers for the written work using rounded numbers if necessary. They should then compare their estimations with their calculated answers to determine weaknesses in their reasoning processes.
- Work individually with the *slower learners*, having them use the procedure in box A with several selected examples. Develop the generalization from the Background and try to have them work the remaining examples mentally.
- If *more capable pupils* complete their work early, you may have them work Supplementary Activities from the end of the chapter.

* You may want pupils to estimate each answer before finding the product. This is a good way for pupils to check themselves on the placement of the decimal point.

Products to Ten-Thousandths

[O]

1. Box A. Explain the work shown for finding 0.5×1.23. The product of the denominators, 10 and 100, is 1,000 so the denominator for the product is 1,000. Is $\frac{615}{1,000}$ equal to 0.615? Yes

A

$$0.5 \times 1.23 = \frac{5}{10} \times \frac{123}{100}$$
$$= \frac{615}{1,000}$$
$$= 0.615$$

2. Box B. Explain how to find 0.5×1.23. To place the decimal point think, $\frac{1}{10} \times \frac{1}{100} = \frac{1}{1,000}$, *so the decimal for the product will show* thousandths.

B

Multiply
1.23
0.5
0.615

3. At the board, work each of Ex. a–d as in box B. Explain your work and the placing of the decimal point in the numeral for the product.

a. 0.7×1.35 0.945 **b.** 0.2×0.83 0.166 **c.** 0.4×5.07 2.028 **d.** 4.7×3.86 18.142

4. The decimal 2.42 shows hundredths and 3.62 shows hundredths. $\frac{1}{100} \times \frac{1}{100} = \frac{1}{10,000}$, so the decimal for the product of 2.42 and 3.62 will show ten-thousandths. At the board, find 2.43×3.62. 8.7966

5. Solve without using paper and pencil.

a. $n = 0.2 \times 0.12$ 0.024 **b.** $n = 0.3 \times 0.022$ 0.0066 **c.** $n = 1.12 \times 0.4$ 0.448

6. For each of Ex. 7–16, tell whether the digit farthest to the right in the decimal for the product will be in hundredth's place, thousandth's place, or ten-thousandth's place. Ex. 7, 10, 13, hundredth's; Ex. 8, 9, 12, 14, 15, thousandth's; Ex. 11, 16, ten-thousandth's

[W]

* Ex. 7–16. Copy and multiply.

7. 5.3 × 3.1 = 16.43	8. 3.45 × 2.7 = 9.315	9. 23.67 × 0.8 = 18.936	10. 5.55 × 23 = 127.65	11. 4.173 × 2.2 = 9.1806
12. 3.728 × 9 = 33.552	13. 2,352 × 0.71 = 1,669.92	14. 623.7 × 0.03 = 18.711	15. 0.888 × 27 = 23.976	16. 12.72 × 6.63 = 84.3336

Ex. 17–22. Solve.

17. $n = 0.006 \times 254$ 1.524 **18.** $n = 3.9 \times 4.427$ 17.2653 **19.** $n = 0.46 \times 0.5$ 0.23

20. $n = 1.25 \times 0.4$ 0.5 **21.** $x = 5.08 \times 7.55$ 38.354 **22.** $x = 56.78 \times 13.7$ 777.886

◆ **Extra Examples.** Set 116.

Using the Factors-Product Relationship

[o]

A
$10.81 = 2.3 \times 4.7$
$10.81 = 4.7 \times 2.3$
$10.81 \div 2.3 = 4.7$
$10.81 \div 4.7 = 2.3$

1. Do the four sentences in box A illustrate the factors-product relationship? Yes ^ For each sentence, 10.81 names the _?_ and 2.3 and 4.7 name the _?_. product, factors

2. Using 1.8 and 2.57 for the factors and n to represent the product, write four mathematical sentences which illustrate the factors-product relationship.
$n = 1.8 \times 2.57$; $n \div 1.8 = 2.57$; $n \div 2.57 = 1.8$; $n = 2.57 \times 1.8$

B
$n \div 0.5 = 3.7$
$n = 3.7 \times 0.5$
$= 1.85$

3. In $n \div 0.5 = 3.7$, does n represent the product or a factor? product ^ What operation is used to find the product when the factors are known? Multiplication ^ Explain the work in box B.

4. At the board, solve each of Ex. a–c.

a. $n \div 23 = 4.13$ ∨ 94.99 **b.** $67 = n \div 3.3$ ∨ 221.1 **c.** $n \div 1.42 = 0.005$ 0.0071

5. In placing the decimal point in the numeral for the product, we can add the numbers of decimal places shown by the numerals for the factors. For 3.24×0.28 think, *3.24 shows 2 decimal places and 0.28 shows* _?_ 2 *decimal places, so the decimal for the product will show* _?_ 4 *decimal places, or ten-thousandths.* At the board, find the product. Use this method to explain the placing of the decimal point for each of Ex. 6–10.

[w]

Ex. 6–15. Copy and multiply.

6. 23.8 × 3.4 = 80.92	7. 87.77 × 2.06 = 180.8062	8. 3.882 × 0.28 = 1.08696	9. 0.0046 × 0.89 = 0.004094	10. \$46.88 × 23 = \$1,078.24
11. 2,893 × 0.9 = 2,603.7	12. 1.7002 × 7.7 = 13.09154	13. 23.32 × 7.538 = 175.78616	14. 28,902 × 5.004 = 144,625.608	15. 0.77354 × 0.24 = 0.1856496

Ex. 16–24. Find the number represented by the letter.

16. $n = 3.74 \times 8.52$ ^ 31.8648
17. $n \div 6.7 = 5.841$ ^ 39.1347
18. $x \div 0.46 = 3.5$ ^ 1.61
19. $n \div 37 = 7.8$ ^ 288.6
20. $278 \times 4.39 = x$ ^ 1,220.42
21. $x \div 55 = 582.83$ ^ 32,055.65
22. $38 = x \div 0.44$ 16.72
23. $n = 39.67 \times 19$ 753.73
24. $n \div 27 = 3.56$ 96.12

♦ **Extra Examples.** Sets 117–118.

Teaching Page 238

Pupil's Objective

To apply the factors-product relationship with decimals.

Background

The material presented on this page is an extension of previous work with whole numbers and rational numbers. When the two factors are known, we multiply to find the product; when the product and one factor are known, we divide to find the unknown factor. Division with decimals will be presented on page 240. Consider:

(a) $3\frac{1}{5} \times 2\frac{2}{5} = \frac{16}{5} \times \frac{12}{5}$
$= \frac{192}{25}$, or $7\frac{17}{25}$

(b) $3.2 \times 2.4 = \frac{32}{10} \times \frac{24}{10}$
$= \frac{768}{100}$, or 7.68

Both (a) and (b) express the same factors-product relationship, one in fraction form, the other in decimal form.

The material presented on the previous teaching page is applicable to the topics presented here. The same generalizations for finding the number of places in a product shown by a decimal are used.

Pre-Book Lesson

- Review expressing the factors-product relationship with fractions, decimals, and numerals for whole numbers. Use several examples, such as that from the Background, where the same mathematical sentence is expressed with both fractions and decimals.
- Review the material on pages 232, 234, and 236 together with the generalizations for showing products when multiplying by tenths, hundredths, and thousandths.

Using the Text Page

- Ex. 1–2. Pupils should not attempt division with decimals since this topic has not yet been presented.
- Ex. 4. Even though the symbol ÷ appears in each example, the sentences all require multiplication for their solution. This is a good illustration of the need for flexibility in expressing the factors-product relationship. The commutative property is applicable here.
- Ex. 5. Again, use the idea that when thinking of the decimals as fraction forms, the denominators (as powers of ten) determine the number of decimal places in the numeral for the product. Establish the use of this generalization in the solution of multiplication examples.
- Ex. 6–15. If pupils are confused by the zeros in an example such as Ex. 9, consider the possibility of treating it as the product of two counting numbers and apply the procedure discussed for Ex. 5.
- Ex. 16–24. Again, pupils should realize that the solution of these examples requires only multiplication.

Individualizing Instruction

- *More capable pupils* should consider whether there are exceptions to the generalizations discussed in the Pre-Book Lesson and for Ex. 5. (There are none.) They should consider as many different possibilities as is practical.
- *Slower learners* may need to work closely with the teacher in working step-by-step solutions for the exercises with the teacher working each step if necessary. The teacher, by careful direction, may help pupils to use rules and generalizations.

Pupil's Objectives

(a) To solve problems requiring multiplication with decimals; and (b) to work with the concept of specific gravity in a problem-solving situation.

Background

The specific gravity of a substance is its weight in air under normal atmospheric pressure compared to the weight in air of an equal volume of pure water at 4°C. under the same conditions of atmospheric pressure. The density of water is used as the standard for specific gravity and other substances are compared to it. At 4°C. one cubic foot of water weighs 62.4 pounds and 1 cubic centimeter of water weighs 1 gram. When the specific gravities of two liquids are compared, the substance with the lesser specific gravity will float on the substance with the greater specific gravity provided that the substances are not miscible. Any substance having a specific gravity less than 1.00 will float on water. There are several different varieties of hydrometers (instruments for measuring specific gravity), for example, the lactometer (specific gravity of milk), the urinometer (specific gravity of urine), and the alcoholmeter (specific gravity of alcohol).

If the specific gravity of gold is 19.3, and the weight of a certain volume of water is 4.2 pounds, the weight of the same volume of gold can be expressed by the mathematical sentence $n = 4.2 \times 19.3$, $n = 81.06$ or 81.06 pounds.

Pre-Book Lesson

• Ask pupils if they can think of reasons for knowing the different amounts of chemicals in various substances. Ask if they can think of instances or industries where this would be useful.

• Ask pupils if they have ever seen a car battery being tested in a garage with a hydrometer. Explain that this is an instrument which indicates the makeup of certain substances by measuring specific gravities. Each substance has a different weight according to its composition, and either floats or sinks when placed on water, so its weight could be compared to the weight of the same amount of water. Explain the meaning of specific gravity in terms of the comparison of these weights and use material from the Background where suitable. Establish the idea that the weight of water is used as the standard for specific gravity.

Using the Text Page

• If possible, bring a hydrometer to class and demonstrate its use with a container of alcohol and water. Refer to a science book for other possible demonstration exercises. Perhaps samples of substances could be obtained and their weights compared with the weight of some volume of water.

• Ex. 2–6. You may need to express the relationships verbally such as "the measure of weight of the substance = the specific gravity × the measure of weight of water."

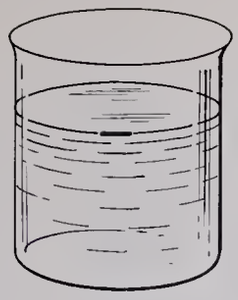
Individualizing Instruction

• *More capable pupils* could do research work on the characteristics and uses of mercury and sulfuric acid. They can find information in a chemistry book or general reference work.

These pupils may wish to investigate the buoyancy of certain substances heavier than water and relate it to the ability of a ship to float on water. Material can be found in general reference works or science books.

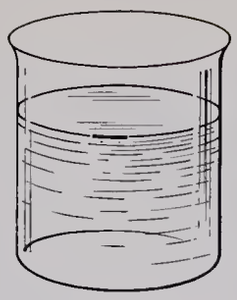
• If you have been able to obtain samples of substances to weigh, *all pupils* should compare these weights with the weight of the same volume of water. If the water weighs more, the substance will float; if the water weighs less, the substance will sink. These pupils may wish to investigate the various types of hydrometers used for special purposes, and report to the class.

• Once the *slower learners* have become accustomed to the vocabulary, they may work the examples with greater ease. Verbal expression of the relationship together with individual help in setting up the mathematical sentence for each problem should prove very beneficial.



Water
5.5 lb.

Fig. 1



Gasoline
_?_ lb.

Fig. 2

Mercury
_?_ lb.

Fig. 3

Substance	Specific Gravity
Water	1.0
Aluminum	2.7
Mercury	13.6
Gold	19.3

Using Mathematics in Science

[O]

If we know the weight of a certain volume of water, we can find the weight of another substance of the same volume by multiplying the measure of the weight of the water by the *specific gravity* of that substance.

Water, 1.0; Aluminum, 2.7; Mercury, 13.6; Gold, 19.3

1. Give the specific gravity for each substance named above.

2. The water shown in Fig. 1 weighs 5.5 lb. If the specific gravity of gasoline is 0.68, then the weight of an equal volume of gasoline (Fig. 2) can be found by multiplying 5.5 _?_ by 0.68. At the board, find 0.68×5.5 3.74 and then give the weight of the gasoline. 3.74 lb.

3. Give a mathematical sentence that could be used to find the weight of the mercury shown in Fig. 3. Write the mathematical sentence on the board and then solve. The weight of the mercury is 74.8 _?_ lb.

$n = 5.5 \times 13.6$

[W]

4. Find the weight of an amount of aluminum if an equal volume of water weighs 42.5 lb. 114.75 lb.

5. Find the weight of an amount of gold if an equal volume of water weighs 75 grams. 1,447.5 grams

6. The specific gravity of sulfuric acid is 1.84. Find the weight of an amount of sulfuric acid if an equal volume of water weighs 13.5 ounces. 24.84 ounces

● **Extra Activity.** Set 172.

* Again, do not immediately state the generalization for the placement of the decimal point. Move from dividing with fractions to dividing with decimals. Encourage pupils to estimate the answer before dividing.

Division with Decimals

Tenths divided by a counting number [O]

1. The Bell Company was hired to resurface the old road from Midville to Scotstown. During the first two days, 0.6 mi. of road was resurfaced. That was a mean average of how many miles per day for that 2-day period?

0.6

* **a.** To find the mean average, we divide _?_ by _?_. Look at the work in box A for finding 0.6 ÷ 2. Before dividing, 0.6 was renamed $\frac{?^{6}}{10}$. Explain the work shown. Give the answer for the problem. 0.3 miles

2

A

$0.6 \div 2 = \frac{6}{10} \div 2$

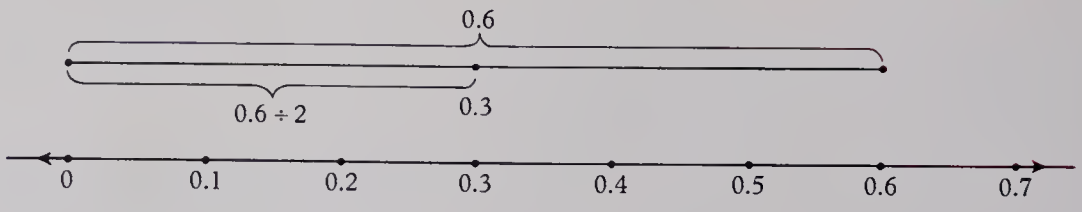
$= \frac{3}{5} \times \frac{1}{2}$

$= \frac{3}{10}$, or 0.3

B

$$\begin{array}{r} 0.3 \\ 2\overline{)0.6} \\ \underline{6} \end{array}$$

b. Box B shows how to find 0.6 ÷ 2 by dividing with decimals in vertical form. Think, *6 tenths divided by 2 is 3 tenths, or 0.3.* Explain how the number-line picture below shows this relationship. The segment for 0.6 is shown partitioned into two segments for 0.3.



2. $0.8 \div 4 = \frac{8}{10} \div 4$

$= \frac{4}{5} \times \frac{1}{4}$

$= \frac{1}{5}$, or 0.2

$$\begin{array}{r} 0.2 \\ 4\overline{)0.8} \\ \underline{8} \end{array}$$

2. Find 0.8 ÷ 4 first as in box A and then as in box B. See left.

[W]

Ex. 3–7. Copy and divide as in box B.

3. $2\overline{)0.8}$ 0.4 **4.** $3\overline{)0.6}$ 0.2 **5.** $3\overline{)0.9}$ 0.3 **6.** $3\overline{)0.3}$ 0.1 **7.** $5\overline{)0.5}$ 0.1

Teaching Page 240

Pupil's Objective

To learn to divide a rational number expressed in decimal form by a counting number.

Background

A division where the known factor is a counting number and the product is named with a decimal can be worked by renaming the product with a fraction.

$9.6 \div 3 = 9\frac{6}{10} \div 3$
$= \frac{96}{10} \div 3$
$= \frac{96}{10} \times \frac{1}{3}$
$= 3\frac{1}{5}$, or 3.2

$64.16 \div 8 = \frac{6,416}{100} \div 8$
$= \frac{6,416}{100} \times \frac{1}{8}$
$= \frac{802}{100}$
$= 8\frac{2}{100}$, or 8.02

An expanded form and the place-value property can also be used to show the work for a division. Division here resembles simple division with whole numbers. $33.36 \div 3 =$
(3 tens + 3 ones + 3 tenths + 6 hundredths) $\div$ 3 = (3 tens $\div$ 3) + (3 ones $\div$ 3) + (3 tenths $\div$ 3) + (6 hundredths $\div$ 3) = (1 ten) + (1 one) + (1 tenth) + (2 hundredths) = 11.12

The division can also be expressed as:

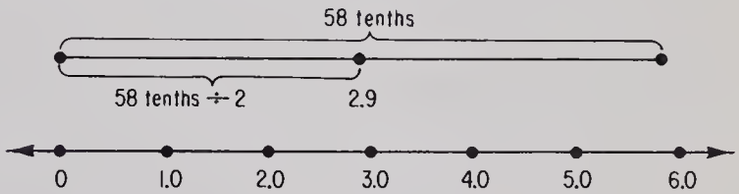
$3\overline{)30 + 3 + 0.3 + 0.06} = 10 + 1 + 0.1 + 0.02$
$= 11.12$

Although distributing division over addition expressed in the example above has not yet been discussed, it can be used intuitively as another expanded form showing place value without detailed explanation to the pupils.

Another approach is possible through an application of the factors-product relationship. In multiplication, if one factor is a counting number, and the other factor shows tenths, the product shows tenths. If the other factor shows hundredths, the product shows hundredths. Consequently, in a division when one factor is a counting number and the product shows tenths, the unknown factor shows tenths; when the product shows hundredths, the unknown factor shows hundredths, and so on. A generalization now becomes evident: When dividing a rational number expressed in decimal form by a counting number, divide as you would whole numbers, but the position of the decimal point in the numeral for the unknown factor is the same as in the numeral for the product.

Pre-Book Lesson

- Review multiplication with decimals stressing the proper placing of the decimal point. Review the factors-product relationship with decimals.
- Present the following: Jim walked 5.8 miles in 2 hours. How far did he walk in 1 hour? Ask pupils how they would solve the problem. Ask a pupil to use a number-line picture at the board to illustrate the example.



The segment representing 5.8 miles can be marked off into two segments having the same length so that each represents 2.9 miles. Pupils can then write a mathematical sentence to illustrate the example. If they think of 5.8 as 58 tenths,

then $2\overline{)58 \text{ tenths}}$ 29 tenths, or 2.9.

Using the Text Page

- Ex. 1. Make sure pupils know how to find the mean average. Use the number-line picture carefully to have pupils intuitively grasp the relationships in the problem. Use material from the Background and relate the work to multiplication with decimals and use of expanded forms.
- Ex. 3–7. Try to have pupils reach the conclusion that in dividing a rational number in decimal form by a counting number, the numeral for the unknown factor has the decimal point in the same position as the numeral for the product.

Individualizing Instruction

Slower learners may need to work Ex. 3–7 using fraction form or expanded notation. Have them generalize about placing the decimal point.

Teaching Page 241

Pupil's Objectives

(a) To practice dividing tenths by a counting number; and (b) to apply this division to problem-solving using mathematical sentences.

Background

This material is an extension of the work on the previous page. The use of the factors-product relationship with the *more capable pupils* will help establish use of the generalization presented on the previous teaching page. Other pupils may need to use expanded forms or number-line pictures as presented in the Pre-Book Lesson for page 240.

Pre-Book Lesson

Review expressing the factors-product relationship four ways with several mathematical sentences. For example:

$3 \times 0.4 = 1.2$	$2 \times 0.3 = 0.6$	$4 \times n = 2.4$
$0.4 \times 3 = 1.2$	$0.3 \times 2 = 0.6$	$n \times 4 = 2.4$
$1.2 \div 3 = 0.4$	$0.6 \div 2 = 0.3$	$2.4 \div 4 = n$
$1.2 \div 0.4 = 3$	$0.6 \div 0.3 = 2$	$2.4 \div n = 4$

Using the Text Page

• Ex. 1. Use material from previous lessons to supplement working through a–d orally. Illustrate each step at the board. After answering c, you may wish to have pupils express 1.2 as 10 tenths + 2 tenths or 12 tenths and work the division as $3\overline{)12 \text{ tenths}}$ (4 tenths), or 0.4.

• Ex. 3. Some pupils may need to express the divisions as shown above for Ex. 1c and Ex. 2 on the pupil's page in order to compute correctly.

• Ex. 4–8. You may wish to have pupils express the factors-product relationship for each example four ways using mathematical sentences. Have pupils tell the position of the decimal point in the numeral for the unknown factor by looking at the numeral for the product.

• Ex. 9–14. The position of the letter representing the unknown number in the mathematical sentences has no bearing upon the method of solution. *Slower learners* may need to be reminded of this fact before working the exercises.

• Ex. 15. Some pupils may need to be reminded about the characteristics of a square.

Individualizing Instruction

• Have *more capable pupils* use the generalization for dividing a rational number by a counting number presented in the previous teaching page and work Ex. 4–15 mentally. They may then verify their work by performing the calculations.

• *Slower learners* may be helped by thinking of the divisions for Ex. 4–15 in an expanded form. In Ex. 3a, $2.8 \div 2 = n$, think of 2.8 as 20 tenths + 8 tenths, or 28 tenths. Then $2\overline{)28 \text{ tenths}}$ (14 tenths), or 10 tenths + 4 tenths, or 1.4.

Looking Ahead

If *more capable pupils* complete their work early, you may wish to anticipate work on division with decimals involving a remainder and the annexing of zeros. You may give them an example such as $5 \times n = 2.6$, or $5\overline{)2.6}$ (_ ? _). There will be a remainder; however, since 2.60 names the same number as 2.6, they can express the division as $5\overline{)2.60}$ so there is no remainder.

More Division with Decimals

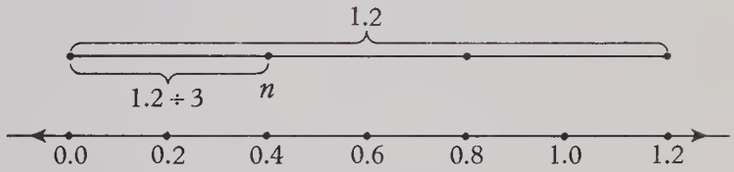
Dividing tenths [O]

1. The perimeter of the equilateral triangle shown at the right is 1.2 in. Find the length of one side.

a. If we use n to represent the measure of one side of the triangle, then does $3 \times n = 1.2$ show the relationship expressed in the problem? Yes Explain. Key idea: It expresses the factors-product relationship.

b. Using the factors-product relationship, can we use $n = 1.2 \div 3$ to show the relationship expressed in the problem? Yes Does n still represent the measure of one side? Yes

c. Explain how the number-line picture at the right shows the relationship that is expressed by $n = 1.2 \div 3$. The segment shown above the number-line picture shows $1.2 \div 3$.



d. From the number-line picture, $n =$ 0.4 , so the length of one side of the triangle is 0.4 in.

2. The box at the right shows how to solve $n = 1.2 \div 3$. Think of 1.2 as 10 tenths + 2 tenths or 12 tenths. $12 \div 3 =$ 4 , so 12 tenths $\div 3 =$ 4 tenths, or 0.4. Why is a 0 written in one's place in the numeral for the answer? To show there are 0 ones.

$n = 1.2 \div 3$

$$3\overline{)1.2}$$

$$\begin{array}{r} 0.4 \\ 3\overline{)1.2} \\ \underline{12} \\ \end{array}$$

$n = 0.4$

3. At the board, solve each of Ex. a–c. Check your work.

a. $2.8 \div 2 = n$ 1.4 **b.** $n = 5.5 \div 5$ 1.1 **c.** $36.6 \div 3 = n$ 12.2

[W]

Ex. 4–8. Copy and divide.

4. $4\overline{)33.2}$ 8.3 **5.** $7\overline{)9.1}$ 1.3 **6.** $5\overline{)4.5}$ 0.9 **7.** $3\overline{)150.6}$ 50.2 **8.** $9\overline{)75.6}$ 8.4

Ex. 9–14. Find the number represented by the letter.

9. $n = 14.4 \div 2$ 7.2 **10.** $x = 38.7 \div 9$ 4.3 **11.** $9.6 \div 2 = x$ 4.8

12. $33.5 \div 5 = n$ 6.7 **13.** $7.2 \div 6 = n$ 1.2 **14.** $x = 504.6 \div 2$ 252.8

15. Find the length of one side of a square whose perimeter is 4.8 cm. 1.2 cm.

♦ **Extra Examples.** Set 119.

Harder Division with Decimals

Renaming to show hundredths and thousandths [O]

Jim used a micrometer to measure the diameters of three ball bearings. He recorded his measurements as follows: 0.42 in., 0.51 in., 0.33 in.

1. To find the mean average for Jim's data, can we use the sentence $n = (0.42 + 0.51 + 0.33) \div 3$? Yes Explain. Key idea: To find a mean average, find the sum of the measures and divide by the number of measures.

a. At the board, find $0.42 + 0.51 + 0.33$. 1.26

b. To find the number represented by n, divide 1.26 by 3. Box A shows how to find $1.26 \div 3$ by dividing in vertical form. Can we divide ones? No Think of 1.26 as 12 tenths + 6 hundredths.

Divide tenths: $12 \div 3 =$ 4 Write a 4 in tenth's place in the numeral for the quotient.

Divide hundredths: $6 \div 3 =$ 2 Write a 2 in hundredths place in the numeral for the quotient.

c. The mean average for Jim's data is 0.42 in.

A

$$\begin{array}{r} 0.42 \\ 3\overline{)1.26} \\ \underline{12} \\ 6 \\ \underline{6} \end{array}$$

B

$$\begin{array}{r} 1.24 \\ 8\overline{)9.92} \\ \underline{8} \\ 19 \\ \underline{16} \\ 32 \\ \underline{32} \end{array}$$

2. On the board, work each of Ex. a–d as in box A.

a. $5\overline{)5.55}$ 1.11 **b.** $4\overline{)0.84}$ 0.21 **c.** $6\overline{)12.06}$ 2.01 **d.** $9\overline{)0.54}$ 0.06

3. Box B. Explain the work shown for finding $9.92 \div 8$. Key idea: Divide ones, then tenths, then hundredths.

4. On the board, work each of Ex. a–d as in box B.

a. $6\overline{)1.32}$ 0.22 **b.** $8\overline{)6.72}$ 0.84 **c.** $3\overline{)405.57}$ 135.19 **d.** $7\overline{)78.19}$ 11.17

Teaching Pages 242 and 243

Pupil's Objectives

(a) To work more difficult divisions with decimals using the vertical form; and (b) to rename a product in a division example to show hundredths or thousandths as necessary.

Background

Some of the ideas presented on these pages are extensions of material presented previously. The same principles apply to dividing whole numbers and dividing with decimals.

In some division examples with decimals, the answer will show a remainder other than 0, unless zeros are annexed to the right of the numeral for the product to show hundredths, thousandths, and so on. The ideas of renaming a number with decimal forms and many names for a number apply here. An alternative is to show the quotient to a specified place and then round it. This latter procedure may or may not involve annexing zeros to the numeral for the product. For example, $3.4 \div 4 = n$ is $4\overline{)3.4}$ $\begin{matrix}0.8,\ R2\end{matrix}$ but carried to the hundredth's place, it is $4\overline{)3.40}$ $\begin{matrix}0.85\end{matrix}$. Sinee 3.4 and 3.40 name the same number, 0.8, R2 and 0.85 are both answers for 3.4 ÷ 4 but 0.8, R2 is an incomplete answer for the division.

The quotient for the division $7.7 \div 5 = n$ may be rounded to the nearest whole number, which involves no annexing of zeros. $5\overline{)7.7}$ $\begin{matrix}1.5,\ R2\end{matrix}$ is approximately 2 or it may be rounded to the nearest tenth after annexing one zero to the numeral for the product; $5\overline{)7.70}$ $\begin{matrix}1.54\end{matrix}$ is approximately 1.5. Remember that in rounding rational numbers shown in decimal form, no zeros need be shown to the right of the place to which the number is rounded.

Pre-Book Lesson

• Review the idea of naming a number in many ways. Use such examples as:

$2 \rightarrow \frac{20}{10}$	$\frac{200}{100}$	2.0	1 + 1	5 − 3
$3.5 \rightarrow 3\frac{5}{10}$	$\frac{35}{10}$	$\frac{350}{100}$	3.500	$3\frac{500}{1,000}$

• Review briefly more difficult divisions in vertical form with whole numbers. Review division of rational numbers as presented on pupil's pages 240 and 241.

• Using whole numbers, have several pupils work at the board divisions having remainders greater than zero. Have them express the quotients with a mixed form and then express the mixed form with a decimal naming the same number. Use such simple examples as:

$4\overline{)87}$ $\begin{matrix}21,\ R3\end{matrix}$, or $21\frac{3}{4}$, or 21.75

$2\overline{)999}$ $\begin{matrix}499,\ R1\end{matrix}$, or $499\frac{1}{2}$, or 499.5

Using the Text Pages

• Ex. 1. Make sure pupils know how to find the mean average. Review using a micrometer on pages 148, 149, and 171.

Some pupils may have forgotten that finding the unknown number in a mathematical sentence may involve two operations. In Ex. 1, the use of parentheses indicates that the addition (0.42 + 0.51 + 0.33) should precede division by 3.

Pupils should be encouraged to think back to the work in the Pre-Book Lesson and apply the same reasoning as used in harder divisions of whole numbers in vertical form. You may wish to leave on the board for easy reference an example of this type as well as an example showing a simple division with decimals.

• Ex. 2. Before doing the work, as in box A, you may have pupils work an example such as $5\overline{)555 \text{ hundredths}}$ $\begin{matrix}111 \text{ hundredths}\end{matrix}$ = 100 hundredths + 10 hundredths + 1 hundredth, or 1.11.

• Ex. 3. The product 9.92 can be thought of as 8 ones + 16 tenths + 32 hundredths in order to divide more easily.

• Ex. 6. After reading the hint in the example, some pupils may bring up the question of choosing the number of places to which the quotient should be shown. Explain that in most cases, specific

instructions are given to preclude any such question. In "higher" mathematics, instructions may not be given, but the pupils themselves will decide the place to which they show a quotient.

• Ex. 7. Some pupils may find that they are renaming the products to show ten-thousandths. Have them check their divisions carefully for errors if their products show ten-thousandths.

• Ex. 10–36. Assign the written work to be performed independently by pupils at their seats. However, some pupils may need individual assistance in order to become familiar with the process of renaming products and showing the division. Work problems with them explaining and illustrating each step in the solution.

• Ex. 26–31. Have pupils express the factors-product relationship four ways with mathematical sentences: $6 \times n = 5.4$ $\quad n \times 6 = 5.4$
$5.4 \div 6 = n$ $\quad 5.4 \div n = 6$
This should help them remember that $5.4 \div n = 6$ can also be expressed as $5.4 \div 6 = n$. This is a division expression with which they are already familiar.

• Ex. 35–36. Pupils should think of the work on either side of the symbol of inequality separately. Then they should compare the answers for both sides.

Individualizing Instruction

• *More capable pupils* may be encouraged to think of divisions where annexing zeros to the numeral for the product still does not enable them to complete the division. They could then tell how they would proceed to solve such examples. The idea of rounding the quotient to a specified place should be one of their responses.

• Leave several examples of divisions on the board and help work out a step-by-step solution for Ex. 10–36 with those *slower learners* who require more assistance.

NOTES

5. Box C shows how to solve $n = 13.3 \div 5$ by dividing in vertical form. Notice that the product was renamed in order to complete the division. Do both 13.3 and 13.30 name the same number? Yes Explain the work shown. Key idea: 13.3 was renamed 13.30 before the work was done.

C

$n = 13.3 \div 5$

$$\begin{array}{r} 2.66 \\ 5\overline{)13.30} \\ 10 \\ \hline 33 \\ 30 \\ \hline 30 \\ 30 \\ \hline \end{array}$$

$n = 2.66$

6. At the board, solve $n = 6.5 \div 4$ as in box C. (Hint: Rename 6.5 as 6.500.) 1.625

7. Work Ex. a–d on the board. Explain your work. 0.185

a. $5\overline{)6.2}$ 1.24 **b.** $5\overline{)17.23}$ 3.446 **c.** $8\overline{)9.8}$ 1.225 **d.** $6\overline{)1.11}$

8. Explain the work in box D for finding $25.2 \div 24$. Key idea: The product is renamed to show zeros in the numeral for the product.

9. At the board, find $6.816 \div 32$. 0.213 Did you rename the product to show some zeros in the numeral for the product? No

D

$$\begin{array}{r} 1.05 \\ 24\overline{)25.20} \\ 24 \\ \hline 1\,20 \\ 1\,20 \\ \hline \end{array}$$

[W]

Ex. 10–25. Copy and divide. If necessary, rename the product when dividing.

10. $5\overline{)27.5}$ 5.5 **11.** $3\overline{)97.5}$ 32.5 **12.** $5\overline{)3.6}$ 0.72 **13.** $8\overline{)4.6}$ 0.575

14. $9\overline{)17.19}$ 1.91 **15.** $21\overline{)142.8}$ 6.8 **16.** $17\overline{)340.17}$ 20.01 **17.** $92\overline{)113.16}$ 1.23

18. $4\overline{)80.5}$ 20.125 **19.** $15\overline{)46.8}$ 3.12 **20.** $4\overline{)0.052}$ 0.013 **21.** $5\overline{)0.008}$ 0.0016

22. $68\overline{)1.02}$ 0.015 **23.** $33\overline{)0.693}$ 0.021 **24.** $25\overline{)0.015}$ 0.0006 **25.** $46\overline{)4.83}$ 0.105

Ex. 26–31. Find the number represented by the letter.

26. $n = 3.6 \div 5$ 0.72 **27.** $x = 12.7 \div 2$ 6.35 **28.** $6.03 \div x = 4$ 1.5075

29. $3.34 \div n = 5$ 0.668 **30.** $35.1 \div 27 = n$ 1.3 **31.** $18 = 0.612 \div x$ 0.034

32. Find the length of one side of a regular octagon whose perimeter is 9.36 in. 1.17 in.

33. Find the width of a rectangular region whose length is 23 cm. and whose area is 11.96 sq. cm. 0.52 cm.

34. Solve: $n = (4.03 + 3.89 + 0.98) \div 25$ 0.356

Ex. 35–36. Write *T* or *F*.

35. $2.3 < 35.84 \div 16$ F **36.** $3.4 + 5.7 > 72.18 \div 8$ T

♦ **Extra Examples.** Sets 120–121.

* The unknown factor can be found to the nearest hundredth by considering the remainder after dividing hundredths, but the recommended procedure is to carry the division to one more decimal place.

Rounding the Quotient

9.23 [O]

1. In $7 \times n = 9.23$, which numeral names the product? Does n represent the unknown factor? Yes To find the unknown factor, we divide the product by the known factor. $n = 9.23 \div 7$

2. Box A. The division $9.23 \div 7$ has been carried to hundredths. To the nearest tenth, 1.31 is 1.3.

* 3. Box B. To find the unknown factor to the nearest hundredth, the division can be carried to thousandths. For the division $9.23 \div 7$, the unknown factor to the nearest hundredth is 1.32.

4. At the board, find $8.3762 \div 5$ to the nearest hundredth. 1.68 Did you have to carry the division to ten-thousandths? No thousandths? Yes

5. Explain the work in box C for finding $1.1 \div 3$ to the nearest hundredth. Key idea: The division is carried to the nearest thousandth.

6. At the board, find $35.63 \div 32$ to the nearest hundredth. 1.11

[W]

Ex. 7–15. Find the unknown factor to the nearest hundredth. If the remainder is 0, as for $52.5 \div 7$, write 0's in the numeral for the unknown factor to show hundredths.

7. $52.5 \div 7$ 7.50
8. $3.96 \div 6$ 0.66
9. $0.01 \div 2$ 0.005
10. $0.45 \div 5$ 0.09
11. $7.288 \div 21$ 0.35
12. $87.3 \div 52$ 1.68
13. $1.328 \div 63$ 0.02
14. $2.51 \div 93$ 0.03
15. $342 \div 15$ 22.80

Ex. 16–24. Find the unknown factor to the nearest thousandth.

16. $3.5454 \div 14$ 0.253
17. $0.0063 \div 3$ 0.002
18. $5.7996 \div 17$ 0.341
19. $22.23436 \div 5$ 4.447
20. $2.6673 \div 11$ 0.242
21. $25.8594 \div 7$ 3.694
22. $333.6 \div 24$ 13.900
23. $1.0008 \div 9$ 0.111
24. $496.64 \div 19$ 26.139

♦ **Extra Examples.** Set 122.

A

$$\begin{array}{r} 1.31 \\ 7\overline{)9.23} \\ 7 \\ \hline 22 \\ 21 \\ \hline 13 \\ 7 \\ \hline 6 \end{array}$$

B

$$\begin{array}{r} 1.318 \\ 7\overline{)9.230} \\ 7 \\ \hline 22 \\ 21 \\ \hline 13 \\ 7 \\ \hline 60 \\ 56 \\ \hline 4 \end{array}$$

C

$$\begin{array}{r} 0.366 \\ 3\overline{)1.100} \\ 9 \\ \hline 20 \\ 18 \\ \hline 20 \\ 18 \\ \hline 2 \end{array}$$

Teaching Page 244

Pupil's Objective

To learn to round the quotient when dividing with decimals when the remainder for the quotient is a number other than 0.

Background

Periodic decimals are called *terminating* decimals when the only digit at the right of the decimal point which repeats is 0. Decimals which continue infinitely with a sequence of non-zero digits are sometimes called non-terminating decimals. For example, when expressing a rational number with a decimal form, dividing the numerator by the denominator results in a periodic decimal which is either a "terminating" decimal or a "non-terminating" decimal.

$\frac{1}{8} = 8\overline{)1.000}$ $\begin{matrix}0.125\\ \end{matrix}$ or $0.125\dot{0}$ indicates 0 repeats

$\frac{1}{2} = 2\overline{)1.00000}$ $\begin{matrix}0.50000\cdots = 0.5\dot{0} \text{ or } 0.5\\ \end{matrix}$

"terminating" decimals

$\frac{2}{11} = 11\overline{)2.000000}$ $\begin{matrix}0.181818\cdots = 0.\dot{1}\dot{8}\\ \end{matrix}$ $\frac{1}{3} = 3\overline{)1.000000}$ $\begin{matrix}0.333333\cdots = 0.\dot{3}\\ \end{matrix}$

"non-terminating" decimals

In work with measures, it is necessary to retain the zeros in a decimal because they may indicate the unit of precision.

Pre-Book Lesson

- Review material presented on the previous page. Have pupils work examples requiring annexing of zeros to the numeral for the product.
- Review rounding rational numbers in decimal form as first presented on page 123. Remind pupils that no zeros need be shown to the right of the place to which the number has been rounded.

Using the Text Page

- Ex. 2. Explain to pupils that there are divisions where the remainder is always greater than 0 and it is possible to continue the division on and on indefinitely. For example, in $7 \div 11 = n$ the division $11\overline{)7.000000}$ $\begin{matrix}0.636363\\ \end{matrix}$ can go on and on. In such instances, it is necessary to round the quotient so the division is reasonable. Explain that instructions are given in such cases to divide to tenths, or hundredths, or thousandths, and so on. Pupils should note that when the division is carried to hundredths, there is still a remainder of 6.
- Ex. 3. Try to have pupils establish the generalization that rounding a quotient requires dividing one place to the right of the place to which the quotient is being rounded. To round to tenths, divide to hundredths; to round to hundredths, divide to thousandths; and so on.
- Ex. 5. In examples of this type, pupils must annex zeros to the numeral for the product in order to round the quotient to the specified place.
- Ex. 7–24. Pupils should attempt to work these on their own at their seats. If some *slower learners* have difficulty, go over each step in the solution with them individually encouraging them to provide the next step wherever possible. Often when encouraged to think out loud, pupils gain insight into a previously hazy idea.

When the numeral for the unknown factor must show hundredths, stress that the 0's must be shown in the quotient whenever the remainder is 0 as in the case for $52.5 \div 7$. Give other such examples as rounding the quotient to thousandths for $0.45 \div 5$, $3.66 \div 6$, and $4.97 \div 7$.

Individualizing Instruction

Present additional material from the Background to *more capable pupils*. This can be done by having them attempt to complete such division as involved in Ex. 5 and in expressing $\frac{2}{3}$ and $\frac{3}{4}$ with a decimal form. They should note that some divisions produce decimal forms which appear to "terminate" or "end"; that is, the division results in a remainder of zero. Other divisions produce decimal forms which are "non-terminating" or have a sequence of digits which go on and on.

Teaching Page 245

Pupil's Objective

To solve problems involving rounding the quotients from divisions with decimals.

Background

On this page the generalizations, understandings, and skills learned on the previous pages are applied to problem-solving in social situations and situations involving geometric ideas.

It is well to remember various problem-solving situations which require division for their solution, for example:

(a) finding an unknown factor when the product and one factor are known

(b) finding the number of members in a subset when the number of members in the set and the number of subsets are known

(c) finding the price of a single item when the price of a specified number of items is known

(d) finding the number of items which can be bought when the price of one item and the total amount of money available are known.

Although it may not always be obvious, the place to which a number is to be rounded, is usually implied by the relationships in the problem. It is not good practice to round the quotient so that it requires expressing the product or the known factor with decimals having more digits than those given in the problem.

Pre-Book Lesson

• Review a general problem-solving procedure which will facilitate the work. Have pupils list the steps on the board and illustrate them by working two simple problems involving whole numbers. Explain that the same general procedure holds for work with fractions and decimals.

• Review the use of mathematical sentences to express the relationships in a problem. Use several simple problems to illustrate this.

• Have pupils work at the board several division examples similar to those presented on pupil's pages 242–244. Have them briefly state the procedure for rounding rational numbers shown in decimal form.

Using the Text Page

• Ex. 1. Since there are 100 cents in a dollar, problems involving a number of cents usually require dividing to thousandths and rounding the quotient to the nearest hundredth. The same situation is encountered in Ex. 5.

• Ex. 2. Instructions for rounding the quotient may not be given explicitly but they are indicated by the wording of the problem. The class situation may be such that you will want to have pupils examine the problem and then work with them to decide how far the division should be carried.

• Ex. 3–9. Before they work these examples, have pupils write a mathematical sentence to indicate the relationships in each of the problems. Some pupils may be able to complete all work independently while others will require some individual assistance.

• Ex. 6. Review briefly characteristics of a regular pentagon.

• Ex. 0. As review, ask pupils to explain why we talk about area of a rectangular region rather than area of a rectangle.

Individualizing Instruction

• In each of Ex. 6–9, *more capable pupils* may find the unit of precision and greatest possible error for each measurement. They should then work the divisions first using the least possible measure, then using the greatest possible measure as taught on pupil's pages 154–157.

• Help those *slower learners* who have reading difficulties by reading the problems with them. They may need help in setting up the mathematical sentences and in showing the relationships for the problems. Use of number-line pictures and diagrams for Ex. 4–9 may prove very useful.

Solving Problems

[o]

1. It cost Bob's father $2.96 for three water pails to use at camp. To the nearest cent, what was the cost per pail?

a. Does $n = 2.96 \div 3$ show the relationship expressed in the problem? Yes ^ Explain. It expresses the factors-product relationship.

Key idea: The division is carried to the nearest thousandth.

b. Explain the work in the box at the right. ^ Why was the division only carried to thousandths? Key idea: The answer will be to the nearest cent.

$$\begin{array}{r} 0.986 \\ 3\overline{)2.960} \\ 27 \\ \hline 26 \\ 24 \\ \hline 20 \\ 18 \\ \hline 2 \end{array}$$

c. Give the answer for the problem. $0.99 (or 99¢)

2. For each of Ex. 3–9, tell how far the division should be carried for the answer to the problem. 3. hundredths; 4. hundredths; 5. thousandths; 6. thousandths; 7. thousandths; 8. thousandths; 9. thousandths.

[w]

Now work Ex. 3–9.

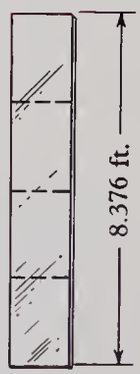
3. A loaf of bread was marked "Net weight: 16.5 oz." To the nearest $\frac{1}{10}$ ounce, what was the weight of one half of that loaf? 8.3 oz.

4. In going from his house to his camp and returning home again, Mr. Davis traveled 23.7 mi. To the nearest $\frac{1}{10}$ mile, what was the distance from Mr. Davis's house to his camp? 11.9 mi.

5. Claire paid $2.95 for 6 yd. of canvas to cover the porch chairs. To the nearest cent, what was the cost per yard? $0.49

6. If the perimeter of a regular pentagon is 15.39 cm., find the length of one side to the nearest $\frac{1}{100}$ centimeter. 3.08 cm.

7. In shop class, Joe had to cut a metal strip into four strips each of the same length. The metal strip was 8.376 ft. long. To the nearest $\frac{1}{100}$ foot, what was the length of each smaller strip? 2.09 ft.



8. Find, to the nearest $\frac{1}{100}$ inch, the width of a rectangular region whose length is 13 in. and whose area is 12.532 sq. in. 0.96 in.

9. Find, to the nearest $\frac{1}{100}$ yard, the length of one side of a square whose perimeter is 4.359 yd. 1.09 yd.

♦ **Extra Problems.** Set 150.

* See page 234.

A	B	C
3.68 10)36.80 30 6 8 6 0 80 80	0.052 100)5.200 5 00 200 200	0.000463 1,000)0.463000 4000 6300 6000 3000 3000

When the Known Factor is a Power of 10

[o]

1. For the divisions shown in boxes A–C, are the known factors powers of 10? Yes Name three other numbers which are powers of 10. Sample answers: 10,000; 100,000; 1,000,000

2. Explain the work in boxes A–C for finding 36.8 ÷ 10, 5.2 ÷ 100, and 0.463 ÷ 1,000. Key idea: For each 0 in the standard numeral for the power of 10, the product digits are shown one place farther to the right.

3. Instead of working as in box B to find 5.2 ÷ 100, we can use the following procedure. This procedure can be used to find the unknown factor when the product is named with a decimal and the known factor is a power of 10.

Step 1: Note the number of 0's in the standard numeral for the power of 10. For 100, the number of 0's is _?_ 2.

Step 2: Write the numeral for the unknown factor using the same digits as in 5.2, only write each digit two places farther to the right. The 5 will be in _?_ hundredth's place and the 2 will be in _?_ thousandth's place. 5.2 ÷ 100 = _?_ 0.052

4. If the power of 10 in Ex. 3 had been 1,000, each digit for 5.2 would have been written _?_ 3 places farther to the right.

5. Use the above procedure to explain the work in box D.

D			
	4.32 ÷ 10 = 0.432	6.98 ÷ 100 = 0.0698	24.6 ÷ 1,000 = 0.0246

*6. Compare the above procedure for dividing by a power of 10 with the procedure for multiplying by a power of 10. Key idea: For each 0 in the standard numeral for the power of 10, the product digits are shown one place farther to the right.

Teaching Pages 246 and 247

Pupil's Objectives

(a) To learn to divide when the known factor is a power of ten and the product is named with a decimal; (b) to find an easier procedure for working such divisions; and (c) to maintain and review skills presented previously.

Background

Division by a power of ten is the inverse of multiplication by a power of ten. On page 234, pupils learned to multiply using the idea of place value. In our system, the value of each consecutive place moving to the left is ten times as great. The value of each consecutive place moving to the right is one tenth as great.

The procedure for multiplying by a power of ten involves noting the number of zeros in the numeral for the power of ten and annexing this number of zeros to the right of the numeral for the other factor. The numeral for the product has the digits farther to the left of the decimal point than the numeral for the factor.

$$\begin{aligned} 100 \times 243 &= 100 \times (200 + 40 + 3) \\ &= (100 \times 200) + (100 \times 40) + (100 \times 3) \\ &= 20{,}000 + 4{,}000 + 300 \\ &= 24{,}300 \end{aligned}$$

Division by a power of ten is the same as multiplication by the reciprocal of the same power of ten.

$$\begin{aligned} &\text{(a) } 243{,}000 \div 100 \\ &= \tfrac{1}{100} \times 243{,}000 \\ &= \tfrac{1}{100} \times (200{,}000 + 40{,}000 + 3{,}000) \\ &= (\tfrac{1}{100} \times 200{,}000) + (\tfrac{1}{100} \times 40{,}000) + (\tfrac{1}{100} \times 3{,}000) \\ &= \tfrac{200{,}000}{100} + \tfrac{40{,}000}{100} + \tfrac{3{,}000}{100} \\ &= 2{,}000 + 400 + 30 \\ &= 2{,}430 \end{aligned}$$

$$\begin{aligned} &\text{(b) } 3.84 \div 1{,}000 \\ &= \tfrac{1}{1{,}000} \times 3.84 \\ &= \tfrac{1}{1{,}000} \times (3 + 0.8 + 0.04) \\ &= (\tfrac{1}{1{,}000} \times 3) + (\tfrac{1}{1{,}000} \times 0.8) + (\tfrac{1}{1{,}000} \times 0.04) \\ &= (\tfrac{1}{1{,}000} \times 3) + (\tfrac{1}{1{,}000} \times \tfrac{8}{10}) + (\tfrac{1}{1{,}000} \times \tfrac{4}{100}) \\ &= \tfrac{3}{1{,}000} + \tfrac{8}{10{,}000} + \tfrac{4}{100{,}000} \\ &= \tfrac{300}{100{,}000} + \tfrac{80}{100{,}000} + \tfrac{4}{100{,}000} \\ &= \tfrac{384}{100{,}000}, \text{ or } 0.00384 \end{aligned}$$

Pupils can apply the principles of multiplication to division in an expanded form, as shown above, and show the resulting fractions in simplest form, or rename the fraction with a decimal.

After working several such examples, a shorter procedure for the division can be worked out. Note the number of zeros in the numeral for the power of ten. Write the numeral for the unknown factor using the same digits as in the numeral for the product, but write each digit as many places to the right as there are zeros in the numeral for the power of ten. In some instances it will be necessary to annex zeros to the right of the numeral for the unknown factor in order for the decimal point to be in the correct place.

Pre-Book Lesson

- Review division of rational numbers as presented on pages 242–244. Have pupils work several examples at the board.
- Have pupils turn to page 234 and review the concepts and skills involved in multiplying one factor by a power of ten. Make sure they understand the concept of place value and the factors-product relationship.
- Review naming fractions in simplest form.

Using the Text Pages

- Ex. 2. The work in boxes A–C can be introduced by using the material from the Background and showing the examples in expanded form. It might be helpful if pupils first work several multiplications by powers of ten, and relate to division the work for each step. Pupils should note that zeros have been annexed to the numeral for the product so that the remainder will be zero.

$$\begin{aligned} &\text{(a) } 36.8 \div 10 \\ &= (30 + 6 + 0.8) \div 10 \\ &= (30 \div 10) + (6 \div 10) + (0.8 \div 10) \end{aligned}$$

$= (30 \times \frac{1}{10}) + (6 \times \frac{1}{10}) + (\frac{8}{10} \times \frac{1}{10})$
$= \frac{30}{10} + \frac{6}{10} + \frac{8}{100}$
$= \frac{300}{100} + \frac{60}{100} + \frac{8}{100}$
$= \frac{368}{100}$, or 3.68

Another way of showing this is

(b) $10\overline{)3 \text{ tens} + 6 \text{ ones} + 8 \text{ tenths}}$
$3 \text{ ones} + 6 \text{ tenths} + 8 \text{ hundredths}$
$= 10\overline{)30 \text{ ones} + 60 \text{ tenths} + 80 \text{ hundredths}}$
$= 3 + 0.6 + 0.08$
$= 3.68$

The product has been renamed so division by 10 results in a remainder of zero.

• Ex. 3. The procedure should not be presented until pupils are thoroughly familiar with the work in the boxes. You may have to work several more examples so pupils note that the same digits appear in the numerals for the unknown factor and the product except that they are farther to the right.

• Ex. 5. You may have pupils use form (b) above from Ex. 2 to facilitate the work. For each example, ask pupils how many places farther to the right the digits will appear.

• Ex. 7–13. Depending upon the varying ability of the pupils, have them use the procedure on page 246 or the generalization from the Background. Some pupils may still need to use an expanded form as for Ex. 2 above. *More capable pupils* may express the factor which is a power of ten in exponent form. They should then note that the digits in the numeral for the unknown factor are as many places to the right as the exponent for the power of ten.

• Ex. 1–8. In the To Keep in Practice section:

For Errors in Ex.	*Review*
1	pages 68–69
2	pages 154–157
3	page 212
4–8	pages 196–197

Individualizing Instruction

More capable pupils may make a chart showing multiplication and division of whole numbers and rational numbers in decimal form by powers of ten. They may also show exponent forms for multiplication examples such as $4.6 \div 100$; $4.6 \div 10^2$; or $4.6 \times \frac{1}{10^2}$.

NOTES

[W]

Ex. 7–12. Find the number represented by n.

7. $n = 8.92 \div 100$ 0.0892
8. $n = 33 \div 10$ 3.3
9. $n = 0.07 \div 1{,}000$ 0.00007
10. $n = 0.0061 \div 1{,}000$ 0.0000061
11. $n = 4.98 \div 10^3$ 0.0498
12. $n = 78.946 \div 10^5$ 0.00078946

E
- a. 64.2
- b. 0.704
- c. 638
- d. 16.007
- e. 6
- f. 1.2

13. Divide each number named in box E by 100; by 10; by 1,000. Use the procedure given on page 246 and write only the answers. 0.642, 0.00704, 6.38, 0.16007, 0.06, 0.012; 0.0642, 0.000704, 0.638, 0.016007, 0.006, 0.0012

14. Multiply each number named in box E by 100; by 10; by 1,000. Use the procedure given on page 234 and write only the answers. 6,420; 70.4; 63,800; 1,600.7; 600; 120 642; 7.04; 6,380; 160.07; 60; 12 64,200; 704; 638,000; 16,007; 6,000; 1,200

◆ **Extra Examples.** Set 123.

Without Paper and Pencil

[O]

Ex. 1–15. Name the number represented by the letter.

1. $n = 5.4 \times 100$ 540
2. $n \times 100 = 0.78$ 0.0078
3. $n \times 6.2 = 62$ 10
4. $x \times 0.48 = 48$ 100
5. $100 \times 0.08 = n$ 8
6. $n \div 10 = 0.46$ 4.6
7. $x = 4.4 \div 20$ 0.22
8. $n \times 26.402 = 26{,}402$ 1,000
9. $10 \times 24 = y$ 240
10. $1{,}000 \times 0.046 = n$ 46
11. $n = 44.8 \div 10$ 4.48
12. $n = 3.2 \times 20$ 64
13. $n \times 4.902 = 4{,}902$ 1,000
14. $n \times 0.48 = 480$ 1,000
15. $100 \times 0.92 = x$ 92

To Keep in Practice

[W]

1. Make a factor tree for 600 and then express 600 as a product of primes using exponent forms. $2^3 \times 3 \times 5^2$

Sample answer:

2. What is the unit of precision for a measurement of $5\frac{0}{8}$ in.? $\frac{1}{8}$ inch

3. Express the reciprocal of $\frac{2}{15}$ with a mixed form. $7\frac{1}{2}$

Ex. 4–8. Copy and multiply.

4. $4\frac{3}{4}$ by 240 — 1,140
5. $10\frac{1}{5}$ by 165 — 1,683
6. $3\frac{2}{7}$ by 15 — $49\frac{2}{7}$
7. $15\frac{7}{12}$ by 480 — 7,480
8. $35\frac{2}{5}$ by \$325 — \$11,505

Ohio Hit Hard by Rain; Crops Severely Damaged

Columbus—May 22—
Heavy rains fell on much of the Midwest yesterday, resulting in flash floods in many areas. In Ohio the State Department of Agriculture has already reported costly damage to farmlands and flooding

A	B
$n = 0.75 \div 0.2$ $= \frac{3}{4} \div \frac{1}{5}$ $= \frac{3}{4} \times 5$ $n = \frac{15}{4}$, or $3\frac{3}{4}$	$0.2\overline{)0.75} = 2\overline{)7.50}$ 3.75 6 15 14 10 10

Dividing with Decimals

Known factor named by a decimal [O]

1. During a rainstorm, rain fell at a rate of 0.2 in. per minute. If 0.75 in. of rain fell, for how many minutes did it rain?

a. Does $n = 0.75 \div 0.2$ show the relationship expressed in the problem? Yes Explain. It expresses the factors-product relationship.

b. Box A. To solve $n = 0.75 \div 0.2$, both the product and the known factor were first renamed with fractions. Explain the renaming and the work shown. Key idea: The reciprocal method is used.

2. To find $0.75 \div 0.2$, instead of renaming with fractions before dividing, we can divide with decimals as in box B.

a. Can $0.75 \div 0.2$ be expressed as $\frac{0.75}{0.2}$? Yes Is it true that $\frac{0.75}{0.2} = \frac{0.75 \times 10}{0.2 \times 10} = \frac{7.5}{2}$? Yes Then do you understand why $0.2\overline{)0.75}$ can be expressed as $2\overline{)7.5}$? Yes

b. Explain the work in box B. Key idea: Product and known factor were multiplied by 10 before the work was done.

For a division, multiplying both the product and the known factor by the same counting number does not change the unknown factor. We usually multiply by a power of 10 so that the unknown factor can be found by dividing by a whole number.

3. At the board, find $8.82 \div 2.1$ by multiplying both the product and the known factor by 10 before dividing. Then find $8.82 \div 2.1$ by renaming both the product and the known factor with fractions and then dividing with fractions. Compare your answers. $2.1\overline{)8.82} = 21\overline{)88.2}$

$$\begin{array}{r} 4.2 \\ 84 \\ \hline 42 \\ 42 \\ \hline \end{array}$$

$n = 8.82 \div 2.1$
$= \frac{441}{50} \div \frac{21}{10}$
$= \frac{441}{50} \times \frac{10}{21}$
$= \frac{441}{105}$, or $4\frac{1}{5}$, or 4.2

Teaching Pages 248 and 249

Pupil's Objective

To learn to divide when both the product and known factor are named with decimals.

Background

Division with two decimals involves multiplying both the known factor and the product by a power of ten so that the known factor is a counting number. Pupils already know how to work divisions where the known factor is a counting number. Two ideas are used to lead to the generalization on the pupil's page:

(a) a number may be named in many ways;

(b) multiplying a number by the identity element for multiplication in any form does not change the number, only its name.

A division can be expressed with a fraction form where the product is the numerator, and the known factor is the denominator. Thus, the division $3\overline{)6}$ can be expressed by the fraction $\frac{6}{3}$.

$$\frac{10}{10} = 1 \qquad 0.75 \div 0.2 = \frac{0.75}{0.2}$$
$$= \frac{0.75}{0.2} \times \frac{10}{10}$$
$$= \frac{7.5}{2}, \text{ or } 7.5 \div 2$$

$$\text{also } 0.75 \div 0.2 = \frac{0.75}{0.2}$$
$$= \frac{0.75}{0.2} \times \frac{100}{100}$$
$$= \frac{75}{20} \text{ or } 75 \div 20$$

The division is now in a form with which pupils re familiar. Another interpretation can be obtained by renaming each number with a fraction.

$$0.75 \div 0.2 = \frac{75}{100} \div \frac{2}{10} \qquad 3\frac{3}{4} = 3 + \frac{3}{4}$$
$$= \frac{3}{4} \div \frac{1}{5} \qquad = 3 + \frac{75}{100}$$
$$= \frac{3}{4} \times 5 \qquad = 3.75$$
$$= \frac{15}{4}, \text{ or } 3\frac{3}{4}$$

Pre-Book Lesson

- Review multiplying by powers of ten.
- Review using the identity element for multiplication and expressing the identity element many ways including using powers of ten ($\frac{10}{10}$, $\frac{100}{100}$, and so on). Have pupils express several numbers with many names. Use decimals, fractions, mixed forms, and numerals for whole numbers. For example, $\frac{1}{2} = \frac{1}{2} \times \frac{4}{4}$ or $\frac{4}{8}$.
- Have pupils express several divisions with decimals as division with fractions, for example

$$0.55 \div 0.5 = \frac{55}{100} \div \frac{5}{10} \qquad 1\frac{2}{20} = 1 + \frac{2}{20}$$
$$= \frac{11}{20} \div \frac{1}{2} \qquad = 1 + \frac{1}{10}$$
$$= \frac{11}{20} \times 2 \qquad = 1.1$$
$$= \frac{22}{20}, \text{ or } 1\frac{2}{20}$$

- Point out to pupils that a division can be expressed with a fraction form where the product is the numerator and the known factor is the denominator. For example

(a) $4\overline{)8} = \frac{8}{4}$
$= 2$
so $8 \div 4 = 2$

(b) $0.75 \div 0.2 = \frac{0.75}{0.2}$
$= \frac{0.75}{0.2} \times \frac{10}{10}$
$= \frac{7.5}{2}$, or $7.5 \div 2$

By using the identity element for multiplication ($\frac{10}{10} = 1$) in (b), the work has been expressed in a form that pupils are familiar with ($7.5 \div 2$).

Have pupils examine a series of divisions having the same unknown factor.

$3\overline{)6}$ $\quad$ $6\overline{)12}$ $\quad$ $12\overline{)24}$ $\quad$ $24\overline{)48}$

In each consecutive example, both the product and known factor have been multiplied by 2 while the unknown factor remains unchanged. $3\overline{)6}$ is the same as $\frac{6}{3}$, and $\frac{2}{2} = \frac{1}{1}$, or 1, so $\frac{6}{3} \times \frac{2}{2} = \frac{12}{6}$ is the same as $\frac{6}{3} \times 1$. Multiplication by the identity element does not change the number.

Likewise,

$0.3\overline{)6} = 6 \div 0.3$
$= \frac{6}{0.3}$
$= \frac{6}{0.3} \times \frac{10}{10}$
$= \frac{60}{3}$

$\frac{60}{3}$ is the same as $3\overline{)60}$

so,

$0.3\overline{)6}$ is the same as $3\overline{)60}$

Using the Text Pages

- Ex. 1–2. Remind pupils to relate the Pre-Book Lesson to these two exercises. The generalization can be justified by re-emphasizing the use of the identity element named as a power of ten.
- Ex. 3. Both answers should be the same. In each division, pupils rename the known factor and the product so that they are always dividing by a counting number.
- Ex. 4. When renaming the known factor and

the product, the choice of the power of ten depends upon the known factor.

If the known factor shows	*multiply the known factor and the product by*
tenths,	10
hundredths,	100
thousandths,	1,000.

• Ex. 6. Review rounding a quotient. Ask what would happen if no instructions were given for rounding.

• Ex. 9–38. You may need to work these examples individually with some pupils. Illustrate the solution by using material from the Pre-Book Lesson.

Individualizing Instruction

You may wish to have *more capable pupils* assist *slower learners* in doing the written work if the class situation permits. *More capable pupils* may also wish to explain to *slower learners* a quicker procedure for division using exponent forms for the powers of ten. They should review exponent forms with these pupils first.

NOTES

4. Box C shows how to find 5 ÷ 0.04 by dividing by a whole number. Both 5, the product, and 0.04, the known factor, were multiplied by _?_. 100 Explain the work shown.

C

$$0.04\overline{)5} = 4\overline{)500}$$

125
4
10
8
20
20

5. At the board, work Ex. a–c as in box C.

a. $0.07\overline{)4.9}$ 70 **b.** $0.02\overline{)0.465}$ 23.25 **c.** $1.25\overline{)20}$ 16

6. Box D. Explain the work shown for finding 0.7589 ÷ 3.6 to the nearest tenth. Why was the division carried to hundredths? Key idea: Product and factor were multiplied by 10. Key idea: So the answer could be given to the nearest tenth.

D

$$3.6\overline{)0.7589} = 36\overline{)7.589}$$

0.21
72
38
36
2

7. At the board, find 0.2314 ÷ 0.22 to the nearest hundredth. 1.05 Explain your work. Key idea: Multiply product and factor by 100. Carry the division to thousandths.

8. For each of Ex. 9–20, tell what power of ten to multiply both the product and the known factor by so that the unknown factor can be found by dividing by a whole number. 9. 10; 10. 10; 11. 10; 12. 100; 13. 100; 14. 100; 15. 1,000; 16. 100; 17. 100; 18. 1,000; 19. 100; 20. 100 [W]

Ex. 9–20. Divide as in boxes B and C.

9. $0.6\overline{)3.6}$ 6 **10.** $0.8\overline{)1.44}$ 1.8 **11.** $0.9\overline{)0.369}$ 0.41 **12.** $0.16\overline{)80}$ 500 **13.** $0.12\overline{)10.8}$ 90 **14.** $0.17\overline{)0.952}$ 5.6 **15.** $0.005\overline{)0.65}$ 130 **16.** $0.15\overline{)4.05}$ 27 **17.** $0.76\overline{)1,824}$ 2,400 **18.** $0.056\overline{)24.08}$ 430 **19.** $0.65\overline{)3.51}$ 5.4 **20.** $0.93\overline{)3.441}$ 3.7

Ex. 21–26. Find the number represented by the letter.

21. $n \times 0.5 = 3.47$ 6.94 **22.** $1.2 \times n = 6.84$ 5.7 **23.** $49.83 \div 1.51 = x$ 33
24. $x = 3.4 \div 0.025$ 136 **25.** $7.5 \times x = 2.4$ 0.32 **26.** $n = 740 \div 0.02$ 37,000

Ex. 27–34. Find the unknown factor to the nearest tenth.

27. $7 \div 6$ 1.2 **28.** $1.48 \div 3.1$ 0.5 **29.** $0.32 \div 1.15$ 0.3 **30.** $285 \div 36.26$ 7.9
31. $8.208 \div 9.9$ 0.8 **32.** $32.62 \div 6$ 5.4 **33.** $3.52 \div 5.7$ 0.6 **34.** $3.8 \div 0.21$ 18.1

Ex. 35–38. Find the unknown factor to the nearest hundredth.

35. $15.5 \div 2.2$ 7.05 **36.** $1 \div 3.2$ 0.31 **37.** $9.75 \div 0.82$ 11.89 **38.** $5 \div 12$ 0.42

♦ **Extra Examples.** Sets 124–127.

Multiply or Divide?

Differentiating M. and D. [O]

For each of Ex. 1–7, tell what operation to use to find the answer for the problem.

1. Mr. Jepsen used 0.75 of a 12-acre field for corn. How many acres did he use for corn? Multiplication; $n = 0.75 \times 12$; 9 acres

2. The 12-acre field was 0.55 of the land that Mr. Jepsen owned. To the nearest $\frac{1}{10}$ acre, how many acres did Mr. Jepsen own? Division; $n = 12 \div 0.55$; 21.8 acres

3. If 1.25 bu. of seed wheat are needed to seed 1 acre, how many acres can be seeded with 7 bu. of seed wheat? Division; $n = 7 \div 1.25$; 5.6 acres

4. At a rate of 16.7 mi. per gallon of gasoline, how many gallons of gasoline are needed for a 208.75-mile trip? Division; $n = 208.75 \div 16.7$; 12.5 gallons

5. Mr. Rose pays 0.15 of his monthly salary for rent. How much does he pay for rent if his monthly salary is \$536.80? Multiplication; $n = 0.15 \times 536.80$; \$80.52

6. Mr. Hobbs only pays 0.12 of his monthly salary for rent. What is his monthly salary if he pays \$54 for rent? Division; $n = 54 \div 0.12$; \$450

7. Find the width of a rectangular region whose length is 5.7 in. and whose area is 17.784 sq. in. Division; $n = 17.784 \div 5.7$; 3.12 in.

[W]

Now write and solve a mathematical sentence to find the answer for each of Ex. 1–7.

♦ **Extra Problems.** Set 151.

Finding Arithmetic Means

[W]

Ex. 1–3. Find the arithmetic mean for the given data. Express the mean to show the same unit of precision as the data.

1. 3.5 cm., 4.7 cm., 4.1 cm., 5.3 cm., 3.9 cm. 4.3 cm.

2. 87.56 mi., 98.50 mi., 94.82 mi., 100.00 mi. 95.22 mi.

3. 0.11 mm., 0.08 mm., 0.08 mm., 0.09 mm., 0.10 mm. 0.09 mm.

Teaching Page 250

Pupil's Objective

To solve problems requiring multiplication or division where both known numbers are expressed with decimals having digits in tenth's, hundredth's or thousandth's place and requiring rounding of the quotient.

Background

In deciding which operation must be used to solve a problem, the following should be considered:

(a) Multiplication can be used to find how many in each group when the numbers of all the groups are equal.

(b) Division is required to find the size of each group when the size of the whole group and the number of groups is known.

(c) Division is required to find the number of groups which can be formed when the total number in all groups and the size of each group are known.

Recall that mathematical sentences can be used to express the factors-product relationship. For any two factors a and b with product c; $a \times b = c$; $b \times a = c$; $a = c \div b$; and $b = c \div a$.

Pre-Book Lesson

- Review expressing the factors-product relationship four ways and have pupils make up one or two verbal problems for which they will express a mathematical sentence.
- Have pupils review division and multiplication with decimals.
- Have some pupils tell about several different problem situations and have other pupils tell how they would differentiate so they could choose the correct operation for solving the problem.
- Review unit of precision and finding the arithmetic mean. Have pupils give the unit of precision and greatest possible error for several measurements listed at the board. Have them find the arithmetic mean for this set of data.

Using the Text Page

- Ex. 1. Make sure pupils are familiar with the units of measurement and their abbreviations. Many problems involve finding "a part of" something. These are multiplication situations.
- Ex. 2–7. Pupils may express the relationships with mathematical sentences in different forms. For example, sentences for Ex. 2 might be $0.55 \times n = 12$, $n \times 0.55 = 12$, $12 \div n = 0.55$, or $12 \div 0.55 = n$. The last sentence is one from which a solution can be directly calculated.
- Ex. 3. Pupils should be reminded that they may need to annex a zero to the numeral for the product in order to have a remainder of 0.

Ask pupils how they would work a division example having a remainder greater than 0 if no instructions are given for rounding. Ask them how far a division must be carried if the problem states that the quotient must be rounded to the nearest tenth or hundredth. Caution pupils that some results need to be rounded even though no such step is specifically indicated in the problem. Such a case would occur when the answer is a number of dollars and cents.

Individualizing Instruction

- *Slower learners* may need help in setting up the mathematical sentences. Help pupils with any reading difficulties. They may also need more help with unit of precision and finding the arithmetic mean.
- *More capable pupils* should have a good understanding of unit of precision. List several measurements on the board such as 3.45 cm., 0.0567 cm., 123.4 cm., 35 cm., and have them compute the mean average. They should realize that the addends must be rounded so they all indicate the same precision as the *least* precise measurement.

These pupils may also use the information from the pupil's page and make up problems which will require addition or subtraction for their solution.

Pupil's Objectives

(a) To solve problems requiring multiplying or dividing with decimals; and (b) to review following directions in exercises involving adding, subtracting, multiplying or dividing with decimals.

Background

These exercises give the pupil experience in solving verbal problems that require multiplying or dividing with decimals. They show a practical use for decimals in scientific situations.

Gravitational force varies in different celestial bodies. Thus, an object on the moon weighs about 0.16 of what it weighs on Earth. If a person weighs 145 pounds on Earth, he would weigh $n = 0.16 \times 145$ or about 23.20 pounds on the moon.

The purpose of the exercises on the lower part of the page is to review following directions in adding, subtracting, multiplying, and dividing with decimals. Some of the examples require two-step solutions. Words such as sum, product, quotient, and remainder suggest which work must be performed before other work. For instance: add the product of 2 and 3 to the sum of 5 and 6. Expressed with a mathematical sentence, this is $n = (2 \times 3) + (5 + 6)$ or, $6 + 11 = 17$.

Pre-Book Lesson

• Review using mathematical sentences to express the factors-product relationship in problem-solving situations involving decimals.

• Write several simple one- and two-step problems involving decimals on the board. Have pupils express the relationships in the problem with a mathematical sentence and then solve.

• Point out that astronauts of the future will be concerned about how their weight will vary on different celestial bodies such as planets and moons. It has been found that while an object on the moon will weigh about 0.16 times as much as it weighs on Earth, on Jupiter it will weigh approximately 2.65 times as much as on Earth.

• Have each pupil write on paper his approximate weight. Choose a weight such as 120 lb. and then write on the board $n = 0.16 \times 120$, or 19.20 pounds. Each pupil should then write on his papers $n = 0.16 \times$ (measure of weight) $=$ _ ? _, which gives an approximation of his weight on the moon.

• Ask the students what other two types of problems might arise from this factors-product application. They should reply that if the weight of an object on the moon is known, the corresponding weight on Earth can be found. Also, if the corresponding weights of an object on Earth and some other planet are known, the decimal factor can be determined for that planet.

Using the Text Page

• Ex. 1–4 (top). Assign the examples to be worked independently by all pupils at their seats.

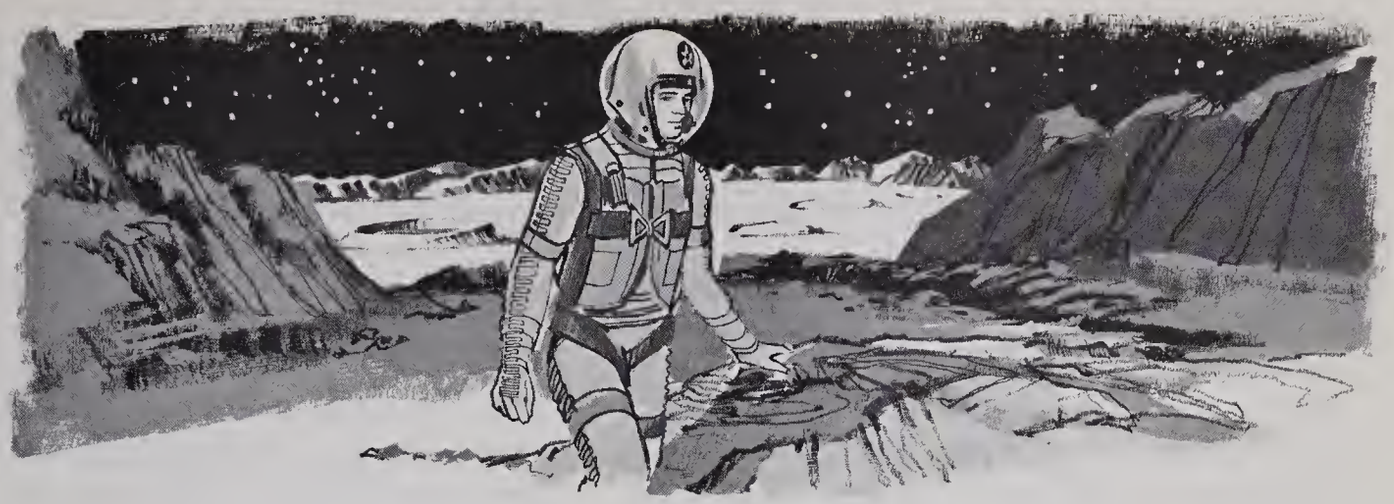
• Ex. 1–5 (bottom). Each exercise should be expressed with a mathematical sentence before any computations are carried out.

Individualizing Instruction

• Have *more capable pupils* try to find information concerning other planets in various reference works or science books and bring to the class problems relating to these facts.

• *Slower learners* will need help in deciding which operations are to be performed first and in setting up the mathematical sentences. They should be given examples similar to those at the bottom of the page, but involving only whole numbers.

If these pupils have reading difficulties, read the problems together with them.



Using Mathematics in Science

[W]

1. An object on the moon would weigh about 0.16 of what it weighs on Earth. A space suit which weighs 5.6 lb. on Earth would weigh about how much on the moon? 0.896 lb.

2. An object weighs 4.7 lb. on the moon. Find, to the nearest pound, the weight of the object on Earth. 29 lb.

3. On Jupiter, an object would weigh about 2.6 of what it weighs on Earth. An object whose weight is 288.6 lb. on Jupiter would weigh about how much on Earth? 111 lb.

4. A man who weighs 185 lb. on Earth would weigh about how much on Jupiter? 481 lb.

Can You Follow Directions?

[W]

1. Add 4.56 to the product of 3.2 and 4.4. 18.64

2. Divide 56.7 by 3 and then multiply the result by 14.2. 268.38

3. Divide 3.78 by the sum of 4.8, 9.2, and 0.6. Carry the division to thousandths and round to the nearest hundredth. 0.26

4. Divide 230,000 by 2.3 and then express 230,000 as a product of 2.3 and a power of 10. Use an exponent form for the power of 10. $230{,}000 = 2.3 \times 10^5$

* Review that m($\overline{AB}$) means "the measure of line segment AB."

Circles

Using $d = 2 \times r$ [O]

Fig. 1

1. Fig. 1. Is the geometric figure shown a simple closed curve? Yes ^ On the board, picture 2 curves which are simple closed curves and 2 curves which are not simple closed curves.

2. Fig. 2. The simple closed curve shown is called a _?_. circle Explain the statement, "A circle is a set of points." Key idea: The set of points of the simple closed curve is the circle.

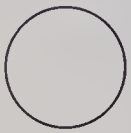


Fig. 2

3. Circle E is shown in Fig. 3. A circle is named by the center for the circle, so point _?_ E is the center for circle E.

a. Is $\overline{AE}$ a radius for circle E? Yes ^ Is $\overline{EB}$? Yes ^ Is $\overline{AB}$? No ^ Give a definition of a radius for a circle. A line segment whose one end point belongs to the circle and whose other end point is the center for the circle.

b. Is $\overline{AB}$ a diameter for circle E? Yes ^ Is $\overline{AB}$ a chord for circle E? Yes ^ Explain why every diameter for a circle is a chord for the circle but not every chord is a diameter. Every diameter is a chord which contains the center for the circle, but not every chord contains the center for the circle.

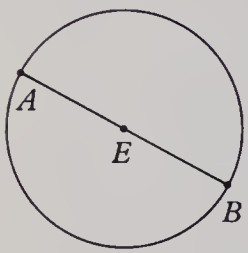


Fig. 3

* 4. All points belonging to a circle are at the same distance from the center for the circle. Does this mean that all the radii for a circle have the same length? Yes ^ Since $\overline{AE}$ and $\overline{EB}$ are radii for circle E, is it true that $m(\overline{AE}) = m(\overline{EB})$? Yes

5. For circle E, is it true that $m(\overline{AB}) = m(\overline{AE}) + m(\overline{EB})$? Yes Is it true that the measure of diameter AB is 2 times the measure of radius AE? Yes ^ Explain. $m(\overline{EB}) = m(\overline{AE})$

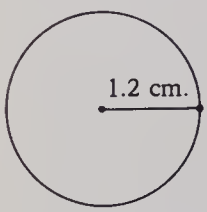


Fig. 4

6. For any circle, the measure of the diameter is 2 times the measure of the radius _?_.

If, for a circle, d represents the measure of the diameter and r represents the measure of the radius, then $d = 2 \times r$.

A
$d = 2 \times r$
$d = 2 \times 1.2$
$= 2.4$
diameter is 2.4 cm.

7. The radius for the circle shown in Fig. 4 is 1.2 cm. Explain the work in box A for finding the diameter for the circle. The measure of the radius is multiplied by 2.

8. At the board, use $d = 2 \times r$ as in box A to find the diameter for a circle if the radius for the circle is

a. 4.6 in. 9.2 in. b. $3\frac{2}{3}$ cm. $7\frac{1}{3}$ cm. c. 0.9 ft. 1.8 ft. d. $\frac{3}{7}$ in. $\frac{6}{7}$ in.

Teaching Pages 252 and 253

Pupil's Objectives

(a) To review the concept of a circle; (b) to review the relationship between a radius and a diameter for a circle; and (c) to apply multiplication and division with decimals to problems related to the measurement of radii and diameters for a circle.

Background

A circle is a simple closed curve in a plane, every point of which is a given distance from a fixed point in the plane. A diameter for a circle is a line segment whose end points lie in the circle and whose midpoint is the center for the circle. Since a chord for a circle is a line segment whose end points lie in a circle, a diameter may be described as a chord which contains the center for a circle. Thus, every diameter is a chord, but not every chord is a diameter. A diameter may also be thought of as being formed by the union of two radii in the same line.

If we let r represent the measure of a radius and let d represent the measure of a diameter, then $d = 2 \times r$ and $r = \frac{1}{2} \times d$.

Pre-Book Lesson

- Review the idea of plane figures and simple closed curves.
- Review expressing mathematical sentences using two letters instead of one. Use examples where one letter represents a known number.
- Draw a large picture representing a circle on the board. Adjacent to this, list all the terms the pupils can recall relating to a circle. Ask them to describe each and have a student mark on the picture, or simply indicate by pointing to the picture, each of the terms as they are discussed. Be sure they include in the list *simple closed curve*, *set of points*, *closed curve*, *circle*, *chord*, *diameter*, and *radius*.
- Review the notation: E for point E, $m(\overline{AB})$ for the measure of $\overline{AB}$, the line segment joining A and B.
- If pupils have much difficulty in multiplying or dividing with decimals, the teacher might wish to review this by working on the board one or two multiplication examples and one or two division examples.

Using the Text Pages

- Ex. 1–7. If the students had no particular difficulty in recalling the vocabulary or notation in the Pre-Book Lesson, they should be able to answer most of Ex. 1–3 without additional help. They will probably need additional practice with the notation in Ex. 4–6.
- Ex. 6. By "the diameter" is meant the length of any diameter for the circle, and by "the radius" is meant the length of any radius for the circle.
- Ex. 8. Each part of this should be worked at the board and accompanied by a complete discussion at each step of the solution.
- Ex. 9. The work in box B is just an application of the factors-product relationship. It should help pupils arrive at a decision as to what operation is to be used.
- Ex. 10–24. Work the first example in each group at the board.
- Ex. 12. Rename 0.8 with a fraction, then compare the measures of the diameters.
- Ex. 24. Explain the meaning of ratio as an expression of the relation between two numbers. Here, pupils should think of the measure of a radius for a circle as compared to the measure of a diameter for the same circle. The ratio is $\frac{1}{2}$.

Individualizing Instruction

More capable pupils may find the unit of precision and greatest possible error for each measurement in Ex. 13–22. They may then compute the greatest and least possible measurements.

NOTES

9. The diameter for the circle shown in Fig. 5 is 3.7 cm. Explain the work in box B for finding the radius for the circle. The measure of the diameter is divided by 2. The radius for the circle is _?_ 1.85 cm.

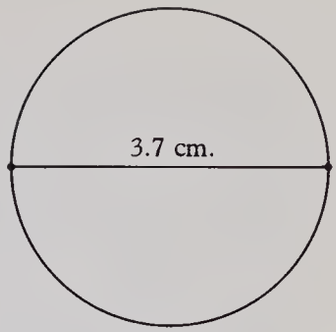


Fig. 5

10. At the board, find the radius for a circle if the diameter for the circle is

a. 15.8 in. 7.9 in. b. $6\frac{3}{5}$ cm. $3\frac{3}{10}$ cm. c. 0.27 ft. 0.135 ft. d. $\frac{3}{10}$ in. $\frac{3}{20}$ in.

11. To find the measure of the diameter, we _?_ multiply the measure of the radius by _?_ 2. To find the measure of the radius, we _?_ divide the measure of the diameter by _?_ 2.

12. Which has the longer diameter, a circle with radius $\frac{3}{4}$ in. or a circle with radius 0.8 in.? The circle with radius 0.8 in. Explain. Key idea: $0.8 > \frac{3}{4}$

[W]

Ex. 13–17. The radius for a circle is given. Find the diameter for the circle. Remember to give the unit of measurement.

13. $\frac{7}{8}$ in. $1\frac{3}{4}$ in. 14. 47 cm. 94 cm. 15. 8.45 yd. 16.9 yd. 16. $5\frac{4}{5}$ in. $11\frac{3}{5}$ in. 17. 0.79 m. 1.58 m.

Ex. 18–22. The diameter for a circle is given. Find the radius for the circle.

18. 45 in. $22\frac{1}{2}$ in. 19. $7\frac{1}{4}$ ft. $3\frac{5}{8}$ ft. 20. 5.78 yd. 2.89 yd. 21. $3\frac{2}{3}$ cm. $1\frac{5}{6}$ cm. 22. 0.34 mm. 0.17 mm.

23. To the nearest $\frac{1}{100}$ inch, find the radius for a circle if the diameter for the circle is 7.257 in. 3.63 in.

24. Express with a fraction the ratio of the measure of the radius for a circle to the measure of the diameter for the circle. $\frac{1}{2}$

B

$d = 2 \times r$

$3.7 = 2 \times r$

$r = 3.7 \div 2$

$2\overline{)3.70}$ → 1.85

2

17

16

10

10

$r = 1.85$

Finding Perimeters of Polygons

Resurvey [O]

Ex. 1–4. Find the perimeter of the polygon.

1. $1\frac{1}{2}$ cm., 5 cm., 5 cm., $1\frac{1}{2}$ cm. 13 cm.

2. 3 in., 3 in., 3 in., 3 in. 12 in.

3. 5 in., 7 in., 4 in., 4 in. 20 in.

4. 6 ft., 10 ft., 10 ft., 6 ft. 38 ft.

* Point out the new symbol ≈ which means "is approximately equal to."

Finding the Circumference of a Circle

[o]

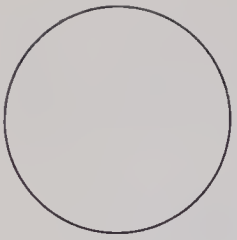


Fig. 1

The distance around a circle, or the length of a circle, is called the *circumference* of the circle.

1. Complete: Circumference is to circle as perimeter is to polygon.

2. To determine the circumference of a circle with diameter 1 in., Jane worked as follows:

Step 1: Jane used a compass to draw a picture of a circle with diameter 1 in. (Fig. 1). What radius did she use? $\frac{1}{2}$ in.

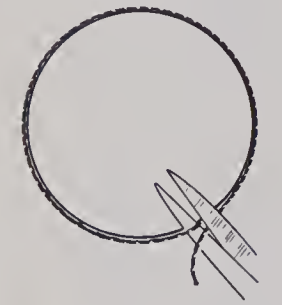


Fig. 2

Step 2: She then covered the model she had drawn with a string and trimmed away the excess string (Fig. 2). Should the length of the string around the model be about the same as the circumference of the circle? Yes

Step 3: Jane then measured the string as shown in Fig. 3. In inches, does the measure of the string appear to be greater than 3? Yes less than $3\frac{1}{4}$? Yes

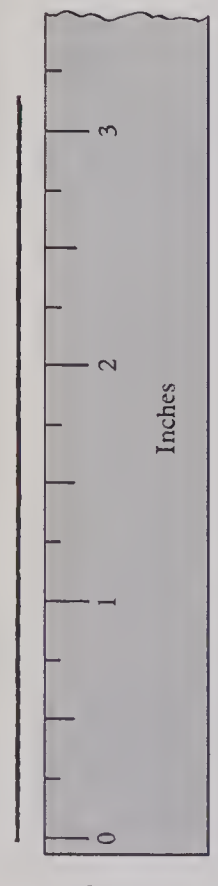


Fig. 3

3. If Jane had been very careful in her work, she would have found that the circumference of the circle was about 3.14 in. The diameter was 1 in. The measure of the circumference of Jane's circle is about 3.14 times the measure of the diameter for the circle. This is true for any circle.

The measure of the circumference of a circle is about 3.14 times the measure of the diameter for the circle.

* 4. For a circle with diameter 3 in., find the approximate circumference by finding 3.14×3. Since the product is not the exact circumference, we may write $C \approx 3.14 \times 3$ where C represents the measure of the circumference and $\approx$ means *is approximately equal to*. $C \approx$ 9.42 **

5. Using $C \approx 3.14 \times d$, if $d = 5$, then $C \approx$ 3.14 $\times$ 5, or 15.7

6. When d is expressed with a fraction, it is often convenient to use $C \approx \frac{22}{7} \times d$. On the board, divide 22 by 7. Carried to hundredths, $22 \div 7 = \frac{22}{7} =$ 3.14

$$7\overline{)22.00}\quad 3.14$$
$$21$$
$$10$$
$$7$$
$$30$$
$$28$$
$$2$$

** "9.42" is the answer, not "9.42 in.," because C represents a measure, not a measurement.

Teaching Pages 254 and 255

Pupil's Objective

To discover a method for finding the circumference of a circle.

Background

The ratio of the measure of the circumference of a circle to the measure of the diameter for the same circle is the irrational number called *pi* (π). Since this number is irrational, it cannot be expressed in fraction form. Pi has now been calculated to over 5,000 decimal places with the aid of high-speed computers. The famous Greek mathematician and inventor, Archimedes, showed that $3\frac{10}{71} < \pi < 3\frac{10}{70}$ as long ago as 250 B.C. After decimal form came into use in the 1600's, mathematicians labored to find an exact value in decimal form. It is now known that this is impossible. The value to 10 decimal places is 3.1415926536. Values which are frequently used to approximate this are $3\frac{1}{7}$, $\frac{22}{7}$, 3.14159, and 3.14. Although there is no exact decimal representation for π, it can be approximated to any required precision, and since measurements are only approximations, this is no serious limitation on our calculations. The measure of the circumference of a circle can be expressed by the form $C = \pi \times d$, or $\pi = \frac{C}{d}$. This means that the measure of the circumference is approximately 3.14 times as great as the measure of the diameter. The diameter for a circle is equal to twice the radius, so we can also compute the circumference in terms of the radius by using $C = 2 \times \pi \times r$. If the radius is given, it is common to first use the relationship $d = 2 \times r$ to find the measure of the diameter, then use $C = \pi \times d$. Later, pupils may confuse the formulas for circumference and area [$C = \pi \times d$, $C = 2 \times \pi \times r$, $A = \pi \times r^2$]. Pupils should understand the difference between the expression $2 \times \pi \times r$ and the expression $\pi \times r^2$.

Pre-Book Lesson

- Review perimeter of a simple closed curve.
- On the board, draw a large picture representing a circle and a chord, radius, diameter, and center for the circle. Have pupils label each of the parts for the circle. Relate circumference of a circle to the perimeter of a polygon. Have pupils suggest ways in which the measure of the circumference might be obtained. The need for a method which is more precise than that of laying out a piece of string or rope should be presented. Some more capable pupils might be encouraged to try to make measurements of the diameter and circumference. Thus, it could be pointed out that a circle with:

diameter	approximate circumference
20 in.	62.8 in.
10 in.	31.4 in.
1 in.	3.14 in.

Bring out that it would be convenient if this relationship could be expressed with the factors-product relationship.

Using the Text Pages

- Ex. 1–3. Discuss these in detail with the class, emphasizing the statement in Ex. 3, "If Jane had been very careful in her work. . . ."
- Ex. 4. Note the introduction of $\approx$ for writing "is approximately equal to" as the first symbolic representation of the relationship, $C \approx 3.14 \times d$. Recall that = may be read, "is another name for." Thus, it might be well to consider using "is approximately equal to" in the same sense.
- Ex. 6–9. While 3.14 can be renamed as $3\frac{14}{100}$ or $3\frac{7}{50}$, the 3.14 is itself an approximation. Using $3\frac{7}{49}$ or $3\frac{1}{7}$ will result in a good approximation, and is more convenient when the measure of the diameter is expressed in fraction form. Compute the measure of the circumference of a circle first using a diameter of $1\frac{1}{2}$, then using $\frac{3}{2}$, finally using 1.5. The results are 4.71 and 4.7 respectively. In completing Ex. 9, pupils should select whichever of the expressions is more convenient. Have them use $C \approx 3.14 \times d$ or $C \approx \frac{22}{7} \times d$. Then discuss the next paragraph which points out that a number does exist such that $C = n \times d$, and that this number is approximated by 3.14, or $\frac{22}{7}$ or other convenient values. Stress that the formula is exact.

• Ex. 9. π is the same as the number, but 3.14 is only an approximation for the number; therefore, when we use π, as in $C = \pi \times d$, the $=$ is correct. When we use 3.14 as in $C \approx 3.14 \times d$, $\approx$ is correct because the result is an approximation.

• Ex. 12. Note the use of the formula $C = \pi \times d$ in the first step in the box since the next step shows $C \approx 3.14 \times 10$. It is vital that pupils know we are writing expressions that approximate the measure of the circumference and not the actual value.

Individualizing Instruction

• *More capable pupils* should be asked to consider the circumference of a circle when the radius is given. They should be asked if there is any other type of problem that might be solved using this relationship; that is, given the circumference, can the diameter or radius be approximated?

• *Slower learners* might need to have $\approx$ and $=$ explained in each example where it appears.

NOTES

* Do not expect complete understanding of the concepts of π. Try to develop a sound background so that meaningful concepts will develop as pupils mature.

7. Using $C \approx \frac{22}{7} \times d$, if $d = 14$, then $C \approx$ _?_ $\times$ 14, or _?_. 22/7, 44

8. We may rename $\frac{22}{7}$ with the mixed form $3\frac{1}{7}$, so $C \approx \frac{22}{7} \times d$ may be expressed as $C \approx$ _?_ $\times d$. $3\frac{1}{7}$

9. At the board, copy and complete each of Ex. a–d. Use either $C \approx 3.14 \times d$ or $C \approx \frac{22}{7} \times d$ and show your work.

a. If $d = 5.2$, then $C \approx$ _?_. 16.328 **b.** If $d = 1{,}000$, then $C \approx$ _?_. 3,140

c. If $d = 2\frac{1}{3}$, then $C \approx$ _?_. $7\frac{1}{3}$ **d.** If $d = 210$, then $C \approx$ _?_. 660

9.
a. $C \approx 3.14 \times 5.2$
≈ 16.328
b. $C \approx 3.14 \times 1{,}000$
$\approx 3{,}140$
c. $C \approx \frac{22}{7} \times 2\frac{1}{3}$
$\approx \frac{22}{3}$, or $7\frac{1}{3}$
d. $C \approx \frac{22}{7} \times 210$
≈ 660

* When d, the measure of the diameter, is multiplied by 3.14, the product is not the exact measure of the circumference. That is why we have used $\approx$ instead of $=$. However, there is a number such that, when d is multiplied by that number, the result is C. The number is usually denoted by the symbol π (pronounced *pi*). Thus, $C = \pi \times d$.

10. Using computing machines, the decimal for the number π has been carried to thousands of decimal places. To five decimal places, the decimal for π is 3.14159. Round π to the nearest ten-thousandth; ∧ 3.1416 to the nearest thousandth; ∧ 3.142 to the nearest hundredth. ∧ 3.14 Do you see why we may use $C \approx 3.14 \times d$? Yes

11. On the board, divide 22 by 7 (carry the division to thousandths). ∧ 3.142 Explain why $\frac{22}{7}$ is not equal to π. Do you see why we may use $C \approx \frac{22}{7} \times d$? By dividing, $\frac{22}{7}$ is equal to 3.142 while $\pi = 3.141+$. ≈ means "is approximately equal to."

$C = \pi \times d$
$C \approx 3.14 \times 10$
$C \approx 31.4$

12. Explain the work in the box for approximating the circumference of a circular flower bed with diameter 10 ft. Key idea: 3.14 is used for π.
To the nearest foot, the circumference is _?_ ft. 31

[W]

Ex. 13–20. The diameter for a circle is given. Find the approximate circumference of the circle. Express your answer in the form "The circumference is about . . . ." Use $C \approx 3.14 \times d$ for each of Ex. 13–16. Use $C \approx \frac{22}{7} \times d$ for each of Ex. 17–20.

13. 17 in. 53.38 in. **14.** 8.91 ft. 27.9774 ft. **15.** 56.28 cm. 176.7192 cm. **16.** 0.06 in. 0.1884 in.

17. $3\frac{3}{7}$ in. $10\frac{38}{49}$ in. **18.** $\frac{7}{8}$ ft. $2\frac{3}{4}$ ft. **19.** $32\frac{1}{2}$ cm. $102\frac{1}{7}$ cm. **20.** 700 m. 2,200 m.

● **Extra Activity.** Set 173. ♦ **Extra Examples.** Set 128.

* Point out that when expressing the relationship between the circumference and a diameter of a circle, we use "=" with the symbol π, and "≈" when a value for π is given.

Finding the Diameter for a Circle

[O]

1. When we know the measure of the circumference of a circle, we can use $C = \pi \times d$ and the factors-product relationship to find the measure of the diameter for the circle.

*a. In $C = \pi \times d$, does C represent the product? Yes Is π the known factor? Yes Then to find d, the unknown factor, we divide _?_ C by _?_ π. We write, $d = C \div \pi$.

b. Explain the expression "$d \approx C \div 3.14$." Key idea: 3.14 has been used for π.

$d = C \div \pi$
$d \approx 7 \div 3.14$
$d \approx 2.2$

2. A circle with a circumference of 7 in. has a diameter of about how many inches? Explain the work in the box for finding the answer. The diameter is about 2.2 in.

Key idea: The measure of the circumference is divided by 3.14.

3. At the board, work as in the box to find the diameter for a circle whose circumference is 5.5 cm. 1.75 cm. Carry the division to hundredths. $d \approx 55 \div 3.14$ ≈ 1.75

[W]

Ex. 4–11. The circumference of a circle is given. Find the approximate diameter for the circle. Express your answer in the form "The diameter is about . . . ." For each of Ex. 4–7, use $d \approx C \div 3.14$ and express the answer to the nearest $\frac{1}{100}$ unit. Use $d \approx C \div \frac{22}{7}$ for each of Ex. 8–11.

4. 5 ft. 1.59 ft. 5. 26 cm. 8.28 cm. 6. 7.36 in. 2.34 in. 7. 0.67 yd. 0.21 yd.

8. $3\frac{2}{7}$ in. $1\frac{1}{22}$ in. 9. $1\frac{5}{7}$ ft. $\frac{6}{11}$ ft. 10. $\frac{3}{14}$ in. $\frac{3}{44}$ in. 11. $\frac{5}{8}$ m. $\frac{35}{176}$ m.

12. The circumference of a circular table is 16 ft. Find, to the nearest $\frac{1}{10}$ foot, the diameter for the table. 5.1 ft.

13. Eric wishes to picture a circle with a circumference of about 31.4 in. About what radius should he use? 5 in. (Hint: About what diameter should the circle have?)

14. Copy and complete:

The measure of the circumference of a circle is about 6.28 times the measure of the radius for the circle.

♦ **Extra Examples.** Set 129.

Teaching Page 256

Pupil's Objectives

(a) To review the relationship between the circumference and diameter for a circle; and (b) to learn how to use this relationship to approximate the diameter for a circle when the circumference is known.

Background

The measure of the circumference of a circle, the distance around the circle, or length of the circle, is expressed with the formula $C = \pi \times d$. However, the measure of the circumference can at best only be approximated by the use of this formula. Since the number pi, represented by the Greek letter π, is an irrational number, it has no exact decimal representation. The value of π has been computed to over 5,000 decimal places, so it is possible to compute a circumference very precisely when the diameter is known. This formula is an example of a factors-product relationship, so it is also possible to approximate the diameter when the circumference is known. Since the measure of the radius for a circle is one half that of a diameter for the same circle, this will enable the measure of the radius to be found by first dividing the measure of the circumference by the appropriate value of π and then dividing this result by two.

Pre-Book Lesson

• Ask the class about the relationship between the circumference of a circle and the diameter for the circle. Have them describe this in two ways: (1) $C = \pi \times d$; and (2) $C \approx 3.14 \times d$ and/or $C \approx \frac{22}{7} \times d$. Discuss this in terms of the factors-product relationship with the measure of the circumference as the product, and 3.14 or $\frac{22}{7}$ as the known factor. Pupils should then suggest that they are able to approximate the diameter by division if the measure of the circumference is known.

• Review the relationship $d = 2 \times r$. Treat this also as a factors-product relationship with the measure of the diameter as the product, and the 2 as the known factor. Pupils should then conclude that if the measure of the diameter is known, or can be found, the measure of the radius can be found by dividing the measure of the diameter by 2.

Using the Text Page

• Ex. 1–3. Discuss these with the pupils, working each step at the board. Note the use of $=$ in the first step in the box, and the use of $\approx$ in the second step. Be sure pupils understand the need for the introduction of this symbol at this point. [The value 3.14 is only an approximation to the value of π.]

• Ex. 4–11. Be sure pupils divide properly to the nearest unit indicated. Include the proper units of measurement for the answer to each example.

• Ex. 12–14. Students who have difficulty here should reread their work for Ex. 1–3, and be reminded that $d = 2 \times r$.

Individualizing Instruction

More capable pupils may make up a set of simple problems involving circumference, diameter, and radius and, after drawing appropriate diagrams, have *all pupils* in the class work them.

Teaching Page 257

Pupil's Objective

To review the concept of ratio in preparation for application to per cent forms.

Background

The ratio of a number represented by a to a number represented by b can be expressed as $a:b$ or with the fraction form $\frac{a}{b}$. The ratio of 25 to 75 can be expressed as:

$$25:75 = \frac{25}{75} \qquad \frac{1 \times 25}{3 \times 25} = \frac{1}{3} \times \frac{25}{25} \qquad \frac{1}{3} = 1:3$$

Thus, if m_1 represents the measure of the first quantity or set, and if m_2 represents the measure of the second quantity or set, the ratio of the first to the second is expressed as $m_1:m_2$. The ratio of the second to the first is expressed as $m_2:m_1$. We are most often concerned with the comparison, by the use of a ratio, of two quantities, or measures with the same units of measurement. But, the statement that a car is traveling at a rate of 30 miles per hour states a ratio of the measure of distance traveled to the measure of time taken to travel this distance. This may be more evident in the statement, "A car travels a distance of 60 miles in 2 hours. This is a rate of 60 miles for 2 hours."

In verbal problems, a ratio exists between the numbers associated with each quantity. Thus, if the length of a chalkboard is 7 feet while the width is 3 feet, the ratio of the measure of the length to the measure of the width is 7:3 or $\frac{7}{3}$. Hence, we loosely say the ratio of the width to the length of the same chalkboard is 3:7 or $\frac{3}{7}$. Both forms for the ratio are to be used and emphasized as different names for the same ratio. Other statements of the above relationship would be: "The length is $\frac{7}{3}$ of the width, or $2\frac{1}{3}$ times the width." "The width is $\frac{3}{7}$ of the length."

A ratio can be expressed in many ways.

$$\frac{2}{5} = \frac{2}{5} \times \frac{3}{3} \quad \frac{2 \times 3}{5 \times 3} = \frac{6}{15}, \text{ or}$$
$$\frac{2}{5} = \frac{2}{5} \times \frac{20}{20} = \frac{2 \times 20}{5 \times 20} = \frac{40}{100}$$

Ratios which compare the number of members in one set to the number of members in a set having 100 members are expressed with per cent forms. Thus, the above paragraph indicates a method of expressing a ratio with a fraction form as a ratio expressed in per cent form. Thus, $\frac{2}{5} = \frac{40}{100}$ or 40%.

Pre-Book Lesson

- Review an intuitive meaning of the word "ratio" as a comparison of two numbers or two sets expressed in fraction form.
- Mark off a section of the chalkboard to picture segments. Discuss the ratio of the measure of the length of each segment to the measure of the whole segment, and the ratio of the measure of each segment to the measure of the other. Express these ratios with fraction and decimal forms. If the measure of length of one segment is 10 and the measure of length of the whole is 40, the ratio of the measure of the part to the whole may be expressed as

$$\begin{aligned} 10:40 &= \frac{10}{40} \\ &= \frac{10}{40} \div \frac{10}{10} \\ &= \frac{1}{4} \text{ or } 1:4 \end{aligned}$$

Using the Text Page

- Ex. 1–3. Discuss these examples in detail, showing a step-by-step analysis at the board. Use information from the Pre-Book Lesson.
- Ex. 4–21. Some pupils may need to review expressing fractions showing a denominator of 100.

Individualizing Instruction

- *More capable pupils* should rename ratios in which the numerator is other than a whole number when the denominator is 100. Examples such as 20:60, 10:80, 2:3, and so on can be used.
- Give *slower learners* extra practice in expressing ratios with fraction forms where the denominators show 100.

Other simple examples can be given where they are asked to express twice as many, $\frac{1}{2}$ as many, 3 times as many, $\frac{1}{3}$ as many, and so on. This will accustom them to the idea that a ratio can be expressed in the form $\frac{b}{a}$ as well as $\frac{a}{b}$.

Remembering about Ratio

Resurvey; preparation for per cent [O]

1. There are _?6_ chairs and _?8_ children left in the game. Then there are 6 chairs for _?8_ children. The ratio of the number of chairs to the number of children is 6 to _?8_. This ratio can also be expressed as 3 to _?4_; as 9 to _?12_.

2. Can the ratio 3 to 4 be expressed with the fraction $\frac{3}{4}$?Yes ∧ $\frac{6}{8}$?Yes ∧ $\frac{9}{12}$?Yes ∧ Can it be expressed as 3:4?Yes On the board, express the ratio in this way using the numerals 6 and 8; 9 and 12. 6:8; 9:12

3. Explain the work in the box for finding a fraction showing denominator 100 which expresses the same ratio as $\frac{3}{4}$. ∧ Do both $\frac{3}{4}$ and $\frac{75}{100}$ express the same ratio?Yes ∧ the same number? Yes

$$\frac{3}{4} = \frac{n}{100}$$
$$\frac{3}{4} = \frac{3 \times ?}{4 \times 25}$$
$$\frac{3}{4} = \frac{75}{100}$$

Key idea: Numerator and the denominator are multiplied by the number that will give 100 for the denominator.

4. At the board, express the ratio 2 to 5 with a fraction showing denominator 100. Work as in box B. $\frac{2}{5} = \frac{n}{100}$ $\frac{2}{5} = \frac{2 \times 20}{5 \times 20}$ $\frac{2}{5} = \frac{40}{100}$

5. John has 20 marbles in all, 7 of which are multicolored. John said, "The ratio of the number of multicolored marbles to the total number of marbles is ~~28~~ 35 to 100." Was John's statement correct?No ∧ If not, correct the statement.

[W]

Ex. 6–10. Express the ratio with a fraction in simplest form.

6. 20 to 25 $\frac{4}{5}$ 7. 16:64 $\frac{1}{4}$ 8. 44 to 100 $\frac{11}{25}$ 9. $\frac{45}{100}$ $\frac{9}{20}$ 10. 10 to 144 $\frac{5}{72}$

Ex. 11–20. Express the ratio with a fraction showing denominator 100.

11. 2 to 25 $\frac{8}{100}$ 12. 17:50 $\frac{34}{100}$ 13. 30 to 40 $\frac{75}{100}$ 14. $\frac{13}{20}$ $\frac{65}{100}$ 15. 37 to 100 $\frac{37}{100}$
16. 6 to 200 $\frac{3}{100}$ 17. 500:1,000 $\frac{50}{100}$ 18. 9 to 10 $\frac{90}{100}$ 19. 29:50 $\frac{58}{100}$ 20. $\frac{36}{400}$ $\frac{9}{100}$

21. In Bob's coin collection, the ratio of the number of Indian-head pennies to the number of Lincoln pennies is 4 to 5. Express this ratio with a fraction showing denominator 100. $\frac{80}{100}$

◆ **Extra Examples.** Set 130.

* Depending on the time available, either all or part (pages 266-269) of this per cent unit may be considered optional.

Fig. 1

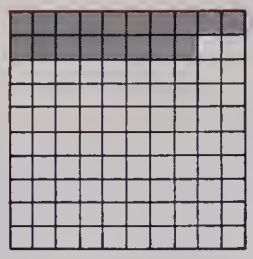


Fig. 2

18%

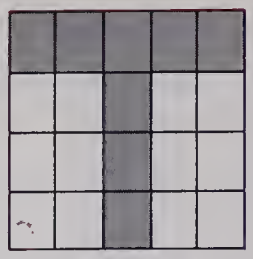


Fig. 3

40%

Fig. 4

40%

* Per Cent

[o]

1. Look at the diagram in Fig. 1. How many congruent regions are shown? 4 How many are shown with color? 1

a. The ratio of the number of regions shown with color to the total number of regions shown is 1 to _?4_. This ratio can also be expressed as 25 to _?100_.

b. Is $\frac{1}{4}$ of the diagram colored? Yes Can we say that $\frac{25}{100}$ of the diagram is colored? Yes

c. Since $\frac{25}{100}$ of the diagram is colored, we may say that twenty-five per cent (usually written *25%*) of the diagram is colored. Is it true that $\frac{75}{100}$ of the diagram is not colored? Yes Then _?75_% of the diagram is not colored. A ratio which compares the number of members in one set to the number of members in a set containing 100 members is called a *per cent*. We may express per cents with *per cent forms* such as 25% and 75%.

** 2. To express a ratio with a per cent form, you can first express the ratio with a fraction showing denominator 100. Then, for the per cent form, write the numeral for the numerator and the symbol "%". The ratio 3 to 5 may be expressed with the fraction $\frac{?60}{100}$, so a per cent form for this ratio is _?60_%.

3. What per cents are suggested by the diagrams in Fig. 2–4? On the board, write a per cent form for each. See Fig. 2-4.

4. At the board, copy and complete.

a. $\frac{4}{5} = \frac{?80}{100} =$ _?80_% b. $\frac{1}{4} = \frac{?25}{100} =$ _?25_% c. $\frac{7}{10} = \frac{?70}{100} =$ _?70_%

** Notice that "per cent forms" are numerals which are comparable to decimals, fractions, standard numerals, and so on.

Teaching Pages 258 and 259

Pupil's Objective

To learn to apply the concept of ratio to per cent forms.

Background

Ratios can be expressed first with one fraction form and then with another fraction form. For example:

$$4:5 = \frac{4}{5} \quad \frac{4}{5} \times \frac{5}{5} = \frac{20}{25} \quad \frac{20}{25} \times \frac{4}{4} = \frac{80}{100}$$

A ratio which compares the number of members in one set to the number of members in a set containing 100 members is called a per cent. Per cents are named from fraction forms which show 100 in the denominator. The per cent form is obtained by writing the numeral for the numerator and the symbol "%." To express a ratio with a per cent form, first express the ratio with a fraction. Then, write the fraction which shows 100 in the denominator. Write the numeral for the numerator and the symbol "%." The ratio 4:5 can be expressed as $\frac{80}{100}$, and $\frac{80}{100}$ can be expressed as 80%.

The number 1 can be expressed with a fraction in the form: $\frac{4}{4}$, or $\frac{8}{8}$, or $\frac{13}{13}$, or $\frac{100}{100}$. Therefore, 100% represents the whole of something since we must be comparing all of the members of a set to 100 members of a set which contains 100 members.

Pre-Book Lesson

• Write the ratio 3:4 on the board and have pupils express it with fraction forms such as $\frac{3}{4}$, $\frac{6}{8}$, $\frac{75}{100}$, and so on. Discuss these in terms of their being measures of width and length of a rectangle. Have pupils verbalize about the ratio and the fraction forms by saying, "The width contains $\frac{3}{4}$ as many units of measurement as does the length." "The length contains a number of inches which is $1\frac{1}{3}$ times the number of inches in the width," and "If the length contained 100 units of measurement, the width would contain 75 units of measurement."

• Use segments marked off into various lengths to show comparisons of ratios of 1 to 5, 7 to 10, 20 to 2, and so on. Have pupils rename these with a fraction showing a denominator of 100.

Using the Text Pages

• Bring to class a clipping from a newspaper or magazine which shows per cent forms and use this with the class to illustrate everyday uses in business, industry, and advertising of per cent forms.

• Ex. 1–3. Discuss these slowly and in detail with students. Perhaps an overhead projector could be used to good advantage in this lesson. Prepare overlays which depict diagrams similar to those in the text in which rectangular shapes are partitioned to show fifths, tenths, and hundredths. Use a rectangular shape of the same dimensions to show $\frac{2}{5}$, $\frac{4}{10}$, $\frac{40}{100}$, and so on. This will help emphasize the different forms for a ratio or a fraction. Point to the various regions as the students follow the discussion in the text diagrams.

Explain that a per cent can mean a comparison to 100 of something, and that it is a quick way to show relationships.

On the board, write out an expanded form for the answers in each section and discuss a general method for expressing ratios with different forms. For instance, in Ex. 1, write

$1:4 = \frac{1}{4} \quad \frac{1}{4} \times \frac{25}{25} = \frac{25}{100} \quad \frac{25}{100} = 25\%$, colored
$3:4 = \frac{3}{4} \quad \frac{3}{4} \times \frac{25}{25} = \frac{75}{100} \quad \frac{75}{100} = 75\%$, not colored.

• Ex. 4. Pupils should now be able to go directly from $\frac{4}{5}$ to $\frac{80}{100}$ to 80%. However, it might prove advantageous for some pupils to show the intermediate step as follows: $\frac{4}{5} = \frac{4}{5} \times \frac{20}{20} = \frac{80}{100}$.

For b and c, ask pupils how the number 1 must be expressed to show the denominator 100 in each case.

• Ex. 5–6. Refer to the pictures on page 258 and ask pupils how many regions in each diagram would have to be colored to show that 100% of the figure is colored. Do the same for 50% in Fig. 1–4. Make up examples illustrating the nature and use of per cent. For example, if an article is sold at a discount of 15% from the list price, ask what this would mean in terms of the amount of money saved (15¢ on an item listed at 100¢, or $1.00; 30¢ on a $2.00 item); and the amount of money they would now have to pay (85¢ on the $1.00 item, and so on). Use a statement of a batting "average" and have some

of the baseball fans state what it means on the basis of 100 times at bat.

• Ex. 7. You may wish to use set notation in working out these examples.

• Ex. 9–20. Some pupils should be able to write the correct per cent forms for the answers directly from the given forms in the text. This should be permitted, and encouraged.

Those having difficulty should be directed to write all the steps on their papers. For instance, in Ex. 20b, $5:25 = \frac{5}{25}$, or $\frac{20}{100}$, or 20%. Some pupils may even need to show the step $\frac{5}{25} \times \frac{4}{4} = \frac{5 \times 4}{25 \times 4}$.

Individualizing Instruction

• *More capable pupils* should consider ratios expressed with mixed forms such as $3\frac{1}{2}$ to 7 or $\frac{3\frac{1}{2}}{7}$, and $5\frac{3}{4}$ to $3\frac{1}{4}$ or $\frac{5\frac{3}{4}}{3\frac{1}{4}}$. They should then rename the numerators and denominators so they are whole numbers, such as $3\frac{1}{2}:7 = \frac{3\frac{1}{2}}{7} \times \frac{2}{2}$ $\frac{7}{14} = \frac{1}{2}$ or 1:2.

These pupils should realize that this same procedure can be used to find an unknown in such examples as $1:3 = n:100$. $1:3 = 33\frac{1}{3}:100$ by using the same procedure as finding an unknown numerator or denominator for an "equivalent" fraction number. Hence, the ratio 1:3 can be expressed as $33\frac{1}{3}\%$. Likewise, $1:8 = n:100$, $12\frac{1}{2}:100 = 12\frac{1}{2}\%$, can be obtained by asking, "By what must I multiply 8 to show 100 in the ratio?"

• *Slower learners* could represent polygons such as triangles and rectangles, and find the ratio of the measure of each side to the perimeter. Have them express this ratio with a fraction and then rename the ratio with a fraction that shows a denominator of 100.

• Have *all pupils* find the following for Ex. 7:

1. W = {whole numbers from 4 through 13}
   (W = {4, 5, 6, 7, 8, 9, 10, 11, 12, 13})
2. P = {prime numbers in set W}
   (P = {5, 7, 11, 13})
3. E = {even numbers in set W}
   (E = {4, 6, 8, 10, 12})
4. Number of members in set W (10) in set P (4) in set E (5)

5. $W \cup P$ 6. $W \cap P$ 7. $E \cup P$
8. $E \cap P$ 9. $E \cup W$ 10. $E \cap W$

NOTES

5. Fig. 5. How many congruent circular regions are shown? 8 How many are shown with color? Then $\frac{?}{8}$ or $\frac{?}{100}$ or _?_% of the diagram is colored. When we mean the whole of something, we may say, _?_%.

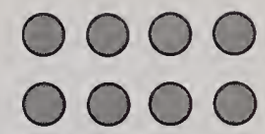


Fig. 5

6. Does a test score of 100% mean that all the examples on the test were worked correctly? Yes What does a test score of 50% mean? One half the examples on the test were worked correctly.

7. On the board, list the whole numbers from 4 through 13. 4, 5, 6, 7, 8, 9, 10, 11, 12, 13

a. How many numbers did you list? 10

b. How many of the numbers listed are primes? 4 Then $\frac{?}{10}$ or $\frac{?}{100}$ or _?_% of the numbers listed are primes.

c. What per cent of the numbers listed are even numbers? 50% Explain. Key idea: Five, or one half, of the numbers listed are even numbers.

d. What per cent of the numbers listed are counting numbers? 100%

8. Give a sentence using "per cent" which tells about something that you have done or something that you own. Answers will vary.

[W]

Ex. 9–18. Express the ratio with a per cent form.

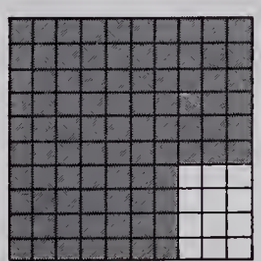
9. 4 to 100 4% 10. 4 to 50 8% 11. 3:10 30% 12. $\frac{19}{100}$ 19% 13. $\frac{7}{20}$ 35%

14. 39:50 78% 15. 3 to 4 75% 16. 25:50 50% 17. 140:200 70% 18. $\frac{135}{500}$ 27%

19. On your paper, show 8 congruent triangular regions and color 25% of the models. Sample answer:

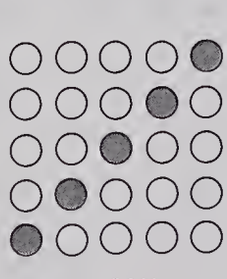
20. For each of Ex. a–e, write a per cent form to express what part of the diagram is colored.

a.



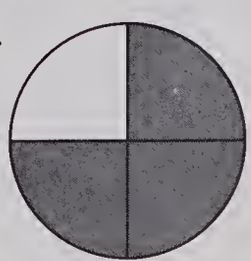
88%

b.



20%

c.

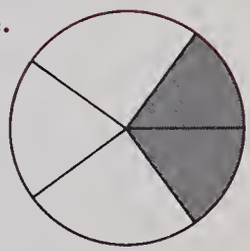


75%

d.

60%

e.



40%

♦ **Extra Examples.** Sets 131–132.

Using Per Cent Forms to Name Rational Numbers

[O]

1. Fractions are sometimes used to express ratios. Give an example in which a fraction expresses a ratio. Answers will vary.

2. Per cent forms are used to name numbers as well as to express ratios or per cents. That is, a per cent form may be used as a numeral. For example, 20% and $\frac{20}{100}$ both express the same ratio, and also, they both are names for the same rational number. On the board, write a decimal for this number. 0.2

3. $8\% = \frac{?8}{100} = \frac{?2}{25}$ Is it true that $8\% = \frac{2}{25}$? Yes Explain. They both name the same rational number.

A	$8\% = \frac{8}{100}$ $= 0.08$
B	$0.12 = \frac{12}{100}$ $= 12\%$

4. Explain the work in box A for naming 8 per cent with a decimal. Key idea: The per cent is renamed with a fraction showing denominator 100 and then with a decimal.

5. At the board, rename the following numbers with decimals as in box A.

a. 15% 0.15 **b.** 29% 0.29 **c.** 1% 0.01 **d.** 80% 0.8 **e.** 99% 0.99

6. Explain the work in box B for renaming 0.12. Key idea: The decimal is renamed with a fraction showing denominator 100 and then with a per cent form.

7. At the board, rename with per cent forms.

a. 0.75 75% **b.** 0.33 33% **c.** 0.07 7% **d.** 0.01 1% **e.** 0.89 89%

8. $100\% = \frac{?100}{100}$, so 100% names the whole number _?1_.

[W]

Ex. 9–16. Copy and complete.

9. $45\% = \frac{?45}{100}$ 10. $51\% = \frac{?51}{100}$ 11. $3\% = \frac{?3}{100}$ 12. _?81_% $= \frac{81}{99}$

13. $\frac{2}{5} =$ _?40_% 14. 0.67 = _?67_% 15. 1 = _?100_% 16. 0.99 = _?99_%

Ex. 17–21. Copy and complete. Ex. 17 is done for you.

	Fraction in simplest form	Fraction showing denominator 100	Decimal	Per cent form
17.	$\frac{7}{25}$	$\frac{28}{100}$	0.28	28%
18.	$\frac{3}{5}$	_?60/100_	_?0.6, or 0.60_	_?60%_
19.	_?11/25_	$\frac{44}{100}$	_?0.44_	_?44%_
20.	_?17/25_	_?68/100_	0.68	_?68%_
21.	_?13/20_	_?65/100_	_?0.65_	65%

Teaching Page 260

Pupil's Objective

To learn to use per cent forms to name rational numbers named with fractions or decimals.

Background

Rational numbers may be named with fractions or decimals. A fraction also expresses a ratio. Since per cent forms can express ratios, they can also name a rational number.

The ratio of the number 3 to the number 4 is named by the form 3:4. This may be expressed as $\frac{3}{4}$ or $\frac{75}{100}$, or the per cent form 75% may be used. These symbols are also names for the same rational number.

Since the fraction $\frac{75}{100}$ and the decimal 0.75 name the same rational number, and 75% names the same rational number as $\frac{75}{100}$, therefore,

$$3:4 = \tfrac{3}{4} = \tfrac{75}{100} = 0.75 = 75\%, \text{ and}$$
$$3 \div 4 = \tfrac{3}{4} = \tfrac{75}{100} = 0.75 = 75\%.$$

Although an analogy may be drawn in terms of the numerals, we should not write $3:4 = 3 \div 4$, since the first emphasizes ratio and the second emphasizes rational number. As numerals, the decimal, fraction, and per cent forms may be used interchangeably to express either ratios or rational numbers.

Pre-Book Lesson

• Write the ratio 3:4 on the board and have pupils express it with other forms. Although pupils have not used the decimal form with ratios, ask them if the decimal form 0.75 can be used to express the rational number named by $\frac{75}{100}$.

• Practice expressing with different forms a more complex ratio such as $13:25 = \frac{13}{25} = \frac{13}{25} \times \frac{4}{4} = \frac{52}{100} = 52\%$. Make a statement to the effect that just as the decimal form 0.52 is used as another name for the rational number named by $\frac{52}{100}$, the decimal form 0.52 may be used to express the ratio $\frac{52}{100}$. Since this is true, it seems reasonable that the per cent form may be used to express either the rational number or the ratio. Therefore, we can write $\frac{52}{100} = 0.52 = 52\%$ because they all name the same number.

Using the Text Page

• Ex. 1–3. It should be emphasized that although we are using the same forms now as before, we are dealing with the naming of rational numbers with per cent forms, whereas we were previously emphasizing the ratio idea.

• Ex. 4–5. Perhaps $\frac{2}{25}$ should be shown on the board and expressed as $\frac{4}{50}$ and $\frac{8}{100}$, then as 0.08, and finally as 8%. Refer to box A, encircling the last three forms on the board.

• Ex. 6–7. Start with a fraction form and express the number in various ways.

$$\tfrac{3}{25} = \tfrac{6}{50} = \tfrac{12}{100} = 12\%$$
$$\tfrac{12}{100} = 0.12.$$

Hence, $0.12 = \frac{12}{100} = 12\%$.

• Ex. 9–21. If the work for Ex. 6–7 is left on the board, students should be able to complete these exercises by referring to it. It might help if the work is discussed using each different step as a starting point. Thus:

$$\tfrac{6}{50} = \tfrac{12}{100} = 12\%, \text{ and}$$
$$12\% = \tfrac{12}{100} = \tfrac{6}{50} = \tfrac{3}{25}$$
$$12\% = \tfrac{12}{100} = 0.12, \text{ and so on.}$$

Individualizing Instruction

• Ask *more capable pupils* to consider more difficult exercises such as

$$\tfrac{1}{8} = \tfrac{12\frac{1}{2}}{100} = 0.12\tfrac{1}{2} = 12\tfrac{1}{2}\%$$

• *All pupils* should be asked to work a sheet containing such exercises as:

	Fraction in simplest form	Fraction showing denominator 100	Decimal form	Per Cent form
1.	$\frac{3}{8}$	- ? -	- ? -	- ? -
2.	- ? -	$\frac{33\frac{1}{3}}{100}$	- ? -	- ? -
3.	- ? -	- ? -	$0.16\frac{2}{3}$	- ? -
4.	- ? -	- ? -	- ? -	$87\frac{1}{2}\%$

Pupil's Objectives

(a) To review naming numbers with fraction, decimal, and per cent forms; and (b) to learn how to add and subtract with per cent forms.

Background

Recall that ratios and rational numbers can be expressed with fraction forms. A decimal form can be written from a fraction form showing a power of ten in the denominator. This power is indicated in the decimal form by the position of the decimal point.

The basic unit for the per cent form is 100, the second power of ten, hence a per cent form is expressed by writing a fraction showing a denominator of 100, then writing the numeral naming the numerator and the "%" sign.

Thus, $\frac{7}{25} = \frac{7}{25} \times \frac{4}{4} = \frac{28}{100} = 0.28 = 28\%$

Decimal and fraction forms can be used to compare numbers named with per cent forms.

$5\% = \frac{5}{100}$ $13\% = \frac{13}{100}$, and since $\frac{5}{100} < \frac{13}{100}$, it must follow that $5\% < 13\%$.

Likewise, $\frac{5}{100} = \frac{1}{100} + \frac{4}{100}$, hence it must follow that $5\% = 1\% + 4\%$.

Pre-Book Lesson

• Write on the board the fraction form $\frac{7}{25}$. Ask pupils to name the number with several other numerals. Be sure they list $\frac{28}{100}$, 0.28, and 28% among others. Discuss how each form can be derived from each of the other forms. Give special attention to the following examples.

Starting with per cent, compare $12\% = \frac{12}{100}$ to $12\% = 0.12$

Starting with a fraction, compare $\frac{12}{100} = 12\%$ to $\frac{12}{100} = 0.12$

Starting with a decimal, compare $0.12 = \frac{12}{100}$ to $0.12 = 12\%$

• Have a general review of the comparison of decimal, per cent, and fraction forms using the material from the Background.

• To add 5% and 3%, write on the board:

$$n = 3\% + 5\%$$
$$= \frac{3}{100} + \frac{5}{100} = \frac{8}{100} = 8\%$$
or
$$= 0.03 + 0.05 = 0.08 = 8\%$$

To subtract 5% from 13%, write

$$n = 13\% - 5\%$$
$$= \frac{13}{100} - \frac{5}{100} = \frac{8}{100} = 8\%$$
or
$$= 0.13 - 0.05 = 0.08 = 8\%.$$

Using the Text Page

• Ex. 1–5. Explain Ex. 1–2 at the board as in box A; then have pupils do Ex. 3 as in box A. Have them then do Ex. 3 again, this time as in box B. Discuss how each of Ex. 3 would be done using the method in box C and discuss Ex. 4–5.

• Ex. 6–17. Encourage the use of the method illustrated by boxes C and D. Remind pupils of the addends-sum relationship and the question they must answer before they decide what operation to use; "Is the sum given, or is the sum to be found?"

• Ex. 15. Some pupils may need to be reminded that 1 may be named by 100%.

• Ex. 18–26. Review the use of the symbols $=$, $<$, and $>$. Some pupils may need to name all the addends and the sum with the same form, while others may be able to work with many forms.

Individualizing Instruction

• *More capable pupils* should be asked to consider per cent forms which contain a fraction as part of the numeral. Have them make up exercises in which they must rename from these per cent forms to fraction and decimal form. Have them use such exercises as

$33\frac{1}{3}\% - 16\frac{2}{3}\% = n$
$12\frac{1}{2}\% + 37\frac{1}{2}\% = n$, and so on.

• *Slower learners* might have to revert to the decimal or fraction form in some of Ex. 6–17 to be convinced that the procedure in box D can always be applied. The method is indicated in box C; the algorithm is in box D.

A. and S. with Per Cent Forms

[O]

1. Box A shows one way to solve the mathematical sentence $n = 3\% + 52\%$. Before adding, both addends were renamed with fractions showing denominator _?_ 100. Explain the work shown. Key idea: The addends were renamed with fractions, the addition was done and the sum was renamed with a per cent form.

2. At the board, solve $n = 16\% + 37\%$ as in box A. $\frac{53}{100}$, or 53%

3. Box B. Before adding, both addends were renamed with _?_ decimals. Explain the work shown. Solve Ex. a and b as in box B. Key idea: The sum was found, and the decimal was renamed with a per cent form.

a. $n = 10\% + 72\%$ 0.82, or 82% **b.** $n = 3\% + 57\%$ 0.6, or 0.60, or 60%

4. Explain the work in box C. Which steps in boxes A and B are done mentally in box C? the renaming Which method seems best, that of box A, box B, or box C? Box C

5. Box D. To solve $n = 60\% - 42\%$, think, $60 - 42 = \_?\_$ 18; so $60\% - 42\% = \_?\_$ 18%. At the board, find $60\% - 42\%$ in two ways, first by renaming the sum and the known addend with fractions showing denominator 100 and then by renaming with decimals.

A

$n = 3\% + 52\%$
$= \frac{3}{100} + \frac{52}{100}$
$= \frac{55}{100}$, or 55%

B

$n = 3\% + 52\%$
$= 0.03 + 0.52$
$= 0.55$, or 55%

C

$n = 3\% + 52\%$
Think: $3 + 52 = 55$
$n = 55\%$

D

$n = 60\% - 42\%$
$= 18\%$

$n = 60\% - 42\%$
$= \frac{60}{100} - \frac{42}{100}$
$= \frac{18}{100}$, or 18%

$n = 60\% - 42\%$
$= 0.60 - 0.42$
$= 0.18$, or 18%

[W]

Ex. 6–17. Solve. Express the answer with a fraction, a decimal, and a per cent form.

6. $36\% + 35\% = n$ $\frac{71}{100}$; 0.71; 71%
7. $n = 55\% - 27\%$ $\frac{28}{100}$; 0.28; 28%
8. $n - 89\% = 7\%$ $\frac{96}{100}$; 0.96; 96%
9. $n = 25\% + 67\%$ $\frac{92}{100}$; 0.92; 92%
10. $n + 45\% = 78\%$ $\frac{33}{100}$; 0.33; 33%
11. $14\% + n = 63\%$ $\frac{49}{100}$; 0.49; 49%
12. $n = 40\% + 0.42$ $\frac{82}{100}$; 0.82; 82%
13. $n = \frac{78}{100} - 51\%$ $\frac{27}{100}$; 0.27; 27%
14. $n = 100\% - 37\%$ $\frac{63}{100}$; 0.63; 63%
15. $n = 1 - 39\%$ $\frac{61}{100}$; 0.61; 61%
16. $n = 6\% + 31\% + 17\%$ $\frac{54}{100}$; 0.54; 54%
17. $n = 0.85 - (43\% + 12\%)$ $\frac{30}{100}$; 0.3; 30%

Ex. 18–26. Copy and complete by writing $=$ or $<$ or $>$.

18. $36\% \_?\_ \geq 33\%$
19. $46\% \_?\_ \leq 47\%$
20. $0.67 \_?\_ = 67\%$
21. $\frac{19}{100} \_?\_ \geq 18\%$
22. $29\% + 33\% \_?\_ = 62\%$
23. $0.78 \_?\_ \leq 1 - 12\%$
24. $89\% \_?\_ = 1 - 0.11$
25. $0.45 \_?\_ \leq 40\% + 6\%$
26. $\frac{36}{100} + \frac{64}{100} \_?\_ = 100\%$

◆ **Extra Examples.** Set 133.

Using the Factors – Product Relationship

[o]

1. Is a factor or the product unknown in $n = \frac{17}{100} \times 29$? product in $n = 17\% \times 29$? product ^ When both factors are known, what operation is used to find the product? Multiplication

2. Box A. To solve $n = 25\% \times 16$, the factor 25% was renamed $\frac{?}{100}^{25}$ and then $\frac{?}{4}^{1}$. Explain the work for solving $n = 25\% \times 16$. $n = \_?\_^{4}$ Key idea: The per cent was renamed with a fraction in simplest form, and the multiplication was done.

3. In box B, the factor 25% was renamed with the decimal $\_?\_^{0.25}$. Explain the work shown. Key idea: The per cent was renamed with a decimal and the product was found.

4. In $4\% \times 75 = n$, does n represent the product or a factor? product ^ At the board, rename as in box A and find the number represented by n. See below.

5. Solve $4\% \times 75 = n$ by renaming the factor 4% with a decimal. Compare your answer with the answer for Ex. 4. $n = 0.04 \times 75 = 3$ They are the same.

6. Solve Ex. a–c first as in box A and then as in box B.
a. $n = 30\% \times 45$ 13.5, or $13\frac{1}{2}$ b. $45\% \times 20 = n$ 9 c. $n = 50\% \times 31$ 15.5, or $15\frac{1}{2}$

7. In $15\% \times n = 30$, does n represent the product or a factor? factor ^ When the product and a factor are known, what operation is used to find the unknown factor? Division ^ To find the number represented by n, we divide $\_?\_^{30}$ by $\_?\_^{15\%}$. Explain the work in box C for solving $15\% \times n = 30$ by renaming 15% with a fraction. Key idea: The per cent was renamed with a fraction and the division was done.

8. At the board, solve $15\% \times n = 30$ by renaming the known factor with a decimal and then dividing. What is the divisor? 0.15 ^ By what number do you multiply both the known factor and the product so that the unknown factor can be found by dividing by a whole number? 100

9. At the board, for each of Ex. a–c, rename the per cent with a fraction and then solve. Then, rename the per cent with a decimal and solve.
a. $20\% \times n = 55$ 275 ^ b. $470 = 47\% \times n$ 1,000 ^ c. $25\% \times n = 142$ 568

A

$n = 25\% \times 16$
$= \frac{25}{100} \times 16$
$= \frac{1}{4} \times 16$
$n = \frac{16}{4}$, or 4

B

$n = 25\% \times 16$
$n = 0.25 \times 16$

$$\begin{array}{r} 16 \\ 0.25 \\ \hline 80 \\ 32 \\ \hline 4.00 \end{array}$$

$n = 4$

C

$15\% \times n = 30$
$n = 30 \div 15\%$
$= 30 \div \frac{15}{100}$
$= 30 \div \frac{3}{20}$
$= 30 \times \frac{20}{3}$
$n = 200$

$n = 30 \div 15\%$
$= 30 \div 0.15$
$= 3{,}000 \div 15$
$n = 200$

Ex. 4

$n = 4\% \times 75$
$= \frac{4}{100} \times 75$
$= \frac{1}{25} \times 75$
$n = \frac{75}{25}$, or 3

Teaching Pages 262 and 263

Pupil's Objectives

(a) To review and practice expressing numbers with fraction, decimal, and per cent form; and (b) to learn the use of per cent forms with the factors-product relationship.

Background

Any form for a number can be expressed with any of the other forms. The one exception would occur with numbers named with decimals which are infinite non-repeating decimals which cannot always be expressed in the fraction form $\frac{a}{b}$ where a and b are integers and $b \neq 0$. These numbers are called irrational numbers.

Any rational number can be expressed with either a finite decimal or an infinite decimal which shows a repeating pattern in its digits. This pattern can be replaced by a fraction in the decimal form. An example of this is the well-known rational number named in fraction form by $\frac{1}{3}$. As a decimal form this can be written $0.33\frac{1}{3}$. However, there is no finite decimal form. It can be expressed as $0.333 \cdots$, where the three dots signify that the digit 3 repeats itself indefinitely. Two other common forms to denote this are $0.\dot{3}$, and $0.\overline{3}$ where the dot or the bar indicates that the 3 repeats indefinitely.

A similar situation exists with the rational number named by $\frac{41}{333}$. If this is expressed with a decimal form, the result is $0.123123 \cdots$ or, using the other two forms, $0.\dot{1}2\dot{3}$ and $0.\overline{123}$. This example also shows why the latter two forms are preferred. For each of these examples, a decimal and per cent form can be written. The first would be $0.33\frac{1}{3}$ and $33\frac{1}{3}\%$, and the second would be $0.123\frac{41}{333}$, or $0.12\frac{104}{333}$ and $12\frac{104}{333}\%$. These latter forms, although somewhat cumbersome in some instances, give us a numeral with which to operate.

The irrational numbers, which will not be used at this level, cannot be expressed with a finite decimal, an infinite decimal which shows a repeating sequence of digits, or a fraction. The number pi, represented by the Greek letter π, is an example of an irrational number and was discussed earlier on Teacher's Pages 254 and 255. Another example of an irrational number is the number which when multiplied by itself gives a product of 2. It is called the square root of 2 and is expressed in the form $\sqrt{2}$. It has an approximate value of 1.414214, and like π, can be expressed to any desired precision, but cannot be expressed exactly with a decimal form.

Recall that the factors-product relationship may be expressed by four related mathematical sentences:

$$a \times b = c \qquad b \times a = c$$
$$c \div a = b \qquad c \div b = a$$

If the product is known, division is required to find the unknown factor; if the product is to be found, multiplication is required. When per cent forms are involved with the factors-product relationship, there are three basic situations involved:

(1) the product is unknown,
(2) the factor named by the per cent form is unknown, or
(3) the product and the factor named by the per cent form are known and the other factor is unknown.

These may be represented by the following mathematical sentences:

(1) $n = 20\% \times 15$
(2) $3 = n \times 15$
(3) $3 = 20\% \times n$.

If the per cent form is known, express it with either the fraction or decimal form and proceed as in previous work. If the per cent is unknown, previous methods are used to find the unknown factor using either the fraction or decimal form. This is then expressed with the per cent form.

Thus, in (1) $n = 20\% \times 15$, we write
$n = \frac{20}{100} \times 15 = \frac{1}{5} \times 15 = 3$, or
$n = 0.20 \times 15 = 3.00 = 3$.

In (2) we have

$3 = n \times 15$, we write
$n = 3 \div 15 = \frac{3}{15} = \frac{1}{5} = \frac{20}{100} = 20\%$

$$n = 3 \div 15 = 15\overline{)3.0} = 0.20 = 20\%$$

In (3) we write

$3 = 20\% \times n$

$3 = \frac{1}{5} \times n$

$n = 3 \div \frac{1}{5} = 3 \times \frac{5}{1} = 15$, or

$n = 3 \div 0.20 = 3.00 \div 0.20 = 300 \div 20$

$n = 15$

Remember that each of the three situations listed above can be expressed by four different mathematical sentences. Thus, the first can be written: $n = 20\% \times 15$ $n = 15 \times 20\%$ $n \div 20\% = 15$ $n \div 15 = 20\%$

Pre-Book Lesson

• On the board, write $a \times b = c$ and ask the class to represent this relationship with three other mathematical sentences. Discuss the question, "Is the product known, or is the product to be found?"

• Write on the board a problem such as $n = 50\% \times 60$. Ask what operation is required to find the value for n. Do the same for $30 = n \times 60$ and $30 = 50\% \times n$. Having decided that either multiplication or division is necessary, they must find methods for using per cent forms with these operations. They should suggest that since they know how to perform operations with decimals or fractions, all that is necessary to solve mathematical sentences involving per cent is to express the per cent with a decimal or a fraction. Leave the solution for the examples on the board for easy reference.

Using the Text Page

• Ex. 1–5. Discuss these using the material in the Pre-Book Lesson and the Background.

• Ex. 7–8. Write on the board 8 mathematical sentences expressing the factors-product relationship when the per cent is the unknown factor and the known factor. Have pupils label the product and the factors and leave this on the board for easy reference.

• Ex. 10–11. These should be done in detail at the board. Do Ex. 10, using the fraction form:

$$n = 16 \div 64 = \frac{16}{64} = \frac{1}{4} = \frac{25}{100} = 25\%$$

• Ex. 12–29. Ask pupils to summarize for the examples whether the decimal or fraction form seemed easier. At the same time, emphasize that either method is correct.

• Ex. 1–20 (page 263). If pupils make errors in these examples, refer them to pupil's pages 258–261 where renaming is first discussed.

Individualizing Instruction

• *More capable pupils* should make up problems similar to those on the page but involving more difficult per cent forms and try them out on each other. Use such examples as

(a) $n = 16\frac{2}{3}\% \times 126$

(b) $4 = n \times 6$, expressing the unknown factor as a per cent, with a decimal, and with a fraction, respectively.

(c) $7 = 12\frac{1}{2}\% \times n$.

• *Slower learners* will need extra help and practice with those examples involving division with decimal forms. Review with them division with decimals and renaming rational numbers, then relate these ideas to division involving per cent.

10. For some problems, the unknown factor may be expressed with a per cent form. Explain the work in box D for solving $n = 16 \div 64$ and the renaming of the answer with a per cent form. Key idea: The unknown factor is named with a decimal, a fraction with denominator 100, and then a per cent form.

D

$n = 16 \div 64$

$$64\overline{)16.00}\quad 0.25$$

$12\,8$

$3\,20$

$3\,20$

$n = 0.25$, or $\frac{25}{100}$, or 25%

11. At the board, solve Ex. a–d using either fractions or decimals. Express the answer for Ex. b and Ex. c with a per cent form.

a. $n = 35\% \times 60$ 21 **b.** $32 = n \times 160$ 20%

c. $n \times 100 = 15$ 15% **d.** $40\% \times 25 = n$ 10

[W]

Ex. 12–17. Rename the per cent with a fraction and then solve. Express the answer in simplest form.

12. $n = 50\% \times 44$ 22 **13.** $x = 15\% \times 125$ $18\frac{3}{4}$ **14.** $25\% \times n = 3\frac{1}{4}$ 13

15. $n = 75\% \times \frac{1}{2}$ $\frac{3}{8}$ **16.** $10\% \times 60 = n$ 6 **17.** $75\% \times x = 20$ $26\frac{2}{3}$

Ex. 18–23. Rename the per cent with a decimal and solve.

18. $n = 23\% \times 45$ 10.35 **19.** $n = 13\% \times 25.6$ 3.328 **20.** $3\% \times 97 = n$ 2.91

21. $n = 697 \times 63\%$ 439.11 **22.** $1\% \times n = 2.5$ 250 **23.** $75\% \times x = 765$ 1,020

Ex. 24–29. Solve. For Ex. 28 and 29, express the answer with a per cent form.

24. $n = 24\% \times 35$ $8\frac{2}{5}$, or 8.4 **25.** $90\% \times 48 = n$ $43\frac{1}{5}$, or 43.2 **26.** $51 = x \times 75\%$ 68

27. $40\% \times 695 = n$ 278 **28.** $x \times 100 = 8$ 8% **29.** $10 = n \times 200$ 5%

Without Paper and Pencil

[O]

Ex. 1–10. Rename with a per cent form.

1. 0.10 10% **2.** 0.37 37% **3.** 0.55 55% **4.** $\frac{4}{100}$ 4% **5.** $\frac{87}{100}$ 87%

6. 0.7 70% **7.** $\frac{3}{4}$ 75% **8.** $\frac{4}{5}$ 80% **9.** 1 100% **10.** $\frac{3}{25}$ 12%

Ex. 11–20. Rename with a fraction in simplest form.

11. 13% $\frac{13}{100}$ **12.** 90% $\frac{9}{10}$ **13.** 50% $\frac{1}{2}$ **14.** 40% $\frac{2}{5}$ **15.** 5% $\frac{1}{20}$

16. 75% $\frac{3}{4}$ **17.** 80% $\frac{4}{5}$ **18.** 10% $\frac{1}{10}$ **19.** 55% $\frac{11}{20}$ **20.** 25% $\frac{1}{4}$

* Have pupils make a table as below for each per cent problem. Notice that the ratio factor, base factor, and product are first described in words, represented by numerals, and then the relationship for the problem is expressed in a mathematical sentence.

Ratio Factors and Base Factors

[o]

* **1.** Because of the Dutch elm disease, 25% of the 12 American elm trees in City Park had to be cut. How many elm trees had to be cut?

a. The mathematical sentence $25\% \times 12 = n$ shows the relationship expressed in the problem.

Ratio Factor		Base Factor		Product
Per cent for trees cut		Elm trees in park		Number of trees cut
25%		12		n
25%	$\times$	12	=	n

b. The situation is *based* on the number of American elm trees in the park, and that number is a factor in the mathematical sentence $25\% \times 12 = n$. So, we call that basic number the *base factor*. In the box at the left, what description is written under *Base Factor*? Elm trees in park What number is the base factor? 12

c. The per cent form 25% expresses the ratio for the problem. That is, 25% expresses the ratio (or per cent) for the number of trees cut. What description is written in the box under *Ratio Factor*? Per cent for trees cut

** **d.** When the two factors, the base factor and the ratio factor, are known, what operation is used to find the product? Multiplication What description is written in the box under *Product*? Number of trees cut Notice that the ratio factor and the product both refer to *trees cut*.

e. At the board, solve the sentence given in the box for the problem. $n =$ _?3_ How many American elm trees were cut? 3

** Point out how the ratio factor directly refers to the product. This can be emphasized by the word description in the charts.

Teaching Pages 264 and 265

Pupil's Objectives

(a) To learn new vocabulary relating to the use of per cent forms in a mathematical sentence; (b) to learn to apply a mathematical sentence with a per cent form to the solution of verbal problems; and (c) to review and practice solving mathematical sentences involving ratio factors and base factors.

Background

The factors-product relationship can be expressed by any one of four mathematical sentences. The two forms which are especially useful when a per cent form is involved are $a \times b = c$, and $b \times a = c$. For the purposes of this lesson, since the commutative property justifies one form on the basis of the other, these are considered as one. In $50\% \times 40 = n$, 50% represents a ratio, and is given the name *ratio factor*.

Recall that per cents are ratios which compare the number of members in one set to the number of members in a set containing 100 members. Rewriting the sentence above as $50\% = n \div 40$, or $50\% = \frac{n}{40}$, we see that $\frac{50}{100} = \frac{n}{40}$, thus the ratio factor and the product, n, refer to the same quantity. 100 and 40 refer to the total numbers of members in the sets being compared. Since this number, 40, is *basic* to the ratio in the problem and is a factor in the mathematical sentence, it is given the name *base factor*. An example will illustrate the use of the vocabulary.

> Jim has a bag of marbles which contains 40 marbles of different colors. His father says that 50% of them are blue. How many of these marbles are blue?

The mathematical sentence that shows the relationship for the problem is $50\% \times 40 = n$. The problem is *based* on the number of marbles in the bag, 40, which is also a *factor* in the mathematical sentence. Thus, 40 is the *base factor*.

The per cent form, 50%, names the *ratio* for the number of marbles which are blue out of 100 marbles and is a *factor* in the mathematical sentence. Thus 50% is called the *ratio factor*.

Note that 50% and n refer to the number of blue marbles in each set, whereas 100 and 40 name the total or base number of marbles in each set respectively. Since the two factors, the base factor and the ratio factor are known, multiplication is required to find the product, n. Thus, the solution would follow as

$$\begin{aligned} 50\% \times 40 &= n \\ \tfrac{50}{100} \times 40 &= n \\ \tfrac{1}{2} \times 40 &= n \\ 20 &= n \end{aligned} \quad \text{or,} \quad \begin{aligned} 50\% \times 40 &= n \\ 0.50 \times 40 &= n \\ 20.00 &= n \\ 20 &= n \end{aligned}$$

Hence there are 20 blue marbles in the bag.

If the known factor is the ratio factor, the solution merely calls for renaming this per cent form with a fraction or decimal form to carry out the division. If the known factor is the base factor, the division is carried out in decimal form or expressed with a fraction form, and this result is then renamed with a per cent form.

Pre-Book Lesson

- Review the idea of per cent as a ratio and have pupils express several ratios with per cents.
- Write on the board:

> John has 12 pennies. 25% of them are new. How many of the pennies are new?

Ask a pupil to represent the relationship with a mathematical sentence on the board. He should write $25\% \times 12 = n$. Point out that the problem is *based* on the total number of pennies and that 12 is a *factor* in the mathematical sentence, thus it is called the *base factor*. Also point out that 25% names the *ratio* for the problem, so 25% tells the *ratio*, or per cent, for the number of new pennies. It is also a *factor* in the mathematical sentence, and is therefore called the *ratio factor*. Label the factors in the mathematical sentence and leave it on the board for later reference. Summarize this information as shown below.

Ratio Factor		Base Factor		Product
per cent for new coins		total number of coins		number of new coins
25%		12		n
25%	$\times$	12	$=$	n

Discuss the summary using material from the Background. Solve the problem:

$$25\% \times 12 = n \quad \text{or} \quad 25\% \times 12 = n$$
$$\tfrac{25}{100} \times 12 = \qquad 0.25 \times 12 =$$
$$\tfrac{1}{4} \times 12 = \qquad 3.00 =$$
$$3 = n \qquad 3 = n$$

Hence John has 3 new pennies.

• Use the problem about the marbles from the Background and have pupils make a table at the board and solve the problem. Leave the work on the board for easy reference.

Using the Text Pages

• Ex. 1. Make a table for Ex. 1 similar to the one on the board. Discuss each step of this example in detail, stressing the new vocabulary as discussed in the Pre-Book Lesson and the solution on the board.

• Ex. 2. Analyze this in the same manner as Ex. 1, but without making a table.

• Ex. 3. Have pupils answer this for each of Ex. 4–7 before working any of them.

• Ex. 4–7. Have pupils make a complete solution as outlined at the board for Ex. 1.

• Ex. 5. Remind pupils not to show the dollar sign in mathematical sentences.

Individualizing Instruction

• *All pupils* should be asked to make up an example involving per cent from their own experiences similar in nature to those in the lesson.

• *Slower learners* will need individual help in filling in the items in the table for Ex. 4–7 and in writing the mathematical sentences. If they have further difficulty solving these sentences, they should turn to pages 262 and 263 to rework some of those exercises.

• *More capable pupils* should make up problems similar to these, but containing per cent forms which are more difficult to work with and try them out on each other. Use such examples as

Sue has eight blouses, $37\frac{1}{2}\%$ of which are long sleeved. How many of Sue's blouses have long sleeves?

Sue has eight blouses, 3 of which have long sleeves. What per cent of her blouses are long sleeved?

ANSWERS NOT SHOWN ON PAGE 266

	Ratio Factor	Base Factor	Product
3.	Per cent for examples worked correctly n	Number of examples on test 16	Number of examples worked correctly 12
4.	Per cent for games won n	Number of games 25	Number of games won 19
5.	Per cent for games Jeff played n	Number of games 25	Number of games Jeff played 7
6.	Per cent for what Mr. Baumer received n	Number of price 250	Number of what Mr. Baumer received 50
7.	Per cent for reduction n	Number of price 6.4	Number of reduction 4

Ratio Factor: 4. Per cent for examples worked incorrectly: 15% | Base Factor: Examples on test: 20 | Product: Number of examples worked incorrectly: n

Ratio Factor: 5. Per cent for amount saved: 32% | Base Factor: price of dress: 28.75 | Product: Number of amount saved: n

2. David gave away 75% of the 60 seashells that he had collected. How many seashells did he give away?

a. The situation is based on the number of seashells in the collection so the base factor is _?_60.

b. On the board, copy and complete the table at the right, writing the missing descriptions and numerals.

Ratio Factor	Base Factor	Product
Per cent for seashells given away	seashells	Number of seashells given away
_?_	in collection	_?_
75%	_?_60	_?_n

75% × _?_60 = _?_n

c. What is similar about the descriptions for the ratio factor and the product? Both refer to seashells given away

d. What is a mathematical sentence for the problem? 75% × 60 = n Since the base factor and the ratio factor are known, we multiply to find the product. $n =$ _?_45

e. How many shells did David give away? 45 shells

3. For Ex. 4–7, both the base factor and the ratio factor are known. Name the base factor and the ratio factor for each problem.

	base factor	ratio factor
4.	20	15%
5.	28.75	32%
6.	650	42%
7.	35	40%

[W]

Ex. 4–7. Analyze the problem by writing a description and either *n* or a numeral for each of the ratio factor, the base factor, and the product. Then write and solve a mathematical sentence to find the answer for the problem. See above and below.

4. Alice worked 15% of the 20 examples on the test incorrectly. How many examples did she work incorrectly? 15% × 20 = n; 3

5. In 3 months, Edith saved 32% of the price of a new dresser for her room. If the price of the dresser was $28.75, how much had Edith saved? 32% × 28.75 = n; $9.20

6. At noon, Mr. Rogers started on a 650-mile trip. When he stopped for supper, he had completed 42% of the trip. How far had he gone that afternoon? 42% × 650 = n; 273 miles

7. One stormy day, 40% of a class of 35 pupils were absent. How many pupils were absent that day? 40% × 35 = n; 14

Ratio Factor: 6. Per cent for miles completed: 42% | Base Factor: miles of trip: 650 | Product: Number of miles completed: n

Ratio Factor: 7. Per cent for pupils absent: 40% | Base Factor: pupils in the class: 35 | Product: Number of pupils absent: n

Finding the Ratio Factor

[o]

1. Mr. Parker has a 25-acre farm. He used 3 of those acres for tomatoes. What per cent of the 25 acres did he use for tomatoes?

A

Ratio Factor	Base Factor	Product
Per cent for acres used for tomatoes	Number of acres in farm	Number of acres used for tomatoes
n	25	3

$n \times 25 = 3$

B

$n \times 25 = 3$

$n = 3 \div 25$

$$25\overline{)3.00}\quad 0.12$$

$$\begin{array}{r} 25 \\ \hline 50 \\ 50 \end{array}$$

$n = 0.12$, or 12%

a. On the board, copy and complete box A by writing the missing descriptions and numerals.

b. The ratio is unknown, so n represents the ratio factor. The base factor and the product are known, so we divide to find the ratio factor.

c. Explain the work in box B. Key idea: The unknown factor is found and renamed with a per cent form.

2. For each of Ex. 3–7, name the base factor and the product.

	Base factor	Product
3.	16	12
4.	25	19
5.	25	7
6.	250	50
7.	6.40	4.00

[w]

Ex. 3–7. Analyze the problem by writing a description and either n or a numeral for each of the ratio factor, the base factor, and the product. See Teacher's Page 265. Then write and solve a mathematical sentence to find the answer for the problem. Express the answer with a per cent form.

3. Miss Kasson gave an arithmetic test of 16 examples. Eric worked 12 examples correctly. What per cent of the 16 examples did Eric work correctly? $n \times 16 = 12$; 75%

4. Casco High won 19 of 25 baseball games. What per cent of the 25 games did they win? $n \times 25 = 19$; 76%

5. Jeff played in 7 of the 25 games. That was what per cent of the games? $n \times 25 = 7$; 28%

6. For selling a $250 rug, Mr. Baumer, a salesman, received $50. What per cent of the selling price of the rug did Mr. Baumer receive? $n \times 250 = 50$; 20%

7. At a book shop, the price of a book was reduced from $6.40 to $4.00. The new price is what per cent of the former price? $n \times 6.4 = 4$; $62\frac{1}{2}$%

Teaching Page 266

Pupil's Objective

To work problems which require solving a mathematical sentence in which the ratio factor is to be found and named with a per cent form.

Background

In a mathematical sentence such as $25\% \times 12 = n$, 25% is called the ratio factor, 12 is the base factor, and n is the product. "Fred has 12 baseball cards. 25% of the cards are for players in the National League. How many of Fred's cards are for the National League players?"

This problem is based upon the total number of cards in Fred's collection, 12. 25% tells the ratio, or per cent, for the number of National League players. The product is the number of cards for players in the National League and is unknown. Since the two factors, the ratio factor and the base factor, are known, multiplication is required to find the product. This is accomplished by renaming the per cent with a fraction or a decimal.

Another related problem is "Fred has 12 baseball cards. Three of the cards are for players in the National League. What per cent of his cards are for National League players?"

The mathematical sentence that represents the factors-product relationship of this problem is $n \times 12 = 3$. The problem is based upon the total number of cards, 12, hence 12 is the base factor. The ratio factor, n, is unknown, and 3 is the product which names the number of National League players for which Fred has cards. Because the product is known, division is required to find the unknown ratio factor.

Pre-Book Lesson

- Review the work from pages 264 and 265.
- On the board write:

Sally has 25 jacks, 3 of which are red. What per cent of Sally's jacks are red?

Tell the class that in order to represent this problem with a mathematical sentence we write,

Ratio Factor		Base Factor		Product
per cent for red jacks		total number of jacks		number of red jacks
n		25		3
n	$\times$	25	$=$	3

$n = 3 \div 25$

$= \frac{3}{25}$ or $= 3.00 \div 25$

$= \frac{3}{25} \times \frac{4}{4}$ $\quad 0.12$

$= \frac{12}{100}$ $\quad = 25\overline{)3.00}$

$\quad 25$

$\quad 50$

$\quad 50$

$= 0.12$

$n = 12\%$

Leave this work on the board for later reference.

Using the Text Page

- Ex. 1. Discuss each step of this in detail, referring to the completed example at the board and material from the Background. Make sure pupils use the table in the text when working this exercise.
- Ex. 3–7. The students should show a complete solution as indicated by the example on the board. Some pupils may need to make a table for each of the examples and may need to label the ratio and base factor in each of their mathematical sentences.
- Ex. 6–7. Remind pupils that the dollar sign is not shown in the mathematical sentences.

Individualizing Instruction

- *More capable pupils* may be able to write the mathematical sentence without filling in the entries to the table each time. This should be allowed, but only after they have completed one or two exercises. When these pupils work Ex. 6, you may wish to discuss the idea of commission with them.
- *Slower learners* will need more practice naming the base factor, ratio factor, and the product. Make sure they label each mathematical sentence. They might be helped by referring to pages 264 and 265.

Pupil's Objective

To solve problems which require solving a mathematical sentence with a per cent form and in which the base factor is to be found.

Background

Most verbal problems which involve a per cent form are examples which show a factors-product relationship. The mathematical sentence then takes the form $a \times b = c$, in which a is written with a per cent form, or is unknown.

a is the ratio factor, b is the base factor, and c is the product. The type to be considered in this lesson is that in which the ratio factor and the product are known, and it is desired to find the base factor. Since the product is known, division is required to complete the solution. This is accomplished by naming the ratio factor with a fraction or decimal form.

The main difficulty for students lies in recognizing the base factor and the ratio factor. Recall that in a verbal problem the base factor is that number upon which the problem is based and with practice is, in general, not difficult to recognize from the wording of the problem. The ratio factor is easily selected, especially if it is written in a per cent form, which is usually the case.

Pre-Book Lesson

• Review the material from pupil's pages 262–266 prior to turning to page 267. Make sure pupils have the concepts from these pages firmly established.

• On the board write,

> David has 33 foreign stamps and this represents 22% of the total number of stamps in his collection. How many stamps has he in all?

Ask the class to suggest a method to help them decide the mathematical sentence to use to show the factors-product relationship in this problem. Have them explain the meaning of ratio factor, base factor, and product for this particular problem. Write on the board

Ratio Factor		Base Factor		Product
per cent for foreign stamps		total number of stamps		number of foreign stamps
22%		n		33
22%	$\times$	n	$=$	33

The pupils should suggest the items in completing this table and steps in the solution. Ask, "Is the product known or is the product to be found?" Since the product is known, division is required in the solution.

$$\begin{aligned} n &= 33 \div 22\% \\ &= 33 \div \tfrac{22}{100} \quad \text{or} \quad 33 \div 0.22 \\ &= 33 \times \tfrac{100}{22} \qquad = 3{,}300 \div 22 \\ &= 150 \qquad\qquad = 150 \end{aligned}$$

150 is the total number of stamps in David's collection. Ask the class how they could check the validity of this answer. Leave the table and the solution on the board so they can be used in the discussion of the first example in the text.

Using the Text Page

• Ex. 1. Discuss each step in detail, using the completed example from the Pre-Book Lesson and material from the Background.

• Ex. 2. The procedure here should be followed before the pupils are allowed to proceed with the solution of any of Ex. 3–5.

• Ex. 3–5. A complete solution as illustrated on the board for Ex. 1 should be required of all pupils for these problems. Some pupils may need to use tables for each of the examples and their solutions.

Individualizing Instruction

Slower learners who are having difficulty naming the base factor should be given problems with no numbers and from which the words relating to the base factor are to be selected.

Then, give them problems with numbers, but do not require them to find the answer. Use them only to practice naming the base factor and setting up a mathematical sentence.

* Emphasize the relationship between the ratio factor and the product. The most common error will be mistaking the product for the base factor in this type of problem.

*Finding the Base Factor

[O]

1. Mary sold 22% of all the tickets that were sold for the spring play. She sold 33 tickets. How many tickets were sold in all?

a. What is the ratio factor? 22% To what does the ratio factor refer? Per cent for tickets Mary sold

b. The ratio factor is the per cent for the number of tickets that Mary sold. Describe in words what the product represents. Number of tickets Mary sold The numeral for the product is _?_ 33

c. Is the base factor known? No Let n represent the base factor. Complete the blanks in box A and then give a mathematical sentence for the problem. 22% × n = 33

d. The ratio factor and the product are known, so to find the base factor we divide the product by the ratio factor _?_.

e. Explain the work in box B. On the board, solve $22\% \times n = 33$ by renaming the ratio factor with a decimal and then dividing. See right. How many tickets were sold in all? 150

A

Ratio Factor	Base Factor	Product
Per cent for tickets Mary sold	Number of tickets sold in all	Number of tickets Mary sold
_?_ 22%	_?_ n	_?_ 33

B

$$22\% \times n = 33$$
$$n = 33 \div 22\%$$
$$= 33 \div \frac{22}{100}$$
$$= 33 \times \frac{100}{22}$$
$$n = 150$$

n = 33 ÷ 0.22
= 3,300 ÷ 22
= 150

2. Ex. 3–5. Write a description and either n or a numeral for the ratio factor, the base factor, and the product. Then write a mathematical sentence for each problem.

[W]

Ex. 3–5. Now solve the mathematical sentence to find the answer for each problem.

3. After driving 68 miles, Joan's father said that they had completed 80% of their trip. When their trip is completed, they will have driven how many miles? 80% × n = 68; 85

4. Harry said that 62% of the pupils in his class are girls. There are 31 girls in Harry's class. In all, how many pupils are there in Harry's class? 62% × n = 31; 50

5. Jim spent 75¢. That was 20% of his earnings for raking leaves. How much had he earned raking leaves? 20% × n = 75; 375¢, or $3.75

2.

	Ratio Factor	Base Factor	Product
3.	Per cent for miles completed 80%	Number of miles in trip n	Number of miles completed 68
4.	Per cent for girl pupils 62%	Number of pupils n	Number of girl pupils 31
5.	Per cent for earnings spent 20%	Number of earnings n	Number of earnings spent 75

* Insist on a table for each problem, so that the problem analysis will be correct.

Per Cent Problems

[O]

* 1. Ex. 2–7. Give a description and either n or a numeral for the ratio factor, the base factor, and the product. Then give a mathematical sentence for the problem and then tell what operation to use to solve the sentence. See Teacher's Page 270.

[W]

Now, for each of Ex. 2–7, solve the mathematical sentence to find the answer for the problem.

2. Last year, 480 people attended the first basketball game of the season but this year only 384 attended. The attendance this year was what per cent of last year's attendance? n × 480 = 384; Division; 80%

3. Instead of receiving a fixed salary, Mr. Thomas receives 15% of his total sales. One week, his total sales were $984.40. How much did he receive that week? 15% × 984.40 = n; Multiplication; $147.66

4. Last week, Mrs. Olson paid 60¢ per dozen for eggs. This week, the price is 45¢ per dozen. The price this week is what per cent of last week's price? n × 60 = 45; Division; 75%

5. Jerry traded 96 stamps from his stamp collection for some old coins. The 96 stamps were 12% of the stamps in his collection. How many stamps did he have before he traded for the coins? 12% × n = 96; Division; 800

6. Because she works at a shoe store, Mrs. Jarvis only pays 85% of the selling price for a pair of shoes. What was the selling price of a pair of shoes that cost Mrs. Jarvis $10.54? 85% × n = 10.54; Division; $12.40

7. Mr. Kern, a contractor, owes $846.50 to Kip Lumber Company. If he pays within 30 days, that amount will be reduced by 2%. How much will he save if he pays within 30 days? 2% × 846.50 = n; Multiplication; $16.93

For each of Ex. 8–13, write whether the ratio factor or the base factor is unknown and then solve.

8. $32\% \times n = 147.84$ (base factor; 462)
9. $n \times 64 = 16$ (ratio factor; 25%)
10. $5\% \times n = 3.2$ (base factor; 64)
11. $15\% \times n = 10.8$ (base factor; 72)
12. $12\frac{1}{2}\% \times n = 8.5$ (base factor; 68)
13. $n \times 55 = 11$ (ratio factor; 20%)

Teaching Page 268

Pupil's Objective

To solve problems which require selecting the base factor, ratio factor, and product.

Background

To solve a problem which contains a per cent form or asks for a per cent form, it is best first to determine the base factor, the ratio factor, and then product.

Recall that the base factor is the number on which the problem is based. The ratio factor names the ratio for the problem. The product refers to the number of the same thing referred to by the ratio factor.

> "Mrs. Simmons bought a coat at a saving of 15% on the selling price. The selling price of the coat was \$24.00. How much did Mrs. Simmons save?"

The number on which the problem is based is the original selling price, 24, hence 24 is the base factor. The ratio factor is indicated with 15% and states the ratio for the amount of money saved. The product is the number which indicates the amount of money saved and is unknown.

Once these selections are made, the mathematical sentence which represents the factors-product relationship is not difficult to write. Thus for this problem we have $15\% \times 24 = n$ and, since the two factors are known, multiplication is required and is accomplished by renaming the per cent form with a fraction or decimal form.

Pre-Book Lesson

- Review pages 262–267 by working random examples at the board.
- On the board write:

  Base Factor Ratio Factor Product

Discuss these terms in a general way eliciting from pupils what each means and what each can show in a problem. Ask for a mathematical sentence showing the relationship of these three terms and how to decide whether division or multiplication is required to find the unknown.

Using the Text Page

- Ex. 1. Students should find it helpful to list the answers in a table for reference.

	Base Factor	Ratio Factor	Product	Operation
Ex. 2.	----	----	----	----
Ex. 3.	----	----	----	----

- Ex. 2–7. Write and leave on the board a complete solution for Ex. 2, including a mathematical sentence and a table.
- Ex. 8–13. Pupils should notice that since the product is given in each of these, division is required for their solution.

Individualizing Instruction

- *Slower learners* who continue to have difficulty selecting the base factor should refer to examples on previous pages, not solving them, but merely naming the base factor in each. If there are any reading difficulties, read through problems with the pupils.
- *More capable pupils* should bring to class advertisements from recent publications in which a per cent form is involved, make up problems based on the information listed, and try them out on each other.

Teaching Page 269

Pupil's Objective

To solve problems which require solving mathematical sentences with a per cent form.

Background

Recall that when a per cent form is involved in the factors-product relationship, the mathematical sentence takes the form $R \times B = P$ where P represents the product, R the ratio factor, and B the base factor. Thus, verbal problems arise in which either the product, ratio factor, or base factor is to be found. These exercises give the pupil experience in deciding what is unknown and how to proceed in finding it.

Pre-Book Lesson

• Review the factors-product relationship as it applies to problems with a per cent form. On the board write:

*p*roduct	=	*r*atio factor	×	*b*ase factor
P	=	R	×	B

Have the pupils express the factors-product relationship illustrated here with three other mathematical sentences.

• Ask the pupils how they decide which numbers in a verbal problem refer to each of the product, ratio factor, and base factor. Ask the class how they decide which operation to use.

Using the Text Page

• Have the class decide, orally, what is to be found in each problem before they start work.

• Ex. 1. This should be worked in detail at the board, using the same procedure as that on pages 262 and 263.

• Ex. 2–8. The relationship in each problem should be expressed as a mathematical sentence before performing any calculations for that problem.

Individualizing Instruction

• Have *all pupils* bring to the class clippings from recent publications in which per cent forms are used in situations similar to these.

• *More capable pupils* could be asked to consider problems in which per cent forms are involved such as the following: 110%, 150%, 200%, 0.5%, $\frac{3}{4}$%, and so on.

They should compose and present to the class problems containing these forms.

• *Slower learners* may need help in understanding the problems. In that case, read the problems with them and make sure they understand the meaning of such words as *satellite*, *orbit*, *space probe*, *cloud formation*, *launch vehicle*, *missile*, *thrust*, and *first stage*. They may need to write out a table for each example before writing the mathematical sentence.

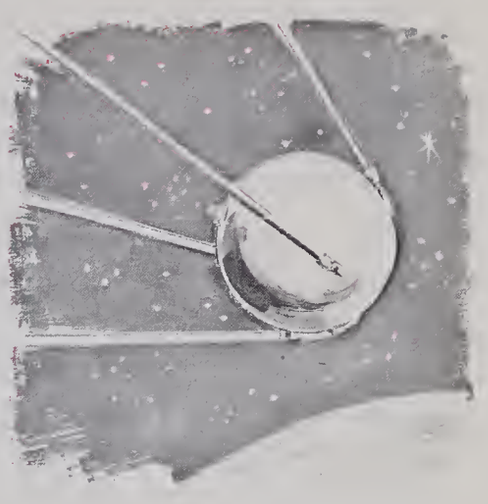
Looking Ahead

In anticipation of the lesson on pupils' page 276, have pupils keep track of the amount of money spent on candy and the amount of milk consumed each day for a week. This data will be tabulated and used in making graphs. The personal preparation of the materials will motivate the lesson.

* Encourage the collecting of scientific data, so pupils can formulate problems of this type.

* Using Mathematics

Per Cents [W]



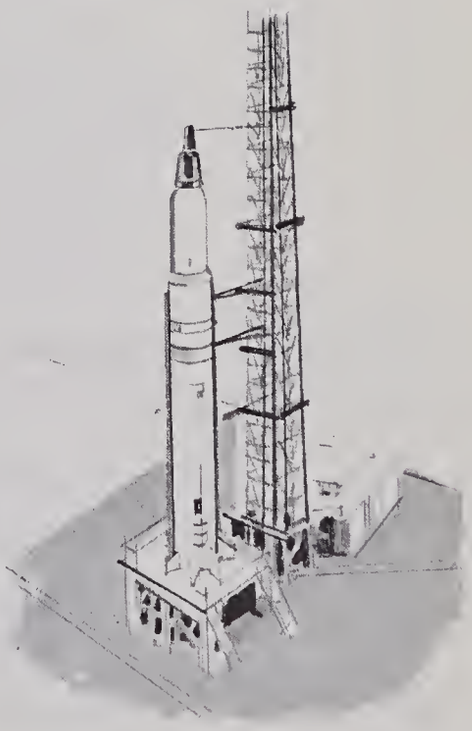
1. Sputnik I was the first man-made Earth satellite. When in orbit, the least distance of Sputnik I from Earth was about 25% of its greatest distance. Find the least distance if its greatest distance was 560 miles. 140 miles

2. The least distance of a satellite from Earth is 533 miles and its greatest distance is 650 miles. The least distance is what per cent of the greatest distance? 82%

3. A satellite can make an orbit in 80% of the time that it takes Mr. Powers to drive 105 mi. to Chicago. If it takes Mr. Powers 2 hr. 5 min. to drive to Chicago, how long does it take the satellite to make 1 orbit? (Hint: 2 hr. 5 min. is how many *minutes*?) 1 hr. 40 min., or 100 min.

4. The outer shell of a space probe weighs 126 pounds. What is the weight of the space probe if the weight of the outer shell is 24% of the total weight? 525 pounds

5. Of 250 pictures taken during a space flight, 60% were of various cloud formations. How many pictures were of cloud formations? 150



6. The launch vehicle shown at the right is 140 feet tall. The length of a 7-ft. spacecraft is what per cent of the height of the launch vehicle? 5%

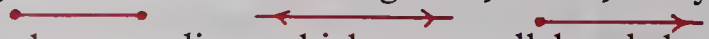
7. The first stage of a missile has a thrust of 300,000 lb. The second stage has a thrust of 90,000 lb. The thrust of the second stage is what per cent of the thrust of the first stage? 30%

8. The thrust of the first stage of a missile is 425,000 lb. Find the thrust of the second stage if it is 35% of the thrust of the first stage. 148,750 lb.

Remembering about Geometric Figures

Resurvey [O]

1. A line segment has how many end points? 2

2. On the board, draw a model of a segment; a line; a ray.
Sample answers: 

3. On the board, show two lines which are parallel and then explain what parallel lines are. Sample answer:
Non-intersecting lines in the same plane

4. Describe, as precisely as you can, each of the following: See below.

a. A rhombus b. A parallelogram c. An angle

d. A regular polygon e. A scalene triangle f. A trapezoid

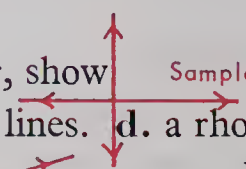
5. Explain how to find the area of a rectangular region. Find the product of the measure of the length and the measure of the width.

6. A simple closed curve partitions a plane into what three sets of points? Those interior to the curve, exterior to the curve, and belonging to the curve

7. On the board, show two different rays, not in the same line, which have a common end point. What kind of geometric figure is formed by the union of the two rays? ; angle, or line

8. In degrees, what is the sum of the measures of the three angles of a triangle? 180

[W]

9. On your paper, show Sample answers: 

a. two perpendicular lines. d. a rhombus which is not a square.

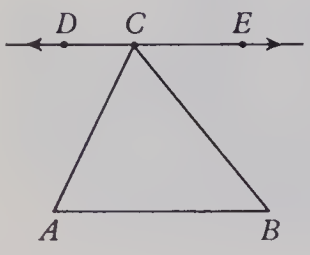
b. an acute angle. e. a scalene triangle.

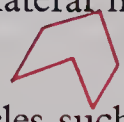
c. an obtuse angle. f. three parallel segments.

Ex. 10–15. Refer to the figure at the left below.

10. $\overleftrightarrow{DE} \cap \overline{AC} =$ _?_ {C} 11. $\overleftrightarrow{DE} \cap \triangle ABC =$ _?_ {C} 12. $\overline{CE} \cap \overline{AB} =$ _?_ { }

13. $\overline{AC} \cap \overline{BC} =$ _?_ {C} 14. $\overrightarrow{CE} \cup \overline{CE} =$ _?_ $\overrightarrow{CE}$ 15. $\angle ACB \cap \overline{AB} =$ _?_ {A,B}



16. On your paper, show an equilateral hexagon which is not a regular hexagon. Sample answer 

17. On your paper, show two circles such that one of the circles appears to be a subset of the interior for the other circle.

4. Sample answers: a. equilateral quadrilateral; b. quadrilateral with opposite sides parallel; c. union of two rays, not in the same line, with a common end point; d. equilateral and equiangular polygon; e. triangle with no two sides congruent; f. quadrilateral with exactly one pair of parallel sides.

Teaching Page 270

Pupil's Objective

To review some of the concepts, vocabulary, and symbols related to geometric figures.

Background

The purpose of this review is to determine if pupils have any weaknesses in geometric concepts to which the questions pertain. This will enable them to restudy the concepts in which weakness is revealed.

Pre-Book Lesson

You may wish to use this page for testing purposes rather than review. In that case, have pupils work several examples similar to these at the board as a brief general review. Assign the examples to be worked independently by the pupils.

Using the Text Page

- Ex. 1–4. Discuss these examples at the board. Stress understanding and use of vocabulary terms. Then have pupils write the answers.
- Ex. 5–17. Before assigning these, discuss the symbols $\leftrightarrow$, $-$, $\cup$, $\cap$, and $\angle$.

Individualizing Instruction

- If this page has been used as a test, group *all pupils* having similar weaknesses for reteaching purposes. You may wish to have them reread appropriate pages in the book. Then they may work selected exercises from these pages for remedial work.
- *More capable pupils* may work Supplementary Activities after they have completed the work on the page.

ANSWERS NOT SHOWN ON PAGE 268

	Ratio Factor	Base Factor	Product
2.	Per cent for people who attended this year n	Number of people who attended last year 480	Number of people who attended this year 384
3.	Per cent for what Mr. Thomas received 15%	Number of total sales 984.40	Number of what Mr. Thomas received n
4.	Per cent for this week's price n	Number of last week's price 60	Number of this week's price 45
5.	Per cent for stamps traded 12%	Number of stamps in collection n	Number of stamps traded 96
6.	Per cent for amount paid 85%	Number of selling price n	Number of amount paid 10.54
7.	Per cent for reduction 2%	Number of amount owed 846.50	Number of reduction n

Teaching Pages 271, 272, and 273

Pupil's Objectives

To take the regular end-of-chapter tests including: (a) Test of Information and Meaning 6; (b) Diagnostic Test 6; (c) Problem Test 6; and (d) Computation Test 6.

Background

These test results should serve not only as a measurement of each pupil's ability, but also as a means of diagnosing and remedying each pupil's weaknesses. The teacher may wish to refer to previous Background sections prior to a general chapter review with special emphasis on material not understood by some pupils.

Pre-Book Lesson

Instructions for using the test pages should be given at this time. Suggest to *more capable pupils* that they may work the Supplementary Activities after they complete the tests.

Using the Text Page

- These tests may be given in any order, but no more than two should be given in a day. Pupils should be encouraged to check their work wherever possible, and follow a general problem-solving procedure when working problems.
- Have pupils scan each test to be sure they understand the directions.
- After each test have the pupils explain the reasons for their responses. For the problems in Problem Test 6, have the pupils explain their choice of operation used in solving each problem.

Individualizing Instruction

- After the papers have been scored, they should be immediately returned to *all pupils*. They should correct the errors and do additional exercises and problems of the type in which they made the error. You may wish to have them work preselected examples from the text or make up other exercises on your own for them to use.

To visualize areas of individual and class weakness more readily, you may want to prepare error-analysis charts showing the examples each pupil failed to answer correctly and the total number of pupils who failed on each example.

- If a sizable number of pupils show weakness in any area, the concept should be retaught and the pupils retested in order to be sure that they understand the material.
- *More capable pupils* will probably not need to have reteaching experiences and may work the Supplementary Activities which follow.

Table of Per Cents for Chapter 6 Scores

Problem Test		Computation Test					
Score	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*
1	11	1	4	11	42	21	81
2	22	2	8	12	46	22	84
3	33	3	11	13	50	23	88
4	44	4	15	14	54	24	92
5	56	5	19	15	58	25	96
6	67	6	23	16	62	26	100
7	78	7	27	17	65		
8	89	8	31	18	69		
9	100	9	35	19	73		
		10	39	20	77		

Do You Understand?

Test of Information and Meaning 6

Ex. 1–3. Copy and complete by writing $=$ or $<$ or $>$.

1. $0.45 \_?\equiv 4.5 \div 10$ 2. $3 \times 1.2 \_?\geq 3$ 3. $0.6 \_?\geq 0.2 \times 0.3$

Ex. 4–9. Rename with a per cent form.

4. 0.67 67% 5. $\frac{89}{100}$ 89% 6. 0.03 3% 7. $\frac{1}{2}$ 50% 8. 1 100% 9. $\frac{3}{5}$ 60%

10. Using 4.7 and 9.2 for the factors and n to represent the product, write four mathematical sentences which illustrate the factors-product relationship. $n = 4.7 \times 9.2$; $n \div 4.7 = 9.2$; $n = 9.2 \times 4.7$; $n \div 9.2 = 4.7$

11. Which property does the following mathematical sentence illustrate? $4 \times (3.52 + 8.1) = (4 \times 3.52) + (4 \times 8.1)$ Distributive Property of Multiplication over Addition

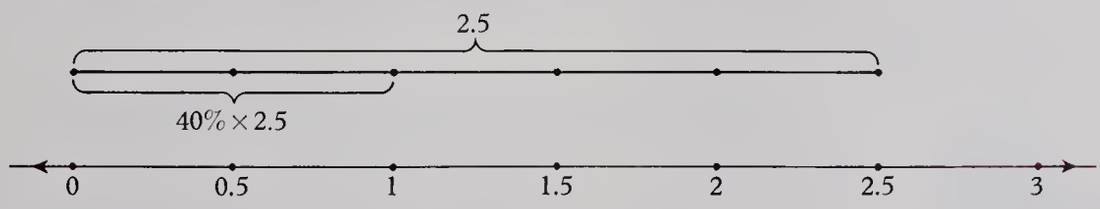
12. Express the ratio 3 to 5 first with a fraction, then with a decimal, and then with a per cent form. $\frac{3}{5}$; 0.6; 60%

Ex. 13–15. Tell whether n represents the product or a factor and then solve.

13. $n \div 34.8 = 7.9$ product; 274.92 14. $n \times 4.7 = 38.54$ factor; 8.2 15. $n = 783 \times 0.035$ product; 27.405

16. Which has a longer diameter, a circle with circumference 46 cm. or a circle with circumference 45 cm.?

17. Write a mathematical sentence which shows the relationship expressed by the following number-line picture. $n = 40\% \times 2.5$



18. How many decimal places should be shown in the decimal for the product of 7.2 and 13.607? 4

19. For $82 \div 4.67$, tell what number to multiply both the product and the known factor by so that the unknown factor can be found by dividing by a whole number. 100

*For problems dealing with per cents, suggest that pupils make tables to aid themselves in the problem analysis.

Do You Make Mistakes?

Diagnostic Test 6

	a	b	c	d	Study Pages	Practice: Use Sets
1. Multiply	3.8 × 7 = 26.6	40.82 × 18 = 734.76	0.021 × 9 = 0.189	4.505 × 75 = 337.875	228-231	110-112
2. Multiply	4.7 × 3.3 = 15.51	7.39 × 0.37 = 2.7343	57.9 × 0.88 = 50.952	162.09 × 0.772 = 125.13348	232-234, 236-238	115-117
3. Divide	$4\overline{)4.88}$ = 1.22	$5\overline{)6.5}$ = 1.3	$32\overline{)0.64}$ = 0.02	$7\overline{)9.03}$ = 1.29	240-243	119-120
4. Divide	$2.3\overline{)9.66}$ = 4.2	$5.1\overline{)66.3}$ = 13	$0.8\overline{)28}$ = 35	$0.16\overline{)40}$ = 250	248-249	122-126
5.	Find the unknown factor to the nearest tenth: $3 \div 7_{0.4}$ $5 \div 3.8_{1.3}$ $0.7 \div 3_{0.2}$ $6.9 \div 1.9_{3.6}$				244, 249	122, 127

Can You Solve Problems?

Problem Test 6

* Let n represent the number which will answer the question in the problem. Write a mathematical sentence for the problem and then solve to find the answer for the problem.

1. Find the perimeter of a regular pentagon if each side measures 5.67 in centimeters. $n = 5 \times 5.67$; 28.35 cm.

2. Find the length of one side of a square if the perimeter of the square is 4.056 ft. $n = 4.056 \div 4$; 1.014 ft.

3. The cost of food for an over-night camping trip was shared equally by 15 boys. The cost was $27.75. How much was that per boy? $n = 27.75 \div 15$; $1.85

4. Of the 15 boys on the camping trip, 80% went fishing after supper. How many boys went fishing after supper? $n = 80\% \times 15$; 12

Supplementary Activities

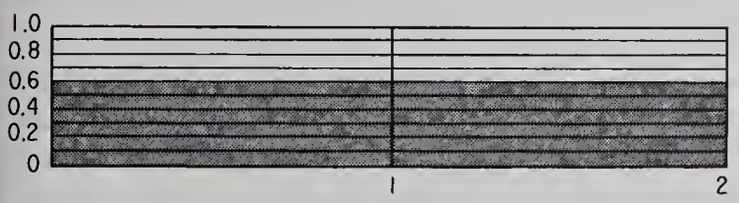
These activities may be used in a variety of ways. The teacher may assign a few of them to pupils who finish the end-of-chapter tests early. *More capable pupils* may work some activities while the teacher is reviewing or reteaching ideas which the tests have indicated need additional study by some pupils. These activities may also be used for review or enrichment at any time throughout the year.

• Have *all pupils* develop the work in box C, page 229, as follows using the distributive property:

$$\begin{aligned} 3 \times 1.4 &= 3 \times (1.0 + 0.4) \\ &= (3 \times 1.0) + (3 \times 0.4) \\ &= 3.0 + 1.2 \\ &= 4.2 \end{aligned}$$

Now have each pupil work Ex. 10–14 on page 229 using the same development.

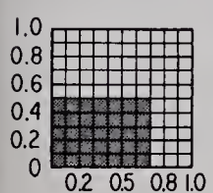
• *All pupils* may explain how the following diagram may be used to represent the mathematical sentence $0.6 \times 2 = n$.

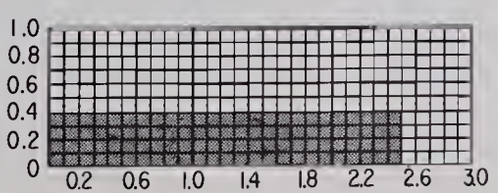


Have them show how the picture may be used to find the product 1.2 (or 1 + 2 tenths, or 12 tenths). They should draw other diagrams and find the products for these examples:

(a) 4×0.3 (b) 2.1×3.0 (c) 1.7×2

• For each diagram below, pupils should write four true mathematical sentences expressing the factors-product relationship. They should then find the product by counting the models of small regions.





• *Slower learners* should find the product for 2.4×3.2 as follows using the distributive property:

$$\begin{aligned} 2.4 \times 3.2 &= (2 + 0.4) \times (3 + 0.2) \\ &= (2 \times 3) + (2 \times 0.2) + (0.4 \times 3) + (0.4 \times 0.2) \\ &= 6 + 0.4 + 1.2 + 0.08 \\ &= 7.68 \end{aligned}$$

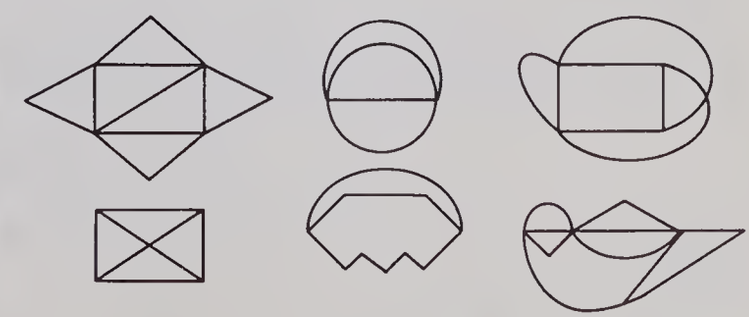
In the same way, they should do Ex. 5–9 on pupil's page 236.

• The function concept is an important mathematical idea. Guide the pupils to discover the relationship between the measure of a circumference and diameter when the diameter is doubled, tripled, and quadrupled. Have them copy the chart and complete the work.

When d is	C is	Double d	The new C is	Ratio of 2d C to 1st C
2	6.28	4	12.56	$\frac{12.56}{6.28} = 2$
3	(9.42)	(6)	(18.84)	$(\frac{18.84}{9.42} = 2)$
5	(15.7)	(10)	(31.4)	$(\frac{31.4}{15.7} = 2)$
10	(31.4)	(20)	(62.8)	$(\frac{62.8}{31.4} = 2)$

Have the pupils copy and complete this sentence: For any circle, when the measure for the diameter is doubled, the measure for the circumference for that circle is (doubled).

• *All pupils* may draw pictures of the figures below without retracing any portion of the figure. They should begin the drawing at any point and may cross any segment. Only the first three figures can be drawn in this manner. Pupils should attempt to describe the characteristics of the figures which cannot be drawn this way. (If a figure has more than two vertices where an odd number of segments or curves meet, the figure cannot be drawn as described.)



• On Fig. 1, a model of a 16 inch square-shaped region is placed in the interior for the model of the 18 inch square-shaped region so the four shaded quadrilateral regions have the same area. What is the area of each? (18 sq. in.)

• In Fig. 2, triangles *ABF* and *CDE* are congruent. The area of triangular region *ABF* is 12 square inches. If the length of rectangle *BCEF* is 20 inches, and the width is 6 inches, what is the area of trapezoidal region *ABDEF*? ($m(\angle ABF) + m(\angle CDE) + m(BCEF) = m(ADEF)$; (12) + (12) + (120) = 144, or 144 square inches)

Fig. 1

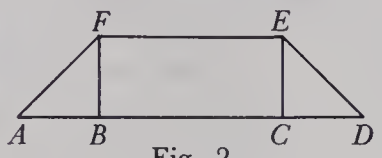
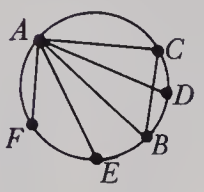


Fig. 2



• *More capable pupils* may make a model representing a circle as shown above. They should picture a diameter *AB*, and with *C* as any point in the circle, they should picture $\overline{AC}$ and $\overline{CB}$. Using their protractors, have them measure $\angle ACB$. Have them locate other points *D*, *E*, *F* in the circle and picture segments *AD*, *AE*, *AF*, *DB*, *EB*, and *FB*. Again, let them use their protractors to find the measure of $\angle ADB$, $\angle AEB$, and $\angle AFB$. Now have them find the measure of the circumference of the circle to the nearest hundredth.

Copy and record the data in a table as below:

	$\angle ACB$	$\angle ADB$	$\angle AEB$	$\angle AFB$
Approximate measure in degrees				

Now make a guess concerning the measure of any angle whose vertex belongs to a circle and whose sides pass through the ends of a diameter.

• Have *all pupils* answer the following:

Suppose you read this statement as part of an advertisement in the newspaper: "The Bang Independent Testing Service found that Bunko Double-coated Pills cured common colds faster in 50% of the cases tested."

a. Is this a strong valid argument for using Bunko Pills? Give reasons for your answer. (No—you don't know how many people were tested or how sick they were.)

b. How many people tried the pills before the statement in the advertisement was made? (Not enough information given.)

c. Does the 50% indicate 1 out of 2, 5 out of 10, 150 out of 300 or some other ratio were helped by the pills? (Yes, but it does not indicate how many were actually helped.)

d. Is it possible that a number of samples of people were tried with poor results before one sample showed that 50% of the people tested were helped? (Yes)

In some cases the use of per cent makes statements seem more convincing than they actually should be. The ratio factor, 50%, is used without the base factor or product. The statement would have been improved if it had contained either the base factor or the product along with the ratio factor.

What is wrong with each of these statements?

a. The number of students in Hillsdale School increased 200% from 1959 to 1963. The enrollment in 1959 was 500 and in 1963 it was 1,000. (There is a 100% increase.)

b. The value of Mr. Jones' property in 1957 was $12,000. In 1963 the value was $36,000. This represents an increase of 300% in the value of the property. (There is a 200% increase.)

c. Last year Dave earned $500. This year he earned 125% of that amount. He hopes to increase his earnings the same rate next year. What does he expect to earn next year? ($781.25)

5. During a basketball game, Jim took 25 shots and made baskets on 9 of them. He made baskets on what per cent of the 25 shots? $n = 9 \div 25$; 36%

6. There were 475 people at the basketball game. That was only $62\frac{1}{2}\%$ of the mean-average attendance for the previous year. What was the mean-average attendance for the previous year? $62\frac{1}{2}\% \times n = 475$; 760 people

7. Find the arithmetic mean for the following data: 3.7 cm., 4.1 cm., 4.2 cm., 3.8 cm., 3.9 cm. $n = (3.7 + 4.1 + 4.2 + 3.8 + 3.9) \div 5$; 3.94 cm.

8. Find the length of a rectangular region whose width is 3.8 ft. and whose area is 22.23 sq. ft. $n = 22.23 \div 3.8$; 5.85 ft.

9. The number 16.78 is how much greater than the product of 4.03 and 3.92? $n = 16.78 - (4.03 \times 3.92)$; 0.9824

How Well Can You Compute?

Computation Test 6

Ex. 1–5. Multiply the number by 4.5.

1. 78 351 2. 8.72 39.25 3. $3\frac{1}{2}$ 15.75 4. $\frac{3}{5}$ 2.7 5. 7.0093 31.54185

Ex. 6–10. The diameter for a circle is given. Find the circumference of the circle to the nearest $\frac{1}{10}$ unit. Use $C \approx 3.14 \times d$.

6. 6 cm. 18.8 cm. 7. 49 in. 153.9 in. 8. 7.3 ft. 22.9 ft. 9. 0.24 mm. 0.8 mm. 10. 1.726 yd. 5.4 yd.

Ex. 11–18. Divide. Express the unknown factor to the nearest hundredth.

11. $2.5 \div 3$ 0.83 12. $3.8 \div 7.2$ 0.53 13. $0.735 \div 7$ 0.11 14. $9 \div 3.3$ 2.73

15. $7.7 \div 5.1$ 1.51 16. $0.9 \div 4.3$ 0.21 17. $1.352 \div 3.8$ 0.36 18. $14 \div 2.18$ 6.42

Ex. 19–26. Find the number represented by the letter.

19. $x = 3.7 \times 6.9$ 25.53

20. $n \times 5 = 6.02$ 1.204

21. $n \div 13 = 0.64$ 8.32

22. $x = 35\% \times 673$ 235.55

23. $n = 2.7 \times (6.7 + 9.9 + 5.1)$ 58.59

24. $x = 15\% \times (78 + 892 + 27)$ 149.55

25. $x = 23\% \times 926.73$ 213.1479

26. $n = (53\% + 13\%) \times 639$ 421.74

7

	Number of Leaves Found			
	Elm	Maple	Oak	Others
Monday	15	8	6	7
Tuesday	12	10	4	8
Wednesday	10	22	8	5
Thursday	14	16	7	10
Friday	8	19	5	6

Elm Maple Oak

Reading a Table and a Pictograph

One symbol representing 1 item [O]

Mr. Harlow's hobby is identifying tree leaves. Last fall, he marked off a circular region on his front lawn and each day for five days he collected and identified the leaves that he found there. The table above shows, for each of the five days, how many elm leaves, maple leaves, oak leaves, and other leaves Mr. Harlow found in that region.

1. How many elm leaves were found on Monday? 15 Friday? 8

2. For the five-day period, find, at the board, the total number of elm leaves that were found. 59 Then do the same for maple leaves and for oak leaves. 75 maple; 30 oak

3. How many leaves were found that were neither elm, maple, nor oak leaves? 36

4. In all, how many leaves were found? 200 What per cent of the total number of leaves were oak leaves? 15%

5. Name two different kinds of leaves that might have been recorded under "Others." Answers will vary.

Overview -- Chapter 7

• Chapter 7 presents work with graphs including construction; interpretation; and use of pictographs, line graphs, bar graphs, double-line graphs, double-bar graphs, and circle graphs. Strands from modern mathematics include the relationship between geometry and rational numbers; the use of statistics with various types of graphs; and the area of triangular and parallelogram regions.

Geometric concepts involving space figures are presented: polyhedrons; prisms and their parts—bases, lateral faces; triangular prisms; rectangular prisms; lateral area; and total area of a right prism. Other modern mathematics strands include the introduction of Venn diagrams with set notation and number properties, and the application of set operations and terminology to geometry. Solid figures are introduced, and the concept of volume explored. The concept of negative numbers is introduced as an intuitive extension from positive numbers.

• **Problem-Solving.** Throughout the chapter, problem-solving is presented concurrently with new topics and strands. Many social and mathematical applications are investigated including using statistics from graphs to show relationships; and solving problems involving science, time, weather, and business. New principles are used in solving problems involving geometry and measurement. Pupils are led to evolve generalizations and understandings pertaining to relationships among geometric figures.

• **Maintenance.** On almost every page, new skills are extended from previously learned skills, and immediately and continuously reinforced. Some topics maintained are operations on rational numbers expressed with decimals, fractions, and per cent forms; rounding numbers; work with data and the arithmetic mean, mode, and median; areas of plane regions; place value and expanded form; work with bases other than ten; other numeration systems; and set notation and number properties.

• **Enrichment.** There are enrichment suggestions in the lesson plans and in the Supplementary Activities at the end of the chapter. In addition, the chapter contains more enrichment topics than any of the previous chapters as shown below.

	Page
(a) area of a polyhedron	305
(b) polyhedral solids	307
(c) cylindrical and conical solids	308
(d) spheres and spherical solids	309
(e) volume of rectangular solids	311
(f) integers	313
(g) constructing congruent segments	314
(h) constructing congruent angles and triangles	315
(i) constructing the perpendicular bisector of a segment	316
(j) constructing an angle bisector	317

• **Tests.** Maintenance and pages entitled, "To Keep In Practice" may be used as tests. The end-of-chapter tests are designed to evaluate the extent to which skills and understandings in Chapter 7 have been mastered. Pupil's understanding of the use and meaning of the several types of graphs presented in the chapter is tested. Other topics covered include Venn Diagrams, measurement of geometric figures, set notation, problems involving geometry and social situations, computation involving decimal, fraction, and per cent form.

Teaching Pages 274 and 275

Pupil's Objective

To read and interpret data represented with a table and with a pictograph.

Background

The purpose of the exercises is to show how a summary of data can be presented in table form and

in the form of a pictograph, and to give the pupils experience reading a table and a pictograph. The pictograph shown on the pupil's page is the type in which one symbol represents one item. Statistics affords an opportunity to involve the pupils in interesting, challenging, and enjoyable mathematical activities both inside and outside the classroom.

The pupils may enjoy presenting the same facts in different ways to create very different impressions. The statements "The gas tank is half full" and "The gas tank is half empty" give the same fact, but the stress is different in each case. This can also be done with graphs. Two different graphs can be designed to state the same facts, but one graph may create a different impression from the other graph.

Pupils may select and discuss data for preparing graphs from any field such as science, geography, and social studies.

Pre-Book Lesson

• Involve pupils in the collection and presentation of data. The object of such a pre-lesson activity would be to stimulate the pupils' interest in statistics.

• You may have each pupil write on a slip of paper the name of his favorite food. (Do not have them put their names on the slips.) Collect these slips, and write on the board the favorite foods with a tally mark (/) for each pupil who chose that food. The results will vary. Tastes may be very diversified, in which case there will be a long list, or many pupils may choose the same food. Point out that generalities can be made from the data collected. For instance, it may be possible to say that there is no person in the class whose favorite food is spinach. Also, point out that the favorite foods of the class as a whole can be found without knowing the favorite foods of the specific pupils. Leave the data on the board for an Individualizing Instruction exercise.

Using the Text Pages

• During the oral exercises it is important to point out that a table is very useful for presenting data because a great amount of data can be presented in a very concise form; a table of data such as that on the text page can contain so much information that it may be difficult to interpret the information quickly and find the information desired; a pictograph does not present data in such a concise manner as does a table, but relationships are seen more quickly among the data from the pictograph than from the table.

• Ex. 6. Be sure pupils understand the meaning of the term *key*.

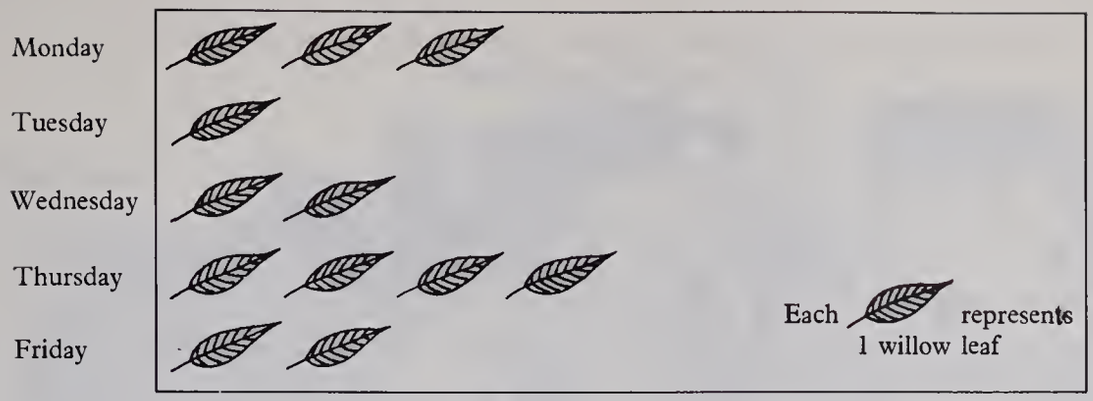
Individualizing Instruction

• Although the procedure for reading the table on page 274 may seem obvious, be sure that *slower learners* understand the procedure for locating a fact on the table.

• Have *all pupils* try to find and bring to class pictographs found in newspapers and magazines. Some pictographs may have one symbol represent one item while other pictographs may have one symbol represent a given number of items.

Have *all pupils* refer to the data on the board, collected in the Pre-Book Lesson. Discuss the possibilities of representing the data with a table or a pictograph. Perhaps, the data is too diversified to successfully represent with either one of these forms. On the other hand, perhaps the three favorite foods of the class could be represented with a pictograph.

• *More capable pupils* may be given the task of representing with pictographs the data summarized in the table on page 274. One pictograph will have to be made for each day of the week or one pictograph made for each type of leaf. This will make very clear the relative advantages and disadvantages of using a table over using a pictograph to present data of this type.



Willow Leaves Found During a Five-Day Period

6. For the number of willow leaves he found each day, Mr. Harlow made the pictograph above. To read the pictograph, we first read the *key* which indicates what each symbol represents. The key for the pictograph above is in the lower right corner of the graph. Read the key.

7. How many willow leaves did Mr. Harlow find on Monday? 3 Wednesday? 2 How many did he find in all during the five-day period? 12

8. On what day did he find the greatest number of willow leaves? Thursday the least number? Tuesday

9. The number of willow leaves found on Friday is what per cent of the number of willow leaves found on Thursday? 50%

[W]

10. The number of willow leaves found on Monday is what per cent of the total number of willow leaves found? 25%

11. The number of willow leaves found on Wednesday and Thursday is what per cent of the total number of willow leaves found? 50%

12. Suppose that each represented 5 willow leaves instead of 1 willow leaf. Then, how many willow leaves would be shown by the graph as having been found on Monday? 15 Wednesday? 10 Thursday? 20

13. Name two things that could be shown by pictographs and make a key for each. Answers will vary.

● **Extra Activity.** Set 174.

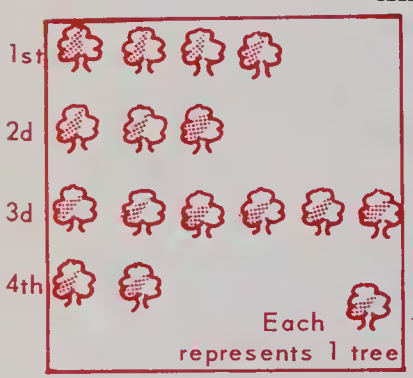
*Point out to pupils that an important part of the graph is the title.

Making a Pictograph

[O]

1. Last summer, during a four-week period, the members of the Junior Conservation Club planted trees in the new park. They planted 4 trees the first week, 3 the second week, 6 the third week, and 2 the fourth week.

Sample answers 1 b.



Trees Planted by the Juniar Canservation Club

a. On the board, make a table to show how many trees they planted each week. See below.

b. Using the information from your table, make a pictograph. Use to represent 1 tree. Use "Trees Planted by the Junior Conservation Club" as the title for your graph.

[W]

*Ex. 2–5. For each exercise, make a table and then make a pictograph using the data from the table. Answers will vary.

2. The number of cents that you have spent for candy each day during a five-day period. Use to represent 1¢.

3. The number of glasses of milk that you drank each day during 1 week. Use to represent 1 glass of milk.

4. The number of pupils absent from class each day during 1 school week (Monday through Friday). Use to represent 1 pupil.

5. The number of runs scored in each World Series game last year by the team that won the Series. (You can find this information in a world almanac.) Use to represent 1 run.

Sample answer 1 a.

Number of trees planted			
1 st Week	2 d Week	3 d Week	4 th Week
4	3	6	2

Pupil's Objectives

(a) To learn to summarize data in table form; and (b) to represent data with a pictograph.

Background

These exercises give the pupils experience in making pictographs. A pictograph often presents information in a more interesting and vivid way than other types of graphs. A pupil is likely to take a greater interest in the learning activity if the data pertains to his "world." Most of the data should be collected by the pupils themselves.

Pre-Book Lesson

Have the pupils write on a slip of paper the brand of toothpaste (or some other item) they use. Select a pupil to collect and read the slips to another pupil who will list on the board the brand names, and write a tally mark for each pupil using that brand. After the tallying is completed, ask for suggestions as to how this tally could easily be shown by a pictograph. Use a "stick figure" of a man for a symbol to represent one pupil, explain the key, and give the graph a title.

Using the Text Page

• If there is any topic of current interest to the pupils which is suitable for representing with a pictograph, feel free to add it to the exercises or substitute it for any of the written exercises on the text page.

• Ask pupils if it is necessary for each symbol in the pictograph to be the same size. (The pictograph depends on the idea of using the same size symbol to represent the same number of each item being compared.)

Individualizing Instruction

• Ask *more capable pupils* to list types of data which would be unsuitable for representing with a pictograph. Examples of such types of data are temperature readings, changes in body weight, heights, distances, velocities, and data expressed with per cent forms.

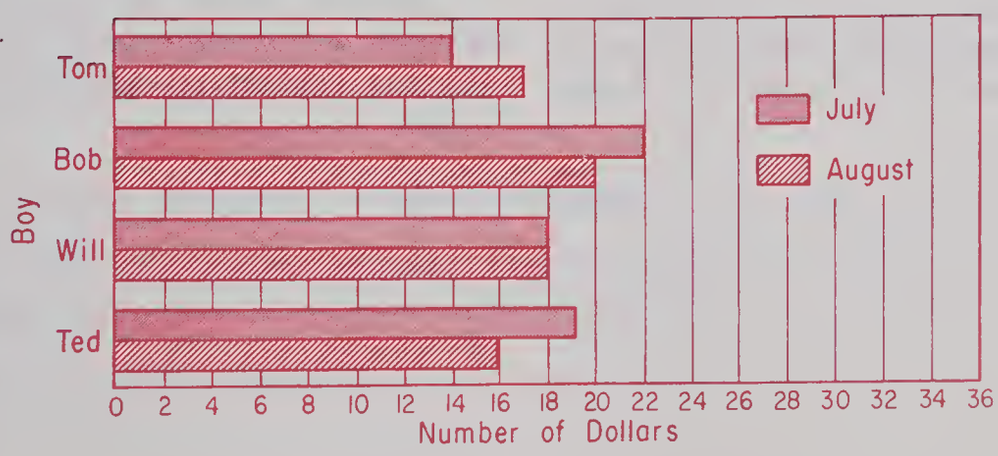
• Have *all pupils* make a pictograph pertaining to data recently acquired in some other course such as social studies or science. It may be necessary to have one symbol represent more than 1 item.

• Give *slower learners* help in writing the title for their pictographs if they have difficulty. Remind them to indicate the key on each pictograph, and emphasize the importance of having an organized table of information before drawing the pictograph.

These pupils may need help in finding the information necessary for Ex. 5. Show them how to look up the information, read it, and select only the pertinent facts.

ANSWERS NOT SHOWN ON PAGE 280

4.



Earnings of Four Boys in July and August

Pupil's Objective

To practice reading and making pictographs.

Background

These exercises give the pupils experience reading pictographs in which one symbol represents more than 1 item. In pictographs of this type, a part of a symbol may be used. In the pictograph at the top of the page, each symbol represents 2 houses. In the pictograph at the bottom of the page, each symbol represents 2 boys. To represent one house or one boy, only one half of a symbol is drawn. Pictographs in which a symbol represents more than one item are far more common than those having one symbol represent 1 item.

Pre-Book Lesson

Again, have pupils bring to class examples of pictographs found in newspapers and magazines. Have them point out any differences they see between these pictographs and those they have studied in the text. Lead them to realize that in most pictographs, one symbol is used to represent more than 1 item, so it may be necessary to use parts of a symbol.

Using the Text Page

• Ex. 1. Point out the importance of the key if one symbol is being used to represent more than 1 item. Remind the pupils to display the key prominently in the pictograph.

• Ex. 2. Have pupils consider a key in which a "stick figure" represents 6 boys. Point out that all but one of the rows would be represented by parts of a symbol. Ask pupils why this would be a poor pictograph.

• Ex. 3. Since there is only one written exercise on this page, you may wish to supplement it with additional written exercises if time permits.

Individualizing Instruction

• Either you or the *more capable pupils* may obtain pictographs in which each symbol represents a great number of items. In making such pictographs, it is common practice to round numbers for the data for convenience before representing it with the pictograph. Have pupils represent the data below with a pictograph, first rounding the numbers to the nearest ten million. Let each "stick figure" symbol represent ten million people.

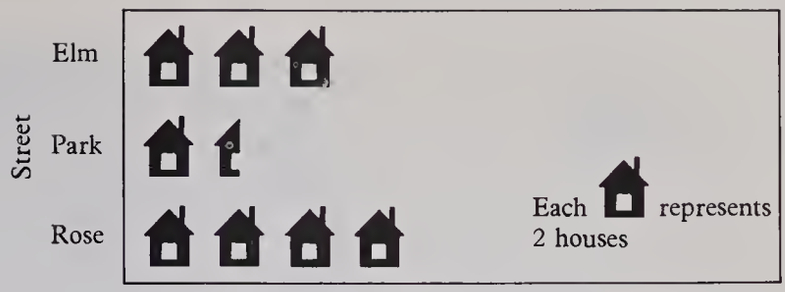
Population of the United States 1900–1960

1900	75,994,575
1910	91,972,266
1920	105,710,620
1930	122,775,046
1940	131,669,275
1950	150,697,361
1960	179,323,175

• Have *all pupils* make a pictograph showing the total number of children under 18 years of age in the families of the pupils in each row in the classroom. Have the pupils discuss and decide among themselves how many children each symbol in the pictograph should represent. Ask pupils why it is best to have all the information before deciding how many children each symbol should represent.

• Show the *slower learners* examples of pictographs found in texts, magazines, and newspapers in which one symbol is used to represent more than one item. Have them read and interpret the pictographs.

		Number of New Houses Built During 1 year
Street	Elm	6
	Park	3
	Rose	8



New Houses Built on Three Streets in Eastvale During 1 Year

Reading and Constructing Pictographs

[O]

1. The table and the pictograph above both show how many new houses were built on three streets in Eastvale during one year.

a. In the pictograph, how many houses does each picture of a house represent? 2

b. From the table, _?6 houses were built on Elm Street, so _?3 🏠 's are shown in the pictograph. Why? Key idea: $3 \times 2 = 6$

c. Looking only at the pictograph, tell how many houses were built on Rose Street. 8 Compare your answer with the information shown in the table.

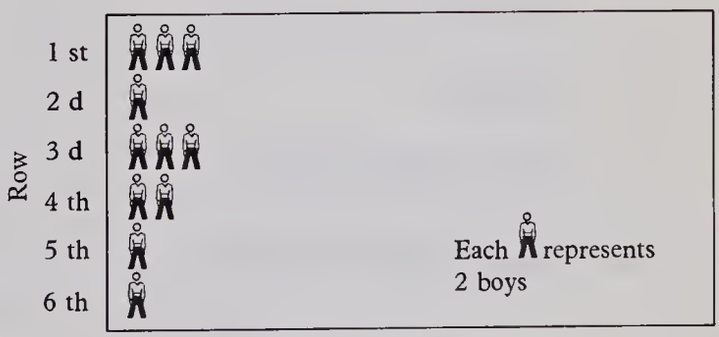
d. In the pictograph, each ◔ represents one half of what each 🏠 represents. Each 🏠 represents _?2 houses, so each ◔ represents how many houses? 1

e. Explain why $1\frac{1}{2}$ 🏠 's are shown for Park Street. Key idea: $2 \times 1\frac{1}{2} = 3$

2. Study the pictograph at the right.

a. How many boys are there in the 3d row in Room 100? 6

b. How many boys are there in all in Room 100? 22

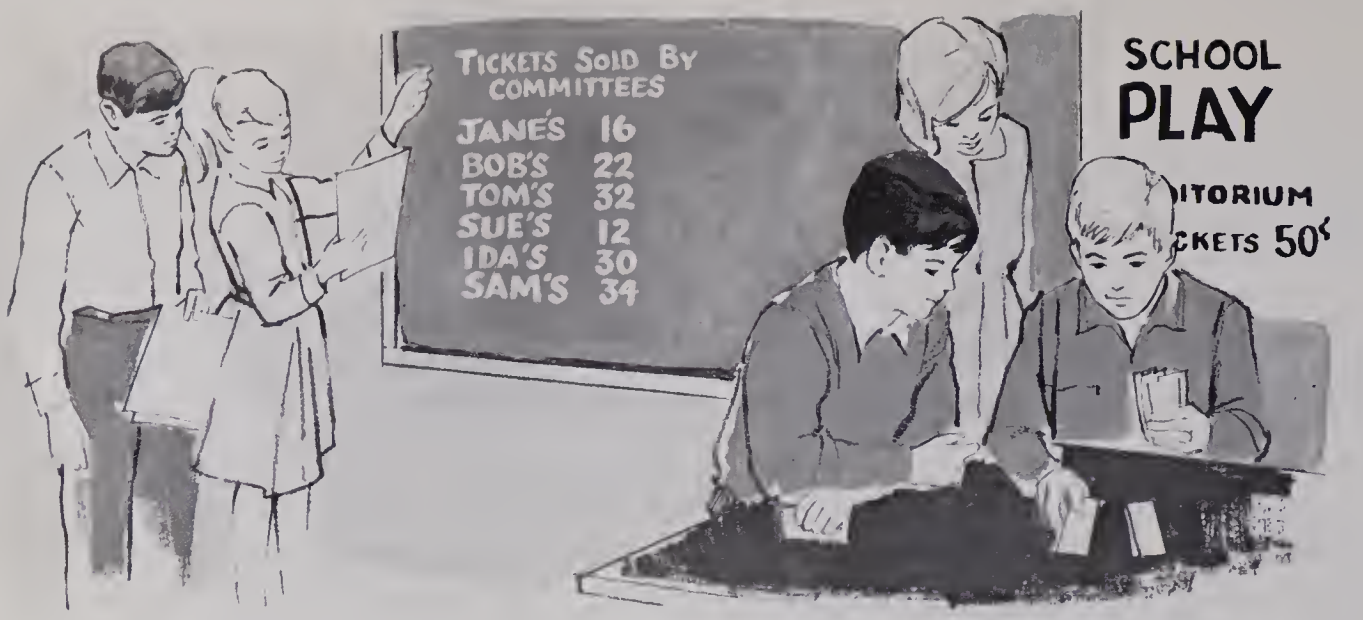


Boys in Room 100

[W]

3. Make a pictograph to show how many hours you studied each day for one week. Use ⌛ to represent 1 hour. Then make another pictograph using ⌛ to represent 2 hours. Answers will vary.

● **Extra Activity.** Set 175.

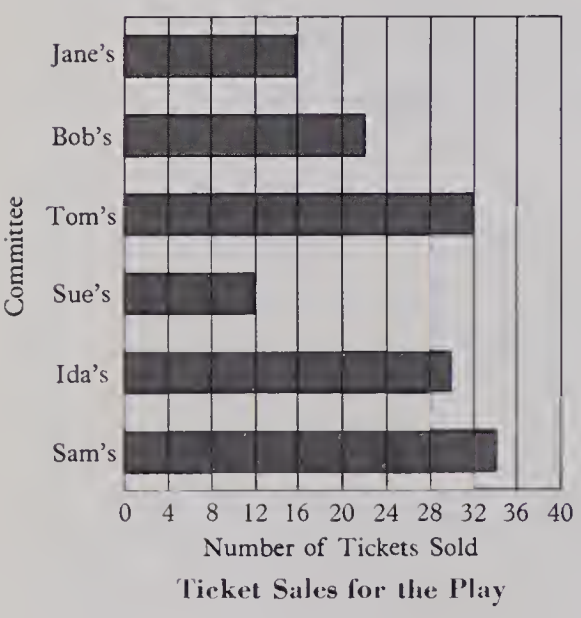


Bar Graphs

[o]

1. From the table in the picture, how many tickets for the play were sold by Jane's committee? 16 by Sam's committee? 34

The graph at the left shows the same information as the table in the picture. Such a graph is called a *bar graph*. For each committee, a bar is drawn. The length of the bar indicates how many tickets were sold by that committee.



Ticket Sales for the Play

2. Looking only at the bar graph, tell how many tickets were sold by Bob's committee; 22 Tom's committee; 32 Sue's committee; 12 Ida's committee. 30 Compare your answers with the data in the table.

3. By looking at the bar graph, is it easy to tell which committee sold the greatest number of tickets? Yes Explain. Sample answer: The longest bar can be seen easily.

4. The bar graph above is a *horizontal-bar graph*. Tell why *horizontal* is a good word to use to describe this graph. The bars are horizontal. Explain the labeling on the horizontal scale. Each unit represents 4 (tickets sold).

Teaching Pages 278 and 279

Pupil's Objective

To learn how to read and make bar graphs.

Background

Like the pictograph, the bar graph can be used to compare quantities, but it has additional uses. It can be used to represent data which are very difficult to represent with pictographs. For example, data dealing with temperatures, heights, distances, weights, velocities, air pressure, and rainfall are not easily depicted with pictographs because it is difficult to draw a symbol which can be recognized easily as a unit. In a bar graph, a unit is not pictured; rather, the various lengths of the bars represent the variations in the data.

It is easier to recognize relationships among the data in a bar graph than in table form. The longest bar in a bar graph obviously represents the greatest quantity. In a table, the greatest number is not so readily apparent. This ease of comparing data is what makes graphs so useful.

One problem in making bar graphs is determining what scale to use. One method is to note the greatest number that is to be represented and then determine how many units on the graph would be most convenient to represent this number. The greatest number that is to be represented on the graph is divided by the number of units to be used in representing this number. For example, in representing the height of several children with a bar graph, if the greatest height is 66 in. and it is convenient to represent that height with a bar about 7 units in length, then dividing 66 by 7 results in a quotient of about 9.4. If 9.4 is used as the unit, the scale would be awkward. To avoid confusion, 9.4 should be rounded to the next greater whole number, and therefore each unit on the graph should represent 10 on the scale.

Reading bar graphs often requires more estimating than reading pictographs. No estimating is required in reading data from a table. If it is necessary that the numerical values associated with the data be exact, it should be presented in table form. However, if comparison is important, the bar graph is often the best representation of the data.

Pre-Book Lesson

Have the pupils determine the height of ten of their classmates. Have them record the heights in table form on the board. Discuss the possibility of representing this data with a pictograph and lead them to realize that the pictograph is not suitable for representing certain types of data, because data cannot always be represented clearly by symbols. Show the need for having another way of expressing such data.

Using the Text Pages

- Ex. 1–6. Emphasize that the length of each bar corresponds to a number of tickets sold. One unit of the graph represents 4 tickets sold as indicated by the scale across the bottom of the graph. Point out that the width of the bar has no significance. To avoid confusion, each bar on the graph should be the same width.
- Ex. 7–9. Emphasize that when making a bar graph, several things should be kept in mind: horizontal or vertical bars may be used; a suitable scale must be determined so that the data can be represented by bars of convenient length; and a properly labeled scale must be used.
- Be sure pupils understand the meaning of *horizontal* and *vertical* as used in reference to bar graphs. It is common practice to refer to any segment parallel to the top and the bottom of a chalkboard or sheet of paper as being *horizontal*, and any segment forming a right angle with a horizontal segment in the plane of the chalkboard or sheet of paper as being *vertical*.

Individualizing Instruction

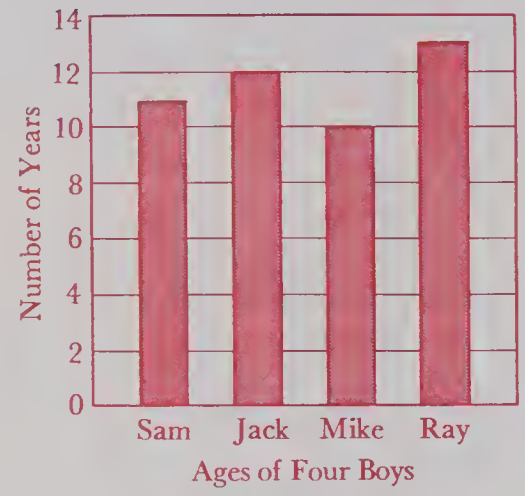
- Have the *more capable pupils* make a bar graph to represent the data given under the Individualizing Instruction section of the previous page, title it "Population of the United States 1900–1960." Let each pupil decide what scale would be appropriate and, after the graphs are completed, display them for comparison.
- Have *all pupils* summarize the possible reasons for using a bar graph to present data instead of using a pictograph or a table.

Have these pupils bring to class examples of bar graphs found in newspapers and magazines. Give them practice in interpreting such bar graphs.

• *Slower learners* may need additional help in reading and in making bar graphs when the length of the bar does not correspond to a whole unit of the scale. (i.e., Sue's height on the graph, pupil's page 279) Give these pupils additional oral exercises, if necessary.

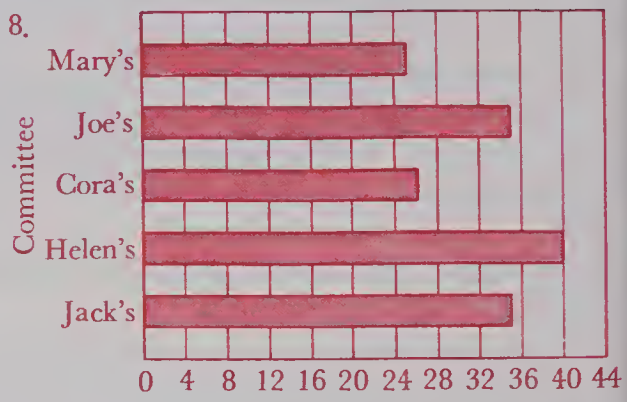
ANSWERS NOT SHOWN ON PAGE 279

7.



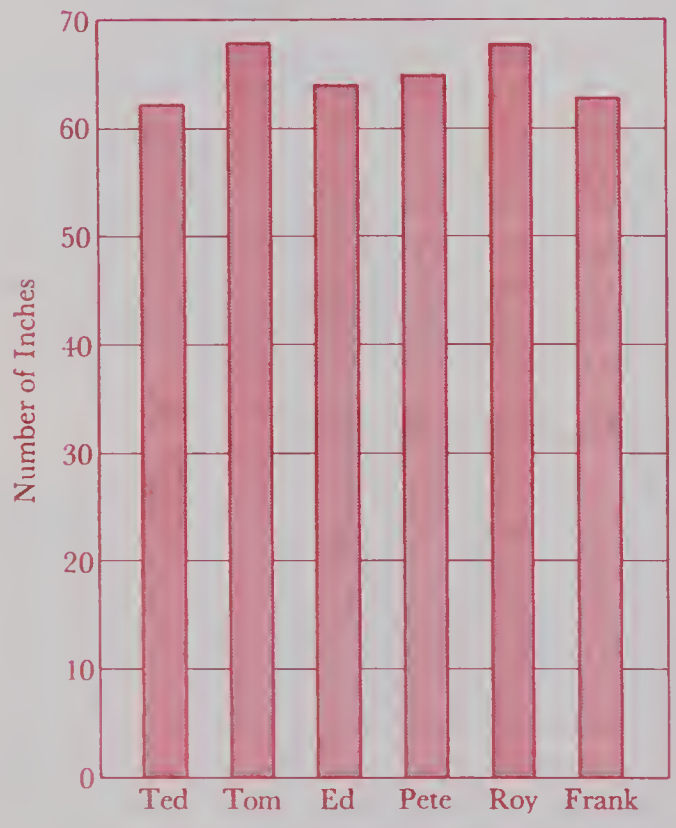
Ages of Four Boys

8.



Ticket Sale For Play

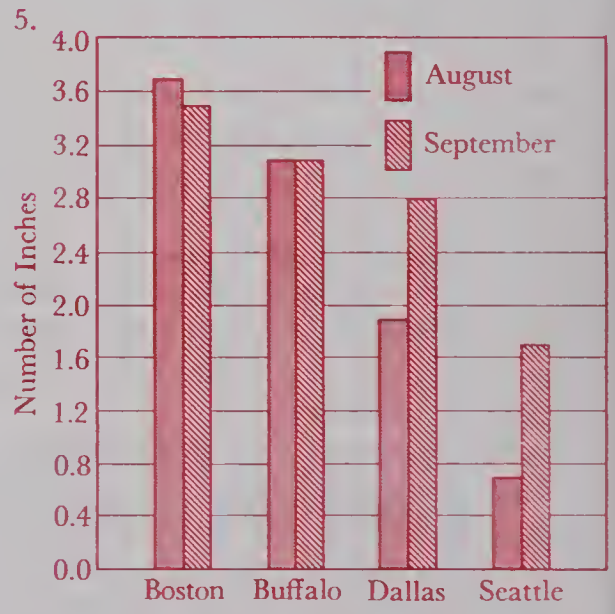
9.



Heights of Boys Belonging to Hiking Club

ANSWERS NOT SHOWN ON PAGE 280

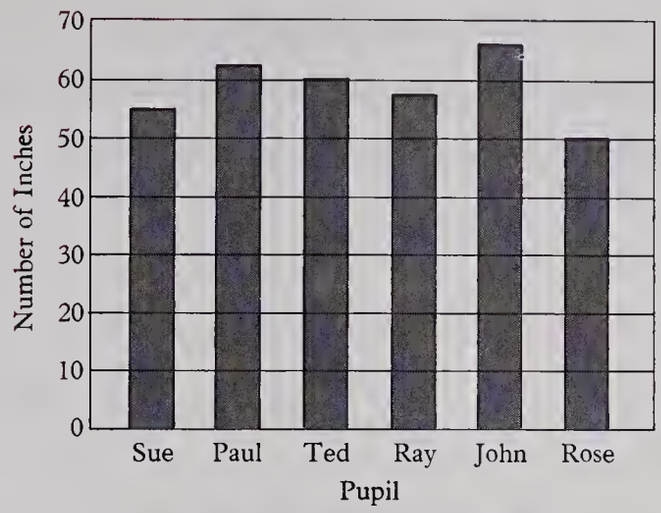
5.



Normal Precipitation for August and September for Four Cities

5. The table and the vertical-bar graph below both show the heights of the pupils in Sue's row. Why is the graph called a *vertical-bar graph*? The bars are vertical.

	Height
Sue	55 in.
Paul	62 in.
Ted	60 in.
Ray	57 in.
John	66 in.
Rose	50 in.



Heights of Pupils in Sue's Row

6. Looking only at the bar graph, can you easily tell Ted's height? Yes Ray's height? No We often estimate when reading a bar graph. Can you easily tell who is the tallest pupil of those listed? Yes

7. On the board, make a vertical-bar graph to go with the table at the right. Label the vertical scale so that each unit represents 2 (years). See Teacher's Page 279.

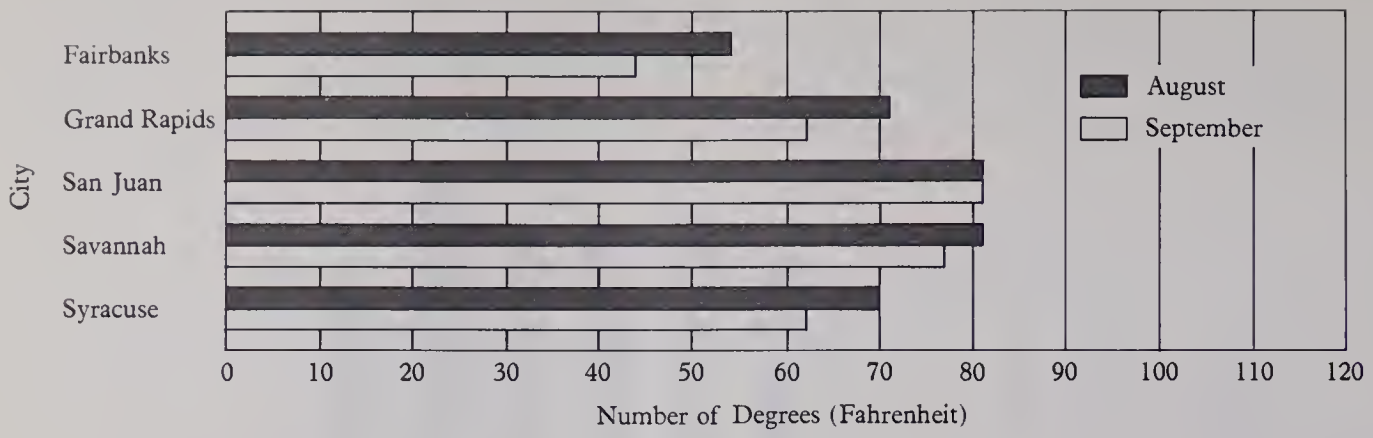
	Age
Sam	11 yr.
Jack	12 yr.
Mike	10 yr.
Ray	13 yr.

[W]

8. Last year, only five committees were formed to sell tickets for the school play. The names of the committees and the number of tickets sold were: Mary's, 25; Joe's, 35; Cora's, 26; Helen's, 40; Jack's, 35. Make a horizontal-bar graph to show this information. Label the horizontal scale so that each unit represents 4 (tickets). See Teacher's Page 279.

9. The heights of the boys that belong to the hiking club are: Ted, 62 in.; Tom, 68 in.; Ed, 64 in.; Pete, 65 in.; Roy, 68 in.; Frank, 63 in. Make a vertical-bar graph to show this information. Label the vertical scale so that each unit represents 10 (inches). See Teacher's Page 279.

♦ **Extra Problems.** Set 152. ● **Extra Activity.** Set 176.



Normal Temperatures for August and September for Five Cities

Double-Bar Graphs

[O]

1. The graph above is called a horizontal *double-bar graph.* For each city, there are _?2 bars. What is indicated by the length of a colored bar?^ normal temperature for August a white bar? normal temperature for September

2. For September, which of the five cities has the lowest normal temperature?^ Fairbanks the highest normal temperature? San Juan

3. Which two of the five cities have the same normal temperature for August?^ San Juan and Savannah for September? Grand Rapids and Syracuse

[W]

4. Make a horizontal double-bar graph to go with the table at the left below. Label the scale so that each unit represents 2 (dollars). See Teacher's Page 276.

5. Make a double-bar graph for the table at the right below. Use vertical bars and label the vertical scale so that each unit represents 0.4 (inches). See Teacher's Page 287.

Earnings

Boy	July	August
Tom	$14.00	$17.00
Bob	$22.00	$20.00
Will	$18.00	$18.00
Ted	$19.00	$16.00

Normal Precipitation (inches)

City	August	September
Boston	3.7	3.5
Buffalo	3.1	3.1
Dallas	1.9	2.8
Seattle	0.7	1.7

Teaching Page 280

Pupil's Objective

To learn to read and to construct a double-bar graph.

Background

There are certain types of information which can be presented clearly by use of a double-bar graph. The graph at the top of the pupil's page shows the normal temperature for five cities for August and also for September. This graph presents an easy comparison of the normal temperature for August among the five cities; the normal temperature for September among the five cities; and the normal temperature for August with the normal temperature for September for any one of the five cities.

Two separate graphs could have been used to present the information contained in this one graph, but a double-bar graph is more efficient. For example, the fact that the normal August temperature for San Juan is the same as the normal September temperature is more easily determined from the double-bar graph than from two regular bar graphs.

Pre-Book Lesson

Draw Fig. 1–3 on the board.

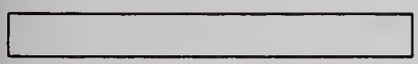


Fig. 1

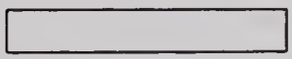


Fig. 2

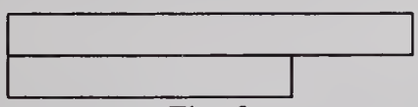


Fig. 3

Ask pupils to compare the bar in Fig. 1 with the bar in Fig. 2. Can the difference in length be determined easily by looking at the bars? Lead pupils to see that it is easier to compare the length of these bars in Fig. 3 than in Fig. 1 and 2.

Using the Text Page

• Ex. 1–3. When doing the oral exercises, be sure the pupils know what types of information are best suited for double-bar graphs.

After doing these exercises, ask *more capable pupils* how to find which city had the lowest normal temperature for both August and September. (It will be necessary to add the temperatures for August and September for each city, and divide by two to find the average temperature for August and September in each city.) Ask these pupils if this double-bar graph is the best method of comparing the normal temperatures for both August and September in each city. (A regular bar graph in which each single bar represents the normal temperature for both August and September would be the clearest representation of the comparison.)

• Ex. 4–5. Before assigning the written work, indicate how the pupils are to make the bars representing July earnings in Ex. 5 different from the bars representing August earnings. In darkening the bars, different colors may be used, different designs may be used, or the bars may be labeled.

Individualizing Instruction

• Have *more capable pupils* make a double-bar graph comparing the 1950 and 1960 populations of five cities, five states, or five countries. The data may be found in a world almanac.

• Show *slower learners* a double-bar graph that has been located in a text, magazine, or newspaper and have them determine pertinent information from the graph.

Teaching Page 281

Pupil's Objectives

(a) To maintain previously learned skills and abilities with operations involving whole numbers and rational numbers expressed in fraction, decimal, and per cent form; and (b) to review mean, median, and mode.

Background

The exercises on this page give pupils practice in computing and in solving mathematical sentences requiring one or more of the four operations.

At the bottom of the page are exercises for a review of mean, median, and mode.

Pre-Book Lesson

- Review the meaning of the term *data*.
- Summaries for a set of data are measures of central tendency often referred to as averages. Briefly review three measures of central tendency among data: mean, median, and mode. The measure that occurs most frequently in any set of data is called the *mode*. When the numbers in a set of data are arranged in order of increasing or decreasing value, the number in the middle is called the *median*. To find the *arithmetic mean* or just *mean*, we add the measures for a set of measurements and divide this sum by the number of measurements in the set.
- In some applications, it is possible for there to be two modes in a set of data. If the highest frequency is common to three or more numbers, we say there is no mode. If there is an even number of items in the data, there is no middle number, and the median is one half of the sum of the two middle numbers.
- The three measures of central tendency for any one set of data may differ widely. These measures are used to emphasize different relationships in a set of data.

Using the Text Page

- The exercises on the pupil's page may be used for review, for reteaching, or for a test. You may want to do several examples with the class and have them do the rest as a test. After correcting the work, group pupils for reteaching according to weaknesses revealed by the test.
- Ex. 6. In Ex. 6 of the review exercises, the relationship among the measure of the total weight t, the measure of the mean weight m, and the number of springs n, may be expressed by the factors-product relationship as

$m = t \div n$ so $t = n \times m$, then $t = 50 \times 2.32$.

Individualizing Instruction

- Use the test scores of the last test given to the class as a set of data. Have *all pupils* find the mean, median, and mode for the data. Discuss the effect, if any, of very high or very low scores on each of these averages.

Have *all pupils* find the effect on the mean, median, and mode of the data in Ex. 3 if 550 is changed to 970. Have them state as a generalization the effect on the mean, median, and mode of changing one of the higher or one of the lower numbers in a set of data.

- Have *more capable pupils* attempt to devise a set of data for which the mean, median, and mode are each the same number.
- Below are three statements made by an employer referring to the salaries paid his employees. Have *slower learners* determine the mean, median, and mode of the salaries: "The most common salary paid to my employees is $5,500 a year (mode). Half of my employees earn a salary of $5,800 or more (median). My employees receive a mean salary of $6,675 (arithmetic mean)."

To Keep in Practice

[W]

Ex. 1–5. Copy and multiply.

1.	2.	3.	4.	5.
235	5,068	$273.55	2.06	0.0586
43	509	33	1.7	0.27
10,105	2,579,612	$9,027.15	3.502	0.15822

Ex. 6–10. Round to the nearest hundredth.

6. 4.703 4.70 7. 0.077 0.08 8. 37.925 37.93 9. 8.008 8.01 10. 0.5061 0.51

Ex. 11–14. Find the unknown factor to the nearest hundredth.

11. $6.732 \div 5 \wedge$ 1.35 12. $23 \div 17 \wedge$ 1.35 13. $0.067 \div 4 \wedge$ 0.02 14. $1.3 \div 0.8$ 1.63

Ex. 15–26. Find the number represented by the letter.

15. $n = 4.5 \times 8.2$ 36.9

16. $x - 3.91 = 12.36$ 16.27

17. $x + 0.46 = 1.83$ 1.37

18. $n = 3\frac{1}{2} \times 5\frac{1}{3}$ $18\frac{2}{3}$

19. $n = \frac{4}{5} \div 2\frac{3}{4}$ $\frac{16}{55}$

20. $x = 45\% \times 75$ 33.75

21. $n = 3.7 + 6.2 + 9.9$ 19.8

22. $x = 15\% \times (456 - 233)$ 33.45

23. $x = (253 + 587 + 1{,}305) \div 5$ 429

24. $n \div 1\frac{4}{5} = 550$ 990

25. $n = 62\% \times 800$ 496

26. $685 \times x = 68{,}500$ 100

Working with Data

Resurvey; mean, mode, median [W]

Ex. 1–3. Find the arithmetic mean for the data.

1. 3.4 cm., 3.8 cm., 3.3 cm., 3.7 cm., 3.3 cm. 3.5 cm.

2. 12.2 in., 12.1 in., 11.9 in., 11.8 in. 12.0 in.

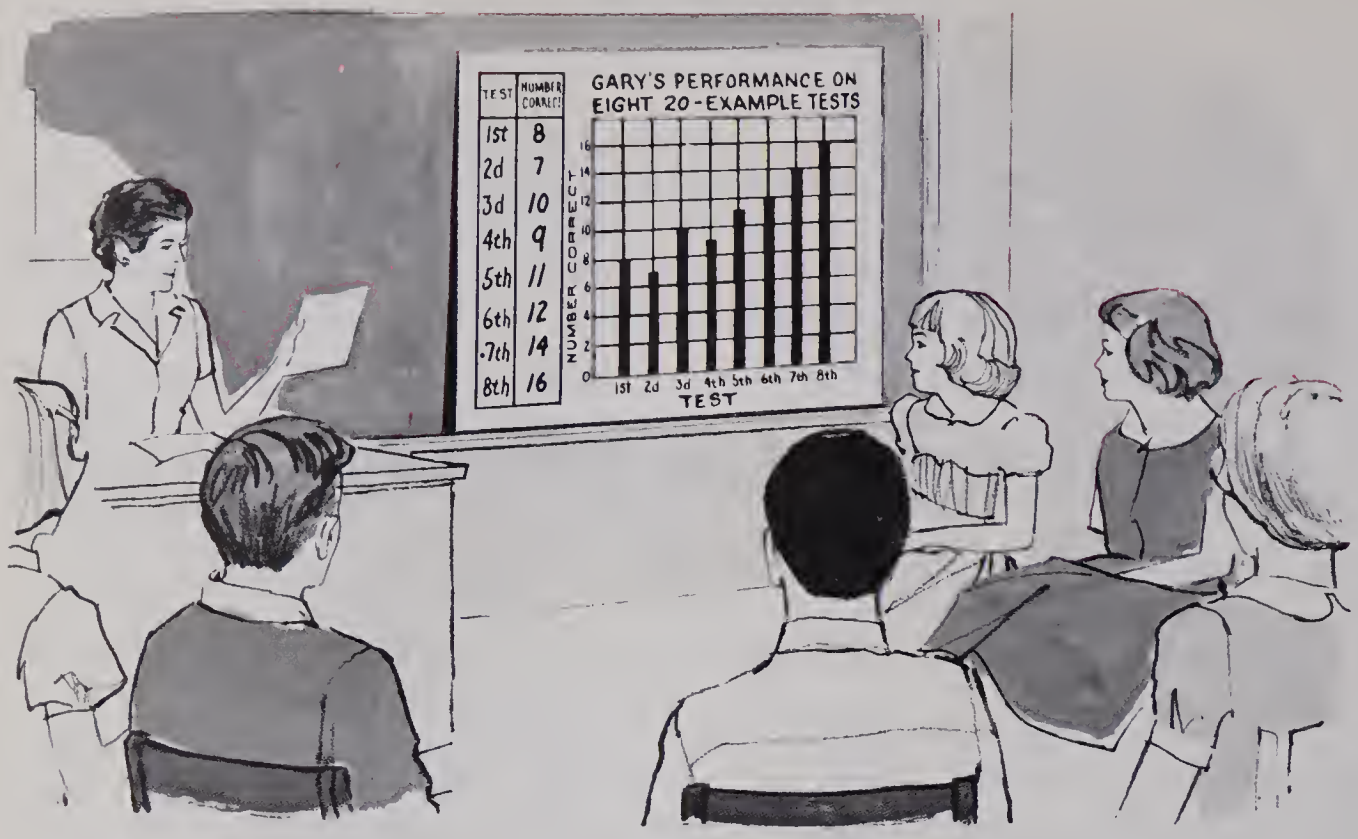
3. 550 mi., 488 mi., 501 mi., 524 mi., 523 mi., 498 mi. 514 mi.

4. Find the mode for the data in Ex. 1. 3.3 cm.

5. Find the median for the data in a. Ex. 1. 3.4 cm. $\wedge$ b. Ex. 2. 12.0 in.

6. Fifty small metal springs were weighed and the mean weight was found to be 2.32 gm. What was the total weight of the fifty springs? 116 gm.

*Emphasize that certain types of graphs are used to picture certain types of relationships.



* Line Graphs

[o]

Each month, for eight months, Miss Barr gave a 20-example mathematics test to her class. The table and the vertical-bar graph in the picture above both show Gary's performance on each of the eight tests.

1. How many examples did Gary work correctly on the 1st test? 8 That was what per cent of the 20 examples? 40%

2. On which test did Gary work 60% of the examples correctly? 6th Explain your reasoning. 60% × 20 = 12

3. On the 5th test, Gary worked _? 11 examples correctly and on the 6th test, he worked _? 12 examples correctly. Was that a change for the better or for the worse? better

4. Looking only at the bar graph, tell whether the change in Gary's performance was for the better or for the worse on the 2d test; worse on the 3d test; better on the 8th test. better

Teaching Pages 282 and 283

Pupil's Objective

To learn to read, interpret, and construct line graphs.

Background

These exercises introduce the pupils to the concept of representing a set of data with a line graph.

Most collections of data can be represented by more than one type of graph, but different graphs have different advantages. When deciding what kind of graph to use for representing data, both the nature of the data and the advantages of the different graphs should be kept in mind.

The pictograph vividly represents comparisons among quantities of objects using easily recognized picture-symbols.

The bar graph can be used to represent any type of data represented with a pictograph, yet the bar graph is usually easier to construct. The bar graph has the advantage of being useful for representations which are difficult or impossible with a pictograph (i.e., comparisons of temperatures heights, distances).

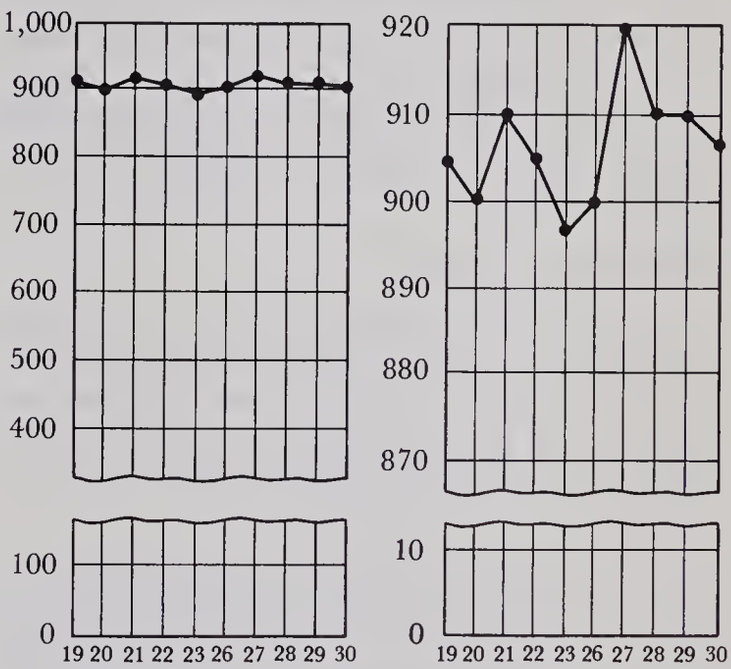
When the purpose of the graph is to show the direction of change of a set of data, the line graph is the type of graph which should be used. Line graphs can be used to show trends, cycles, changes in a variable, or relationships between two variables.

One of the exercises on the text page involves the pupils bringing to class some line graphs. It is probable that some pupils will bring line graphs in which a portion of the graph is omitted. This can create a false impression by exaggerating a rate of change in the data, and the pupils should be aware that the slope of the line would be different if the missing portion of the graph had not been omitted.

Omitting a portion of a graph is sometimes an indication that an attempt is being made to create a false impression. However, omitting a portion of a graph is sometimes useful when the purpose of the graph is to show a trend or cycle in a variable and the variation in per cent form is very small. Graphic representation of stock-market prices is an excellent example. The variation in day-to-day prices of stocks listed on the New York Stock Exchange may be quite small on a basis of per cent increase or per cent decrease. Yet the trend is extremely important.

Below are two graphs representing the Dow Jones Index for stocks of industrial firms listed on the New York Stock Exchange. This index is a number calculated by dividing the price of stocks of certain firms listed on the exchange by a fixed number. This fixed number is selected so that the index will be a number convenient to use, and it is changed to reflect such things as stock splits so that day-to-day changes in the Dow Jones Index is a meaningful indication of the day-to-day changes in stock-market prices in general. There is a Dow Jones Index for stocks of industrial firms, for railroads, for utilities, and for other such firms. They are sometimes called the Dow Jones Averages.

Dow Jones Industrial Average



In the first graph, the Dow Jones Industrial Average is shown for a ten-day period. Saturdays and Sundays are not shown as there is no trading on the stock exchanges on those days. The entire graph is shown.

In the second graph, the Dow Jones Industrial Average is shown for the same period as in the first graph. However, in this second graph, a large portion has been omitted and the vertical scale is

much greater than in the first graph. The second graph shows in much better detail the day-to-day trend in prices of stocks. In using graphs of this type, care must be taken because there is always the danger of creating a false impression among those people who fail to realize that a portion of the graph has been omitted and that the vertical scale is much greater than it would be if the entire graph were shown.

Pre-Book Lesson

Write on the board a table of total class attendance for the past five days. Have pupils begin to construct a vertical-bar graph to represent this data, but instead of having them draw the bars, have them just draw the marks which would form the upper end of the bars, each bar representing one day of attendance. Ask them if they can read the graph just as well without the bars being drawn in. Then have them erase each mark and replace it with a dot in what would have been the center. Ask them if they could read the graph just as well with only the dots being used to represent the data. Then ask for suggestions as to what could be added to the graph to make the direction of change in the data more prominent, leading them to the concept of a line graph.

Using the Text Pages

• Ex. 1–5. When doing the oral exercises, emphasize that the line graph shows trends and direction of change. Have the pupils realize that making a line graph is easier than making a bar graph to represent the same set of data. Review briefly the meaning of per cent.

• Ex. 6–7. The pupil is given a line graph that is already partially constructed so that he can have experience in making a line graph without having to decide about horizontal and vertical scales. In Ex. 7, the pupil has to make the entire graph, but he can use the graph at the top of the page as a guide.

Individualizing Instruction

• Have *more capable pupils* find (or you may obtain for them) a line graph in which a portion of the graph is omitted, such as that shown in the Background section. Have them show what the graph would be like if the missing part had not been omitted. Have them explain the purpose of omitting a portion of the graph. Use the material from the Background if you wish.

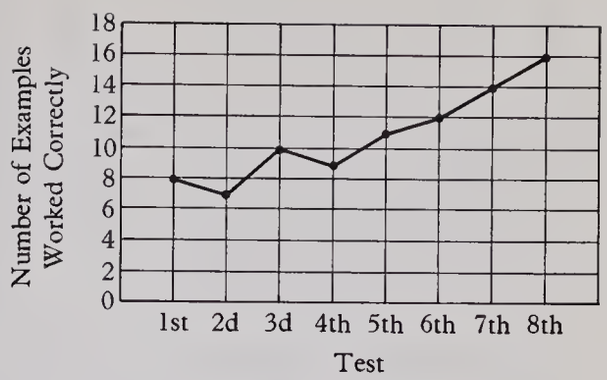
• Have *all pupils* collect data such as temperature changes, test scores, or attendance and represent the data with a line graph. Give them guidance in selecting appropriate horizontal and vertical scales if necessary.

Give *all pupils* a set of data and have them represent it first with a pictograph, then with a bar graph, and then with a line graph. Have them compare the graphs, and discuss briefly the type of data for which each type of graph is best suited. Point out that the most suitable graph depends largely on what characteristics of the data are to be stressed.

• Point out to *more capable pupils* that, although the points on the line between the dots in the graph at the bottom of page 283 do not represent exact data, they may be used in estimating data. For example, the graph shows that at 9:30 A.M. the temperature was *about* 5°F. Estimation of this type is done a great deal in analyzing scientific data by graphing it on such a line graph. Estimation of this type cannot be done with a line graph of the type at the top of the text page. Discuss why.

• Be sure to point out to *slower learners* that, in doing Ex. 7, they can use the graph at the top of the page as a guide.

The graph at the right shows more clearly the changes in Gary's performance. Such a graph is called a *broken-line graph* or, simply, a *line graph*. Instead of making vertical bars, dots are placed where the tops of the bars would have been. Then the dots are connected.

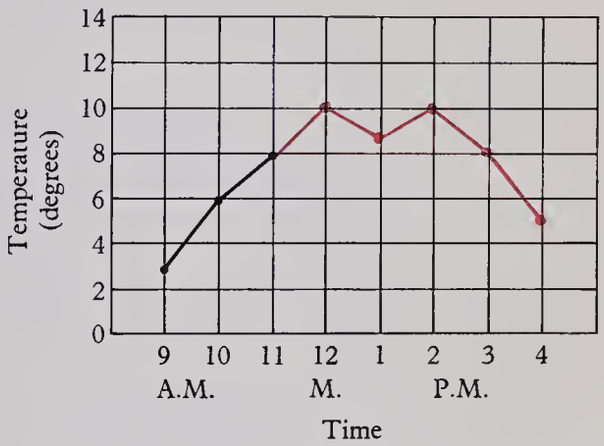


Gary's Performance on Eight 20-Example Tests

5. Explain how the line graph at the right above shows the changes in Gary's performance. Key idea: The line is shown going up where Gary's performance changed for the better and going down when his performance changed for the worse. [W]

6. One very cold day, Ted recorded the temperature every hour from 9 A.M. to 4 P.M. Copy the graph at the right below and complete it using the information in the table below.

Time	Temperature	Time	Temperature
9 A.M.	3°	1 P.M.	9°
10 A.M.	6°	2 P.M.	10°
11 A.M.	8°	3 P.M.	8°
12 M.	10°	4 P.M.	5°

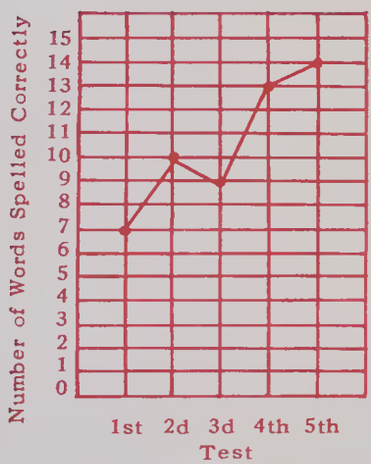


Hourly Temperature from 9 A.M. to 4 P.M.

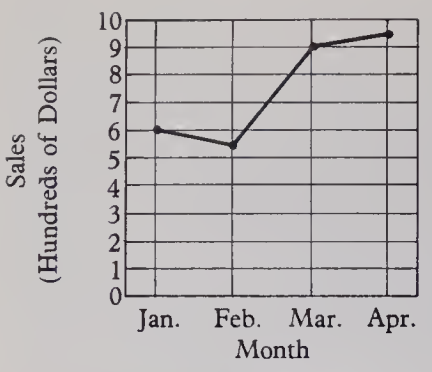
7. There were 15 words on each of five spelling tests. The number of words that Gary spelled correctly on each test was as follows: 1st test, 7; 2d test, 10; 3d test, 9; 4th test, 13; 5th test, 14. Make a line graph to show this information.

8. Find some line graphs and bring them to class. Be prepared to explain the graphs.

♦ **Extra Activity.** Set 177.

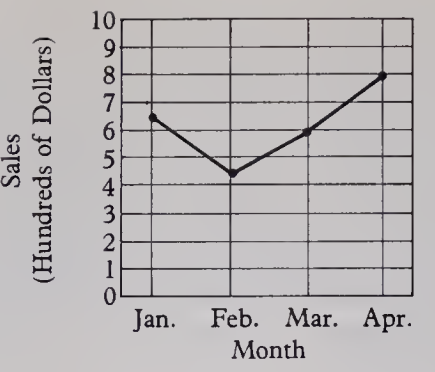


Gray's Performance on Five 15-Word Spelling Tests



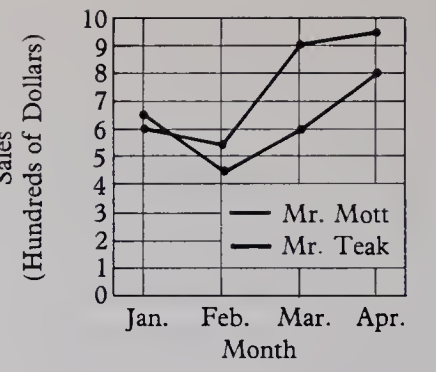
Mr. Mott's Monthly Sales

Fig. 1



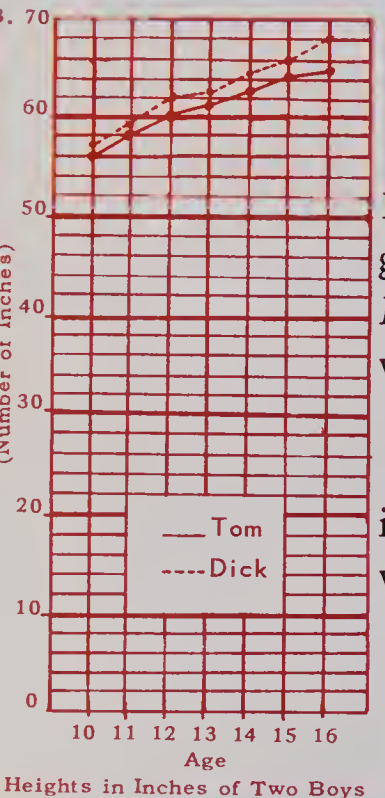
Mr. Teak's Monthly Sales

Fig. 2



Monthly Sales of Two Salesmen

Fig. 3



Heights in Inches of Two Boys

Two Graphs Using One Set of Scales

Double-Line graph [O]

1. Since the scales are labeled the same in the graphs in Fig. 1–2, the data shown in the two graphs can be shown in just one graph as in Fig. 3. Who had the greatest total sales during March, Mr. Mott or Mr. Teak? Is it easier to compare data when both graphs are shown on one set of scales? Yes

[W]

2. Copy the graph shown in Fig. 3 and then show the following information using the same scales. Mr. McCabe's total sales were: January, $800; February, $500; March, $600; April, $1,000.

Ex. 3–4. Make a double-line graph for each table.

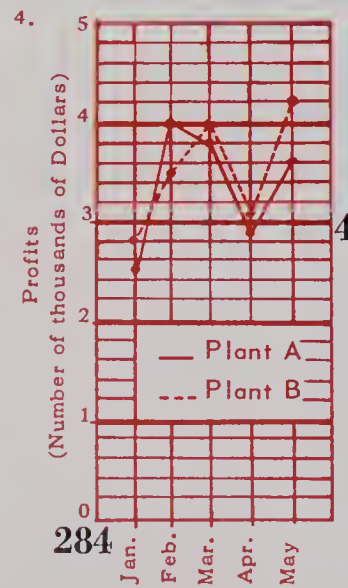
3.

Heights in Inches of Two Boys							
Boy Age:	10	11	12	13	14	15	16
Tom	56	58	60	$61\frac{1}{2}$	$62\frac{1}{2}$	64	65
Dick	57	59	62	63	$64\frac{1}{2}$	66	68

4.

Profits in Thousands of Dollars of Three Plants					
Plant	Month				
	January	*February*	*March*	*April*	*May*
A	2.5	4	3.8	2.9	3.6
B	2.8	3.5	4	3.1	4.2

4.



Profits in Thousands of Dollars of Two Plants

Teaching Page 284

Pupil's Objectives

To learn to read and construct double-line graphs.

Background

On page 280, pupils were introduced to the idea of the double-bar graph. They learned that the double-bar graph represents two sets of related data, and by plotting both sets of data on one graph, relationships could be more easily recognized than if the data were plotted on two separate graphs. On this page, pupils learn that the data from two graphs can be represented on one double-line graph—two graphs using the same scales. If two sets of data are so related that they can both be plotted on the same scales, not only is a comparison of the data easier than if the data had been plotted on separate graphs, but also the relationships between trends, cycles, and directions of change are much more readily apparent.

Pre-Book Lesson

Review the purpose of the double-bar graph and its advantages over the use of two regular bar graphs for representing sets of related data. Ask if there is any way that the concept of the double-bar graph could be applied to the use of line graphs for representing data. Lead pupils to the realization that two sets of related data can be plotted on one line graph providing that the set of scales applies to both sets of data.

Using the Text Page

- Ex. 1. Point out that when two line graphs are plotted on one set of scales, the sizes of the data can easily be compared; and trends, including both direction and rate of change, can easily be recognized.
- Ex. 2. When both line graphs are plotted using the same scales, the lines must be shown so they can be easily distinguished from each other. They may each be shown in a different color or one shown as a solid line and the other as a dotted line or as a line of dashes. A key should be shown on the graph as indicated in Fig. 3 on the text page.
- Ex. 3–4. In plotting data of this type, the graph is usually easier to read if the quantity being measured is represented by the vertical scale and time intervals represented by the horizontal scale.

Individualizing Instruction

- Have *all pupils* try to find and bring to class line graphs consisting of two graphs using the same scales. Have the pupils compare sizes, trends, and rates of change in direction of the data.
- Help *slower learners* in doing the written exercises by guiding them in selecting the horizontal scale and vertical scale, in making the two lines distinct, and in showing the key on the graph.
- Have *more capable pupils* collect two sets of related data suitable for plotting on a line graph using the same scales and have them construct the graph.

Pupil's Objective

To learn to round greater numbers for making graphs.

Background

When information is presented in table form, comparison of the numbers is often much easier if the numbers are first rounded. For example, comparing 7,549,276 with 7,459,118 is more difficult than comparing 7.55 million with 7.46 million.

Since the advantage of graphs is that information can be compared easily, and since rounded numbers are easier to compare than those that are not rounded, we usually round great numbers when making graphs.

Rounded numbers are frequently used when presenting information in graph or table form simply because exact data is either impossible to find or unnecessary. For example, an exact count of the population of the United States is impossible to obtain, and approximation is adequate for purposes of comparison of data.

Another reason why rounded numbers are used in making graphs is that it is often physically impossible to plot the exact data on a graph of convenient size. To plot the exact data pertaining to the yearly budget for the United States government for each year for the past 10 years would require a graph of enormous size. If the information consisted of numbers rounded to billions of dollars, it could be presented on a graph of convenient size.

Pre-Book Lesson

• Write on the board a table of data consisting of 5-place numerals. Begin to construct a graph for the data on the board. Attempt to select a proper scale. Discuss the problem that arises, and lead the pupils to recognizing the necessity of rounding the numbers before attempting to construct the graph.

• It may be necessary to review rounding numbers, particularly for *slower learners*. You may wish to give pupils practice rounding numbers to the nearest million before doing the written exercises.

Using the Text Page

• Ex. 1. Be sure that pupils understand the meaning of the expression, "paid attendance."

Slower learners may need to be told that the numerals in the table represent thousands as is indicated by the label at the top of the table.

• Ex. 2. Discuss the meaning of the term, "poultry."

Review the concept of a horizontal-bar graph, and a horizontal scale.

Individualizing Instruction

• Have *more capable pupils* find the population from the last five census years in a world almanac or other sources. Rounding each number to the nearest million, have them show the information on a line graph.

• Obtain from a world almanac or other sources the total area of the United States at ten-year intervals, 1790–1960. Have selected pupils put this information on the board. *Slower learners* may make a new table from this information, rounding the numbers to the nearest hundred thousand square miles.

• Have *all pupils* make a line graph to represent this information, using the rounded numbers. You may have *more capable pupils* compare population growth and growth in area of the United States during any given period.

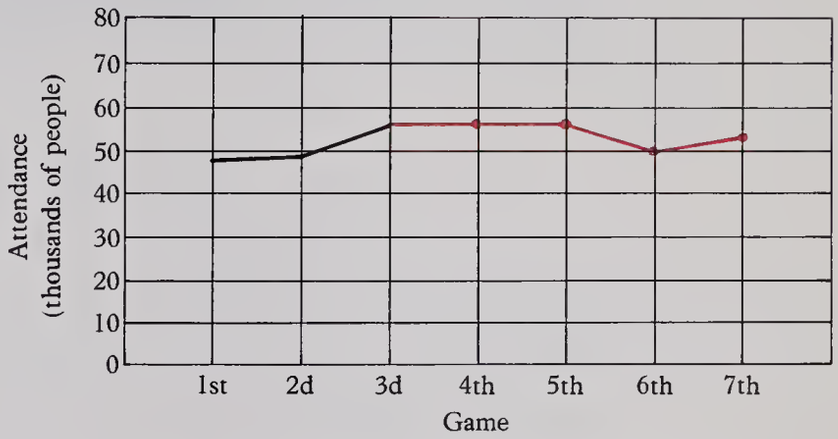
Graphing, Using Greater Numbers

[O]

1. For a recent World Series, the paid attendance at each game was as follows: 1st game, 47,797; 2d game, 48,700; 3d game, 55,934; 4th game, 55,920; 5th game, 55,801; 6th game, 49,578; 7th game, 50,596.

a. We can graph this information more easily if we first round each of the numbers to the nearest thousand. 47,797 rounded to the nearest thousand is *48,000* or _?48 *thousands*. On the board, copy and complete the table at the left below by rounding each number to the nearest thousand.

Game	Attendance (thousands)
1st	48
2d	_?49
3d	_?56
4th	_?56
5th	_?56
6th	_?50
7th	_?51



Paid Attendance During a Recent World Series

b. Now, using the information in your table, copy and complete the line graph at the right above.

[W]

2. During one voyage, the number (rounded to the nearest thousand) of pounds of each of seven items used by the ocean liner Delta was as follows: meat, 72,000; poultry, 25,000; fish, 12,000; potatoes, 60,000; fresh vegetables, 34,000; frozen vegetables, 9,000; sugar, 12,000. Make a horizontal-bar graph to show this information. Let each unit on the horizontal scale represent 5 (thousands of pounds). See Teacher's Page 287.

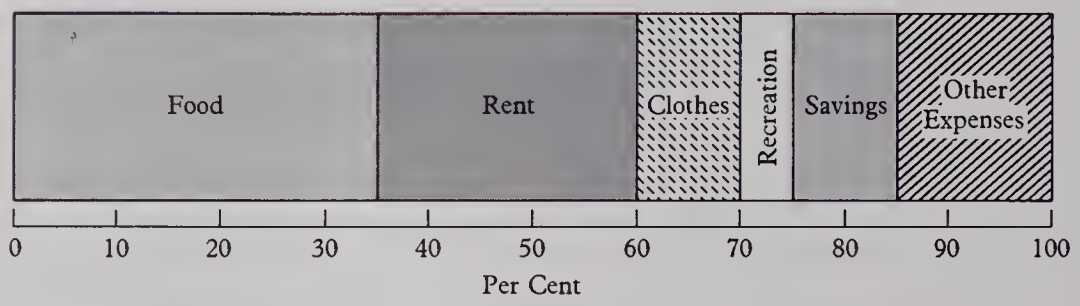
♦ **Extra Activity.** Set 178.

Divided-Bar Graphs and Circle Graphs

[o]

Each month, Mr. Jackson uses 35% of his income for food, 25% for rent, 10% for clothes, 5% for recreation, 10% for savings, and 15% for other expenses.

1. The graph below shows the above information. Such a graph is called a *divided-bar graph*. Tell why this is a good name for such a graph. The informotion is shown with o divided bor.

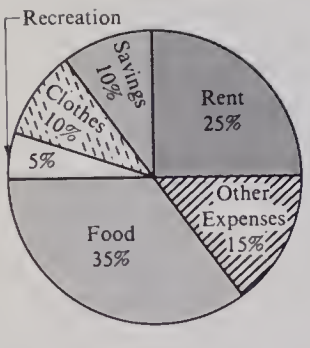


How Mr. Jackson Uses His Monthly Income

2. For the above graph, each unit on the horizontal scale represents _?_ 10 per cent. If the scale were not shown, could you still tell for which item Mr. Jackson uses the greatest part of his income? Yes Explain. The lorgest section of the divided bar shows for which item Mr. Jockson uses the greotest part of his income.

3. To find *how much* Mr. Jackson uses for each item, what information would you need? Mr. Jackson's monthly income

4. If Mr. Jackson's monthly income is $460, find, at the board, how much he uses each month for **a.** food. $161 **b.** recreation. $23



5. The graph at the left also shows how Mr. Jackson uses his income. Such a graph is called a *circle graph*. The region shown labeled "Food 35%" is _?_ 35% of the circular region. Give similar statements for the other regions shown. Express each part with a fraction. Food, $\frac{7}{10}$; Recreation, $\frac{1}{20}$; Clothes, $\frac{1}{10}$; Sovings, $\frac{1}{10}$; Rent, $\frac{1}{4}$; Other Expenses, $\frac{3}{20}$

6. If the per cent forms were not shown in the circle graph, could you still tell for which item Mr. Jackson uses the greatest part of his income? Yes

Teaching Pages 286 and 287

Pupil's Objectives

(a) To learn to read and to construct divided-bar graphs; and (b) to learn to read circle graphs.

Background

Up to this point pupils have studied five types of graphs:

(1) Pictograph—small easily recognized pictures are used to represent various amounts of objects.

(2) Bar graph—the length of the bars may represent the same type of data as the pictograph, but the bars may also represent quantities not conveniently represented with a pictograph.

(3) Double-bar graph—the bars are used to represent two sets of related data on one graph in such a way that the data in one set is easily compared with data in the other set.

(4) Line graph—a line is used to show the direction of change of a set of data. The line graph shows trends, cycles, change in a variable, or relationships between two variables.

(5) Two line graphs using one set of scales—the two lines are used to compare the trends in one set of data with the trends in a second set of data.

Another use of graphs is to show the relationship between the parts of a whole and the whole itself, such as the relationship between the items that make up a budget and the total budget. The two types of graphs used for this purpose are the *divided-bar graph*, which is sometimes called the rectangular-distribution graph, and the *circle graph*. In each of these types of graphs, area is used to represent the data.

If a circle graph is used to represent a family budget, and 20% of the budget is for rent, then 20% of the circular region must be used to represent the rent portion of the budget. To do this, central angles are pictured from the center for the circle. The sum of the measures of all such angles will be 360 in degrees. The angle used to form the portion of the circular region must be 20% of 360° or 72°. An angle of 72° is drawn with the use of a protractor at the center for the circle so that it intersects the circle to form a region whose area is 20% of the area of the entire circular region. Although the exercises do not require the pupils to construct a circle graph, you may wish to assign such an activity to some of the *more capable pupils*.

Partitioning a region in this way to represent data also applies to the divided-bar graph. If a rectangular region is used to represent a total budget, then an item which is 20% of the budget will be represented by a region whose area is 20% of the area of the entire given rectangular region.

If the data for either of these types of graphs is not given in per cent form, it is usually best to first rename the data with per cent forms by methods described on pages 258–261 before attempting to construct the graph.

One advantage of these two types of graphs is that relationships between the parts of a whole and the whole itself are readily apparent even when the scale or the per cent forms are not indicated on the graph.

Pre-Book Lesson

• Review briefly the use of per cent forms and renaming whole numbers and rational numbers with fractions and decimals. Have pupils practice renaming in per cent form.

• Write on the board the items of a typical family budget indicating with per cent forms the part of the entire budget that each of the items constitute. Draw a rectangular region to represent the entire budget. Ask pupils how the data on the board could be represented by the use of this one rectangular region. Lead them to the concept of the divided-bar graph. Use the one presented on the text page. Repeat this procedure using a circular region to represent the entire budget, leading pupils to the concept of a circle graph like the one presented on the text page.

Using the Text Pages

• Ex. 1–9. By doing the oral exercises before beginning the written exercises, pupils should have learned: (a) that the divided-bar graph and the circle graph are used to compare a whole and any of its parts; (b) how to determine the quantity represented by each part of the divided-bar graph

or the circle graph when the total quantity represented by the entire graph is given and the parts are named with a per cent; (c) how to construct a divided-bar graph when the part of the whole formed by each item is given in per cent form; and (d) how to construct a divided-bar graph when the measure of each part is given.

• Ex. 10–15. Notice that in the written exercises pupils are not required to construct a circle graph. However, they should be able to read and interpret such a graph.

If the number of written exercises is excessive for *slower learners*, assign only Ex. 10–12 for written work.

Individualizing Instruction

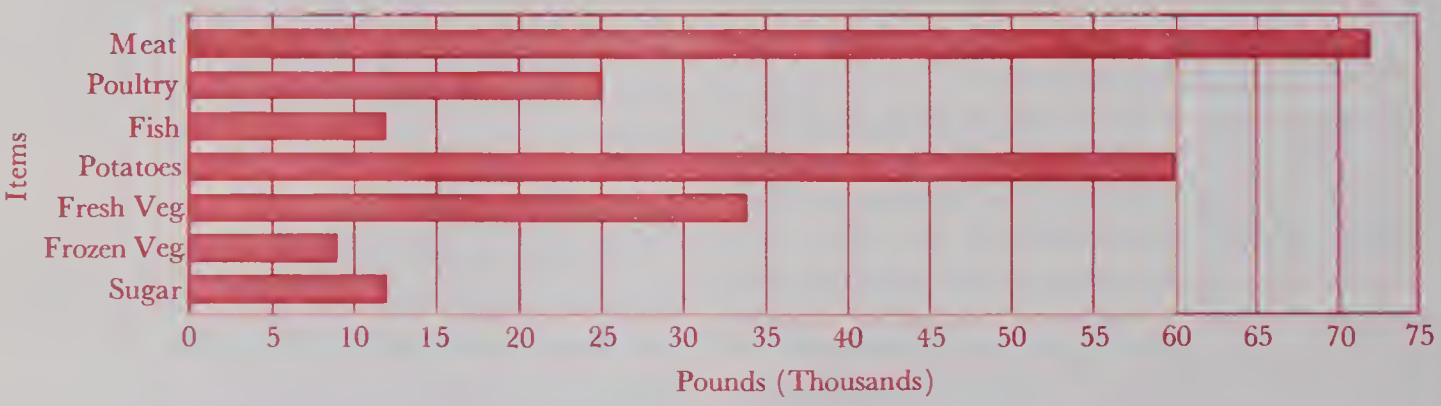
• Have *more capable pupils* construct a circle graph using the information given in Ex. 13.

• Have *all pupils* bring to class examples of divided-bar graphs and circle graphs found in magazines and newspapers. Have them read and interpret these graphs.

Discuss with *all pupils* the possible advantages of the circle graph over the bar graph and vice versa for representing a whole and its parts.

• *Slower learners* may try to find and interpret divided-bar and circle graphs in other texts such as social studies or science.

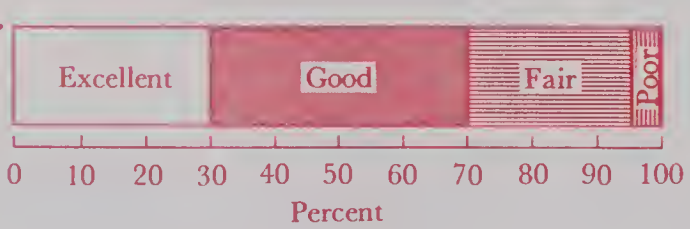
ANSWERS NOT SHOWN ON PAGE 285



Number of Pounds of Each of Seven Items used by the Ocean Liner Delta During One Voyage

ANSWERS NOT SHOWN ON PAGE 287

10.

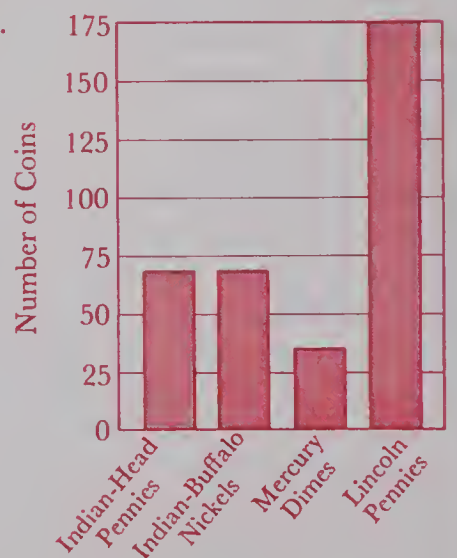


How Twenty Mathematics Papers were Marked

11.

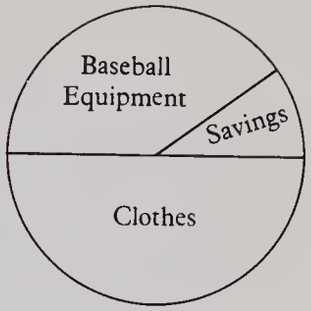
How Ted Spends a Typical Day

14.



David's Coin Collection

7. What information does the circle graph at the right show? How Jim used $50 What per cent of the $50 did Jim use for clothes? 50% Explain your reasoning. One half, or 50%, of the circular regian is labeled "Clothes."



How Jim Used $50

8. Jim used 10% of the $50 for savings. Give a fraction in simplest form which tells what part of the circular region is shown labeled "Savings." $\frac{1}{10}$

9. Explain the work in the box for finding how much Jim used for savings. The per cent farm is shown with a fraction and the wark is done. [W]

$n = 10\% \times 50$
$= \frac{1}{10} \times 50$
$= \frac{50}{10}$, or 5
Jim used $5.00 for savings.

10. Of 20 mathematics-test papers, 30% were marked Excellent, 40% were marked Good, 25% were marked Fair, and 5% were marked Poor. Make a divided-bar graph to show this information. Let each unit on the horizontal scale represent 10 (per cent). See Teacher's Page 287.

11. Ted made the table at the right to show how he spends a typical weekday. Make a divided-bar graph to show this information. See Teacher's Page 287.

Sleeping	10 hr.
Eating	$2\frac{1}{2}$ hr.
School	6 hr.
Reading and studying	$1\frac{1}{2}$ hr.
Working	2 hr.
Playing	2 hr.

12. Make a table like Ted's to show how you spend a typical weekday. Then make a divided-bar graph to go with your table. Answers will vary.

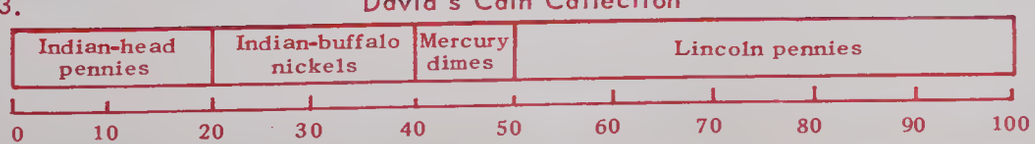
13. Of the 350 coins in David's coin collection, 20% are Indian-head pennies, 20% are Indian-buffalo nickels, 10% are Mercury dimes, and 50% are Lincoln pennies. Make a divided-bar graph to show this information. See below.

14. Make a table to show *how many* of each kind of coin David has in his coin collection (see Ex. 13.). Then make a vertical-bar graph to go with your table. See Teacher's Page 287.

15. Ted's father wishes to make a circle graph to show his expenses for a business trip. His expenses were as follows: transportation: $264; 60% room: $110; 25% business expense: $44; 10% food: $22. 5% Tell the per cent for each item.

● **Extra Activity.** Set 179.

13.



*See pages 21–23 for a review of the base-five system of numeration.

Remembering about Place Value

Resurvey [O]

1. In a base-ten system of numeration, the place values are powers of _?10. In 358, the 3 in its place indicates the number _?300. The 5 in its place indicates the number _?50.

2. An expanded form for 358 is (3 × _?100) + (5 × _?10) + (8 × _?1). On the board, write an expanded form for 2,653. (2 × 1,000) + (6 × 100) + (5 × 10) + (3 × 1)

*3. In a base-five system, the place values are powers of _?5. In the base-five numeral 213, the 3 is in _?one's place, the 1 is in _?five's place, and the 2 is in _?twenty-five's place. At the board, rename 213_{five} with a base-ten numeral. 58

4. In a base-ten system, the value of a place is _?10 times the value of the place to the right. The value of a place is how many times the value of the place to the right in a base-five system?5 times in a base-seven system?7 times in a base-four system? 4 times

5. Is the Roman numeral system a place-value system?No Explain. Numerals name the same number in any place. On the board, write the standard numeral for the number CXXIV;124 MMXIX;2,019 DCCLX;760 CDXXV;425 CMLIV. 954

Renaming Numbers

Resurvey [W]

Ex. 1–5. Write a base-five numeral for the number.

1. 12 22_{five} 2. 37 122_{five} 3. 26 101_{five} 4. 9 14_{five} 5. 130 1010_{five}

Ex. 6–10. Write a base-eight numeral for the number.

6. 16 20_{eight} 7. 64 100_{eight} 8. 65 101_{eight} 9. 32 40_{eight} 10. 77 115_{eight}

Ex. 11–19. Write a base-ten numeral for the number.

11. 41_{five} 21
12. 222_{three} 26
13. 26_{eight} 22
14. 62_{seven} 44
15. 302_{four} 50
16. 27_{eight} 23
17. XXXVIII 38
18. CCXIX 219
19. DCVIII 608

20. Copy and complete: 22 is a base-_?four numeral for the number ten.

Teaching Page 288

Pupil's Objective

To compare the base-ten system to systems with bases other than ten and the Roman-numeral system with special emphasis on place value.

Background

The exercises on this page review ideas related to place value in various bases; writing numerals in expanded form; naming numbers with numerals in other bases; and expressing Roman numerals with base-ten numerals. The purpose of the review is to help pupils maintain skills and understandings related to place value introduced at the beginning of the book.

Pre-Book Lesson

• Review expressing numbers in exponent form to prepare for work with bases other than ten.

• Quickly review the basic concepts of place value, asking such questions as:

(1) What do we mean when we say a system of numeration is a place-value system? (Each place in a numeral has a value different from any other place.)

(2) Name a system of numeration which is not a place-value system. (The Egyptian system.)

(3) How could a set of 24 objects be grouped so as to illustrate the base-five numeral and the base-eight numeral for the number property of the set? (4 fives and 4 ones; 3 eights)

Using the Text Page

• The oral exercises should be sufficient preparation for most pupils for working the written exercises. However, you may use one or two of the written exercises for oral review before assigning the remaining exercises as written work. After correcting the exercises, you may wish to go over difficulties with individual pupils. Place value and systems of numeration are taught on pages 13–25 of Chapter 1.

• Ex. 5. For any numeral in a place-value system, every digit in that numeral has two numbers associated with it: the number named by the digit, and the number for the "place" (that is, place value). Therefore, the Roman-numeral system is not considered to be a place-value system.

• Ex. 1–20. Remind pupils that numerals are base-ten numerals unless indicated otherwise.

Individualizing Instruction

• Have *all pupils* express each of the following numbers with a numeral in the base indicated. Have them first rename the given number with a base-ten numeral; and then express this with a numeral in the base indicated:

a. Rename 34_{five} with a base-eight numeral.

b. Rename 1000000_{two} with a base-four numeral.

c. Rename 2002_{five} with a base-twelve numeral.

• If *slower learners* have difficulty with Ex. 20, express the question in the form $22 = (2 \times \_ ? \_) + (2 \times 1)$ as an aid in analyzing the problem.

Teaching Page 289

Pupil's Objective

To review some ideas associated with sets of numbers including designating sets, union of sets, intersection of sets, and subsets.

Background

The main purpose of this review page is to prepare pupils for an introduction to the concept of Venn diagrams on the next page. Some relevant generalizations which pertain to sets are given below so that you may refer to them while conducting this review.

(1) Intersection of sets. The intersection of set A and set B, denoted by "$A \cap B$," is the set of all members belonging to both set A and to set B. For a member to be in the intersection of two sets, it must be common to both sets.

(2) Union of sets. The union of set A and set B, denoted by "$A \cup B$," is the set of all members that are members of set A or set B or both sets.

(3) Even number. If n is any whole number, then $(2 \times n)$ is an even number. Any whole number that has 2 as a factor is an even whole number. Zero is an even number.

(4) Odd number. If n is any whole number, then $(2 \times n) + 1$ is an odd number. Any whole number that does not have 2 as a factor is an odd whole number.

(5) Multiples. If n is a counting number and k is a counting number, then $(k \times n)$ is a counting-number multiple of n. The set of whole-number multiples of a number is formed by multiplying the given number by each whole number in turn.

(6) Factor. A counting number, f, is called a factor of another counting number, n, if n is a counting-number multiple of f.

(7) Subset. If every member of set A is also a member of set B, then set A is a subset of set B. The empty set is a subset of every set and every set is a subset of itself.

Pre-Book Lesson

Briefly review the concepts listed in the Background.

Using the Text Page

• Ex. 7. In this example, pupils may confuse "list the factors of 24" with "express 24 as a product of primes." In both instances, pupils are to find factors of 24, but in the first instance they are to show every number that is a factor of 24. (1, 2, 3, 4, 6, 8, 12, 24) In the second instance they are to find the set of prime numbers whose product is 24. $(2 \times 2 \times 2 \times 3)$ If any such confusion becomes evident when pupils are doing this exercise, give them additional work so that they will be able to differentiate between the instructions.

• Ex. 9–14. After the oral exercises are completed, you may use the written exercises for testing purposes if you wish.

• After returning corrected exercises, group pupils for reteaching in order to remedy any weaknesses. A review of pages 2–9 dealing with ideas related to sets may be helpful. Give *slower learners* additional written exercises.

Individualizing Instruction

Have *more capable pupils* describe, using braces, the total number of subsets for each of these sets. (Make sure they don't forget that the empty set is a subset of every set; and every set is a subset of itself.)

(1) {3, 5, 7, 8}

(2) {0, 4, 10, 12}

(3) {△, ○, □}

(4) The set of subjects you are taking in school this year.

(5) { }

Have these pupils describe verbally union and intersection of sets, subsets, even and odd numbers, and factor.

Sets of Numbers

Resurvey [O]

1. $\{1, 2, 3, \cdots\}$ is the set of _?△(counting) numbers.

2. $\{0, 1, 2, 3, \cdots\}$ is the set of _?△(whole) numbers. Which whole number is not a counting number? 0

3. $\{0, 2, 4, 6, \cdots\}$ is the set of _?△(even) whole numbers.

4. $\{1, 3, 5, 7, \cdots\}$ is the set of _?△(odd) whole numbers. What is the intersection of this set and the set described in Ex. 3? $\{\ \}$

5. What is the union of the sets described in Ex. 3–4? $\{0, 1, 2, 3, \cdots\}$

6. $\{0, 5, 10, 15, \cdots\}$ is the set of whole-number multiples of _?5_. On the board, use braces to describe the set of counting-number multiples of 5. $\{5, 10, 15, \cdots\}$

7. On the board, list within braces the factors of 24; of 30; of 50. $\{1, 2, 3, 4, 6, 8, 12, 24\}$; $\{1, 2, 3, 5, 6, 10, 15, 30\}$; $\{1, 2, 5, 10, 25, 50\}$

8. Is each of the sets described in Ex. 1–7 a subset of the set of rational numbers?∧(Yes) Is $\frac{1}{2}$ a rational number?∧(Yes) Is $\frac{2}{3}$?∧(Yes) Is 5.7? Yes

[W]

9. List within braces the factors of 24 which are even numbers. $\{2, 4, 6, 8, 12, 24\}$

10. List within braces the factors of 24 which are also factors of 30. $\{1, 2, 3, 6\}$

11. List within braces the members of $\{0, \frac{2}{5}, 5, 8\frac{1}{2}, 12, 35\}$ which are

a. counting numbers. $\{5, 12, 35\}$

b. whole numbers. $\{0, 5, 12, 35\}$

c. multiples of 5. $\{0, 5, 35\}$

d. multiples of 3. $\{0, 12\}$

e. rational numbers. $\{0, \frac{2}{5}, 5, 8\frac{1}{2}, 12, 35\}$

f. greater than 8.4. $\{8\frac{1}{2}, 12, 35\}$

Ex. 12–14. Write *T* or *F*.

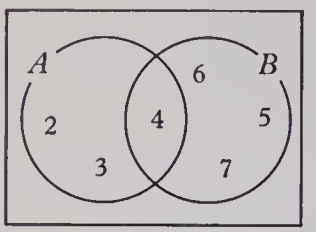
12. The set of counting numbers is a subset of the set of whole numbers. T

13. $\{3, 5, 7\}$ is a subset of the set of odd numbers. T

14. The set of factors of 18 is a subset of $\{1, 3, 5, \cdots\}$. F

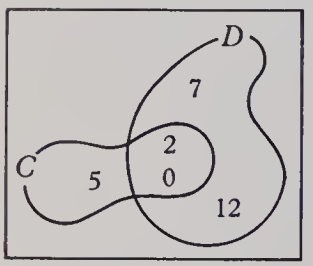
Venn Diagrams

Extension [O]



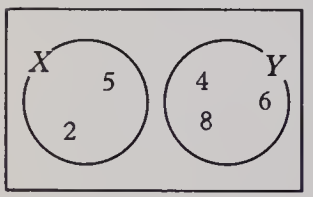
1. Set relationships and operations can be pictured by diagrams like the one at the left. Such diagrams are called *Venn diagrams*. In a Venn diagram, the members of a set are shown in the interior for a simple closed curve.

The members of set A are 2, 3, and 4. The members of set B are 4, 5, _?6, and _?7.

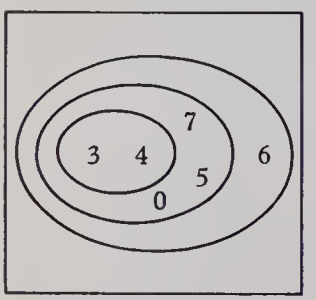


2. Is $\{4\}$ the intersection of $\{2, 3, 4\}$ and $\{4, 5, 6, 7\}$? Yes
How is this shown in the Venn diagram for Ex. 1? In the intersection of the simple closed curves

3. The simple closed curves used in a Venn diagram do not have to be circles. Study the Venn diagram for sets C and D. How many members are there in set C?3 set D? 4 $C \cap D$?2 $C \cup D$?5 On the board, list within braces the members in set C; set D; $C \cap D$; $C \cup D$. {0, 2, 5}; {0, 2, 7, 12}; {0, 2}; {0, 2, 5, 7, 12}



4. On the board, draw a Venn diagram to show $\{5, 7, 9\}$ and $\{2, 3, 7, 9, 15\}$. $\{5, 7, 9\} \cap \{2, 3, 7, 9, 15\} = \{$_?7, _?9$\}$ See below.

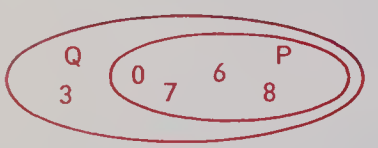


5. The intersection of $\{2, 5\}$ and $\{4, 6, 8\}$ is _?{_.} How does the diagram at the left show this? How many members are there in $X \cup Y$? 5
Key idea: The curves do not intersect.

6. Let P denote $\{0, 6, 7, 8\}$ and let Q denote $\{0, 3, 6, 7, 8\}$.

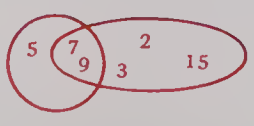
a. Is every member of P also a member of Q? Yes Then is P a subset of Q? Yes Explain what it means to say that one set is a subset of another. Every member of one set is a member of another set.

b. On the board, draw a Venn diagram to show that P is a subset of Q. See left.



7. Is $\{3, 4\}$ a subset of $\{0, 3, 4, 5, 7\}$? Yes Is $\{0, 3, 4, 5, 7\}$ a subset of $\{0, 3, 4, 5, 6, 7\}$? Yes How is this shown in the Venn diagram at the left? Key idea: One simple closed curve shown in the interior for another simple closed curve. From your answers, why is $\{3, 4\}$ a subset of $\{0, 3, 4, 5, 6, 7\}$? Every member of $\{3, 4\}$ is a member of $\{0, 3, 4, 5, 6, 7\}$.

4.



Teaching Pages 290 and 291

Pupil's Objectives

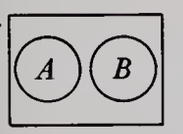
(a) To learn to read and construct Venn diagrams; and (b) to use Venn diagrams to illustrate set relationships.

Background

The members of a set may be represented in the interior for a simple closed curve. When more than one set is being represented in this way, a rectangular region is often used to represent the universal set, the total set of numbers or things about which the diagram is concerned.

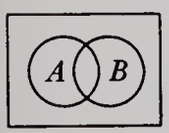
The Swiss mathematician, Leonhard Euler, introduced the idea of using circles to represent sets, and John Venn, a British logician of the 19th century, introduced further refinements of the idea. The pictorial representations of sets by the use of circular regions shown within a rectangular region are referred to as Euler circles or *Venn diagrams*. Simple closed curves other than circles have now come into use for representing sets.

There are five possible representations of a universal set and two sets within the universal set:

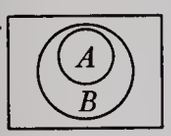
1. 

In Fig. 1, set A and set B are *disjoint*.

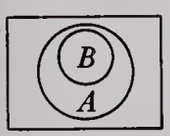
Their intersection is the empty set.

2. 

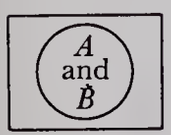
In Fig. 2, set A and set B have an *intersection* but are not proper subsets of each other.

3. 

In Fig. 3, set A is a *proper subset* of set B.

4. 

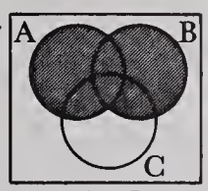
In Fig. 4, set B is a proper subset of set A.

5. 

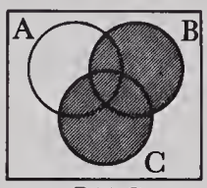
In Fig. 5, set A and set B are *equal sets*.

They are subsets of each other.

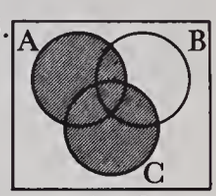
Venn diagrams may be used to indicate set operations. Venn diagrams showing three subsets of the universal set, with several examples of union and intersection illustrated by shaded areas are shown below.

1. 

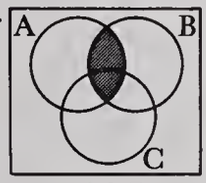
$A \cup B$

2. 

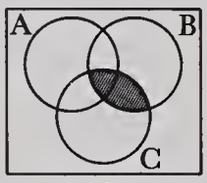
$B \cup C$

3. 

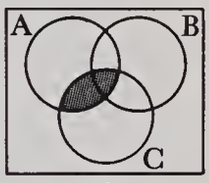
$A \cup C$

4. 

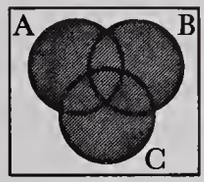
$A \cap B$

5. 

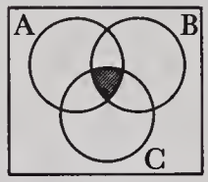
$B \cap C$

6. 

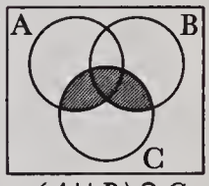
$A \cap C$

7. 

$A \cup B \cup C$

8. 

$A \cap B \cap C$

9. 

$(A \cup B) \cap C$

Diagrams are very useful in analyzing problems involving relationships among sets and operations with these sets, and they provide a clear pictorial representation of sets that might be difficult to visualize otherwise.

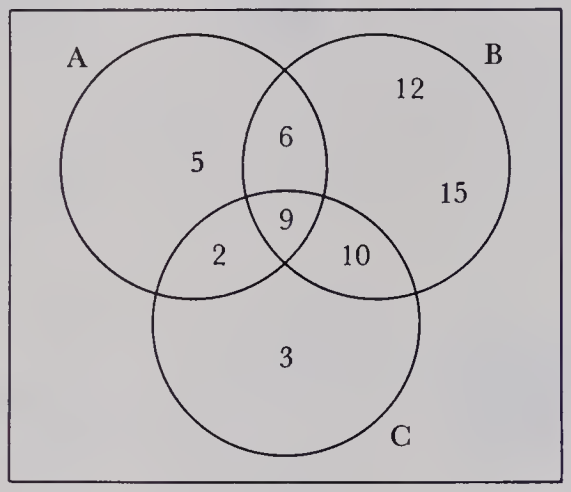
Pre-Book Lesson

Introduce the lesson by asking pupils about social situations involving sets having members in common. Review the meaning of the terms and symbols for *intersection* ($\cap$), *union* ($\cup$), and *subset*. Describe verbally an intersection of two sets. Emphasize the advantages of using diagrams to record the idea.

Using the Text Pages

- Ex. 1–10. The purpose of Ex. 1–10 is to teach pupils how to read and interpret a Venn diagram.
- Ex. 6–7. Have pupils describe the various types of subsets including the empty set and the given set itself. Have them picture several subsets after completing Ex. 7.
- Ex. 11–13. If pupils have difficulty constructing a Venn diagram, it may be helpful to outline a procedure which they can use as a guide. For

example, in Ex. 10, a sample procedure would be: (1) Show the rectangular region and one simple closed curve for each set. (2) Determine if there are any members common to all three sets. If so, represent them on the drawing. (3) Determine if there are any members common to any two sets. If so, represent them on the drawing. (4) Now, indicate on the diagram the members which are not common to any two sets.



Individualizing Instruction

• If you wish to give *all pupils* additional exercises, the following may be used:

Draw a Venn diagram to show $A = \{1, 3, 10, 14, 17\}$, $B = \{3, 5, 7, 14\}$, $C = \{3, 7, 9, 10, 12\}$.

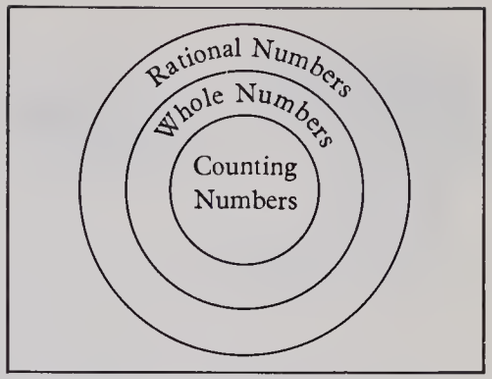
List within braces the elements in set C if $B = \{11, 13, 21, 27\}$, $B \cap C = \{13, 21\}$, and $B \cup C = \{7, 11, 13, 17, 21, 27\}$.

• You may wish to confine the written exercises for *slower learners* to exercises involving only two sets. Have them represent with a Venn diagram set A and set B when

a. set A and set B have no intersection.
b. set A and set B have an intersection but are not proper subsets of each other.
c. set A is a proper subset of set B.
d. set B is a proper subset of set A.
e. set A and set B are equal sets.

NOTES

8. What does the Venn diagram at the right show? The set of counting numbers is a subset of the sets of whole numbers and rational numbers; the set of whole numbers is a subset of the set of rational numbers.

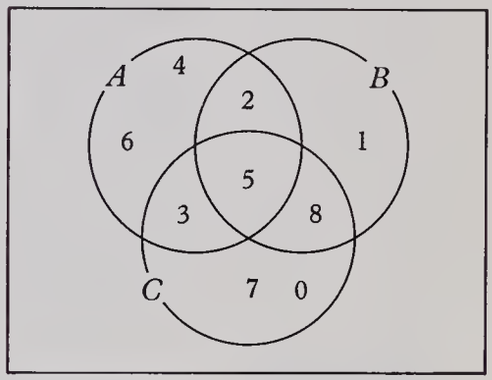


a. Name four members of the set of counting numbers; of the set of whole numbers. Sample answers: 1, 2, 3, 4 ; 0, 1, 2, 3

b. What whole number is not a counting number? 0

c. Name four rational numbers two of which are whole numbers. See below. Name two more rational numbers that are not whole numbers. $\frac{1}{3}$, $\frac{3}{4}$

9. Sets A, B, and C are shown in the Venn diagram at the right. How many members are there in set A? 5 in set B? 4 in set C? 5 On the board, list within braces the members of



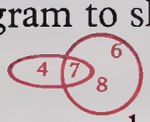
a. $B \cap C$. {5,8} **b.** $A \cap C$. {3,5} **c.** $A \cup C$. {0,2,3,4,5,6,7,8}

10. On the board, draw a Venn diagram to show {2, 5, 6, 9}, {6, 9, 10, 12, 15}, and {2, 3, 9, 10}.

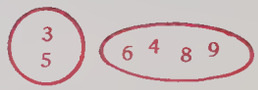
[W]

11. Draw a Venn diagram to show

a. {4, 7} and {6, 7, 8}.

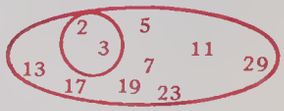


b. {3, 5} and {4, 6, 8, 9}.

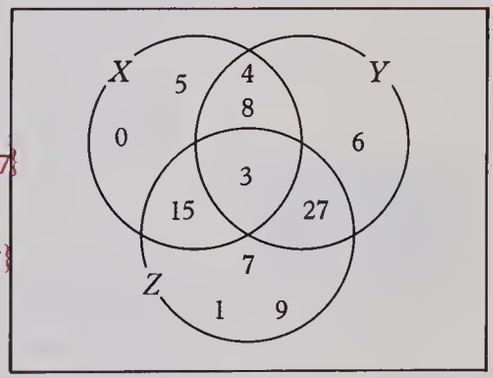


12. Draw a Venn diagram which shows that the set of even numbers and the set of odd numbers have no members in common.

13. List within braces the prime factors of 24. {2,3} Then draw a Venn diagram to show that this set is a subset of the set of prime numbers less than 30.



Ex. 14–19. List within braces the members of the set. Refer to the diagram at the right.



14. $X \cap Y$ {3,4,8} **15.** $X \cup Y$ {0,3,4,5,6,8,15,27} **16.** $Y \cap Z$ {3,27}

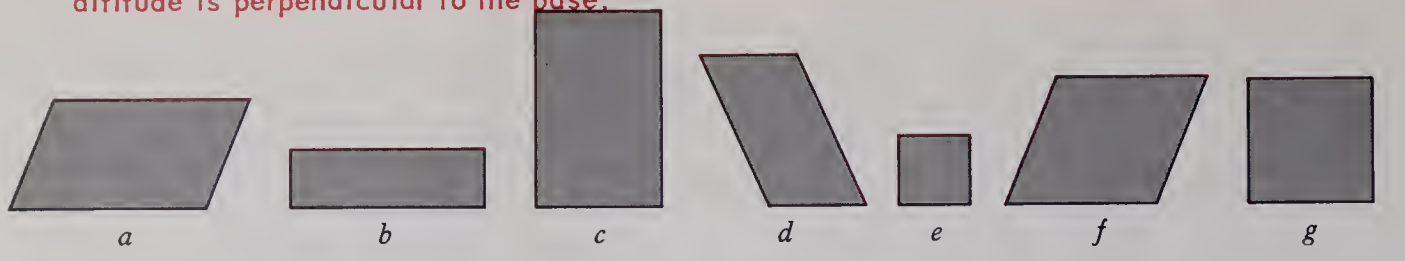
17. $Y \cup Z$ {1,3,4,6,7,8,9,15,27} **18.** $X \cap Z$ {3,15} **19.** $X \cup Z$ {0,1,3,4,5,7,8,9,15,27}

20. List within braces the number which is a member of each of the three sets X, Y, and Z. {3}

● **Extra Activity.** Set 180.

8. c. Sample answers: $\frac{1}{2}$, 1, $2\frac{1}{2}$, 2

* Point out that an altitude for a parallelogram is not necessarily one of the sides. An altitude is perpendicular to the base.



Parallelogram Regions

Resurvey and extension; area [O]

1. The regions shown above are *parallelogram regions*. A parallelogram region is the union of a _?^ parallelogram and its interior. Which regions shown above appear to be rectangular regions?^ b, c, e, g Which appear to be square regions? e, g

2. Is every rectangular region also a parallelogram region?^ Yes Is the converse true, that is, is every parallelogram region also a rectangular region?^ No Explain. Key idea: Every parallelagram is not also a rectangle.

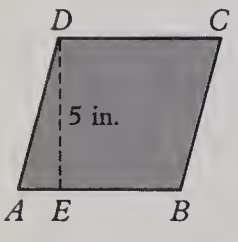


Fig. 1

* 3. Any one of the four sides of a parallelogram region may be thought of as the *base* of the region. Think of $\overline{AB}$ as the base of the parallelogram region shown in Fig. 1. Segment DE is perpendicular to the base. Does $\overline{DE}$ have one end point in the line that contains the base and the other end point in the side opposite the base?^ Yes Such a segment is called an *altitude*.

4. Fig. 2. For the rectangular region shown, think of $\overline{PQ}$ as the base. Then, is $\overline{SP}$ an altitude?^ Yes Is $\overline{RQ}$?^ Yes Name two segments which are altitudes when $\overline{SP}$ is thought of as the base. $\overline{SR}$ and $\overline{PQ}$

Fig. 2

5. The length of an altitude for a parallelogram region is the *height* of the region. Thinking of $\overline{AB}$ as the base, what is the height of the parallelogram region shown in Fig. 1? 5 in.

6. Fig. 3. For the rectangular region, if $\overline{MN}$ is the base, the height is _?3_ in. If $\overline{PM}$ is the base, the height is _?7_ in.

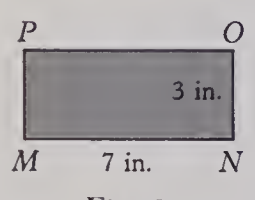


Fig. 3

7. To find the area of a rectangular region, you know that we can use $A = l \times w$. Tell what the letters A, l, and w represent. measure of area; length; width Could we also use $A = b \times h$ where b represents the measure of the length of the base of the rectangular region and h represents the measure of the height?^ Yes Explain. Key idea: For a rectangle, length and width are the same as base and height.

Teaching Pages 292 and 293

Pupil's Objectives

(a) To learn the meaning of base and height for a parallelogram region; and (b) to practice finding areas of parallelogram regions.

Background

The meaning of terms that pupils should be familiar with are given below so that you may refer to them when developing the concepts introduced on these pages.

(1) Simple closed curve—a closed curve that does not cross itself.

(2) Polygon—a simple closed curve that is formed by line segments only.

(3) Quadrilateral—a polygon having four sides.

(4) Parallelogram—a quadrilateral with two pairs of opposite sides parallel (opposite sides, therefore, have the same length).

(5) Rectangle—a parallelogram all of whose angles are right angles.

(6) Square—a rectangle all of whose sides have the same length.

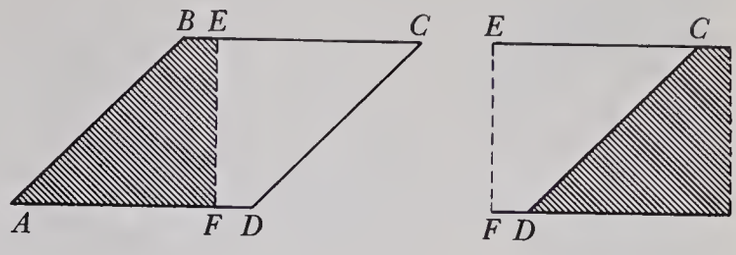
(7) Region—the union of a simple closed curve and its interior.

(8) Parallelogram region—the union of a parallelogram and its interior.

(9) Area of a rectangular region—the measure of area of a rectangular region is equal to the product of the measures of its length and width (length and width must be expressed in the same units of measurement).

The exercises on page 176 introduced the concept of finding the area of a rectangular region. This concept is now applied in developing a method for finding the area of a parallelogram region.

A model for a parallelogram region which is not a rectangular region can be changed so that it becomes a model representing a rectangular region. Picture a segment that joins two opposite sides of a parallelogram in such a way that it is perpendicular to these sides, such as $\overline{EF}$. $ABEF$ is cut off and placed along $FECD$ so that $\overline{AB}$ and $\overline{DC}$ coincide. The model for $ADCB$ now represents a rectangular region.



The measure of the area of a rectangular region is the product of the measures of the length and width. Since the length of the rectangle is the same as the base of the parallelogram, and since the width of the rectangle is the same as the height of the parallelogram, the measure of the area of the parallelogram region must be equal to the product of the measure of the length of the base and the measure of the height.

Pre-Book Lesson

Review the meaning of the terms from the Background. Pupils should be able to describe these ideas accurately in their own words. Memorization of rigid definitions is not necessary.

Using the Text Pages

• Ex. 2–3. A rectangle is a special type of parallelogram, so every parallelogram region is not a rectangular region. Use material from the Background and make a cardboard model to illustrate the differences and similarities between the parallelogram region and the rectangular region. Make sure pupils know what perpendicular means. Put diagrams representing the figures on the board and label the base, altitude, and so on.

• Ex. 5. Make sure pupils understand all the corresponding "parts" of the parallelogram and the rectangle.

• Ex. 7. Put a model of a parallelogram region and a model of a rectangular region on the board or use a flannel board and cutouts. Use a piece of cloth or a sheet of paper and place it over the model of the rectangular region. Now place the same piece of cloth or paper over the parallelogram region. Pupils should be able to point out that the

same amount of material covers the rectangular region and the parallelogram region.*

• Ex. 8. Review the meaning of congruent. Explain that any model of a parallelogram region can be converted to a model of a rectangular region, both regions having the same area. Ex. 12 will clarify this idea. Do not use the phrase "area of a parallelogram." A parallelogram has no area —it has only length. A parallelogram region has area.

• Ex. 9. You may have to work several more examples before the generalization $A = b \times h$ becomes apparent.

• Ex. 13–18. Some pupils may benefit from using a diagram and labeling the parts for the parallelogram region.

Individualizing Instruction

• Give *all pupils* exercises similar to Ex. 13–18 using different units of measurement. For example, have them find the area of a parallelogram region whose length is 2 ft. and whose height is 8 in.

• If *slower learners* have difficulty with Ex. 19, remind them that $A = b \times h$ is the factors-product relationship. When the product and one factor are given and the missing factor is to be found, division is required.

• The formula $A = b \times h$ may be used to find the measure of the area of *any* parallelogram region. Since every rectangle is a parallelogram and every square is a parallelogram, the formula for the area of a rectangular region and for a square region must be consistent with the formula for the area of a parallelogram region. Have *more capable pupils* explain the relationships among the three formulas:

$A = b \times h$ (area of a parallelogram region)
$A = l \times w$ (area of a rectangular region)
$A = s^2$ (area of a square region)

* See items 1 and 2, page xix.

NOTES

8. Fig. 4. Triangular regions BCE and ADF are congruent. Does that mean they have the same shape? Yes ∧ the same size? Yes ∧ the same area? Yes ∧ Explain why the parallelogram region $ABCD$ has the same area as the rectangular region $ABEF$. See below.

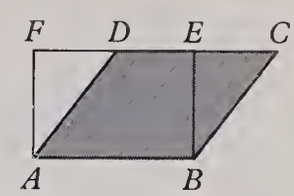


Fig. 4

9. The measure of the area of region $ABEF$ can be found by multiplying $m(\overline{AB})$ by $m(\overline{BE})$. Why? $A = b \times h$; $m(\overline{AB}) = b$; $m(\overline{BE}) = h$ ∧ Region $ABCD$ has the same area as region $ABEF$, so the measure of the area of region $ABCD$ can be found by multiplying $m(\overline{AB})$ by $m(\overline{BE})$. If $\overline{AB}$ is the base of the parallelogram region $ABCD$, then is $\overline{EB}$ an altitude? Yes

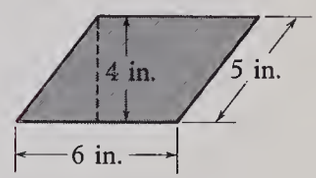


Fig. 5

> **A, the measure of the area of a parallelogram region, is equal to the product of b, the measure of the length of the base, and h, the measure of the height. $A = b \times h$**

10. Explain the work in the box for finding the area of the parallelogram region shown in Fig. 5. The product is found for the measure of the length of the base and the measure of the height.

$$A = b \times h$$
$$A = 6 \times 4$$
$$= 24$$

Area is 24 sq. in.

11. At the board, find the area of the parallelogram region shown in Fig. 6. Express the answer first in terms of square feet and then in terms of square yards. 42 square feet; $4\frac{2}{3}$ square yards

$A = 6 \times 7$
$= 42$

[W]

12. Trace drawing a in Fig. 7. Cut out the model for the parallelogram region and then cut along the dotted line. Then place the model for the triangular region as in drawing b, Fig. 7 to represent a rectangular region.

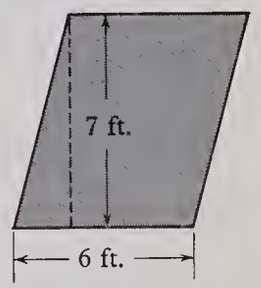


Fig. 6

Ex. 13–18. Find the area of the parallelogram region.

	Length of base	Height		Length of base	Height
13.	13 in.	5 in. 65 sq. in.	16.	2.7 in.	1.6 in. 4.32 sq. in.
14.	25 cm.	15 cm. 375 sq. cm.	17.	5.36 cm.	0.34 cm. 1.8224 sq. cm
15.	$4\frac{1}{2}$ ft.	8 ft. 36 sq. ft.	18.	$3\frac{3}{5}$ yd.	$5\frac{1}{2}$ yd. $19\frac{4}{5}$ sq. yd.

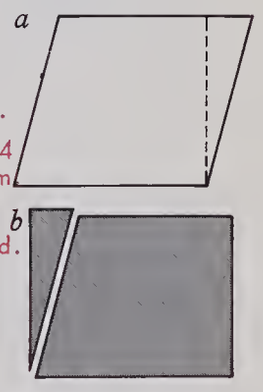


Fig. 7

19. Find the height of a parallelogram region whose area is 45 sq. in. and whose base measures 8 in inches. $5\frac{5}{8}$ in.

◆ **Extra Examples.** Set 134.

8. Key idea: The measure of each area is the sum of the measure of the area of region ABED and of the area of one of the congruent triangles.

Finding the Area of a Triangular Region

Extension [O]

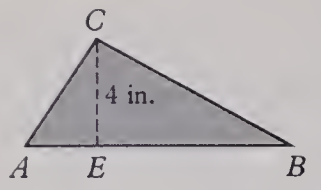


Fig. 1

1. Any one of the three sides of a triangular region may be thought of as the *base* of the region. Think of $\overline{AB}$ as the base of the triangular region shown in Fig. 1. Segment CE is perpendicular to the base. Does $\overline{CE}$ have one end point in the line which contains the base? Yes Name its other end point. C Such a segment is called an altitude.

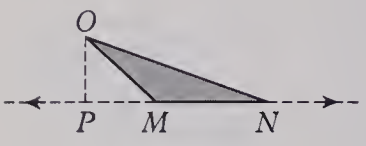


Fig. 2

2. Fig. 2. Line PM contains $\overline{MN}$, and $\overline{OP}$ is perpendicular to $\overleftrightarrow{PM}$. If $\overline{MN}$ is thought of as the base of the triangular region shown, then is $\overline{OP}$ an altitude? Yes Explain. Key idea: $\overline{OP}$ has one end point in the line containing the base and the other in the side opposite.

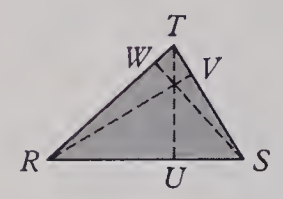


Fig. 3

3. Fig. 3. Name an altitude for the triangular region RST if the base is $\overline{RS}$; $\overline{ST}$; $\overline{TR}$. $\overline{TU}$; $\overline{RV}$; $\overline{WS}$

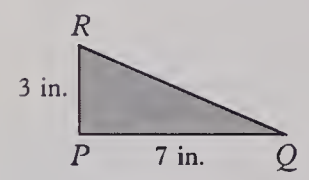


Fig. 4

4. The length of an altitude for a triangular region is the *height* of the region. For Fig. 1, if we think of $\overline{AB}$ as the base, what is the height of the triangular region? 4 in.

5. Fig. 4. Angle RPQ is a right angle, so region PQR is a _? right triangular region. If $\overline{PQ}$ is the base, the height is _? 3 in. If $\overline{RP}$ is the base, the height is _? 7 in.

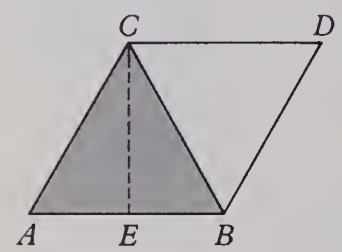


Fig. 5

6. Fig. 5. Region $ABDC$ is a parallelogram region. The triangular regions ABC and BDC are congruent. The measure of the area of triangular region ABC is _? $\frac{1}{2}$ the measure of the area of the parallelogram region.

7. Fig. 5. Let b represent the measure of the base AB. Let h represent the measure of the altitude CE. Is the product $b \times h$ the measure of the area of the parallelogram region $ABDC$? Yes Explain. For a parallelogram region, $A = b \times h$. Then the measure of the area of the triangular region ABC is $\frac{1}{2} \times (b \times h)$. Why? The measure of the area of triangular region ABC is $\frac{1}{2}$ the measure of the parallelogram region.

A, the measure of the area of a triangular region, is equal to one half the product of b, the measure of the length of the base, and h, the measure of the height.

$$A = \tfrac{1}{2} \times (b \times h)$$

Teaching Pages 294 and 295

Pupil's Objectives

(a) To learn how to find the area of a triangular region; and (b) to practice finding areas of triangular regions.

Background

The formula for the area of a triangular region may be derived from the concept that models representing any two congruent triangular regions can be arranged to form a parallelogram region. Therefore, the area of the triangular region is one half the area of the parallelogram region if the measures of base and altitude of each are equal.

The measure of area of a parallelogram region is equal to the product of the measure of the length of the base and the measure of the height.

$$A = b \times h$$

Since the area of the given triangular region is one half the area of the parallelogram region, the area of the triangular region may be found by the relationship

$$A = \tfrac{1}{2} \times (b \times h)$$

Pre-Book Lesson

• Be sure pupils remember the meaning of *congruent region* and triangular region. Review the characteristics of the various types of triangles and triangular regions.

• Have pupils draw models of pairs of congruent triangular regions, including a pair of right isoceles triangular regions, a pair of right scalene triangular regions, and a pair of acute scalene triangular regions. Have them cut the models from paper and arrange the three pairs consecutively to form a square, a rectangle, and a parallelogram. Since two triangular regions form one quadrilateral region, one triangular region is $\frac{1}{2}$ a quadrilateral region. Ask for suggestions as to how to find the areas of the triangular regions based on a knowledge of how to find the area of the square, rectangular, and parallelogram regions formed by a pair of congruent triangles. Explain that today's lesson will demonstrate the relationship between the area of a triangular region and the area of a parallelogram region both having the same base and the same height.

Using the Text Pages

• Ex. 1–3. Make a cardboard model to represent a triangular region. Illustrate the meaning of base of a triangle by rotating the model so it rests on each side. A segment from a vertex of a triangular region perpendicular to the line containing the opposite side is called an *altitude* of the triangular region. Discuss the meaning of the term, "perpendicular." In Fig. 2, notice that the altitude does not have to be in the interior of the triangular region. Since there are three vertices in any triangular region, there are three altitudes, as shown in Fig. 3.

• Ex. 4. Clarify the difference between height and altitude. An altitude of a triangular region is a segment; the height of a triangular region is the length of that altitude. Since a triangular region has three altitudes, it must have three heights. To make clear which of the three heights is being referred to, it must be known which of the three sides of the triangular region is being considered as the base.

• Ex. 5. Note that the side of a triangular region may also be the height of that region.

• Ex. 6. The area of a triangular region is arrived at in terms of the area of a parallelogram. It must be clearly demonstrated that the area of any triangular region is one half the area of a parallelogram region whose base and altitude are the same as the base and altitude of the given triangular region. $\overline{CE}$ is the altitude for parallelogram $ABDC$.

• Ex. 7. Some pupils may wonder if the order of operations for $A = \frac{1}{2} \times (b \times h)$ is important. It is usually most efficient to perform the operation indicated within the parentheses first and then multiply by $\frac{1}{2}$. However, since $A = \frac{1}{2} \times (b \times h)$ means $A = \frac{1}{2} \times b \times h$, and multiplication is associative; it makes no difference which two of the factors are multiplied first.

• Ex. 12–17. Some pupils may need to use diagrams to help them work each of the examples so they can visualize the base and the height.

• Ex. 20 and 22. Point out that each of these figures should be considered as two regions. In Ex. 20, the area for two different rectangular regions should be found and the measures of the areas added: $A = (2 \times 7) + (2 \times 2)$ or $A = (2 \times 5) + (2 \times 4)$. If some pupils were to visualize the rectangular region whose measure of area was 7×4, and subtract the measure of the area of a rectangular region 2×5, they would also be correct. In Ex. 22, the measure of the area for a rectangular region $(3\frac{1}{2} \times 7)$, should be added to the measure of the area of a triangular region, $\frac{1}{2} \times (3 \times 7)$.

• Ex. 23. To find the area of region *EFGCD*, remember that the measures of area may be added or subtracted to find the total area.

Individualizing Instruction

• Give *more capable pupils* several drawings representing different triangular regions each with the same base and with altitudes having the same length. Ask them to compare the areas of the triangular regions. Since each has the same base and height, the measure of areas of all the triangular regions will be equal even though they may not appear to be in a drawing.

• Ask *all pupils* if every parallelogram region can be partitioned into two congruent triangular regions. Have pupils draw models representing parallelogram regions and show a diagonal. Then, cut out the model and cut along the diagonal. Determine if the two resulting triangular regions are always congruent.

Give *all pupils* exercises in finding areas of triangular regions in which the measures of the altitude and the base are not given in the same units of measurement.

• If necessary, have *slower learners* do more exercises finding area of triangular regions for which the length of the base and the height are expressed in the same English unit of measurement and have measures which are whole numbers. Have them work exercises of this type until they understand the concept of finding the area of a triangular region.

Looking Ahead

In anticipation of the next lesson, the teacher should make models, using cardboard, staples, and tape, of some of the figures shown at the top of pupil's page 296.

NOTES

8. Explain the work in the box for finding the area of the triangular region shown in Fig. 6.

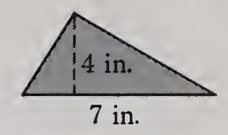


Fig. 6

9. At the board, find the area of the triangular region shown in Fig. 4 on page 294. Think of $\overline{PQ}$ as the base and work as in the box. $A = \frac{1}{2} \times (7 \times 3) = \frac{1}{2} \times 21 = 10\frac{1}{2}$ Area is $10\frac{1}{2}$ sq. in.

$$A = \tfrac{1}{2} \times (b \times h)$$
$$A = \tfrac{1}{2} \times (7 \times 4)$$
$$= \tfrac{1}{2} \times 28$$
$$= 14$$

Area is 14 sq. in.

10. At the board, find the area of a triangular region if the length of its base is 14 cm. and its height is $3\frac{1}{2}$ cm. $A = \frac{1}{2} \times (14 \times 3\frac{1}{2}) = \frac{1}{2} \times 49 = 24\frac{1}{2}$ Area is $24\frac{1}{2}$ sq. cm. [W]

11. Make two tracings of drawing *a* in Fig. 7. Cut out the models for the triangular regions and place them as in drawing *b*, Fig. 7 to represent a parallelogram region.

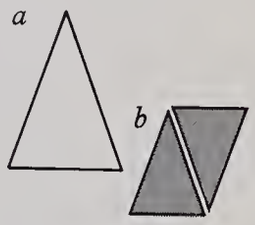


Fig. 7

Ex. 12–17. Find the area of the triangular region.

	Length of base	Height			Length of base	Height	
12.	13 in.	8 in.	52 sq. in.	**15.**	5.5 yd.	4.3 yd.	11.825 sq. yd.
13.	1.4 ft.	5 ft.	3.5 sq. ft.	**16.**	$4\frac{1}{2}$ ft.	$\frac{1}{2}$ ft.	$1\frac{1}{8}$ sq. ft.
14.	230 cm.	27 cm.	3,105 sq. cm.	**17.**	4.25 in.	7 in.	14.875 sq. in.

18. The region shown in Fig. 8 is a right triangular region.

a. Find its perimeter. 12 in. **b.** Find its area. 6 sq. in.

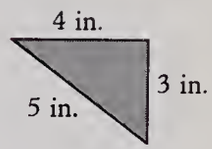
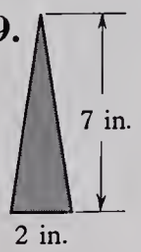


Fig. 8

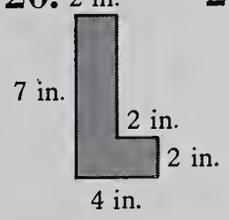
Ex. 19–22. Find the area of the region shown.

19.



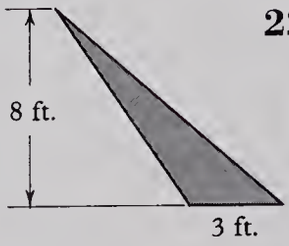
7 sq. in.

20.



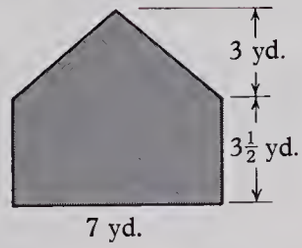
18 sq. in.

21.



12 sq. ft.

22.



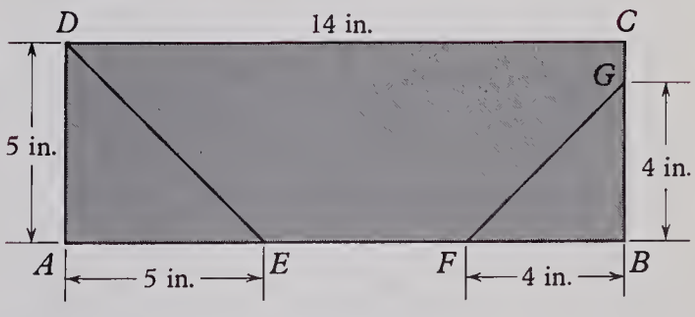
35 sq. yd.

23. Region *ABCD* shown at the right is a rectangular region. Find the area of

a. triangular region *AED*. $12\frac{1}{2}$ sq. in.

b. triangular region *FBG*. 8 sq. in.

c. polygonal region *EFGCD*. $49\frac{1}{2}$ sq. in.



◆ **Extra Examples.** Set 135. ◆ **Extra Problems.** Set 153.

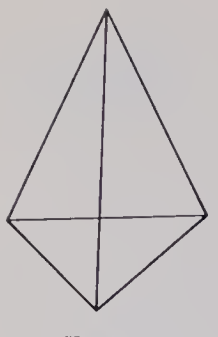


Fig. 1

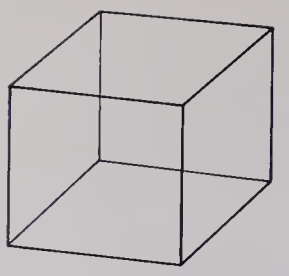


Fig. 2

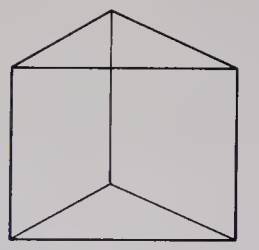


Fig. 3

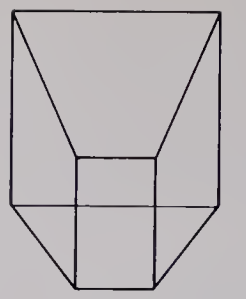


Fig. 4

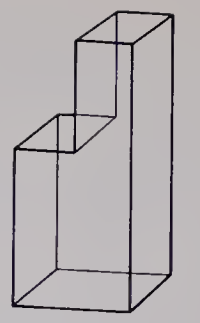


Fig. 5

Space Figures

[o]

1. When all the points in a geometric figure are contained in a plane, the figure is called a plane figure. Is a line segment a plane figure? Yes Is a polygon a plane figure? Yes Name some other kinds of plane figures. Sample answers: angle, circle, line

2. When all the points in a geometric figure do not lie in the same plane, the figure is called a *space figure*. Name some things that may be thought of as models for space figures. Answers will vary.

3. The geometric figures shown above are special space figures called polyhedrons. You know that a polygon is a plane figure that is a simple closed curve. A polygon is the union of line segments which are called the sides of the polygon. A *polyhedron* is a simple closed surface in space which is the union of polygonal regions that are called the *faces* of the polyhedron. The polyhedron shown in Fig. 1 has 4 faces. Give the number of faces of each polyhedron shown in Fig. 2–5. 6, 5, 6, 8

4. Is a polyhedron a set of points? Yes Explain. Key idea: Polyhedrons are the unions of plane regions which are sets of points.

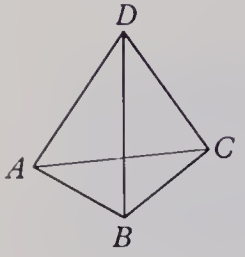


Fig. 6

5. The sides of the faces of a polyhedron are called the *edges* of the polyhedron. The faces of the polyhedron shown in Fig. 6 are triangular regions *ABC*, *ABD*, *BCD*, and _? DAC △. Name the edges of the polyhedron. $\overline{AB}$, $\overline{BC}$, $\overline{CA}$, $\overline{BD}$, $\overline{DA}$, $\overline{DC}$

6. The vertices of the faces of a polyhedron are called the *vertices* of the polyhedron. How many vertices does the polyhedron shown in Fig. 6 have? 4

Teaching Pages 296 and 297

Pupil's Objectives

(a) To learn the concept of space figures and the meaning of polyhedron; and (b) to learn the parts of a polyhedron such as face, edge, and vertex.

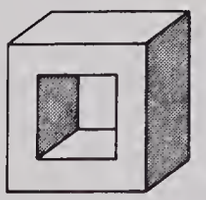
Background

Pupils are now introduced to the concept of a *space figure*. The first space figure to be studied is the polyhedron, which is a simple closed surface in space formed by the union of polygonal regions.

Care must be taken not to confuse a polyhedron with a solid. A polyhedron is a union of regions forming a closed surface while a solid is a union of a polyhedron and its interior. The concept of a solid is introduced later.

On the text pages, the various parts of a polyhedron are described in simple, practical language. Each of the polygonal regions forming the polyhedron is called a *face*. The sides of the faces are called the *edges*, each edge being a segment contained in the intersection of two faces. The vertices of the faces are also the *vertices* of the polyhedron, each vertex being contained in at least three of the faces.

There is an interesting relationship among the numbers of faces, vertices, and edges of a simple polyhedron. This relationship is expressed by Euler's formula $F + V = E + 2$, where F represents the number of faces of the polyhedron, V the number of vertices, and E the number of edges.



Euler's formula is valid only for simple polyhedrons. It does not hold true for more complex polyhedrons such as those which contain holes.

Just as a polygon partitions a plane into three distinct sets of points, so a polyhedron partitions space into three distinct sets of points: (1) the set of points belonging to the polyhedron; (2) the set of points interior to the polyhedron; and (3) the set of points exterior to the polyhedron.

The simplest polyhedron is determined by four points not all in the same plane. (See Fig. 1 on pupil's page 296.) If each point is joined with a segment to each of the other three points, four triangular regions result. The union of these four triangular regions forms a simple closed surface in space, and is, therefore, a polyhedron.

Polyhedrons are classified according to the number of faces. The four-faced polyhedron is called a tetrahedron. Below are names of some of the more common polyhedrons.

five faces—pentahedron
six faces—hexahedron
seven faces—heptahedron
eight faces—octahedron
nine faces—nonahedron
ten faces—decahedron
twelve faces—dodecahedron
twenty faces—icosahedron

A regular polyhedron is a polyhedron all of whose faces are regular congruent polygons and whose polyhedral angles are all congruent. A polyhedral angle is formed by faces of a polyhedron which have a common vertex.

There are five regular polyhedrons: (1) the regular tetrahedron (a pyramid); (2) the regular hexahedron (a cube); (3) the regular octahedron; (4) the regular dodecahedron; and (5) the regular icosahedron.

The faces of the regular tetrahedron, octahedron, and icosahedron are triangular regions. The faces of the regular hexahedron are square regions, and the faces of the regular dodecahedron are pentagonal regions.

Pre-Book Lesson

Select some object in the classroom such as a closed box, and have pupils consider each side (including top and bottom) as being a model of a polygonal region. Have them name the type of region represented by each side. Review the idea of a polygon being the union of line segments which are called sides forming a simple closed curve. Then, ask pupils to describe what would be the meaning of *simple closed surface*. Lead them, through discussion, to the realization that the union of the regions represented by the sides of the box form a simple closed surface. Ask them to point out other examples in the room of objects whose

surface can be considered as models of polygonal regions, the union of the regions forming a simple closed surface.

Using the Text Pages

• Ex. 3. Progress from explaining a polygon as a union of segments, and a polygonal region as the union of a polygon and its interior, to the idea of the union of polygonal regions. Emphasize the two main characteristics of a polyhedron: (1) it is the union of polygonal regions; and (2) it is a simple closed surface in space. Be sure pupils do not confuse the idea of a polyhedron with the idea of a solid. (See the distinction in the Background.) Make models out of cardboard of the figures shown at the top of the pupil's page. Show and take them apart in front of the class to bring out the fact that they are "hollow" and are "shell figures."

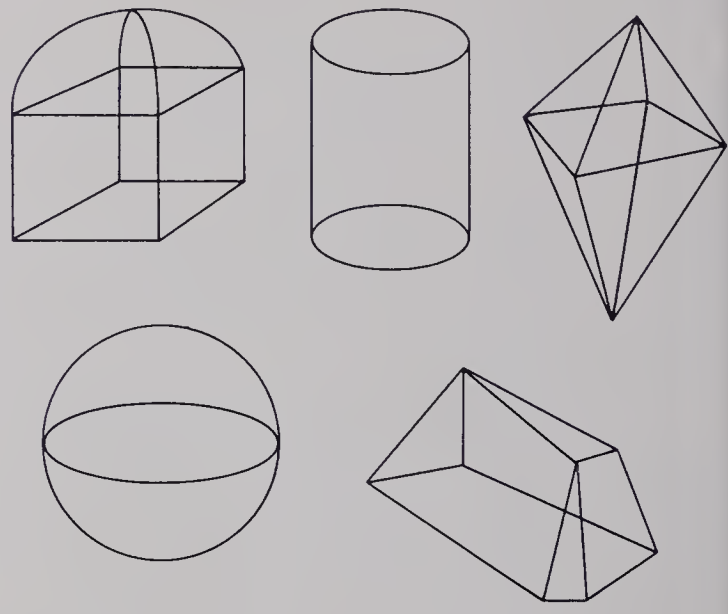
• Ex. 5. Tape pieces of paper naming the edges, faces, and vertices to one large cardboard model of a polyhedron so all pupils can refer to it. Ask pupils what other goemetric figures are contained in the space figure (segments for the faces, points for the vertices, and so on).

• In Ex. 7–8, no attempt is made to present to pupils a definition of interior or exterior of a polyhedron. This is a sophisticated mathematical concept. The difficulty of defining these terms in language appropriate for this grade level is apparent when you consider interiors and exteriors of more complex polyhedrons such as that shown in the Background section. If possible, make a transparent rectangular polyhedron, a type of transparent box, by using strips of cardboard and freezer wrap. Pupils can more easily imagine the points interior and exterior to the figure, and the points belonging to the figure.

• Ex. 16. Review the meaning of *trapezoid*, and trapezoidal region. A trapezoid is a quadrilateral having two, and only two, opposite sides parallel. Some pupils may need to work Ex. 10–20 with the assistance of cardboard models. Construction of such models will help pupils grasp concepts more easily.

Individualizing Instruction

Ask *slower learners* to identify which of the following are polyhedrons by drawing models on the board.



7. A plane which contains a polygon is partitioned into three distinct sets of points. Describe them. The set of points belonging to, in the interior for, and in the exterior for the polygon. Space is partitioned by a polyhedron into three distinct sets of points.

(1) The set of points belonging to the polyhedron

(2) The set of points in the interior for the polyhedron

(3) The set of points in the exterior for the polyhedron

8. Fig. 7. Points A, B, C, D, and E belong to line AB. Which of the labeled points appear to lie in the interior for the polyhedron? D Which appear to lie in the exterior for the polyhedron? A, B Which appear to belong to the polyhedron? C, E

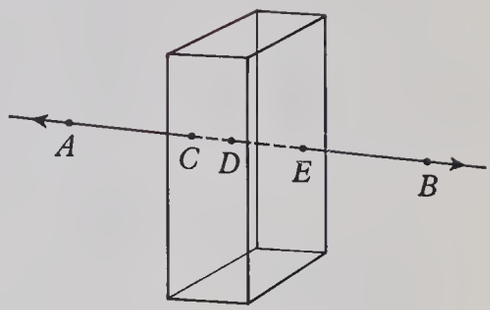


Fig. 7

9. Name some things that may be thought of as models of polyhedrons. Answers will vary.

[W]

For Ex. 10–14, refer to Fig. 8.

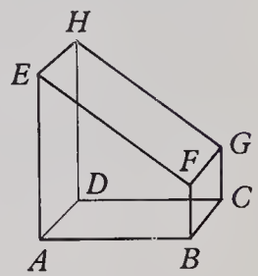


Fig. 8

10. How many faces does the polyhedron have? 6

11. Name the edges of the polyhedron. $\overline{EH}$, $\overline{EF}$, $\overline{FG}$, $\overline{GH}$, $\overline{EA}$, $\overline{AB}$, $\overline{BF}$, $\overline{BC}$, $\overline{GC}$, $\overline{HD}$, $\overline{DC}$, $\overline{AD}$

12. List within braces the vertices of the polyhedron. {A, B, C, D, E, F, G, H}

13. Name a pair of faces which appear to be congruent. EABF and HDCG

14. Name three segments each of which appears to be parallel to $\overline{FG}$. $\overline{CB}$, $\overline{DA}$, $\overline{EH}$

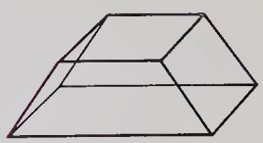


Fig. 9

15. What is the least number of faces that a polyhedron can have? 4

16. How many trapezoidal faces does the polyhedron shown in Fig. 9 have? 2

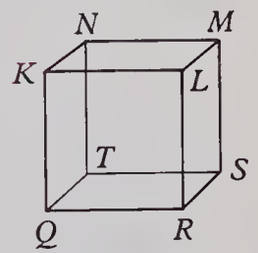


Fig. 10

Ex. 17–20. Write T or F. Refer to Fig. 10.

17. $\overline{KN} \cap \overline{NM} = \{N\}$ T

18. $\overline{RS} \cap \overline{QT} = \{\ \}$ T

19. face $QRST \cap$ face $RSML = \{\ \}$ F

20. face $QRST \cap$ face $KLMN = \{\ \}$ T

* A prism has as many lateral faces as there are sides of one of the bases.

Prisms

[o]

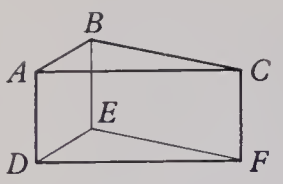


Fig. 1

1. Fig. 1. Do segments AB, BC, and AC appear to be congruent and parallel to segments DE, EF, and DF respectively? Yes When the sides of a polygonal region are congruent and parallel to the sides of another polygonal region, we say that the two regions are congruent and parallel. Then does it appear that faces ABC and DEF of the polyhedron shown are congruent and parallel? Yes

Fig. 2

2. Fig. 2. Which polyhedron appears to have two faces which are congruent and parallel? b

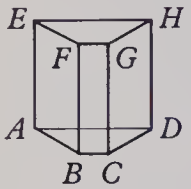


Fig. 3

3. Fig. 3. Two of the faces of the polyhedron shown are congruent and parallel. Name them. Face ABCD and face EFGH How many other faces are there? 4 Do the other faces appear to be parallelogram regions? Yes Polyhedrons such as those shown in Fig. 1, Fig. 2b, and Fig. 3 are called *prisms*.

> A *prism* is a polyhedron with at least two congruent and parallel faces that are called *bases*. The faces which are not bases are called *lateral faces*. Each lateral face is a parallelogram region.

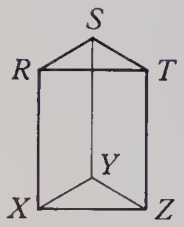
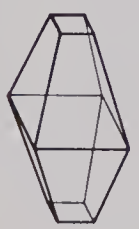


Fig. 4

4. Fig. 4. Name the bases of the prism shown. faces RST and XYZ Name the lateral faces. faces RTZX, STZY, and RSYX

* 5. If the bases of a prism are pentagonal regions, the prism has _5_ lateral faces.

6. Which of the polyhedrons shown in Fig. 1–5** on page 296 are prisms? 2, 3, 5 Which are not? 1, 4



7. Find some things in your classroom which may be thought of as models of prisms. Answers will vary.

8. Does the polyhedron shown at the left appear to have two faces which are congruent and parallel? Yes Explain why this polyhedron is not a prism. Each lateral face is not a parallelogram region.

** Fig. 5 is a prism. Consider the "front" and "back" faces as the bases.

Teaching Pages 298 and 299

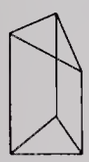
Pupil's Objectives

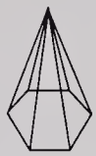
(a) To learn the meaning of *prism*; (b) to learn to classify prisms according to the shape of their bases; and (c) to learn to recognize various kinds of prisms.

Background

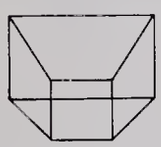
A prism is a polyhedron with at least two congruent and parallel faces, called bases. The faces which are not bases are called lateral faces. Each lateral face is a parallelogram region. Note that the lateral faces are parallelogram regions formed by joining the vertices of the bases, and that there are as many lateral faces as there are edges of one base.

The models below are not prisms. The characteristic which prevents each from being called a prism is indicated.

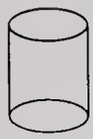




Do not have two congruent and parallel faces.

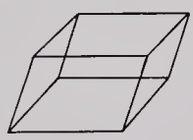


Bases are not congruent. Lateral faces are not parallelogram regions.



Not a polyhedron.

In order for a polyhedron to be classified as a prism, it is necessary that the lateral faces be parallelogram regions. However, most of the prisms shown on the text page have lateral faces which are rectangular regions. The drawing at the right shows a prism whose lateral faces are not all rectangular regions.



Some pupils may be familiar with the term *prism* as it is used to name a glass object whose surface is a model for a triangular prism. It is used in seience demonstrations to disperse sunlight into its spectrum colors. Point out that this is a colloquial use of the term and that only the "outside shape" of the object is a prism.

Pre-Book Lesson

Display models representing several different types of prisms. Then display models representing figures that are not prisms. You may wish to use cardboard strips and freezer wrap to make models for prisms. The transparency of the freezer wrap will stress the concept that a prism is a space figure. If models are not available, make drawings on the board or have *more capable pupils* make the drawings on the board. Point out the polyhedrons which are prisms without telling the pupils that these are called prisms. Tell pupils that these polyhedrons have three characteristics in common, and that the other polyhedrons do not possess all three characteristics. Have pupils try to determine what these three characteristics are, namely: (1) each has at least one pair of faces that are both congruent and parallel; (2) the remaining faces are all parallelograms; and (3) there are the same number of these remaining faces as the number of sides on one of the congruent and parallel faces. Explain that in the text lesson they will learn the special name given to such polyhedrons.

Using the Text Pages

• Ex. 1–3. Be sure pupils fully understand the meaning of congruent regions and the meaning of parallel regions. Congruent regions have exactly the same size and shape. Pupils may cut from paper a shape the same as one of the congruent and parallel sides of the model for the polyhedron. By placing this shape on the opposite side, they will see that it fits exactly and the two sides really are congruent. Parallel regions are contained in parallel planes.

The definition of prism given after Ex. 3, may seem quite formidable to some of the pupils. However, if the activity suggested in the Pre-Book Lesson was carried out, pupils should be able to understand the definition given on the pupil's page. Tape strips of paper to the cardboard models labeling the bases and lateral faces.

• Ex. 6. If cardboard models for the figures on pupil's page 296 are available, let pupils examine

them for the characteristics of a prism before answering the example.

• Ex. 8. The lateral faces of the figure are not parallelogram regions. Also, the number of lateral faces is not equal to the number of sides of one base. Each base of the polyhedron in the drawing has only four sides, while there are eight lateral faces to the polyhedron. Therefore, the polyhedron is not a prism.

• Ex. 10–15. A quadrangular prism is any prism whose bases are quadrangular regions—regions formed by the union of a quadrilateral and its interior. A rectangular prism is a particular type of quadrangular prism, all of whose faces and bases are rectangular regions. It is possible for a polyhedron to have rectangular regions as bases and have lateral faces which are not rectangular regions. Such a polyhedron is not a rectangular prism.

Individualizing Instruction

• Draw the models from the Background on the board and help *all pupils* generalize about the characteristics which prevent each from being a prism.

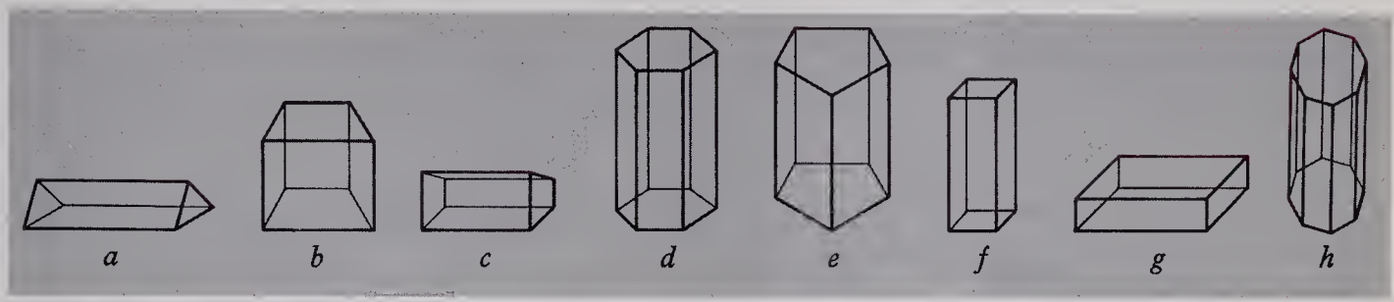
• Have *slower learners* cut from cardboard, models representing square regions, rectangular regions, and parallelogram regions of such dimensions that pupils can assemble them with tape to form prisms. Help them with this project, and have them name the particular type of prism which each model represents.

• Have *more capable pupils* try to make a model from cardboard of a prism for which every face and base is a non-rectangular parallelogram region.

NOTES

* Note that the definition of a rectangular prism is more restrictive in that all faces are rectangular regions.

9. Prisms often are named according to the shape of each base. For example, prism *a* shown in the box below is called a *triangular prism*. Which of the prisms shown are pentagonal prisms? e hexagonal prisms? d Prism *h* is an octagonal ? prism.



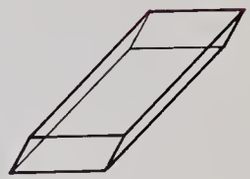
10. If the bases of a prism are quadrangular regions, then the prism is called a *quadrangular prism*. Give a definition of a quadrangular region. The union of a quadrangle and its interior.

11. Which of the prisms shown in the box above are quadrangular prisms? b, c, f, g

* **12.** By *rectangular prism*, we mean not only that the bases are rectangular regions but also, that *all* the faces of the prism are rectangular regions. Which prisms shown in the box above appear to be rectangular prisms? f, g

13. If each face of a prism is a square region, is the prism a rectangular prism? Yes Explain. Squares are rectangles

14. The faces of the prism shown at the right are parallelogram regions which are not rectangular regions. Then, is it a rectangular prism? No Is it a quadrangular prism? Yes



15. Find some things in your classroom which may be thought of as models of rectangular prisms. Answers will vary.

[W]

16. How many lateral faces does a pentagonal prism have? 5

17. How many lateral faces does a triangular prism have? 3

Ex. 18–19. Write *T* or *F*.

18. Every rectangular prism is a quadrangular prism. T

19. A hexagonal prism has 8 lateral faces. F

Right Prisms

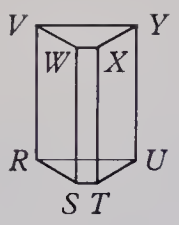


Fig. 1

[o]

1. Fig. 1. Name the bases of the prism shown. faces VWXY and RSTU The lateral faces of this prism are rectangular regions. Such a prism is called a *right prism*.

If the lateral faces of a prism are rectangular regions, then the prism is a right prism.

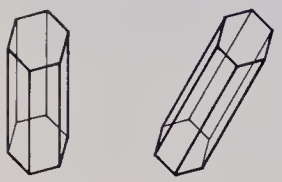


Fig. 2

2. A prism which is not a right prism is called an *oblique prism*. Of the prisms shown in Fig. 2, which one appears to be an oblique prism? b a right prism? a

3. Is every face of a rectangular prism a rectangular region? Yes Is a rectangular prism a right prism? Yes Explain. The lateral faces are rectangular regions.

Fig. 3

4. Fig. 3. Is the prism shown a triangular prism? Yes It is also a right prism, so we call it a *triangular right prism*. Is the prism shown in Fig. 1 a quadrangular right prism? Yes

5. The faces of the prism shown in Fig. 4 are congruent square regions. Such a prism is called a *cube*. Is a cube a rectangular prism? Yes Is a cube a right prism? Yes

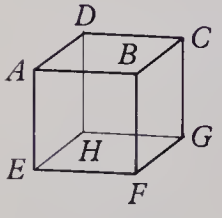


Fig. 4

6. Can faces *ABCD* and *EFGH* be thought of as bases of the cube shown in Fig. 4? Yes Can faces *AEHD* and *BFGC* be thought of as bases? Yes Name two other faces which can be thought of as bases. faces CDHG and BAEF

[w]

7. On your paper, show

a. a cube. See Fig. 4.

b. a triangular right prism. See Fig. 3.

c. a hexagonal right prism. See Fig. d, page 299.

d. an octagonal right prism. See Fig. h, page 299.

Ex. 8–11. Write *T* or *F*.

8. All rectangular prisms are cubes. F

9. All cubes are rectangular prisms. T

10. The edges of a cube are congruent segments. T

11. A cube has six congruent faces. T

Teaching Page 300

Pupil's Objective

To learn the meaning of *right prism*, *oblique prism*, and *cube*.

Background

A *right prism* is a prism whose lateral faces are rectangular regions. The bases must be parallel and congruent, but they may have the shape of any polygonal region. Right prisms are easy to recognize because each lateral face is perpendicular to each of the bases. Right prisms are named for the regions forming the bases, such as triangular right prism, quadrangular right prism, and pentagonal right prism.

A *cube* is a right prism all of whose faces are congruent square regions. A cube is one type of rectangular right prism which in turn is one type of quadrangular right prism.

A prism which is not a right prism is an *oblique prism*. Its faces are not perpendicular to the base.

Pre-Book Lesson

Display models of prisms, some of which are right prisms and some of which are oblique prisms.

If models are not available, draw pictures on the board. Point to those prisms which are right prisms and have pupils determine what characteristic these prisms have in common which the other prisms do not have. Lead them to see the common characteristic of these prisms—all their lateral faces are rectangular regions. This will serve as an introduction to the concept of right prism as presented on the text page.

Using the Text Page

• Ex. 3. On page 299, the distinction was made between a quadrangular prism and a rectangular prism. Prisms are usually named for the regions which form the bases. However, by rectangular prism we mean that not only are the bases rectangular regions, but also every lateral face is a rectangular region. Since any prism all of whose lateral faces are rectangular regions is a right prism, every rectangular prism is a right prism. At this point, you may wish to ask pupils for examples of objects that may be considered as models of rectangular prisms, such as various types of boxes.

• Ex. 4. The glass object used in science demonstrations to disperse sunlight into its spectrum colors usually has the shape of a triangular right prism. If such a glass is available, demonstrate to the class the fact that the surface of this object may be considered as a model of a triangular right prism, but the object itself is not a prism.

• Ex. 7. If necessary, help pupils visualize space and depth when drawing the models. You may wish to use Ex. 7 as an oral exercise at the board for some pupils and as a written exercise for other pupils.

Individualizing Instruction

Have *slower learners* suggest objects which are representations of prisms, telling whether each is a model of an oblique or right prism. Then have them name each of these prisms: triangular, rectangular, hexagonal, and so on.

Pupil's Objective

To learn to find the lateral area of a right prism.

Background

The measure of the area of a polyhedron is equal to the sum of the measures of the areas of the faces of the polyhedron. The exercises on the page pertain to right prisms. In doing these exercises, we are not concerned with finding the area of the right prism, but only in finding the lateral area of a right prism.

The lateral area of a right prism is the union of the areas of the lateral faces only. The measure of this lateral area is equal to the sum of the measures of the areas of the lateral faces. It is important for pupils to understand that the lateral area of a right prism does not include the area of the bases.

Each lateral face of a right prism is a rectangular region. Finding the lateral area of a right prism requires finding the areas of rectangular regions.

A cube is a right prism consisting of six congruent square faces, four of which are lateral faces (two are bases). The measure of the lateral area of a cube is four times the measure of the area of one lateral face.

Pre-Book Lesson

Review finding areas of rectangular regions, giving pupils exercises involving English and metric units of measurement and exercises finding areas of rectangular regions for which the length and width are not given in the same units of measurement. Review finding the area of a square region.

Using the Text Page

• Ex. 1–3. Emphasize that the lateral area of a right prism does not include the area of the bases. If pupils have difficulty finding the measure of length of one of the rectangular regions, remind them that the measures of length of the opposite sides are equal.

• Ex. 4. Have pupils describe other prisms whose lateral faces have equal measures of area.

• Ex. 6. Since a cube is a right prism having four lateral faces which are congruent, to find the measure of the lateral area, we multiply the measure of the area of one lateral face by 4. Emphasize that only 4 of the 6 congruent faces of a cube are lateral faces.

• Ex. 8. Point out that the factors-product relationship can be utilized in setting up a mathematical sentence to solve the problem.

Individualizing Instruction

• Ask *more capable pupils*, "If the lateral area of a cube is 100 sq. in., what is the length of one edge?" The area of each lateral face is 25 sq. in. Since the area of a square region is found by using the relationship $A = s^2$, where s is the measure of length of the square region, the length of one edge is 5 in.

• Have *all pupils* find the measure of lateral area of right prisms for which the length and width of the rectangular regions forming the lateral faces are given in different units of measurement.

• Show *slower learners* physical objects which can serve as models of right prisms and have them indicate the faces of each whose areas form the lateral area of the prism.

Lateral Area of a Right Prism

[O]

1. The triangular prism shown in Fig. 1 is a right prism. Then is each lateral face a rectangular region? Yes

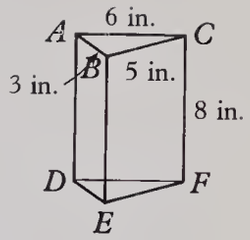


Fig. 1

2. For the prism shown in Fig. 1, the area of face $BEFC$ is $8 \times$ _?5, or _?40 sq. in. What is the area of face $ADEB$? 24 sq. in. of face $ADFC$? 48 sq. in.

3. The measure of the *lateral area* of a right prism is equal to the sum of the measures of the areas of the lateral faces of the prism. Explain the work in the box for finding the lateral area of the prism shown in Fig. 1. The sum of the measures of the areas of the lateral faces is found.

$n = 40 + 24 + 48$
$= 112$

The lateral area is 112 sq. in.

4. Fig. 2. Each of the three lateral faces of the right prism shown has an area of _?21 sq. in., so the lateral area of the prism is $3 \times$ _?21 or _?63 sq. in.

5. At the board, find the lateral area of the right prism shown in Fig. 3. $n = 10 + 30 + 10 + 40$
$= 90$ The lateral area is 90 sq. in.

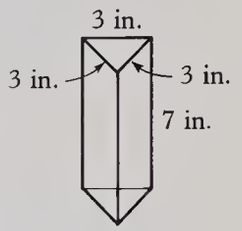


Fig. 2

6. To find the measure of the lateral area of a cube, we can multiply the measure of the area of a face by _?4. Explain. Key idea: The measure of the area of each of the four lateral faces is the same.

[W]

7. What is the lateral area of a cube if the area of each face is $3\frac{1}{4}$ sq. yd.? 13 sq. yd.

8. If the lateral area of a cube is 60 sq. cm., what is the area of each face? 15 sq. cm.

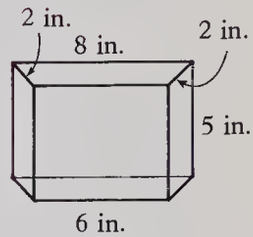
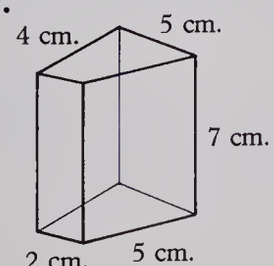


Fig. 3

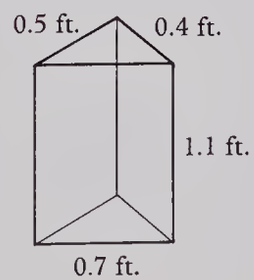
Ex. 9–12. Find the lateral area of the right prism shown.

9.



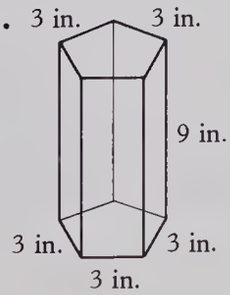
112 sq. cm.

10.



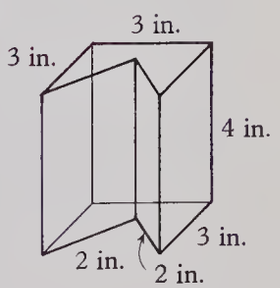
1.76 sq. ft.

11.



135 sq. in.

12.



52 sq. in.

A	B
$n = (7 \times 5) + (7 \times 3) + (7 \times 4)$ $= 35 + 21 + 28$ $= 84$ The lateral area is 84 sq. in.	$n = 7 \times (5 + 3 + 4)$ $= 7 \times 12$ $= 84$ The lateral area is 84 sq. in.

Finding the Lateral Area of a Right Prism

[O]

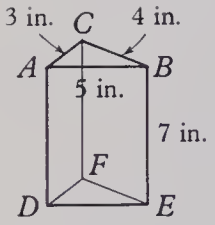


Fig. 1

1. The prism shown in Fig. 1 is a right prism. The intersection of faces *ADEB* and *BEFC* is segment _?BE Segment *BE* is a *lateral edge* of the prism.

The intersection of two adjacent lateral faces of a right prism is an edge of the prism which is called a lateral edge.

2. Fig. 1. Is $\overline{AD}$ a lateral edge? Yes Is $\overline{CF}$? Yes Explain. Each is the intersection of two adjacent lateral faces of a right prism.

3. Are the lateral edges of a right prism congruent? Yes Then do they each have the same length? Yes

4. The length of a lateral edge of a right prism is the *length* of the prism. Fig. 1. The length of the right prism is _?7 in.

5. Give the lengths of the right prisms shown for Ex. 9–12 on page 301. 7 cm.; 1.1 ft.; 9 in.; 4 in.

6. Explain the work in box A for finding the lateral area of the right prism shown in Fig. 1. The measure of the area of each lateral face is found and then their sum is found.

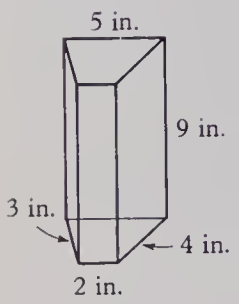


Fig. 2

7. Read the following rule and explain the work in box B.

To find the measure of the lateral area of a right prism, multiply the measure of the perimeter of a base by the measure of the length of the prism. The measure of the perimeter of the base is multiplied by the measure of the length of the prism.

8. At the board, find the lateral area of the right prism shown in Fig. 2 first as in box A and then as in box B.

$n = (9 \times 4) + (9 \times 5) + (9 \times 3) + (9 \times 2)$
$= 36 + 45 + 27 + 18$
$= 126$

$n = 9 \times (5 + 4 + 3 + 2)$
$= 9 \times 14$
$= 126$

The lateral area is 126 sq. in [W]

Using the rule above, find the lateral area of each right prism shown for Ex. 9–12 on page 301. 112 sq. in.; 1.76 sq. ft.; 135 sq. in.; 52 sq. in.

♦ **Extra Examples** Set 136.

Teaching Page 302

Pupil's Objectives

(a) To learn the meaning of *lateral edge* and *length* of a right prism; and (b) to learn an efficient method of finding the lateral area of a right prism.

Background

On the previous page, pupils learned to find the lateral area of a right prism by finding the sum of the measures of areas of the lateral faces of the prism.

To find the lateral area of the right prism shown in Fig. 1 on pupil's page 302, we use the mathematical sentence shown in box A:

$$n = (7 \times 5) + (7 \times 3) + (7 \times 4)$$

This is a sum of products. We can now apply the Distributive Property.

$a \times (b + c + d) = (a \times b) + (a \times c) + (a \times d)$ or, $(a \times b) + (a \times c) + (a \times d) = a \times (b + c + d)$. Therefore, $n = (7 \times 5) + (7 \times 3) + (7 \times 4)$ may also be expressed as $n = 7 \times (5 + 3 + 4)$, where 7 is the measure of length of the prism and $(5 + 3 + 4)$ is the sum of the measures of width of the sides, or perimeter of a base. The sum of the measures of width of all the lateral faces is equal to the measure of the perimeter of a base. Therefore, the measure of the lateral area of a right prism is equal to the product of the measure of length of the prism and the measure of the perimeter of a base.

Pre-Book Lesson

• Review the idea that the sum of the measures of width of the lateral faces of a prism is equal to the measure of the perimeter of one base. Again, stress the fact that the lateral faces of a right prism are rectangular regions.

• Discuss why $n = (7 \times 5) + (7 \times 3) + (7 \times 4)$ and $n = 7 \times (5 + 3 + 4)$ are equivalent mathematical sentences. For *more capable pupils*, this may be discussed in terms of the Distributive Property, as described in the Background.

Using the Text Page

• Ex. 1–3. Pupils are led to the understanding that the lateral edges of a right prism each have the same length. Pupils are not asked to explain why this is true. You may wish to have some of your *more capable pupils* discuss why they have the same length. Review the definition of a prism, stressing the characteristic that the bases of a prism are parallel faces.

• Ex. 7. Some pupils may recognize that the work in box B is an application of the Distributive Property.

Remind *slower learners* that the bases of a prism are congruent. This must be kept in mind when finding the perimeter of the prisms shown in Ex. 9–12 on page 301.

Individualizing Instruction

• *More capable pupils* may be asked to find, without using pencil and paper, the lateral area of right prisms using the methods developed on this page.

Have *more capable pupils* find the length of a cube whose lateral area is 256 sq. in. (length—8 in.)

• *All pupils* may be given the lateral area of a right prism, and the perimeter, and be asked to find the length of the prism. Then they may be given the lateral area and the length of the prism and be asked to find the perimeter. For example:

1. Find the length of a right prism if the lateral area is 60 sq. in., and the perimeter of a base is 12 inches.

2. Find the perimeter of a right prism whose lateral area is 80 sq. in., and whose length is 4 in.

• If necessary, discuss step-by-step with *slower learners* finding the lateral area of the right prisms by the new method for Ex. 9–12 on page 301.

Teaching Page 303

Pupil's Objective

To review finding area of triangular and parallelogram regions.

Background

The purpose of these exercises is to review the concept of finding the area of a triangular region and of a parallelogram region, as a preparation for the study of area of a right prism and of a polyhedron which is introduced on page 304 and 305.

The idea of finding the area of a parallelogram region was developed on pages 292 and 293. Finding the area of a triangular region was introduced and developed on pages 294 and 295.

Pre-Book Lesson

• Review finding the area of a parallelogram region, stressing that (1) A, the measure of the area of a parallelogram region, is equal to the product of b, the measure of the length of the base, and h, the measure of the height. $A = b \times h$. (2) Any one of the four sides of a parallelogram region may be thought of as the *base* of the region. (3) A perpendicular segment from the line containing the base to an opposite vertex is called the *altitude* of the parallelogram region. The length of the altitude for a parallelogram region is called the *height* of the region. The altitude of a parallelogram region may be contained in the interior of the region, in the exterior of the region, or in the parallelogram bounding the region.

• Review finding the area of a triangular region, stressing that (1) A, the measure of the area of a triangular region, is equal to one half the product of b, the measure of the length of the base, and h, the measure of the height. $A = \frac{1}{2} \times (b \times h)$. (2) Any one of the three sides of the triangular region may be considered as the *base* of the region. (3) A segment perpendicular to the base, having one end point in the line containing the base and the vertex opposite the base as the other end point, is called the *altitude* of the triangular region. The length of the altitude for the triangular region is the *height* of the region. The altitude of a triangular region may be contained in the interior of the region, partly in the exterior of the region, or contained in the triangle which is the boundary of the region.

Using the Text Page

Do not assign Ex. 11–18 as written work until you have checked the pupils' work on earlier exercises to be sure they understand Ex. 3–10 and are working them correctly. After explaining the errors to pupils having difficulty, assign additional exercises for practice before having them work Ex. 11–18.

Individualizing Instruction

• *More capable pupils* may be asked to find areas of triangular and parallelogram regions, without using pencil and paper.

• *All pupils* may be given the area and the height of triangular and parallelogram regions and be asked to find the length of the base. Give them the area and the length of the base and ask them to find the height.

• Before beginning the written exercises, *slower learners* may, as an oral exercise, be asked to indicate the height and length of the base in smaller units for each of Ex. 3–18. After handing back the written work, group *slower learners* in order to go over individual difficulties.

* This is a review of finding areas of different regions in preparation for finding the total area of a polyhedron.

* Remembering about Plane Regions

Resurvey; area [O]

1. The length of the base of a triangular region is 8 in. and its height is 7 in. What is the area of the region?

$n = \frac{1}{2} \times (8 \times 7)$

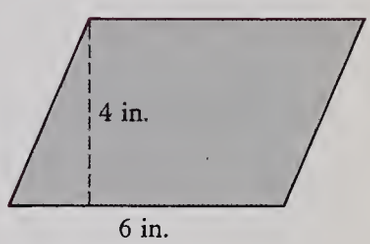
$n = \frac{1}{2} \times 56$

$= 28$

a. On the board, write a mathematical sentence which shows the relationship expressed in the problem.

b. Solve the sentence and then give the answer. 28 sq. in.

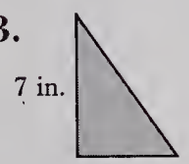
** **2.** Give a mathematical sentence that can be used to find the area of the parallelogram region shown at the right. $n = 4 \times 6$ Mentally find the area of the region. 24 sq. in.



[W]

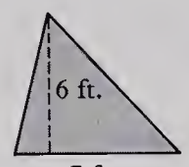
Ex. 3–6. Find the area of the triangular region shown.

3.



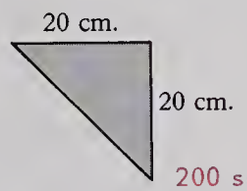
$17\frac{1}{2}$ sq. in.

4.



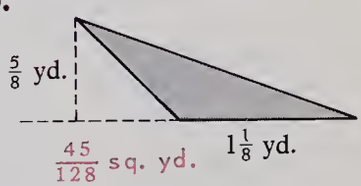
21 sq. ft.

5.



200 sq. cm.

6.



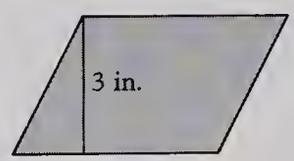
$\frac{45}{128}$ sq. yd.

Ex. 7–10. Find the area of the parallelogram region shown.

7.

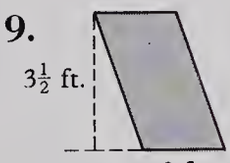
22.4 sq. cm.

8.



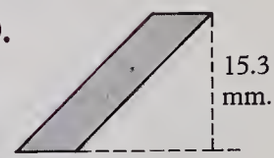
1.105 sq. ft. or 159.12 sq. in.

9.



7 sq. ft.

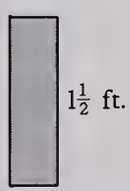
10.



108.63 sq. mm.

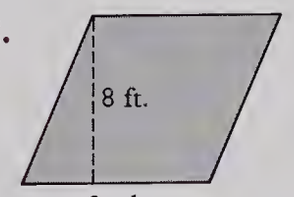
Ex. 11–18. Express the area of the region in terms of the smaller unit.

11.



90 sq. in.

12.

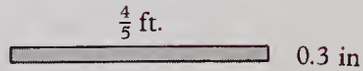


72 sq. ft.

13.

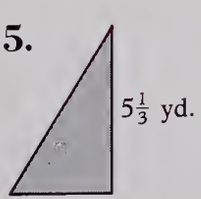
54,000 sq. cm.

14.



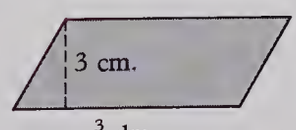
2.88 sq. in. or $2\frac{22}{25}$ sq. in.

15.



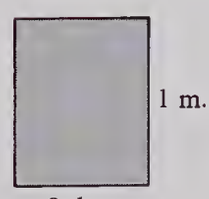
80 sq. ft.

16.



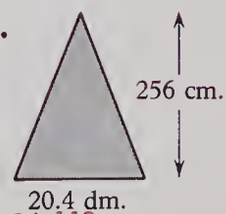
$22\frac{1}{2}$ sq. cm.

17.



80 sq. dm.

18.



26,112 sq. cm.

** Review using $A = \frac{1}{2} \times (b \times h)$ to find the area of a triangular region and $A = b \times h$ to find the area of a parallelogram region.

* Note that the bases are parallelogram regions and that the lateral faces are rectangular since it is given that the polyhedron is a right prism.

Finding the Total Area of a Right Prism

[O]

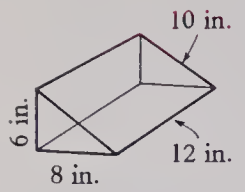


Fig. 1

1. The bases of the right prism shown in Fig. 1 are right triangular regions. What is the area of each base? 24 sq. in. of each lateral face? 120 sq. in., 72 sq. in., 96 sq. in. At the board, find the total area of the prism by finding the sum of the measures of the lateral area and of the area of the two bases. $n = (2 \times 24) + 120 + 72 + 96$ $= 48 + 120 + 72 + 96$ $= 336$ The total area is 336 sq. in.

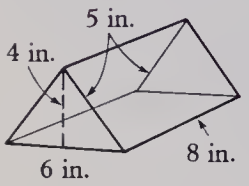


Fig. 2

2. At the board, find the total area of the right prism shown in Fig. 2. Explain your work. $n = (2 \times 12) + 40 + 40 + 48$ $= 24 + 40 + 40 + 48$ $= 152$ The total area is 152 sq. in.

3. Fig. 3. Bases *ABFE* and *DCGH* of the right prism shown are parallelogram regions which are not rectangular regions. Name the lateral faces. faces EHGF, FGCB, ABCD, ADHE Are the lateral faces rectangular regions? Yes Why? It is a right prism. At the board, find the total area of this prism. $n = (2 \times 21) + 32 + 56 + 32 + 56$ $= 42 + 32 + 56 + 32 + 56$ $= 218$ The total area is 218 sq. in. [W]

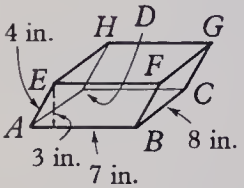


Fig. 3

4. Fig. 4. The area of each base of the right prism shown is about 10.8 sq. in. Copy and complete: The total area of the prism is about _?_ sq. in. 156.6

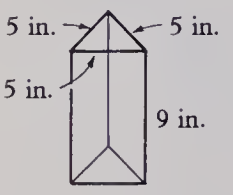


Fig. 4

* 5. The bases of the right prism shown in Fig. 5 are parallelogram regions which are not rectangular regions. The length of the prism is 15 in.

a. Find its lateral area. 420 sq. in. **b.** Find its total area. 500 sq. in.

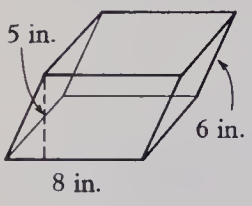
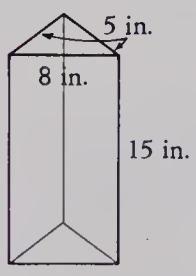


Fig. 5

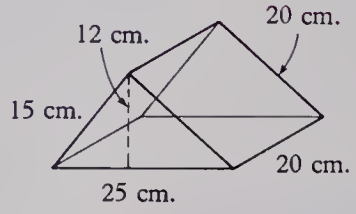
Ex. 6–8. Find the total area of the right prism shown.

6.



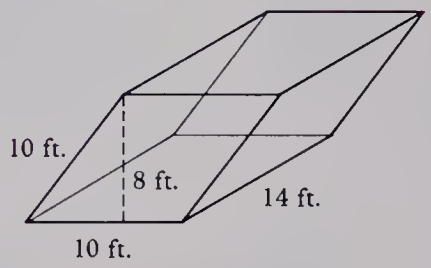
Area of each base is 12 sq. in
294 sq. in.

7.



1,500 sq. cm.

8.



720 sq. cm.

9. Find the area of each base of a triangular right prism whose lateral area is 44 sq. in. and whose total area is 65 sq. in. $10\frac{1}{2}$ sq. in.

Teaching Page 304

Pupil's Objective

To learn to find the total area of a right prism.

Background

To find the *total area* of a right prism, add the measure of lateral area and the measure of the area of the two bases. The exercises on this page pertain to right prisms whose bases are either triangular or parallelogram regions. At this point, pupils would have difficulty finding the total area of other right prisms because they have not learned how to find the area of certain regions, i.e., the hexagonal region. (Remember that rectangular and square regions are parallelogram regions.)

Pre-Book Lesson

Review the idea that any polyhedron partitions space into three distinct sets of points:

a. The set of points in the interior for a polyhedron.

b. The set of points in the exterior for the polyhedron.

c. The set of points in the polyhedron. In finding the total area of a right prism (one type of polyhedron), we are concerned with just one of these three sets of points—the set of points which is the polyhedron. We are finding the area of the simple closed surface forming the right prism. On page 306, the concept of the union of the set of points in the polyhedron and the set of points in the interior of the polyhedron will be introduced.

Using the Text Page

• Ex. 1–3. Be sure pupils realize that finding the lateral area by multiplying the measure of the perimeter of a base and the measure of the length applies only to polyhedrons which are right prisms.

• Ex. 3, 5, and 8. Be sure pupils realize that the lateral sides of these figures are rectangular regions. The figures are given as right prisms so the lateral faces must necessarily be rectangular regions even if an optical impression does not suggest this.

• Ex. 9. The measure of the total area and the measure of the lateral area are given and the measure of one base is to be found. When the measure of the lateral area is subtracted from the measure of the total area, the resulting number is the sum of the measures of both bases. The measure of the area of one base will be one half this number. The addends-sum relationship can be utilized.

Individualizing Instruction

Be sure *slower learners* understand the difference in meaning of lateral area and total area of a right prism. Be sure they understand the relationship among total area, lateral area, and area of the bases as being:

$$\begin{matrix}\text{measure of}\\ \text{total area}\end{matrix} = \begin{matrix}\text{measure of}\\ \text{lateral area}\end{matrix} + \begin{matrix}\text{measure of}\\ \text{area of bases}\end{matrix}$$

NOTES

Teaching Page 305

Pupil's Objective

To learn to find the total area of a polyhedron.

Background

The statement, "The measure of the area of the polyhedron is equal to the sum of the measures of the areas of the faces of the polyhedron" applies to all polyhedrons. However, the oral and written exercises on this page pertain only to polyhedrons whose faces are triangular and rectangular regions.

Pre-Book Lesson

• This enrichment page may be appropriate for *all pupils* if the class situation permits. If not, give *slower learners* more practice in finding the total area of a right prism.

• Review the meaning of the term polyhedron, and discuss the various types of polyhedrons introduced previously in the chapter.

• Have pupils find the lateral area of a right prism using the method described on page 301, and then the method described on page 302. Have pupils find the total area of the right prism. Ask pupils how they would find the area of all the faces of a polyhedron which is not a right prism. Lead them to realize that the measures of area of all the faces must be added. Discuss why the rule, "multiply the measure of the perimeter of the base by the measure of the length of a lateral edge" would not apply to finding the lateral area of polyhedrons that are not right prisms.

Using the Text Page

• Ex. 1–7. Pupils now know three methods of finding the area of right prisms such as that shown in Fig. 1 on the pupil's page.

(1) They may find the area of each face, the area of the prism being the sum of the measures of the faces.

$$\begin{aligned} n &= 8 + 8 + 12 + 12 + 24 + 24 \\ &= 88 \end{aligned}$$

(2) They may find the area of all regions that are congruent by multiplying the number of congruent regions by the measure of area of one of the congruent regions. These products are added to find the measure of area of the prism.

$$\begin{aligned} n &= (2 \times 8) + (2 \times 12) + (2 \times 24) \\ &= 16 + 24 + 48 \\ &= 88 \end{aligned}$$

(3) Since the polyhedron is a right prism, pupils may first find the lateral area using the method introduced on page 302, and find the areas of the bases. The area of the prism is then the sum of the measure of the lateral area and the measures of the areas of the bases.

$$\begin{aligned} n &= [6 \times (4 + 4 + 2 + 2)] + [2 \times (4 \times 2)] \\ &= (6 \times 12) + (2 \times 8) \\ &= 72 + 16 \\ &= 88 \end{aligned}$$

Pupils should be allowed to use any one of these three methods in finding the area of a polyhedron when the polyhedron is a right prism; but they should only consider the first two methods for finding the area of a polyhedron which is not a right prism.

• Ex. 8. In answering Ex. 8b, pupils may simply express the answer to Ex. 8a in square yards, 9 sq. ft. = 1 sq. yd., or they may express 108 sq. ft. as 12 sq. yd. before dividing by the number of faces of the cube.

Individualizing Instruction

Have *more capable pupils* find the area of polyhedrons having triangular and rectangular faces, the measurements of whose lengths and sides of bases are not given in the same unit of measurement.

* Note that "area of a polyhedron" means the total area. That is, the area of all the faces.

Total Area of a Polyhedron

Enrichment [O]

* **The measure of the area of a polyhedron is equal to the sum of the measures of the areas of all the faces of the polyhedron.**

1. Fig. 1. The polyhedron shown is a rectangular prism. Is each face a rectangular region? Yes Tell how to find the area of a rectangular region. Find the product of the measure of the length and the measure of the width.

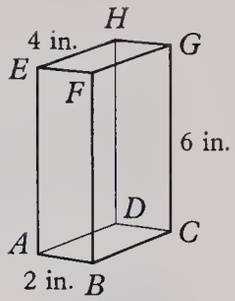


Fig. 1

2. Name the faces of the polyhedron shown in Fig. 1 and give the area of each face. Can we use the mathematical sentence $n = 8 + 8 + 12 + 12 + 24 + 24$ to find the area of this polyhedron? Yes At the board, solve this mathematical sentence and then give the area of the polyhedron. n = 88 88 sq. in.

face ABFE 12 sq. in.
face BCGF 24 sq. in.
face GHDC 12 sq. in.
face HEAD 24 sq. in.
face EFGH 8 sq. in.
face ABCD 8 sq. in.

3. Fig. 2. The polyhedron shown has 1 square face and ? 4 triangular faces. Each triangular face has an area of 12 sq. in. What is the area of the square face? 16 sq. in. What is the area of the polyhedron? 64 sq. in.

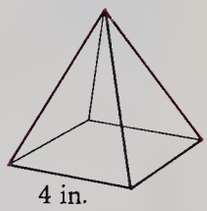


Fig. 2

4. Fig. 3. The polyhedron shown has 2 triangular faces and 3 rectangular faces. Each triangular face has an area of 6 sq. in. At the board, first find the lateral area of the polyhedron and then find the total area. n = 40 + 30 + 50 = 120 The lateral area is 120 sq. in. The total area is 132 sq. in.

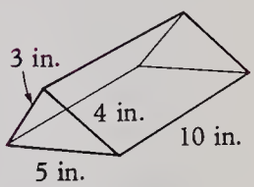


Fig. 3

5. Tell how to find the total area of a cube if the area of one face is known. Multiply the measure of the area of one face by 6.

[W]

6. What is the total area of a cube if each face has an area of $4\frac{1}{3}$ sq. in.? 26 sq. in.

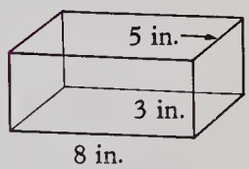


Fig. 4

7. Find the area of the rectangular prism shown in Fig. 4. 158 sq. in.

8. If the area of a cube is 108 sq. ft., what is the area of each face

a. expressed in terms of square feet? 18 sq. ft.

b. expressed in terms of square yards? 2 sq. yd.

◆ **Extra Examples.** Set 137.

Solids

[O]

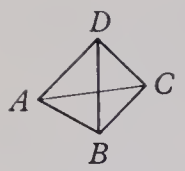


Fig. 1

1. The space figure shown in Fig. 1 is a polyhedron. It is the union of the four triangular regions ABC, _?_ (ABD), _?_ (BCD), and _?_ (ACD). Do the points in the interior for this figure belong to the figure? No

Fig. 2

2. The space figure shown in Fig. 2 is the union of the polyhedron shown in Fig. 1 and the set of points in its interior. Such a space figure is called a *solid figure* or, simply, a *solid*.

> **The union of a simple closed surface in space and the set of points in its interior is called a *solid*. The simple closed surface is called the *surface* of the solid.**

Fig. 3

Give a special name for the surface of the solid shown in Fig. 3. (Rectangular prism) This solid is called a *rectangular solid*. Why is rectangular solid a good name for this figure? Each face is rectangular.

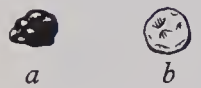


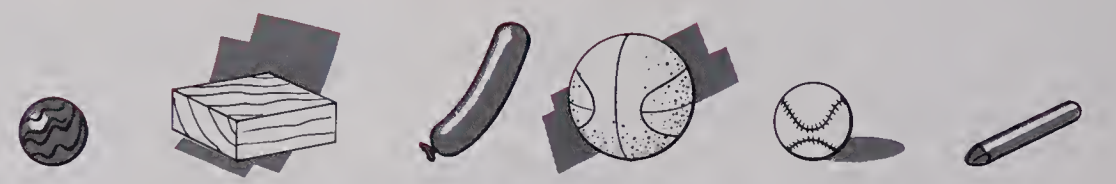
Fig. 4

3. A lump of coal such as the one shown in Fig. 4a would be a good model of a solid. Explain why a dented ping-pong ball (Fig. 4b) would not be a good model of a solid. Key idea: A ping-pong ball is hollow.

Fig. 5

4. An empty closed cardboard box (Fig. 5) would be a good model of a rectangular prism. Would it be a good model of a solid? (No) Explain. Key idea: It is hollow.

5. Which of the items shown below would be good models of solids? (balloon, basketball) Which would not? (marble, block of wood, baseball, chalk)


Marble · Block of Wood · Balloon · Basketball · Baseball · Chalk

[W]

6. Name something that would be a good model of a solid whose surface Answers will vary.

a. is a polyhedron but is not a prism.

b. is a rectangular prism.

c. is a triangular prism.

Teaching Page 306

Pupil's Objective

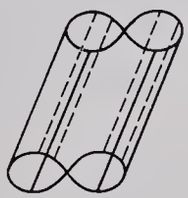
To understand the concept of a solid figure.

Background

Pupils have learned that a polyhedron is a simple closed surface. Any simple closed surface partitions space into three distinct sets of points: (1) the set of points in the interior of the simple closed surface; (2) the set of points in the exterior of the simple closed surface; and (3) the set of points in the simple closed surface.

The union of a simple closed surface and the set of points in the interior is called a *solid*. The name given a solid is often similar to the name of the simple closed surface. For example, the union of a rectangular prism and its interior is called a rectangular solid.

Since the definition of simple closed surface is not given in the text, you may wonder what would be the shape of a space figure that is a closed surface, but is not a simple closed surface. Below is an example of a non-simple closed surface.



Pre-Book Lesson

Review the concept of space figures as presented on pages 296–297. Point out that the only space figures studied so far have been polyhedrons. Ask if space figures could consist of sets of points other than those forming polygonal regions. Lead pupils to the realization that simple closed curved surfaces can have shapes other than those of a polyhedron. Have them give examples of objects whose surfaces would be a good model for a simple closed surface which is not a polyhedron.

Using the Text Page

• Ex. 1–2. It may take a little while for pupils to get used to thinking of a solid in the geometric sense of being a set of points. They are so used to conceiving of a solid as being something physical and hard. Discuss this concept until you feel the pupils understand the concept of solid in the geometric sense.

• Ex. 3. The dents in the Ping-Pong ball are of no significance. A Ping-Pong ball is not a good model of a solid because a solid includes all the points in the interior of the simple closed curve. For the same reason, the balloon and the basketball in Ex. 5 are not good models for solids.

• Ex. 5. Emphasize the different characteristics of space figures and solids. For instance, ask pupils to think about an empty box with a top on it. Then, consider filling the box to the top with sand. The empty box would have characteristics of a space figure, and may be thought of as a shell. The box filled with sand would have characteristics of a solid having both a simple closed surface (the box) and an interior (the sand).

Individualizing Instruction

• If you find Ex. 6 too difficult for *slower learners*, you may give these pupils a list of objects in the room and ask them to classify each as being a good model for a solid or a good model for a simple closed surface.

• Have *all pupils* think of the set of points pictured in Fig. 2 on the pupil's page as:

Set A is the set of all points in the interior for the polyhedron.

Set B is the set of all points in the exterior for the polyhedron.

Set C is the set of all points in the polyhedron.

Ask which of the following is a solid:

$$A \cup B;\ B \cup C;\ A \cup C$$

Teaching Page 307

Pupil's Objectives

(a) To provide an opportunity for pupils to discover more about solids; and (b) to practice multiplying with whole numbers and decimals.

Background

It has been mentioned previously that a solid is often named for the simple closed surface which forms its surface. Two examples of this are given on this text page. A *polyhedral solid* is the union of a polyhedron and its interior, and a *prismoid solid* is the union of a prism and its interior. Since a prism is a special type of polyhedron, a prismoid solid is a special type of polyhedral solid. Below is a list of the descriptions of various space figures which may form the surface of a solid and for which the solid is named.

Polyhedron—a space figure consisting of a simple closed surface which is the union of polygonal regions.

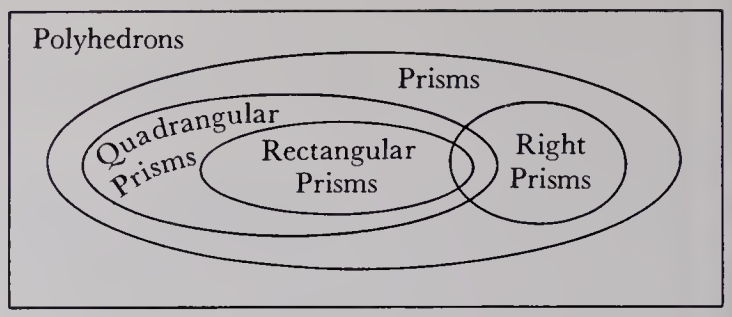
Prism—a polyhedron with at least two congruent and parallel faces, and with each lateral face a parallelogram.

Quadrangular prism—a prism whose bases are quadrangular regions.

Rectangular prism—a quadrangular prism all of whose faces are rectangular regions. It is a special type of right prism.

Right prism—a prism all of whose lateral faces are rectangular regions.

Notice that a right prism is not necessarily a rectangular prism nor is it necessarily a quadrangular prism. The bases of a right prism may be any type of polygonal region. The relationship among these space figures may be summarized by the following Venn diagram.



Pre-Book Lesson

Review the definitions listed in the Background section, emphasizing the relationship among quadrangular prism, rectangular prism, and right prism.

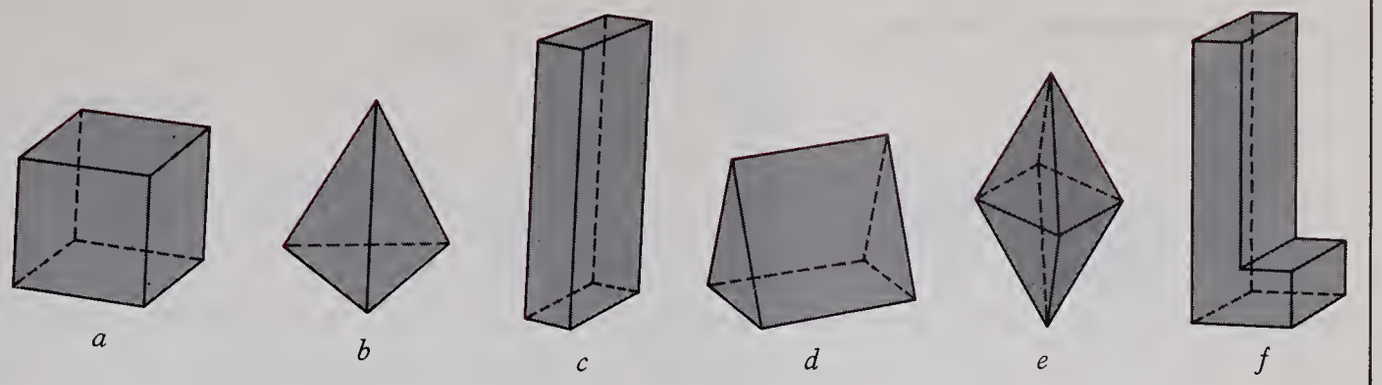
Using the Text Page

The top of the page is designed as enrichment and is most likely appropriate for *more capable pupils*. However, if the class situation permits, all pupils may do the enrichment. If you wish, you may assign "To Keep In Practice" exercises at the bottom of the page to all pupils. Those who complete the exercises correctly may then be permitted to do the enrichment exercises on their own. If *slower learners* display difficulty with the practice exercises, go over difficulties, and assign more practice from Extra Practice Sets at the back of the pupil's book.

Individualizing Instruction

- *All pupils* may be asked to draw a Venn diagram such as that shown in the Background section to show the relationships among the various polyhedrons. Have *more capable pupils* draw a Venn diagram to show the relationships among the various polyhedral solids studied so far.
- Briefly review the characteristics of a solid from page 306 with *slower learners*. This will also serve as an introduction for pages 308–309.

* Figure f is a prismoid solid. Consider the "front" and "back" faces as the bases.

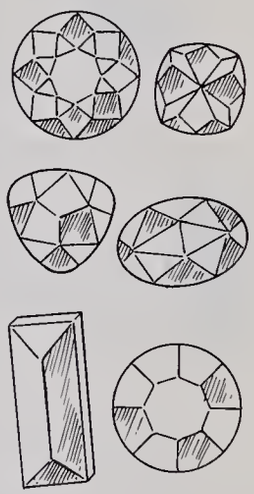


Polyhedral Solids

Enrichment [W]

For each solid shown in the box above, the surface is a polyhedron. Such solids are called *polyhedral solids.*

The union of a polyhedron and its interior is a solid called a *polyhedral solid.*



1. The diamonds pictured at the right are good models of polyhedral solids. Name some other things which may be thought of as models of polyhedral solids. Answers will vary.

2. If the surface of a polyhedral solid is a prism, then the solid is called a *prismoid solid.* Copy and complete: The union of a prism and its interior is a prismoid solid.

* **3.** Which of the solids shown in the box above are prismoid solids? a, c, d, f

Ex. 4–6. Write *T* or *F.*

4. Every rectangular solid is a prismoid solid. T

5. Every prismoid solid is a rectangular solid. F

6. All prismoid solids are polyhedral solids. T

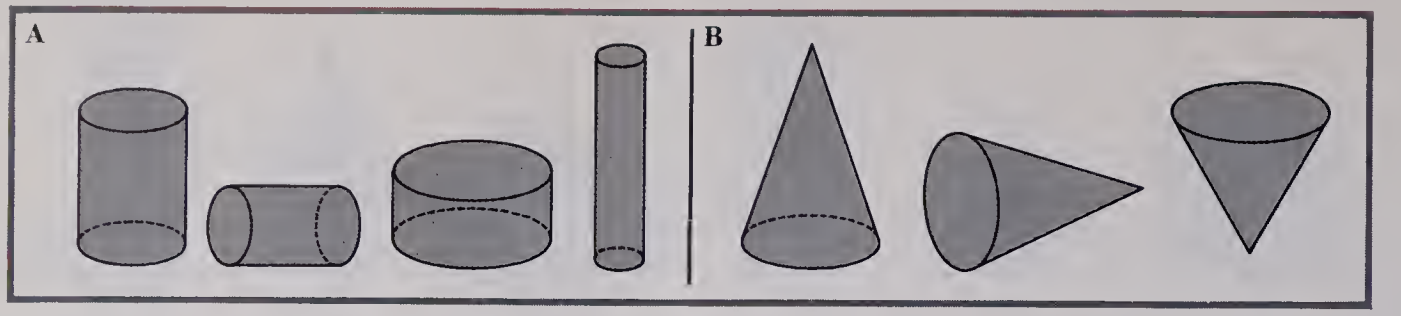
To Keep in Practice

[W]

Ex. 1–5. Copy and multiply.

1.	2.	3.	4.	5.
23,657	5.034	43.68	$107.55	63.01
256	2.3	5.47	8	0.003
6,056,192	11.5782	238.9296	$ 860.40	0.18903

* The region shown in gray in Fig. 1 is a rectangular region. The region shown in gray in Fig. 2 is a parallelogram region. It is not a rectangular region.



Cylindrical and Conical Solids

Enrichment [O]

Fig. 1

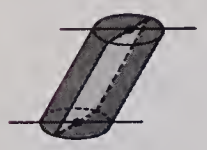


Fig. 2

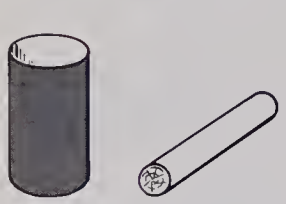


Fig. 3

1. The solids shown in box A are called *right circular cylindrical solids*. The curved part of the surface of a right circular cylindrical solid is part of a geometric figure called a *cylinder*. The other two parts of the surface are congruent circular regions called bases.

2. Fig. 1 shows that the intersection of a right circular cylindrical solid and a plane which contains the centers of the bases is a rectangular region. Explain why the solid shown in Fig. 2 is not a right circular cylindrical solid. An intersection with a plane which contains the centers of the bases is not a rectangular region.

3. Find some things in your classroom which may be thought of as models of right circular cylindrical solids. Answers will vary.

4. Which would be a good model of a right circular cylindrical solid, an empty cardboard container (Fig. 3a) or a wooden dowel (Fig. 3b)? Explain. The empty container is hallow.

5. The solids shown in box B are *conical solids*. The curved part of the surface of a conical solid is part of a geometric figure called a *cone*. The other part of the surface is a circular region.

[W]

6. Stalactites and stalagmites often are good models of conical solids. Name some other things which would be good models of conical solids. Answers will vary.

7. Name some things which may be thought of as good models of right circular cylindrical solids. Answers will vary.

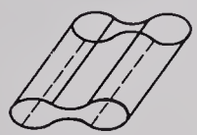
Teaching Page 308

Pupil's Objective

To become familiar with cylindrical solids and conical solids.

Background

A *cylindrical solid* is most often thought of as a solid with a circular base. There is no requirement that the bases of a cylindrical solid be circular as long as they are congruent. Below is a drawing representing a cylindrical solid whose bases are not circular regions.



If the bases of a cylindrical solid are circular regions, the solid is called a circular cylindrical solid.

A cylindrical solid whose lateral surface is perpendicular to the bases is a right cylindrical solid. Otherwise, it is an oblique cylindrical solid. Box A and Fig. 1 show right cylindrical solids. Fig. 2 shows an oblique cylindrical solid.

If the base of a *conical solid* is a circular region, the solid is a circular conical solid. If the segment joining the apex (the point of the cone) to the center of the circular region is perpendicular to the circular region, the solid is a right circular conical solid.

Pre-Book Lesson

• Assign the page to be worked by the entire class.

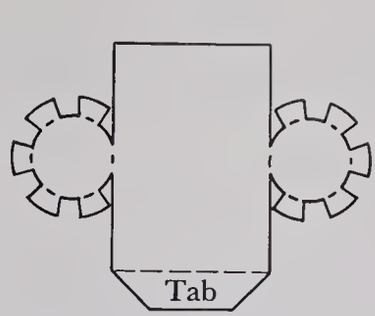
• If possible, make models from drawings like those at the top of the next column. Cut out along the solid lines and fold along the dotted lines.

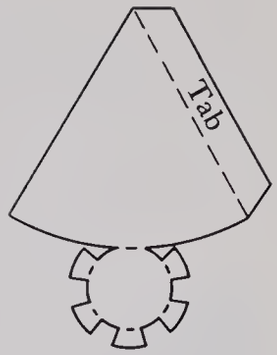
As each model is assembled, ask pupils,

(1) "Can this represent a space figure?"

(2) "Can this represent a simple closed surface?"

(3) "Does the union of the surface and its interior form a solid?"





Display models of cylindrical and conical solids made of wood*, and ask, "How are these better models of the solids than the paper models we made?"

Using the Text Page

• Ex. 1–7. The exercises pertaining to cylindrical and conical solids are non-metric. Pupils are expected to become familiar with models, not to define the concepts.

• Ex. 1 and 5. Point out that the concepts of cylindrical solid and conical solid are similar to the concepts of other solids with which the pupils are already familiar in that a solid is the union of a simple closed curve and its interior.

• Ex. 6. Stalactites are deposits hanging from cavern walls which look like icicles, or similar models of other material. Stalagmites are deposits formed on the floor of caverns, or similar models of other material.

Individualizing Instruction

• Ask *more capable pupils* to describe how the bases of a polyhedron differ from the bases of a cylinder.

• Have *all pupils* describe differences between a circular cylindrical solid and a circular conical solid.

* See item 19, page xix.

Teaching Page 309

Pupil's Objectives

(a) To become familiar with spheres and spherical solids; and (b) to practice renaming numbers.

Background

A *sphere* is a space figure consisting of the set of all points a given distance from a given point called its center. A sphere is a simple closed surface. The union of the sphere and the set of all points in its interior is called a *spherical solid*.

Fig. 2 on the pupil's page shows the plane containing the center of the spherical solid. Such a plane partitions the sphere, the surface of the spherical solid, into three distinct sets of points: (1) the set of all points in the sphere on one side of the plane; (2) the set of all points in the sphere on the other side of the plane; (3) the set of points in the intersection of the plane and the sphere.

Pre-Book Lesson

Ask pupils for examples of solids whose surfaces are curved, but which are neither cylindrical nor conical solids. Display models of solids*.

Ask:

(1) "How many bases does a cylindrical solid have?"

(2) "How many bases does a conical solid have?"

(3) "Can you think of an example of a solid which has a simple closed surface but does not have a base?"

This should lead to the idea of spherical solid. Remember that all points of the surface must be a given distance from the center.

* See item 19, page xix.

Using the Text Page

• You may wish to assign the practice exercises at the bottom of the page first. The pupils completing these exercises can then go on to the enrichment exercises. When the other pupils have finished the practice exercises, you can briefly present the idea of sphere and spherical solid for *all pupils*.

• All or part of the practice exercises may be used for testing purposes. Go over individual difficulties with *slower learners*.

Individualizing Instruction

• Compare the Earth to a true sphere with *more capable pupils*. The Earth is often used as a model of a sphere. Actually, it is not a sphere because its surface is irregular and its equatorial diameter is 7,927 miles while its polar diameter is 7,900 miles. This deviation from a perfect sphere is about $\frac{1}{300}$ or 0.33%. Thus, the earth is practically a sphere.

• *All pupils* making errors consistently in the practice exercises should be assigned remedial work. Help them analyze their errors and give them additional practice exercises as necessary.

* Note that the intersection is a region, not a circle, since we are considering also the points "inside" the sphere.

Spheres and Spherical Solids

Enrichment [O]

1. Is it true that all points belonging to a circle are at the same distance from the center for the circle? Yes ∧ Do they all lie in one plane? Yes ∨ Then, is a circle a plane figure? Yes

2. Think of all the points in space which are at a given distance from a given point. You are thinking of a space figure which is called a *sphere*. Explain why a ping-pong ball would be a good model of a sphere. It represents a set of points in space at a given distance from a given point.

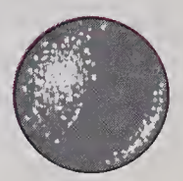


Fig. 1

3. Would a marble be a good model of a sphere? No ∧ Explain. ∨ It is solid.

4. The union of a sphere and its interior is a solid called a *spherical solid* (Fig. 1). Explain why a marble would be a good model of a spherical solid. It is solid. ∧ Explain why a basketball would not be a good model of a spherical solid. It is not solid.

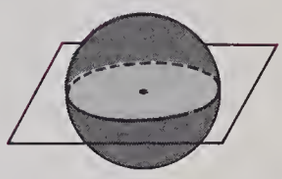


Fig. 2

* 5. Fig. 2 shows that the intersection of a spherical solid and a plane containing its center is a circular region.

[W]

6. Name two things other than those mentioned above which would be good models of **a.** spheres. **b.** spherical solids.

Answers will vary.

To Keep in Practice

Renaming [W]

Ex. 1–5. Rename with a fraction in simplest form.

1. 20% $\frac{1}{5}$ 2. 0.6 $\frac{3}{5}$ 3. 0.75 $\frac{3}{4}$ 4. 15% $\frac{3}{20}$ 5. 0.125 $\frac{1}{8}$

Ex. 6–10. Rename with a decimal.

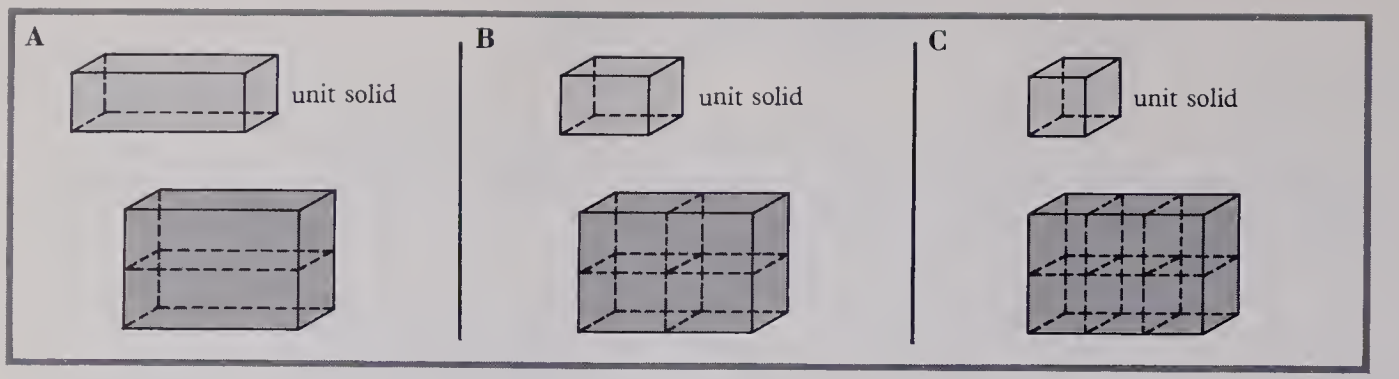
6. 47% 0.47 7. $\frac{4}{5}$ 0.8 8. $\frac{93}{100}$ 0.93 9. $2\frac{3}{4}$ 2.75 10. $\frac{5}{8}$ 0.625

Ex. 11–15. Write a base-five numeral for the number.

11. 8 13_{five} 12. 22 42_{five} 13. 47 142_{five} 14. 76 301_{five} 15. 129 1004_{five}

Ex. 16–20. Write a base-ten numeral for the number.

16. 41_{five} 21 17. 321_{four} 57 18. 66_{eight} 54 19. 34_{five} 19 20. CCXIV 214

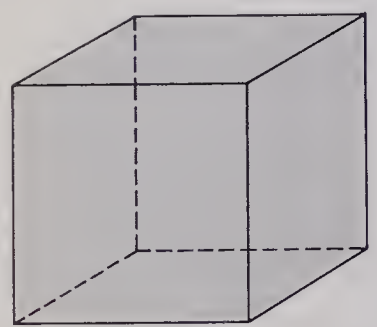


Volume of a Solid

Rectangular solids [O]

1. The *volume* of a solid is the size of the solid. For a unit of volume, we use a unit solid. In each of boxes A–C above, the unit of volume is a rectangular solid. In which box does the unit solid appear to be a cubical solid (the union of a cube and its interior)? Box C

2. Box A. The large rectangular solid is shown partitioned into how many unit solids? 2 Then, what is the volume of the large rectangular solid in terms of the unit of volume shown? 2


cubic inch

3. Give the volume of the large rectangular solid in terms of the unit of volume shown in box B; 4 in box C. 6 Do you see that we need to use certain unit solids as standard units of volume? Yes

4. Cubical solids often are used as standard units of volume. The cubical solid shown at the left is a standard unit called a *cubic inch* (cu. in.). Its volume is 1 cu. in. Cubic foot and cubic yard are two other units of volume. Describe a solid whose volume is 1 cu. ft.; 1 cu. yd. A cubical solid 1 ft. by 1 ft. by 1 ft.; A cubical solid 1 yd. by 1 yd. by 1 yd.

5. What unit of volume is shown at the left? 1 cubic centimeter Name a model of a solid in your classroom which seems to have a volume that is greater than 1 cu. cm.; less than 1 cu. cm. Answers will vary.

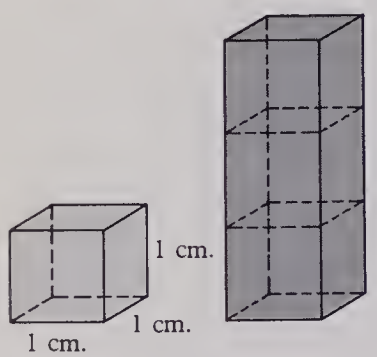


Fig. 1

6. The rectangular solid shown in Fig. 1 appears to be partitioned into how many cubic centimeters? 3 Then its volume is _?_ 3 cu. cm.

Teaching Pages 310 and 311

Pupil's Objective

To understand the concept of volume of a solid, and to express the volume of a rectangular solid in terms of standard-unit solids.

Background

The study of volumes of solids in this text pertains only to the study of volumes of rectangular solids.

Expressing volume of a solid in terms of standard-unit solids is presented in much the same way that expressing area of a region in terms of standard-unit regions was presented on pages 174–176. Note that the concept of volume of a rectangular solid, as presented in these text pages, does not develop a statement or an equation to find volume. The concept is developed so that the pupil will know intuitively how to find the volume of a rectangular solid of the type referred to in the exercises.

Below is a comparison of the main aspects of area of a region and volume of a solid.

Area of a region	*Volume of a solid*
(1) The area of a region is a measurement of the size of the region.	(1) The volume of a solid is a measurement of the size of the solid.
(2) The measure of the area of a region is equal to the sum of the measures of the areas of the regions into which the given region is partitioned.	(2) The measure of the volume of a solid is equal to the sum of the measures of the volumes of the solids into which the given solid is partitioned.
(3) Standard-unit regions are usually square regions.	(3) Cubical solids are used as standard units of volume.
(4) A square region each of whose sides is one inch in length is a standard unit of area called the square inch. Its area is one square inch.	(4) A cube each of whose edges is one inch in length is a standard unit of volume called the cubic inch. Its volume is one cubic inch.
(5) Other standard units of area are square centimeter, square foot, and square yard.	(5) Other standard units of volume are cubic centimeter, cubic foot, and cubic yard.

The volume of a rectangular solid may be found by partitioning the solid into standard units of volume. The measure of volume of the solid will be equal to the sum of the measures of volume of the standard units into which the given solid has been partitioned. The examples of rectangular solids given in the exercises make partitioning into standard units of volume quite simple.

Pre-Book Lesson

Review the development for finding the area of a rectangular region as outlined in the Background section. You may wish to summarize the five steps by writing them on the board as they are reviewed. Have pupils realize that similar steps must be taken to find the volume of a rectangular solid. During the review, stress the importance of having a standard-unit region.

Using the Text Pages

• Ex. 1–4. During the development of the concept of finding the volume of a rectangular solid, you may wish to refer to the analagous steps in the development of the concept of finding the area of a rectangular region.

• Ex. 8–11. Cubical Counting Blocks* can be given to the pupils so that they may "build" rectangular solids from standard-unit regions of one cubic inch.

As mentioned before, the concept of finding the volume of a rectangular solid as presented on these pages does not involve the development of a formula. In Ex. 8, pupils are to find the volume of a rectangular solid by determining the number of standard-unit solids into which the given rectangular solid can be partitioned. In the written exercises 9–11, the rectangular solids have already been partitioned into cubic inches or cubic centimeters.

• Ex. 1–4. In the enrichment exercises at the bottom of the page, the rectangular solids have not been partitioned into standard units of volume. You may suggest that pupils trace the figures for Ex. 1–3 and then draw lines to indicate how each solid could be partitioned. Ex. 1 may be illustrated with the Cubical Counting Blocks, because their measure is one cubic inch.

For Ex. 4 of the enrichment exercises, suggest to pupils that they first determine how many cubic units would be in one layer and then determine

* See item 13, page xix.

how many such layers there would be. The product of these two numbers will be equal to the number of standard units of volume into which the rectangular solid can be partitioned. Illustrate with the "Cubical Counting Blocks."

Individualizing Instruction

• Give *more capable pupils* an exercise such as finding the volume of a rectangular solid whose dimensions are $2\frac{1}{2}$ in. $\times$ 4 in. $\times$ 4 in.

Have *more capable pupils* determine the effect on the volume of rectangular solids of doubling one of the three dimensions, of doubling two of the three dimensions, and of doubling all three dimensions.

• Have *all pupils* determine how many cubic inches are equal in volume to one cubic foot and how many cubic feet are equal in volume to one cubic yard.

• *Slower learners* may use the Cubical Counting Blocks to make rectangular regions, and then give the measure of the volume.

NOTES

* Point out that multiplying the measures of the length, width, and height to find the measure of the volume applies only to rectangular solids.

7. Give the volume of a rectangular solid whose dimensions are **a.** 1 cm. by 1 cm. by 7 cm. 7 cu. cm.

b. 1 in. by 1 in. by 8 in. 8 cu. in.

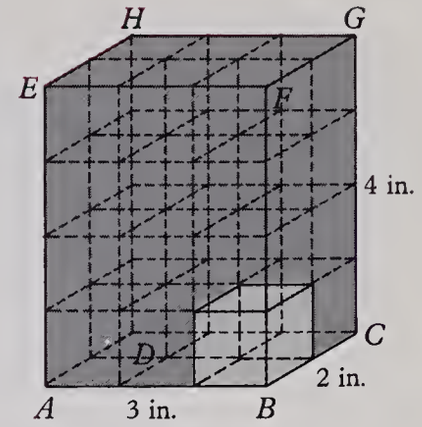


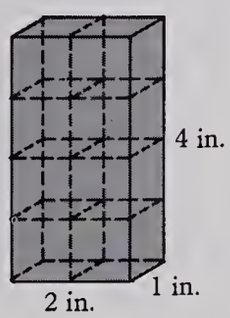
Fig. 2

8. Fig. 2. Give the dimensions of the rectangular solid shown. 3 in. by 2 in. by 4 in. To find the volume of this solid, think, *the solid can be partitioned into four layers of cubic inches with* _?6_ *cubic inches in each layer, so the volume of the solid is* $4 \times$ _?6_, *or* _?24_ *cu. in.*

[W]

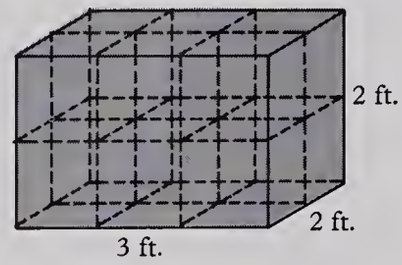
Ex. 9–11. A rectangular solid is shown partitioned into cubic units. Find the volume of the solid. Express your answer in terms of the appropriate unit of volume.

9.



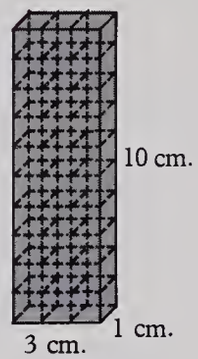
8 cu. in.

10.



12 cu. ft.

11.



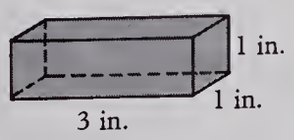
30 cu. cm.

Try These Exercises

Enrichment [W]

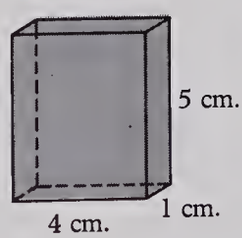
* Ex. 1–3. Think of the rectangular solid as being partitioned into cubic units and then give the volume of the solid shown.

1.



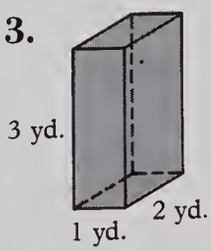
3 cu. in.

2.



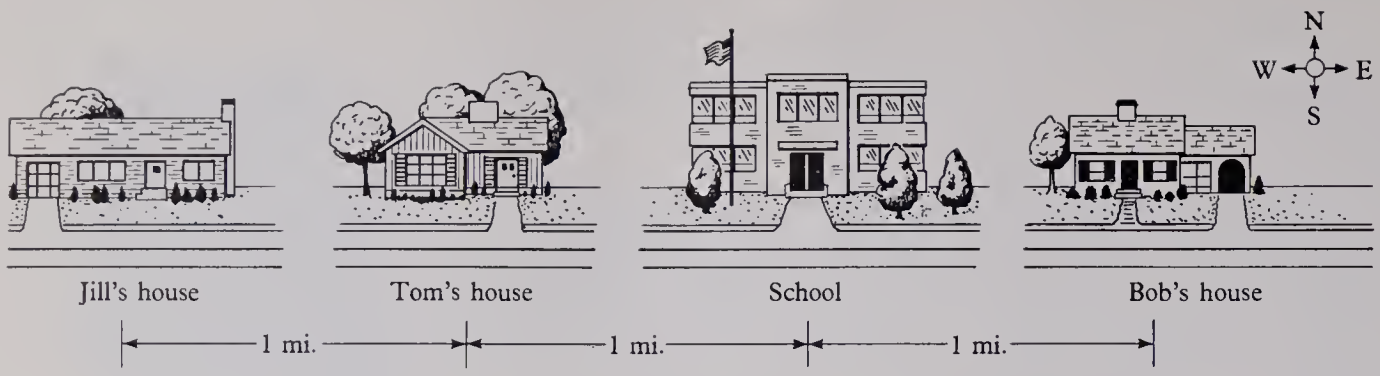
20 cu. cm.

3.



6 cu. yd.

4. Find the volume of a rectangular solid whose dimensions are **a.** 2 in. by 2 in. by 7 in. 28 cu. in. **b.** 3 cm. by 3 cm. by 5 cm. 45 cu. cm.



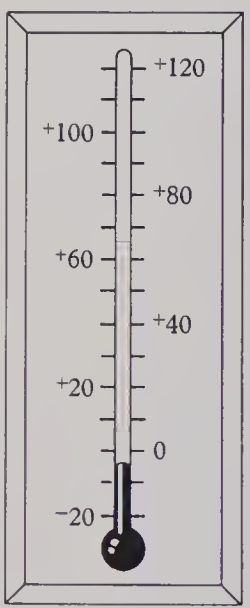
Positive and Negative Numbers

Extension [O]

1. One morning, before leaving for school, Bob noticed that the temperature was below zero. When he returned home from school, he found that the opposite was true, that is, that the temperature was above zero. Describe the opposite of

a. above sea level. below sea level **b.** located east of the school. located west of the school

2. When an amount is involved, the opposite involves the same amount. For example, the opposite of *8° above zero* is *8° below zero*. Then the opposite of *200 feet below sea level* is 200 feet above sea level.



3. We can use the words *positive* and *negative* when describing opposites. For example, if *8° above zero* is considered positive, then its opposite, *8° below zero*, is considered negative. We write $^{+}8$ (read *positive 8*) and $^{-}8$ (read *negative 8*). Explain the scale on the thermometer shown at the left. Temperatures above zero are shown as positive; temperatures below zero are shown as negative.

4. Study the diagram at the top of the page. Bob's house is located how far east of the school? 1 mile Is the location of Tom's house the opposite of the location of Bob's house with reference to the school? Yes Explain. It is 1 mile west of the school.

5. If we use 0 to represent the location of the school and $^{+}1$ to represent the location of Bob's house, then we would use $^{-}1$ to represent the location of Tom's house. Which would we use to represent the location of Jill's house, $^{-}2$ or $^{+}2$? Explain. Since east is positive, west is negative.

Teaching Pages 312-313

Pupil's Objectives

(a) To learn to use the words positive and negative to show opposites; (b) to learn the symbols for positive and negative numbers; and (c) to learn to associate positive and negative numbers with unique points in the number line.

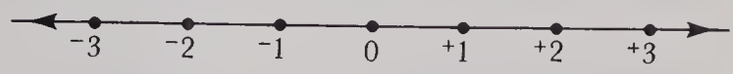
Background

A technical, yet important, reason for extending the number system to include negative numbers is to provide closure under the operation of subtraction. For the mathematical sentence $n = 9 - 13$, there is no counting number that can be substituted for n to make the statement true. To be able to perform subtraction with *any* two numbers, we need a new set of numbers.

There are situations from our everyday living experiences which illustrate the inadequacy of the set of counting numbers, such as situations requiring the use of opposites. If counting numbers are used to indicate distances to the right of a reference point, we need another kind of number to indicate the opposite idea of distances to the left of the reference point. If counting numbers are used to indicate temperature above zero, we need another kind of number to indicate temperatures below zero.

These needs for another kind of number are met by a set of numbers called *integers*. The integers consist of three sets of numbers: (1) a set of numbers corresponding to the counting numbers, each of which may be called a positive number; (2) the set of opposites for each of these numbers, each of which may be called a negative number; and (3) zero.

The positive numbers are designated by a small raised "+" sign written in front of the numeral naming the counting number. The negative numbers are designated by a small raised "−" sign written in front of the numeral naming the counting number. The numeral for the integer positive four is written $^{+}4$, and the numeral for its opposite, negative four is written $^{-}4$. Zero is its own opposite. The set of integers may be represented with set notation as $\{\cdots, {}^{-}3, {}^{-}2, {}^{-}1, 0, {}^{+}1, {}^{+}2, {}^{+}3, \cdots\}$. It may also be represented using a number-line picture.



The set of integers consists of an infinite set of positive numbers, and an infinite set of negative numbers, and zero. Zero is neither positive nor negative. When using integers, zero is used as a reference point. If $^{+}3$ refers to 3 units to the right of zero on the number-line picture, $^{-}3$ refers to 3 units to the left of zero.

The use of positive and negative numbers on thermometer scales points out the fact that zero is used simply as a reference point. It is important for pupils to understand that "0" is not the symbol for nothing, nor the symbol for the empty set. When we say the temperature is 0° F., we do not mean that there is no heat. We are simply using "0" as a reference point. On the centigrade thermometer scale, the 0° C. reference point is an indication of the freezing point of water.

The symbolism for positive and negative numbers sometimes becomes confused with the symbolism for the operations addition and subtraction. This is why it is important to make the "+" and "−" signs in front of the numerals both small and raised. While the symbols for positive and negative numbers are not symbols of operation, the ideas of addition and subtraction and positive and negative are closely related. To simplify the notation, the "+" sign is often not written when designating positive integers. The corresponding counting number is named instead ($^{+}21$ is written as 21).

The question now arises as to what is the difference between the counting number 21 and the positive integer $^{+}21$. While each is contained in a different set of numbers, and these two sets of numbers have different properties, for all practical purposes there is no difference.

Pre-Book Lesson

• Have pupils draw on the board a number-line picture representing the set of whole numbers. Point out that the line extends indefinitely in both directions. Raise the question as to the possibility

of having a set of numbers that would continue indefinitely on one side of zero as well as on the other side of zero.

• Ask for examples of scales of measurement which use numbers which are less than zero.

Using the Text Pages

• Ex. 1–5. Point out that "0" is used as an arbitrary reference point, that $^{+}1$ is used to indicate the measure of an arbitrary distance from 0, and $^{-}1$ is the measure of that same distance from 0 only in the opposite direction.

• Ex. 3. Explain that when the set of integers is used for a vertical scale, the positive numbers are shown above zero and the negative numbers are shown below zero. This is an arbitrary convention, but it is considered to be more logical to have the numbers arranged so they increase in value upward and decrease in value downward. Discuss other examples of opposites in everyday life, i.e., above and below, left and right, before and after, gain and loss.

• Ex. 6. In Fig. 2, the horizontal number-line picture shows the positive integers to the right of zero and the negative numbers to the left of zero. This is also arbitrary, but is a common convention.

• Ex. 1–3. The enrichment exercises at the bottom of page 313 are intended for *more capable pupils*. However, you may wish to use them as written exercises for *more capable pupils* and as oral exercises for the other pupils. Point out in Ex. 2 that on most thermometer scales, the positive integers are renamed with the corresponding counting numbers. Pupils can tell from the set notation shown in Ex. 1. ($\{\cdots, {}^{-}3, {}^{-}2, {}^{-}1, 0, {}^{+}1, {}^{+}2, {}^{+}3, \cdots\}$) that the set of whole numbers is contained in the set of integers, or is a subset of the set of integers.

Individualizing Instruction

• Remind *more capable pupils* that when the set of whole numbers is represented with a horizontal number-line picture, the greater of two numbers is always to the right. If this same concept applies to the set of integers, have them determine whether the following are true or false:

a. $^{-}3 < 0$ b. $0 > {}^{-}11$ c. $^{-}1 > {}^{-}6$

• *All pupils* may name situations in which whole numbers are inadequate. Stress that if the direction from the reference point must be indicated, opposite distances of the same measure should be used.

• Prepare a list such as the following for *slower learners* and have them describe the opposite of each item listed,

a. a decrease of 11 degrees.
b. a gain of $14.
c. 4 minutes early.
d. gain 2 lb.
e. 200 ft. above sea level.
f. a decrease of 10%.
g. positive 5.
h. negative 1.
i. moving to the right 2 ft.
j. shrinking 2%.

6. In a number line, each point corresponds to a number. For Fig. 1, is point A located at the same distance to the left of 0 as the point corresponding to 1 is to the right of 0? Yes The number corresponding to point A is the opposite of the number 1. Is the point corresponding to point B the opposite of the number 2? Yes

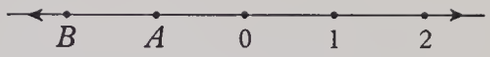


Fig. 1

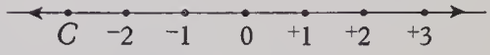


Fig. 2

7. Fig. 2. The numbers 1, 2, and 3 have been renamed $+1$, $+2$, and $+3$. You know that -1 is the opposite of $+1$. Is -2 the opposite of $+2$? Yes How is this shown in the number-line picture? The corresponding points are located the same distance from 0 in opposite directions.

8. Is point C located at the same distance from 0 as $+3$ but in the opposite direction? Yes Then point C corresponds to -3? The opposite of $+3$ is -3.

9. On the board, draw a number-line picture and show each of the following and their opposites.

a. $+4$ **b.** $+7$ **c.** -5 **d.** -10 **e.** $+15$

[W]

10. On your paper, draw a number-line picture and show $+10$, $+20$, $+30$, and $+40$ and their opposites.

Integers

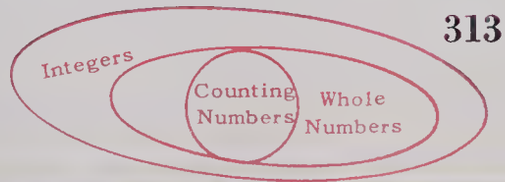
Enrichment [W]

The union of $\{-1, -2, -3, \cdots\}$, $\{0\}$, and $\{+1, +2, +3, \cdots\}$ is a set of numbers called the set of *integers*. To describe this set using braces, we write $\{\cdots, -3, -2, -1, 0, +1, +2, +3, \cdots\}$.

1. Why are three dots shown to the left of -3? The set is infinite.

2. Since the counting numbers 1, 2, 3, and so on may be renamed $+1$, $+2$, $+3$, and so on, the counting numbers are integers. They can be renamed $0, +1, +2, +3, \cdots$ Explain why the set of whole numbers is a set of integers.

3. Draw a Venn diagram to show that the set of counting numbers is a subset of the set of whole numbers and that the set of whole numbers is a subset of the set of integers.

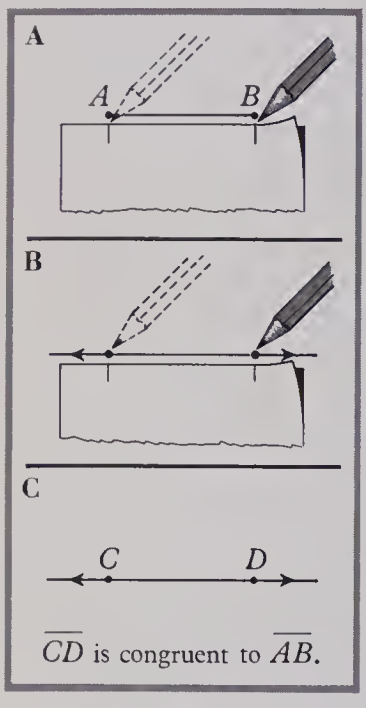


* Notice that the steps and developmental constructions are implied by dashing the shape of the pencil in its previous position and by showing the marks for that particular step in blue.

R S T U

* Constructing Congruent Segments

Enrichment [O]



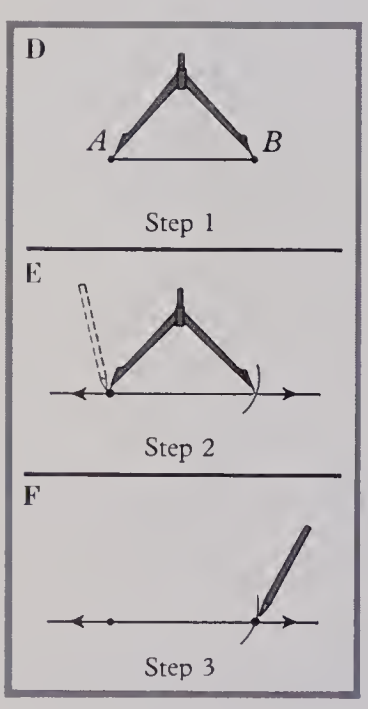
1. Does $\overline{RS}$ appear to be congruent to $\overline{TU}$? Yes What is meant by the statement, "Segments RS and TU are congruent"? They have the same length.

2. Boxes A–C show one way to construct a segment CD that is congruent to a segment AB.

Place a straightedge (for example, an edge of a rectangular sheet of paper) along a picture of $\overline{AB}$ and mark the end points as shown in box A. Then, placing the straightedge along a picture of a line (box B), use the marks to locate two points in the line. Label these points C and D (box C).

a. Does $\overline{CD}$ appear to be the same length as $\overline{AB}$? Yes We say that we have *constructed* a segment that is congruent to $\overline{AB}$.

b. Use the above procedure to show a segment on your paper which is congruent to segment RS.



3. Explain the procedure shown in boxes D–F for using a compass to construct a segment that is congruent to $\overline{AB}$. (Note: When we strike off part of a circle as in step 2, we say that we strike off an *arc*.) Place the compass ends on the end points of $\overline{AB}$. On a picture of a line, make a mark for one end of the compass and strike off an arc with the other to locate 2 points in the line. [W]

4. On your paper, show a segment MN about two inches long. Then, using your compass and the procedure shown in boxes D–F, construct a segment that is congruent to $\overline{MN}$. M N

5. On your paper, show a segment XY. Then using the procedure shown in boxes D–F, construct a segment congruent to $\overline{XY}$. X Y

Teaching Page 314

Pupil's Objective

To construct congruent segments by using a straightedge and by using a compass.

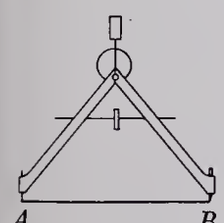
Background

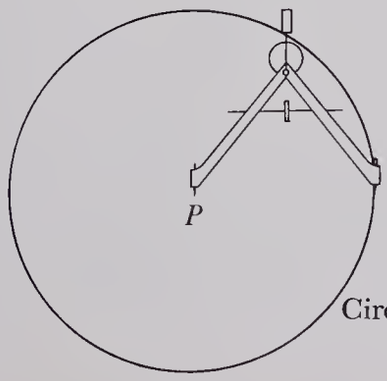
It is important to differentiate between "congruent" and "equal." In mathematics, "equal" means "identical." If we say point A equals point B, we mean that A and B are different symbols for the same point. The statement $\overline{AB} = \overline{CD}$ states that $\overline{AB}$ and $\overline{CD}$ are different symbols for the same line segment. For example, the statement $\overline{AB} = \overline{BA}$ is always true.

When we say that $\overline{AB}$ and $\overline{CD}$ are congruent, we mean that they are different segments but have the common property of having the same length. This may be symbolized as $\overline{AB} \cong \overline{CD}$.

One method of constructing a segment congruent to a given segment is by use of a straightedge as shown on the text page. A model of the given segment is marked off on the straightedge, and this model is used to construct the segment congruent to the given segment.

A second method of constructing a segment congruent to a given segment involves the use of a compass (sometimes called a pair of compasses). This method makes use of the concept of a circle being the set of all points in the plane which are a given distance from a given point called its center. If we set the points of the compass on the end points of the given segment, the distance between the compass points will be equal to the length of the given segment. We can construct a circle which is the set of all points in the plane whose distance from a selected point will be equal to the length of the given segment.





Circle P

If we are only interested in constructing one of these segments, a small arc may be drawn instead of the entire circle.

Pre-Book Lesson

- If the class situation permits, *all pupils* may do this enrichment page.
- Furnish to pupils, or have them bring to class, compasses which can be used for constructing models of circles. Have them practice using the compasses by constructing circles with various radii so they will be familiar with the instrument when doing the text lesson. Remind pupils that all radii of a circle have the same length.

Using the Text Page

- Ex. 1. Emphasize the difference between equal and congruent. Be sure they understand that two congruent segments have the same property of length but not the same property of location.
- Ex. 2. Point out to pupils that if they measured the length of $\overline{AB}$ with a ruler, and measured another segment of the same length, they would be measuring another segment of approximately the same length rather than constructing a congruent segment.
- Ex. 3. Pupils should be familiar with the compass from the Pre-Book Lesson. Explain the use of the term "arc" with construction. Point out that if the arc was extended and a whole circle was shown, a line shown from the center for the circle to any point in the circle would be congruent to $\overline{AB}$. Discuss the material from the Background.

Individualizing Instruction

Have *more capable pupils* explain why the compass, which is an instrument used for constructing circles, is used for constructing congruent segments. Explain to them the rationale of this method as presented in the Background.

Pupil's Objective

To learn to construct congruent angles and to construct congruent triangles.

Background

These topics are offered as enrichment topics for *more capable pupils*. Other pupils will be assigned more suitable work. Pupils will probably not fully understand why the procedure on the text page results in a figure congruent to the given figure, but a mathematical explanation is inappropriate at this time.

Pupils, being unaware of the nature of pure geometric constructions, may argue that you can simply use tracing paper or carbon paper to construct congruent angles and congruent triangles. Explain that the procedure presented on the pupil's page is another way to make models representing geometric figures.

An explanation of these constructions is given below for your own information.

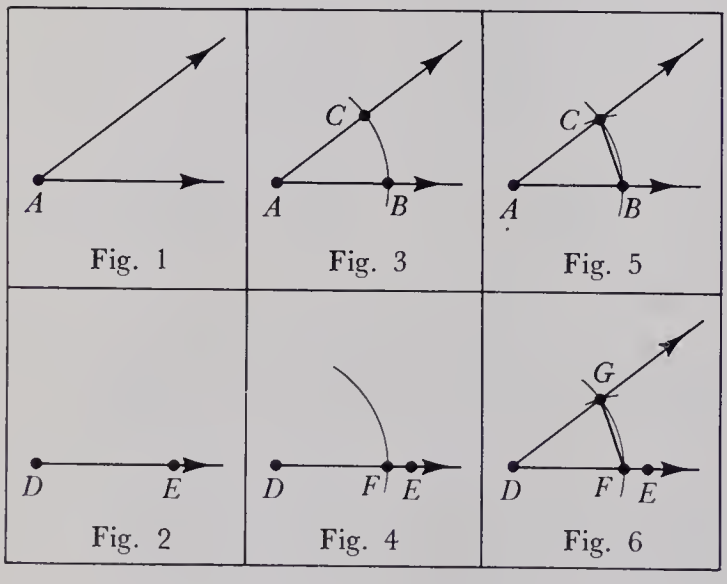


Fig. 1. Given $\angle A$.

Fig. 2. Construct an angle congruent to $\angle A$, with $\overrightarrow{DE}$ as one ray.

Fig. 3. Draw an arc with point A as its center intersecting $\angle A$ at point B and point C so $\overline{AB} \cong \overline{AC}$.

Fig. 4. Draw an arc with point D as its center intersecting $\overrightarrow{DE}$ at point F so $\overline{DF} \cong \overline{AB} \cong \overline{AC}$.

Fig. 5. Draw $\overline{CB}$ to form $\triangle BAC$.

Fig. 6. Construct $\overline{FG}$ so $\overline{FG} \cong \overline{BC}$ and so $\overline{DG} \cong \overline{DF} \cong \overline{AB} \cong \overline{AC}$.

Two triangles are congruent if three sides of one triangle are congruent respectively to the three sides of the other triangle. Because corresponding parts of congruent triangles are congruent, the corresponding angles are congruent.

Pre-Book Lesson

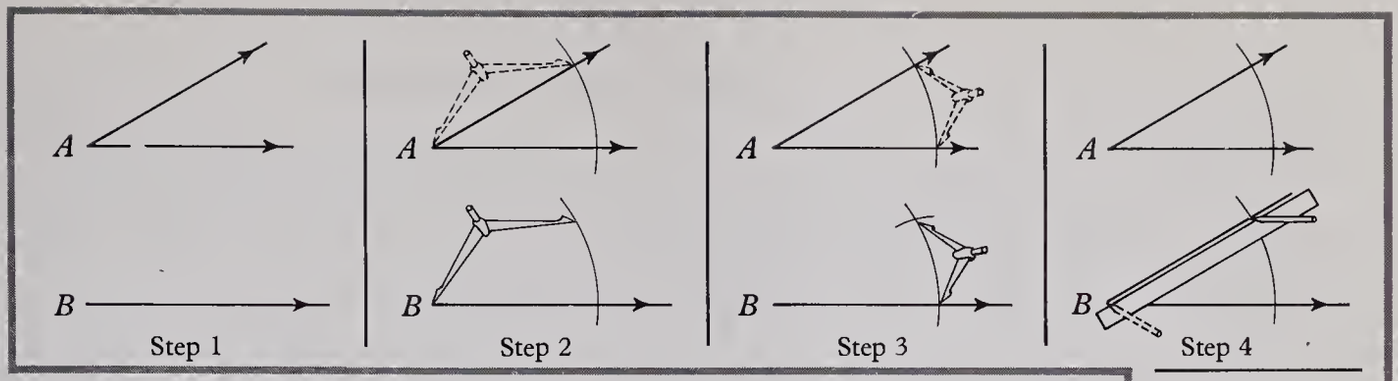
While *more capable pupils* are reading the text page, instruct *slower learners* in some paper-folding activities given under the Individualizing Instruction section. After you have completed the oral work with *more capable pupils* and they are working on their own, help any *slower learners* who cannot cut out the proper shapes by themselves.

Using the Text Page

- Ex. 1. Pupils are expected to understand only the method, not the rationale behind this work. Point out that in step 2, the arc can be marked off any distance from point A as long as the same distance is marked off from the end point of the model of the ray. In steps 3 and 4, the constructed ray will pass through the intersection of both arcs.
- Ex. 2. Review the meaning of "acute angle." Point out to pupils that they may begin with a model of any acute angle for this exercise.
- Ex. 4. Review the meaning of "obtuse angle."
- Ex. 1–2 (bottom). Point out to pupils that constructing a congruent triangle is really constructing one congruent angle and two congruent sides.

Individualizing Instruction

Have *slower learners* cut out of paper models representing various angles. Actually, the union of the angle and its interior will be represented. However, we are only interested in the boundary—the angle. If no scissors are available, they may tear the paper using the edge of a ruler. Then, have them compare these models with a corner of a sheet of paper which represents a right angle. Have them label models that fit exactly with right angles, and label the others either as obtuse or acute angles.



Constructing Congruent Angles

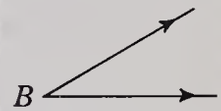
Enrichment [O]

1. Explain the procedure shown above for constructing an angle that is congruent to an angle A. Answers will vary.

2. On your paper, picture an acute angle. Then, using your compass, construct an angle that is congruent to that acute angle. Answers will vary. [W]

3. Use your protractor to picture an angle G which measures about 45 in degrees. Then construct an angle congruent to $\angle G$. Answers will vary.

4. On your paper, picture an obtuse angle and then construct an angle congruent to that obtuse angle. Answers will vary.



$\angle B$ is congruent to $\angle A$.

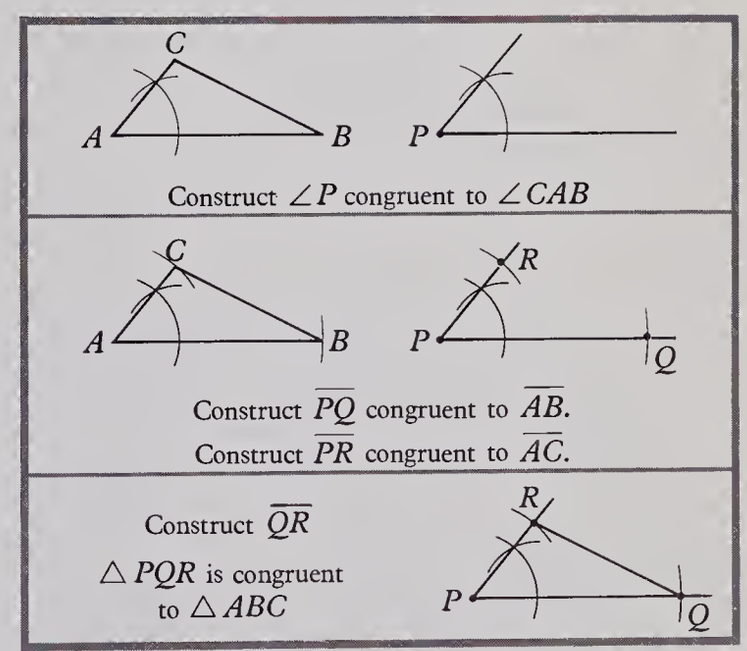
Constructing Congruent Triangles

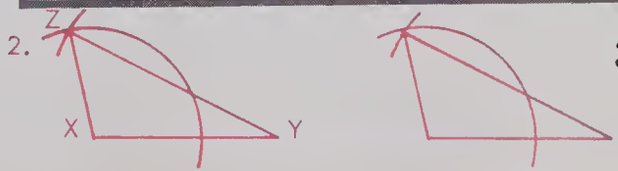
Enrichment [W]

Study the procedure for constructing a triangle congruent to $\triangle ABC$.

1. On your paper, picture a triangle DEF such that each angle is an acute angle. Then, construct a triangle congruent to $\triangle DEF$ by following the procedure at the right.

2. On your paper, picture a triangle XYZ such that one angle is an obtuse angle and then construct a triangle congruent to $\triangle XYZ$.





Constructing Bisectors

Enrichment [O]

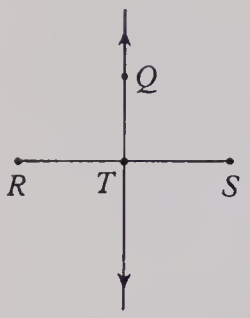


Fig. 1

1. Fig. 1. Point T is the *midpoint* of segment RS. That is, $\overline{RS}$ is partitioned by point T into two congruent adjacent segments. The two congruent adjacent segments are $\_?\overline{RT}$ and $\_?\overline{TS}$.

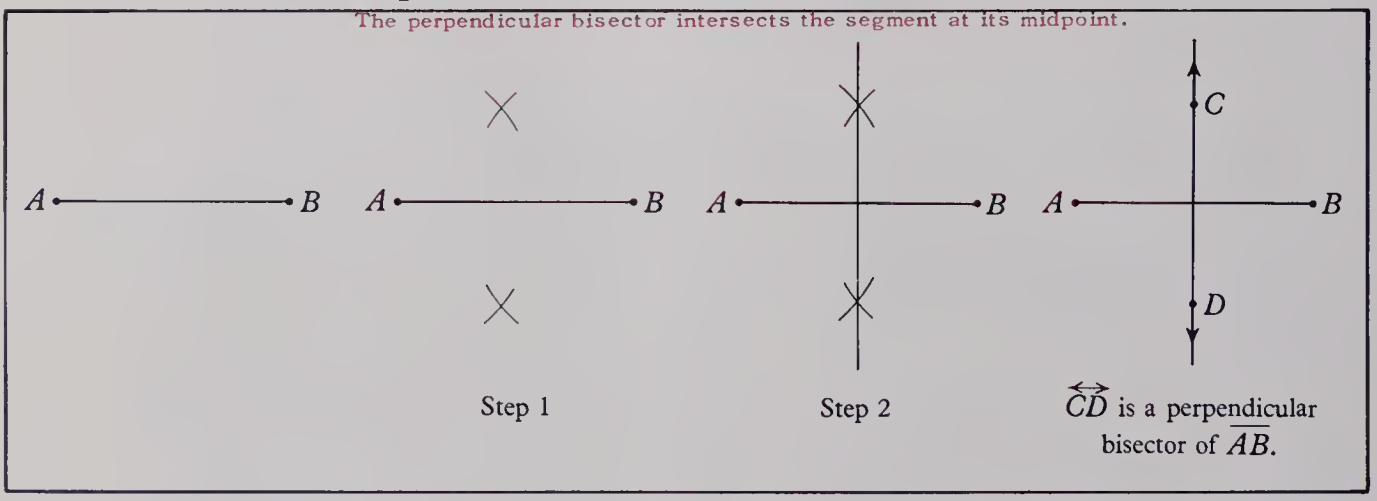
2. Also, line QT is perpendicular to $\overline{RS}$. Tell when a line is perpendicular to a segment. When it is perpendicular to the line containing the segment.

3. $\overleftrightarrow{QT}$ is called a *perpendicular bisector* of $\overline{RS}$. The box below shows how to use a compass in constructing a perpendicular bisector of a segment AB.

Step 1: Using a radius longer than one half of $\overline{AB}$, first, strike off two arcs using point A and then, using point B, strike off two more arcs which intersect the first two arcs. Show the points of intersection of the arcs.

Step 2: Show the line which contains the points of intersection of the arcs. This line is a perpendicular bisector of $\overline{AB}$.

Explain how this construction also locates the midpoint of $\overline{AB}$.
The perpendicular bisector intersects the segment at its midpoint.



4. On your paper, picture a segment about 2 inches long and then construct a line that is a perpendicular bisector of that segment. Show and label the midpoint of that segment. See below.

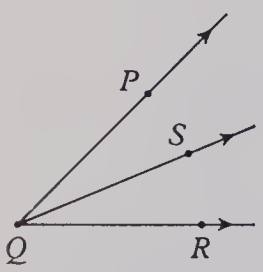
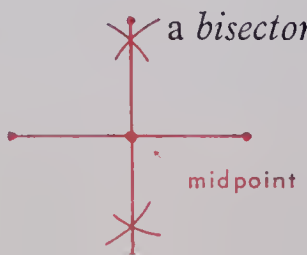


Fig. 2

5. Fig. 2. Angle PQS is congruent to angle SQR. Then what can you say about their measures in degrees? They are the same. Explain why $\angle PQS$ and $\angle SQR$ are adjacent angles. Ray QS is called a *bisector* of angle PQR.
They are two distinct angles with a common side and vertex.


midpoint

Teaching Pages 316 and 317

Pupil's Objective

To learn to construct the perpendicular bisector of a segment, and the "bisector of an angle."

Background

The method of determining the midpoint of a segment by construction may be arrived at by using a compass to construct several points each equidistant from the end points of the segment. This is performed by using a radius longer than one half of $\overline{AB}$, and striking off two arcs first from point A, and then from point B. Do this several times.

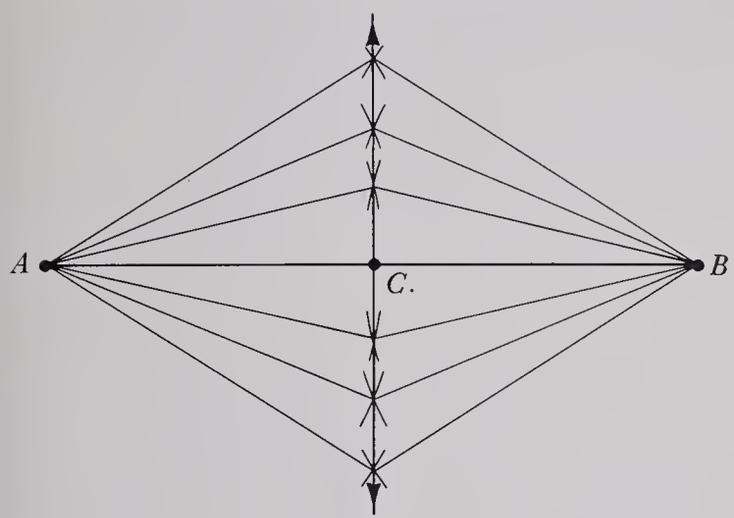


Fig. 1

Such a construction demonstrates the obvious fact that the set of all such points in the intersection of the arcs form a line. Since each point in the line is equidistant from the end points of the segment, we can conclude $\overline{AC} \cong \overline{CB}$. Therefore, point C is the midpoint of $\overline{AB}$ and the segment has been bisected.

The construction can be simplified by not picturing the segments from the end points of the given segment, and by determining only two points equidistant from the end points.

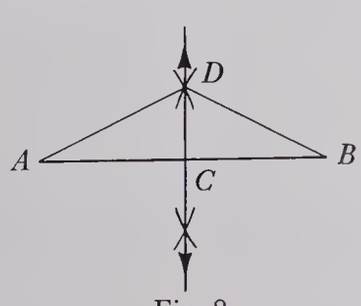
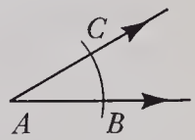


Fig. 2

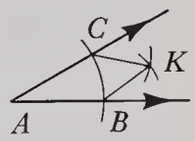
If $\overline{AD}$ and $\overline{BD}$ are drawn, two triangles are formed. It can be proven by formal geometric proof that $\triangle CAD$ is congruent to $\triangle CBD$. Since corresponding angles of congruent triangles are congruent, $\angle ACD \cong \angle BCD$. Since they are equal in measure and their sum is 180°, each angle must have a measure of 90 in degrees. $\overleftrightarrow{DC}$ is perpendicular to the given segment and it also bisects the given segment. Therefore, we call $\overleftrightarrow{DC}$ the perpendicular bisector of the given segment.

Constructing a "bisector of an angle" is based on the idea that a bisector partitions the angle and its interior to form two angles equal in measure, if it is a ray which is in the interior for the angle. To do this, two congruent triangles are constructed.

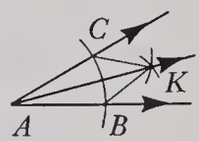
(1) Given $\angle A$, $\overline{AB}$ is marked off on one ray and $\overline{AC}$ marked off on the other ray so that $\overline{AB} \cong \overline{AC}$.



(2) Locate a point which is called point K in the interior for the angle such that $\overline{BK} \cong \overline{CK}$ by marking off two arcs that intersect using point B and then, using point C.



By showing $\overleftrightarrow{AK}$, two triangles are formed. Since $\overline{AB} \cong \overline{AC}$, and $\overline{CK} \cong \overline{BK}$ (and since $\overline{AK}$ is contained in both triangles), the three sides of $\triangle ABK$ are congruent respectively to the three sides of $\triangle ACK$ and the two triangles are congruent to each other. $\angle BAK$ is congruent to $\angle CAK$ because corresponding angles of congruent triangles are congruent. Since the measures of congruent angles are equal, we have "bisected the given angle."



Pre-Book Lesson

- If *all pupils* worked page 315, and the class situation permits, you may wish to have them work these pupil's pages. However, exercises are provided for *slower learners* under Individualizing Instruction.
- Review (for *all pupils*) the meaning of the terms midpoint, perpendicular bisector, and bisector. Give illustrations from the pupil's page. Be sure that pupils are familiar with the meaning of congruent adjacent angles.

Using the Text Pages

• Ex. 1–10. Explain each step in the constructions at the board using a demonstration compass. Have pupils perform each step with their compasses at their desks.

• Ex. 3. After they have constructed the perpendicular bisector of a segment, help pupils verify that the given segment has been partitioned into two congruent smaller segments, and that each angle formed by the partitioned segment and the bisector is a right angle. Use the material from the Background.

• Ex. 7. When constructing an "angle bisector," the fact that the angle and its interior has been partitioned to form two congruent angles can be verified by folding the paper along the bisector, holding the paper up to a light, and seeing that one angle is shown superimposed on the other. When they fit exactly it indicates that they are congruent.

Individualizing Instruction

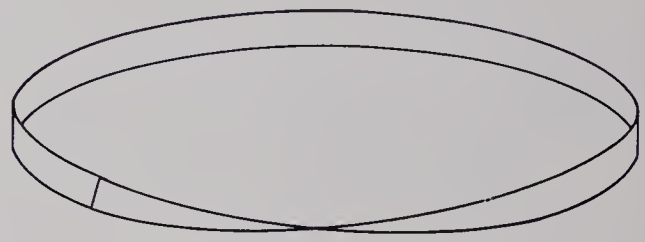
• There are many paper-folding activities which make it possible to introduce mathematical concepts of geometry in a way that *all pupils* can understand, including the *slower learner*. Below are sample paper-folding activities.

(1) The bisector which partitions a given angle and its interior to form two angles: Draw an angle on a piece of paper and fold the paper so that the two rays coincide and the crease passes through the vertex of the angle. The angle and its interior has been partitioned to form two congruent angles and has, therefore, been "bisected."

(2) The perpendicular bisector of a given line segment: Fold the paper so that the end points are superimposed on each other. The crease will represent the perpendicular bisector of the given segment. Have pupils decide whether or not the angles formed are right angles. Have them select any point on the crease and determine whether it is equidistant from both end points.

• The study of the Moebius strip can be fascinating for *all pupils*. You may wish to work this exercise with pupils who are not doing the pupil's pages. First, have them cut strips of paper about $1\frac{1}{2}$ inches wide and 12 inches long. Glue the ends of one strip together to form an ordinary round band. Have them color as much of the strip as they can without lifting the crayon from the paper or crossing an edge, leading them to the realization that both sides cannot be colored unless the crayon is lifted from the paper or an edge is crossed. Have them stick the point of a scissors into one half of the width from the edge of the paper and cut all the way around to form two ordinary round bands.

Now have them take another strip and give one end a half turn (180°) before gluing it to the other end. The result is a Moebius strip.



Have pupils now color as much of the strip as they can without crossing an edge or lifting the crayon from the paper. Pupils will discover that they can color the entire strip, proving it has only one surface.

Stick the point of the scissors in the paper and cut all the way around lengthwise one half of the width from the edge. This time the result will not be two round bands, but one long band with a twist in it. By cutting the band in half, they have made it twice as long! For a different result, have them make another Moebius strip and cut it one third of the width of the strip from an edge. The results will be very interesting.

A ray is a *bisector* of an angle if the ray forms, with the sides of the angle, two congruent adjacent angles.

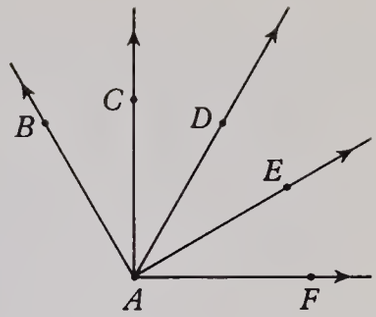


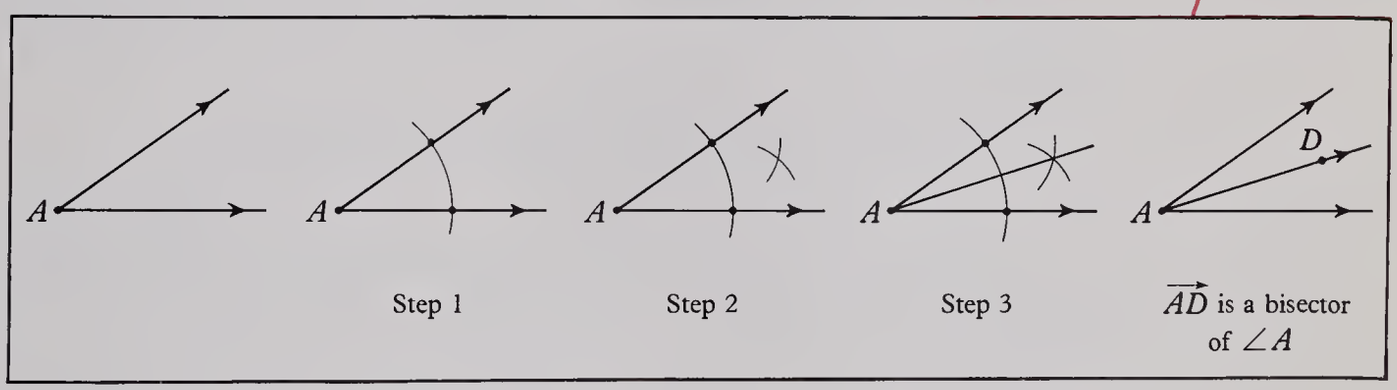
Fig. 3

6. Fig. 3. Angle BAF is shown partitioned into 4 congruent angles. Which ray shown is a bisector of $\angle BAF$? $\overrightarrow{AD}$ of $\angle BAD$? $\overrightarrow{AC}$ of $\angle DAF$? $\overrightarrow{AE}$

7. The box below shows how to use a compass in constructing a bisector of angle A.

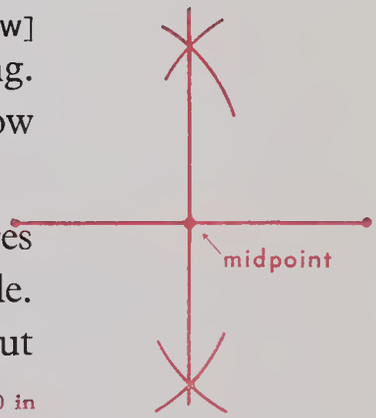
Using any radius, strike off an arc as in step 1 and show the points of intersection of the arc and the sides of the angle. Then using these two points, strike off two arcs which intersect as shown in step 2. Show the point of intersection. Then draw a ray as in step 3.

On your paper, show an acute angle and then, using your compass, construct a bisector of that angle.



[W]

8. On your paper, picture a segment about $1\frac{1}{2}$ inches long. Then construct a perpendicular bisector of that segment. Show the midpoint of the segment.

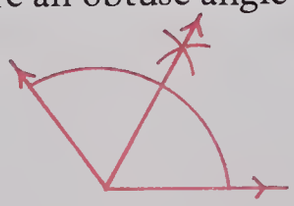


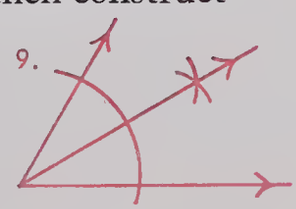
9. Using your protractor, picture an angle which measures about 60 in degrees. Then construct a bisector of that angle. Use your protractor to check your construction. (Hint: About what should the measure of each small angle be in degrees?)

See below.

30 in degrees

10. On your paper, picture an obtuse angle and then construct a bisector of that angle.





● **Extra Activity.** Set 181.

Do You Understand?

Test of Information and Meaning 7

1. For a pictograph, each represents 25 trees. Then 7 's would represent how many trees? 175

2. To show the daily change of your weight over a 20-day period, would it be better to make a bar graph or a line graph?

3. To compare the heights of 4 girls, would it be better to make a bar graph or a line graph?

4. If each unit on the scale of a horizontal-bar graph represents 10 (tickets), then a bar that is $5\frac{1}{2}$ units long would represent how many tickets? 55

5. Which is more important in a bar graph, the length of a bar or the width of the bar?

6. Copy and complete: To find the area of a parallelogram region, we multiply the measure of the length of the base by the measure of the _?_ of the region. height

7. Name something that would be a good model of

a. a rectangular prism. **b.** a rectangular solid. Answers will vary.

8. What is the lateral area of a triangular right prism if the area of each lateral face is $4\frac{5}{6}$ sq. ft.? $14\frac{1}{2}$ sq. ft.

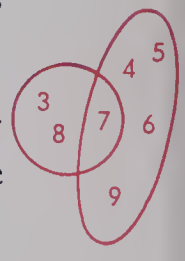
9. If *5 pounds overweight* is represented by $+5$, what symbol would represent *5 pounds underweight*? -5

10. A quadrangular prism has how many

a. lateral faces? 4 **b.** faces? 6

11. Find the area of a triangular region whose base measures 5 in inches and whose height is 7 in. $17\frac{1}{2}$ sq. in.

12. Draw a Venn diagram to show {3, 7, 8} and {4, 5, 6, 7, 9}.



13. If one edge of a cube measures 5 in centimeters, find the lateral area of the cube. 100 sq. cm.

Ex. 14–17. Round to the nearest ten thousand.

14. 18,627 20,000 15. 25,032 30,000 16. 142,618 140,000 17. 775,500 780,000

Teaching Pages 318, 319, 320, and 321

Pupil's Objectives

To take the regular end-of-chapter tests including (a) Test of Information and Meaning 7; (b) Diagnostic Test 7; (c) Problem Test 7; and (d) Computation Test 7.

Background

These tests provide a measurement of the pupil's mastery of the skills and understandings presented in Chapter 7. The results serve as a means of diagnosing and remedying weaknesses through Extra Examples Sets and individual help.

Using the Text Pages

• Before assigning Diagnostic Test 7, be sure to explain how you want the graph made, including the general size and whether or not it is to be made on regular graph paper.

• Before assigning Problem Test 7, remind pupils of the regular problem-solving procedures including: (1) Determine the mathematical relationship in the problem. (2) Express the relationship in a mathematical sentence. (3) Determine what operation is required to solve the mathematical sentence. (4) Estimate the answer. (5) Solve the mathematical sentence and compare the answer to the estimate.

• After the tests are completed, grade the papers and hand them back. Go over each question, asking pupils to explain the why of their response.

Individualizing Instruction

• Give *slower learners* Extra Examples Sets to correct weaknesses revealed by these tests. Have them restudy appropriate text pages.

• *More capable pupils* may proceed with the Supplementary Activities at the end of the chapter, or do Extra Activity Sets.

Table of Per Cents for Chapter 7 Scores

Problem Test		Computation Test			
Score	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*
1	10	1	7	11	78
2	20	2	14	12	86
3	30	3	21	13	93
4	40	4	29	14	100
5	50	5	36		
6	60	6	43		
7	70	7	50		
8	80	8	57		
9	90	9	64		
10	100	10	71		

Supplementary Activities

• Have *all pupils* indicate the table or graph that is best suited for the purpose being described. Have *slower learners* review the beginning of Chapter 7.

1. To show direction of change of a variable over a period of time. (a) circle graph; (b) bar graph; (c) line graph; (d) pictograph. ans. (c)

2. To compare the heights of seven mountain peaks. (a) bar graph; (b) double-bar graph; (c) pictograph; (d) circle graph. ans. (a)

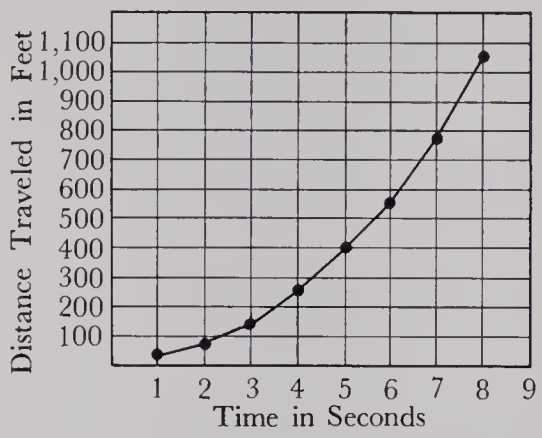
3. To show exact data involving great numbers. (a) pictograph; (b) circle graph; (c) data table; (d) line graph. ans. (c)

4. To show relationships between parts of a whole and the whole, expressed with per cent forms. (a) double-bar graph; (b) line graph; (c) pictograph; (d) circle graph. ans. (d)

5. To compare pupil enrollment in each of the seven elementary schools in a city in 1962 with the enrollment of each of the schools in 1966. (a) double-bar graph; (b) circle graph; (c) divided-bar graph; (d) data table. ans. (a)

• The graph below (which should be considerably enlarged when reproduced) indicates the vertical distance traveled by an object falling to Earth under the force of gravity, neglecting air friction. To eliminate any possibility of confusion, a table is shown at the left with detailed information from the graph.

Sec.	Feet
0	0
1	16
2	64
3	144
4	256
5	400
6	576
7	784
8	1,054



Present this to *more capable pupils* and ask:

1. What is the approximate distance the object has traveled by the end of the sixth second? (580 ft.)

2. What is the approximate distance the object has traveled by the end of the eighth second? (1,050 ft.)

3. About how far did the object travel during the third second? (80 ft.) during the seventh second? (208 ft.)

4. About how long did it take the object to travel 600 feet? ($6\frac{1}{4}$ seconds)

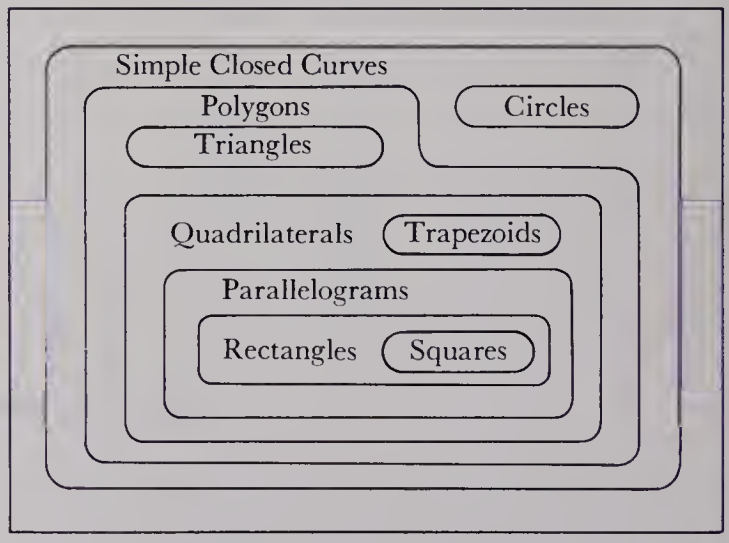
5. About how far had the object traveled by the end of 5.5 seconds? (510 ft.)

• Have *all pupils* prepare graphs involving positive and negative numbers, using "0" as a reference point. For example, use a bar graph to show the number of yards gained or lost in the first ten plays by one team in a football game, or use a line graph to show profit and loss by a business firm over a certain period of years.

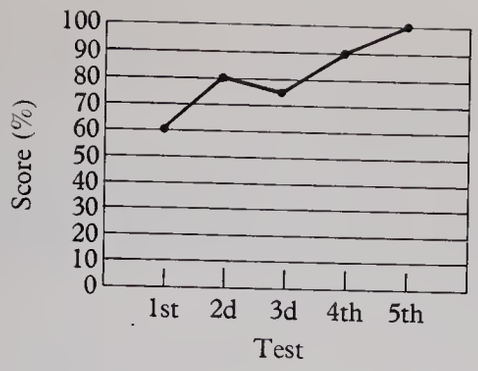
• *More capable pupils* can explore adding with integers. Since the addition $n = 6 + 9$ is performed by beginning at 6 and "counting 9 units" in the direction of greater numbers, the addition $n = {}^{+}6 + ({}^{-}9)$ is performed by beginning at $^{+}6$ and "counting 9 units" in the direction of lesser numbers. Therefore, $n = {}^{-}3$. Have pupils apply this concept in determining what integers n must represent to make true sentences.

	ans.
1. $n = {}^{+}8 + ({}^{-}8)$	0
2. $n = {}^{+}8 + ({}^{-}6)$	$^{+}2$
3. $n = {}^{+}8 + ({}^{-}10)$	$^{-}2$
4. $n = {}^{-}8 + ({}^{-}8)$	$^{-}16$
5. $n = {}^{-}16 + ({}^{+}16)$	0
6. $n = {}^{-}16 + ({}^{-}16)$	$^{-}32$
7. $n = {}^{+}16 + ({}^{-}10)$	$^{+}6$
8. $n = {}^{-}144 + ({}^{-}266)$	$^{-}410$

• *All pupils* can make a Venn diagram to represent the relationship among plane figures—polygons, circles, quadrilaterals, squares, rectangles, parallelograms, and triangles. Show the classifications of plane figures on a Venn diagram similar to the one below. Perhaps you could give pupils the definition of trapezoid (a quadrilateral with exactly one pair of opposite sides parallel) and the definition of rhombus (a parallelogram with the measures of all sides equal) and have them indicate these on the Venn diagram as shown below.



• *More capable pupils* can make a Venn diagram to describe the relationship among solids, polyhedral solids, prismoid solids, rectangular solids,



Meg's Score on Five Mathematics Tests

Fig. 1

Lake	Area (to nearest 1,000 sq. mi.)
Erie	10,000 sq. mi.
Huron	23,000 sq. mi.
Michigan	22,000 sq. mi.
Ontario	8,000 sq. mi.
Superior	32,000 sq. mi.

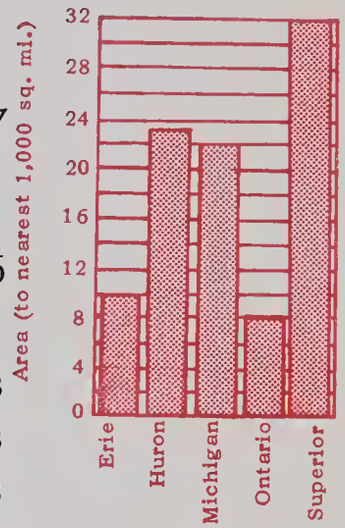
Do You Make Mistakes?

Diagnostic Test 7

1. Fig. 1. On which one of the five tests did Meg have the
   a. greatest score? 5th test **b.** least score? 1st test

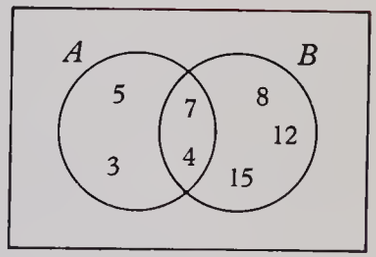
2. Meg's score on the 3d test was 75%. How many of the 16 examples on that test did she work correctly? 12

3. The table above shows the area of each of the Great Lakes of North America. Make a vertical-bar graph to show this information. Label the vertical scale so that each unit represents 4,000 (square miles). Show the length of each bar to the nearest $\frac{1}{4}$ unit.



Area of Great Lakes of North America

4. Copy and complete the following:
   a. $A \cup B =$? {3,4,5,7,8, 12,15} **b.** $A \cap B =$? {4,7}



5. List within braces the members of $\{0, \frac{3}{4}, 2.4, 4, 5\frac{1}{2}\}$ which are **a.** counting numbers. {4} $\{0, \frac{3}{4}, 2.4, 4, 5\frac{1}{2}\}$
   b. whole numbers. {0,4} **c.** rational numbers. ^

6. How many lateral faces does a right prism have if each base has 12 edges? 12 lateral faces

7. Find the height of a parallelogram region whose base measures 7 in inches and whose area is 28 sq. in. 4 in.

For errors in	Ex. 1–3	Ex. 4–5	Ex. 6	Ex. 7
Study pages	278–280, 282–285	289–291	298–299	292–293

Can You Solve Problems?

Problem Test 7

Let n represent the number which will answer the question in the problem. Write a mathematical sentence for the problem and then solve to find the answer for the problem.

1. The total of Mr. Craig's expenses for a business trip was \$225. He wishes to make a circle graph to show how he used the \$225. If he used \$67.50 for "Travel," what per cent of the circular region should be labeled "Travel"? $n = 67.5 \div 225$; 30%

2. For a vertical-bar graph, each unit on the vertical scale represents 50 (people). How many people are represented by a bar that is $7\frac{2}{5}$ units long? $n = 7\frac{2}{5} \times 50$; 370

3. In a horizontal-bar graph, a certain bar is $3\frac{1}{2}$ units long. What does each unit on the horizontal scale represent if that bar represents 14 people? $n = 14 \div 3\frac{1}{2}$; 4 people

4. The length of the base of a triangular region is 27 in. and the height of the region is 2 ft. What is the area of the region? (Hint: Rename one of the measurements.) $n = \frac{1}{2} \times (27 \times 24)$; 324 sq. in. or $n = \frac{1}{2} \times (2\frac{1}{4} \times 2)$; $2\frac{1}{4}$ sq. ft.

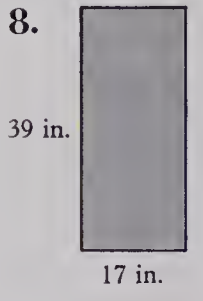
5. What is the lateral area of a right prism if the perimeter of a base is 21.4 cm. and the length of the prism is 14 cm.? $n = 21.4 \times 14$; 299.6 sq. cm.

6. What is the total area of a cube if each face has an area of $15\frac{2}{3}$ sq. in.? $n = 6 \times 15\frac{2}{3}$; 94 sq. in.

7. The volume of a rectangular solid 2 ft. by 2 ft. by 19 ft. is how many cubic feet? $n = 2 \times 2 \times 19$; 76 cu. ft.

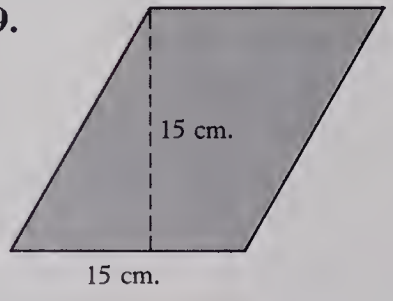
Ex. 8–10. What is the area of the parallelogram region shown?

8.



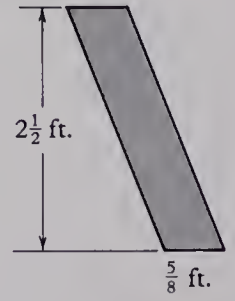
$n = 39 \times 17$; 663 sq. in.

9.



$n = 15 \times 15$; 225 sq. cm.

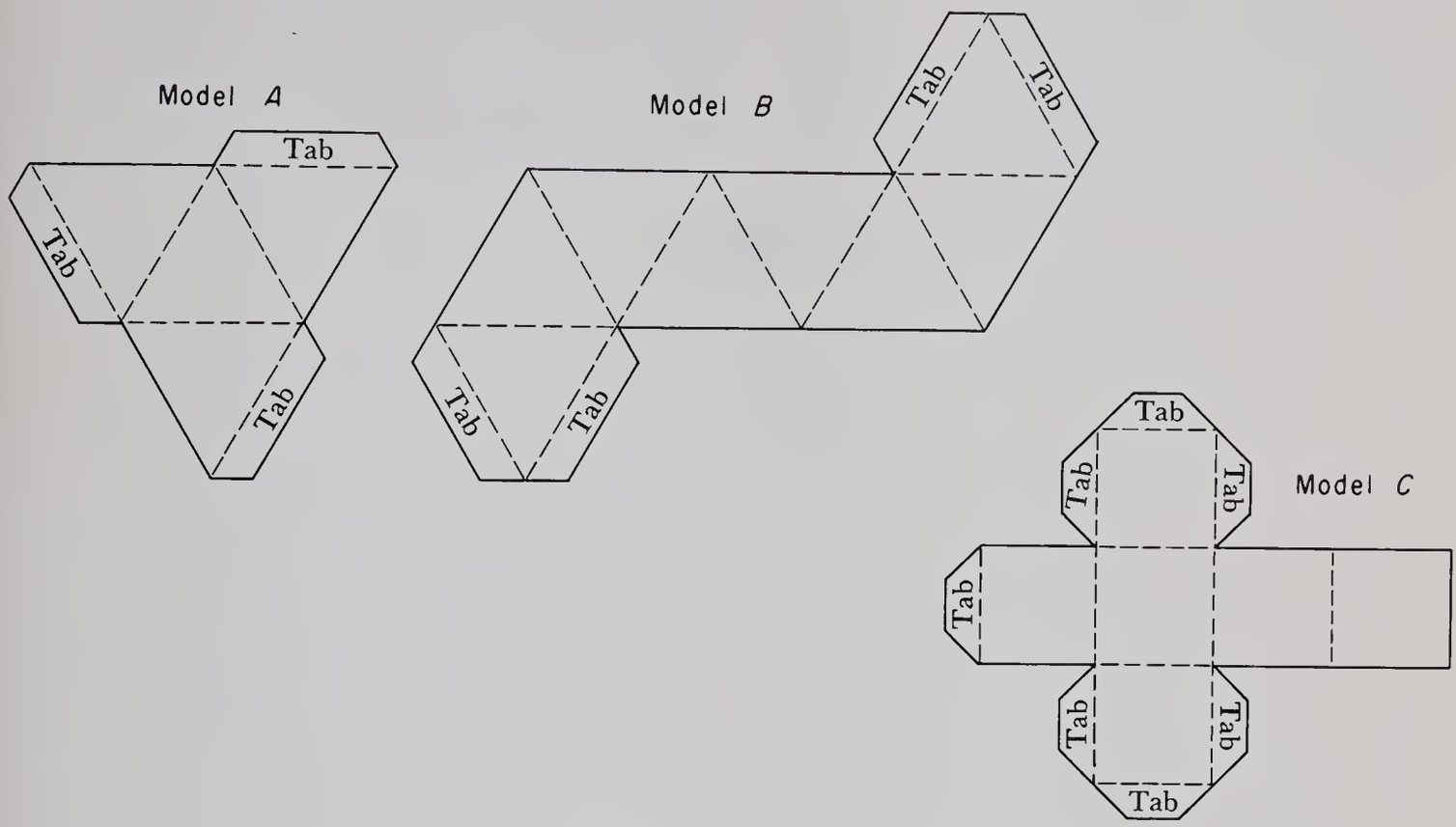
10.



$n = 2\frac{1}{2} \times \frac{5}{8}$; $1\frac{9}{16}$ sq. ft.

conical solids, cylindrical solids, and circular cylindrical solids, and right circular cylindrical solids.

• Instruct *all pupils* to cut out models with tabs and dashes like those indicated in the drawings shown below. Fold along the dotted lines, leaving tabs for gluing the faces together as shown. After the models are completed, have pupils determine the number of faces, the number of vertices, and the number of edges for each of the polyhedrons represented. Pupils may try to design patterns for other models.



• Have *more capable pupils* summarize the information in the chart below. Ask them to determine how the number of edges compares with the sum of the number of faces and the number of vertices for each polyhedron by comparing the data in the fourth column with the data in the third column. This relationship can then be expressed by Euler's formula, $F + V = E + 2$, where F represents the number of faces of the polyhedron, V the number of vertices, and E the number of edges.

	No. of faces	*No. of vertices*	*No. of edges*	*Sum of the No. of faces and vertices*
Model *A*	(4)	(4)	(6)	(8)
Model *B*	(8)	(6)	(12)	(14)
Model *C*	(6)	(8)	(12)	(14)

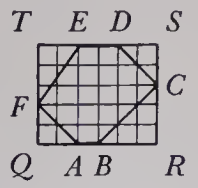
• (1) Dry air contains the following per cents by volume of gases: nitrogen, 78%; oxygen, 21%; argon, 0.93%; and other gases, 0.07%. Find the volume of each of these gases in a cubic foot of dry air. Do you have to compute the answer? (0.78 cubic feet, 0.21 cubic feet, 0.0093 cubic feet, 0.0007 cubic feet) Find the volume of each in a cubic mile of dry air. (0.78 cubic miles, 0.21 cubic miles, 0.0093 cubic miles, 0.0007 cubic miles)

(2) It is estimated that the average annual rainfall in the United States is 1,500 cubic miles of water. This water weighs almost 7 trillion tons. About what is the weight of one cubic mile of water? ($\frac{7,000,000,000,000}{1,500}$ tons is about 5 billion tons.)

(3) One cubic yard of sea water contains about 730 ounces of salt, 32 ounces of magnesium, 10 ounces of calcium, 9.5 ounces of potassium, 1.6 ounces of bromine, and 22 ounces of sulfur. A tank holding 1,250 cubic yards of sea water would contain about how much of these substances? (Hint: add the measures of the substance contained in one cubic yard; and multiply by 1,250.) (a total of about 1,006,375 oz. for 1,250 cu. yd.)

Teacher's Page 320

• Each small square region shown at the right has 1 as the measure of its area. Find the measure of the area of polygon *ABCDEF* by finding the measure of *QRST* and subtracting the measures of areas of certain triangular regions. ($18\frac{1}{2}$ small square regions)



• The following problem involving sets and their relationships is difficult to solve without a Venn diagram, but is clearly illustrated by a Venn diagram. List within braces the elements of each of sets A, B, and C if

$$A \cup B = \{2, 4, 6, 8, 10, 12\}$$
$$B \cup C = \{4, 6, 8, 10, 12, 14\}$$
$$A \cap B = \{4, 10\}$$
$$A \cap C = \{10, 12\}$$
$$B \cap C = \{8, 10\}$$
$$(A \cap B) \cap C = \{10\}$$

(Set $A = \{2, 4, 10, 12\}$; Set $B = \{4, 6, 8, 10\}$; Set $C = \{8, 10, 12, 14\}$)

NOTES

	Height
Tallest tree	367.8 ft.
2d tallest tree	367.4 ft.
3d tallest tree	364.3 ft.
4th tallest tree	356.5 ft.
5th tallest tree	352.6 ft.
6th tallest tree	352.6 ft.

How Well Can You Compute?

Computation Test 7

The six tallest known trees in the world are California redwoods. Refer to the table above in working Ex. 1–5.

1. The height of the tallest tree is how many feet greater than the height of the 6th tallest tree? 15.2 feet

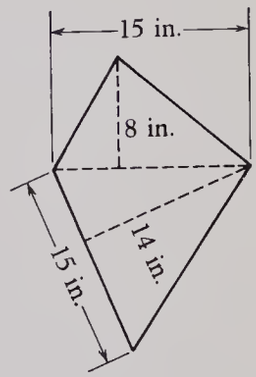
2. The girth (greatest circumference) of the tallest tree is 44 ft. To the nearest 1%, the girth of the tallest tree is what per cent of its height? 12%

3. Find, to the nearest $\frac{1}{10}$ foot, the mean for the data in the table. 360.2 feet

4. What is the mode for the data in the table? 352.6 ft.

5. Find, to the nearest $\frac{1}{10}$ foot, the median for the data in the table. 360.4 ft.

6. Find the area of the region shown at the right. 165 sq. in.



7. Copy and complete:

1 cu. yd. = 27 cu. ft., so $4\frac{1}{3}$ cu. yd. = _?_ cu. ft. 117

Ex. 8–13. Solve.

8. $n - 31.5 = 64.6$ 96.1

9. $n \times 14 = 5{,}294$ $387\frac{1}{7}$

10. $x = 45\% \times 385.3$ 173.385

11. $n = 62.7 + (39 \times 5.3)^{\wedge}$ 269.4

12. $35\% \times n = 15.61$ 44.6

13. $x = \frac{1}{2} \times (57 \times 148)$ 4,218

14. Copy and complete:

30 sq. ft. = _?_ $3\frac{1}{3}$ sq. yd. 2,304 sq. in. = _?_ 16 sq. ft.

Whole Numbers

Set 1

To be used after page 6

The Set of Whole Numbers:
{0, 1, 2, 3, · · ·}

The Set of Counting Numbers:
{1, 2, 3, · · ·}

The *whole numbers* are the numbers 0, 1, 2, 3, and so on. The *even whole numbers* are the numbers 0, 2, 4, and so on. The *odd whole numbers* are the numbers 1, 3, 5, and so on.

1. Which members of {0, 7, 8, 21, 55, 86, 192, 890} are

a. even whole numbers? 0, 8, 86, 192, 890 **b.** odd whole numbers? 7, 21, 55

2. List within braces the even whole numbers less than 51. {0,2,4,6,8,10,12,14,16,18,20,22,24,26,28,30,32,34,36,38,40,42,44,46,48,50}

3. List within braces the odd whole numbers less than 40. {1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31,33,35,37,39}

4. The whole numbers greater than 6 and less than 16 are 7, 8, 9, 10, 11, 12, 13, 14, and 15. List within braces the whole numbers greater than 14 and less than 32. {15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31}

5. Which of the numbers that you listed for Ex. 4 are

a. even whole numbers? 16, 18, 20, 22, 24, 26, 28, 30 **b.** odd whole numbers? 15, 17, 19, 21, 23, 25, 27, 29, 31

6. By the multiples of a number, we mean the whole-number multiples of that number. The multiples of 5 are 0, 5, 10, 15, and so on. List within braces the multiples of 8 that are less than 85. {0,8,16,24,32,40,48,56,64,72,80}

7. List within braces the multiples of 9 that are greater than 20 and less than 50. {27,36,45}

8. The *counting numbers* are the numbers 1, 2, 3, and so on. There is only one whole number which is not a counting number. Name that number. 0

9. List within braces the counting-number multiples of 8 that are less than 85. How does this set differ from the set listed for Ex. 6? {8,16,24,32,40,48,56,64,72,80}; 0 is not a member of this set.

10. Copy and complete each of Ex. a–f.

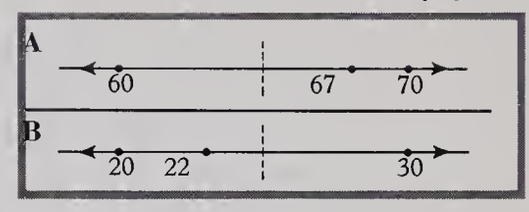
a. $45 = 40 + \_?\ 5$ **b.** $81 = \_?\ 70 + 11$ **c.** $78 = 50 + \_?\ 28$

d. $60 + 50 = \_?\ 110$ **e.** $54 = 9 \times \_?\ 6$ **f.** $200 = \_?\ 4 \times 50$

Rounding

Set 2

To be used after page 11



1. Box A. The number-line picture shows that 67 is nearer 70 than 60. 67 rounded to the nearest ten is _?70_.

2. Round to the nearest ten.

a. 18 20 **b.** 56 60 **c.** 97 100 **d.** 139 140 **e.** 806 810 **f.** 1,156 1,160

3. Box B. The number-line picture shows that 22 is nearer _?20_ than 30.

4. 22 rounded to the nearest ten is _?20_

5. Round to the nearest ten.

a. 21 20 **b.** 33 30 **c.** 51 50 **d.** 11 10 **e.** 182 180 **f.** 5,963 5,960

6. 65 rounded to the nearest ten is 70. In rounding to the nearest ten, if the digit in one's place is a 5, then which do we round to, the lesser multiple of 10 or the greater multiple of 10?

7. Round to the nearest 10: **a.** 45 50 **b.** 62 60 **c.** 85 90

8. To round to the nearest ten, we examine the digit in one's place. To round to the nearest hundred, we examine the digit in _?ten's△_ place.

9. Round to the nearest hundred.

a. 348 300 **b.** 566 600 **c.** 837 800 **d.** 958 1,000 **e.** 6,393 6,400

10. To round to the nearest thousand, we examine the digit in _?hundred's△_ place.

11. Round to the nearest thousand.

a. 1,389 1,000 **b.** 6,504 7,000 **c.** 89,839 90,000 **d.** 5,555 6,000 **e.** 9,521 10,000

12. In rounding to the nearest ten thousand, we examine the digit in _?thousand's△_ place, and each digit to the right of ten-thousand's place is replaced by the digit _?0_.

13. Round to the nearest ten thousand.

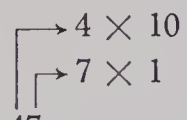
a. 15,301 20,000 **b.** 37,920 40,000 **c.** 38,129 40,000 **d.** 513,567 510,000 **e.** 921,584 920,000

Place Value

Set 3

To be used after page 16

A



1. Box A. In the numeral 47, the 7 is in one's place and the 4 is in ten's place. The 4 in its place indicates the number 4×10, or 40. The 7 in its place indicates the number 7×1, or _?7_. Copy and complete: $47 = 40 + $ 7?_

2. In the numeral 647, the 6 is in hundred's place and indicates $6 \times$ _?100_, or 600. $647 =$ _?600_ $+ 40 + 7$

3. The value of hundred's place is _?10_ times the value of ten's place. $100 =$ _?10_ $\times 10$

4. The value of thousand's place is how many times
   a. the value of hundred's place? 10 times $1{,}000 =$ _?10_ $\times 100$
   b. the value of ten's place? 100 times $1{,}000 =$ _?100_ $\times 10$
   c. the value of one's place? 1,000 times $1{,}000 =$ _?1,000_ $\times 1$

5. For the numeral 93,728, write the digit that is in
   a. hundred's place. 7 b. ten-thousand's place. 9

6. For each of the following, name the number indicated by the digit 6 in its place.
   a. 16 6 b. 3,768 60 c. 962 60 d. 69,702 60,000 e. 127,605 600

B

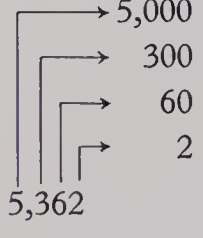
57
25
51,937
35,800
1,589

7. In which of the numerals in box B does the 5 indicate
   a. 5? 25 b. 50? 57 c. 500? 1,589 d. 5,000? 35,800 e. 50,000? 51,937

8. The number 63 can be thought of as 6 tens + 3 ones or as 63 ones. 175 can be thought of as _?17_ tens + 5 ones.

9. Copy and complete.
   a. $86 = 80 +$ _?6_ b. $149 = 100 +$ _?40_ $+ 9$
   c. $594 =$ _?500_ $+ 90 + 4$ d. $3{,}829 =$ _?3,000_ $+ 800 + 20 + 9$

C



10. Study the diagram in box C and then make a similar diagram for each of the following:
    a. 438 b. 888 c. 97 d. 3,129 e. 7,466

11. Copy and complete:
    $489{,}738 = 400{,}000 +$ _?80,000_ $+ 9{,}000 + 700 + 30 + 8$

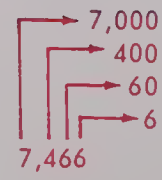
10.a.

b.

c.

d.

e.



Mathematical Sentences

Set 4

To be used after page 29

A	$29 = 8 + 21$
	$5 \times 12 = 60$
B	$23 = 25 - 3$
	$44 \div 4 = 12$

1. The mathematical sentences in box A are true because they express true ideas. The mathematical sentences in box B are false because they express false ideas. For each of Ex. a–f, write *T* if the sentence expresses a true idea and write *F* if the sentence expresses a false idea.

a. $25 + 16 = 41$ T **d.** $34 - 14 = 10$ F

b. $36 = 4 \times 8$ F **e.** $50 \div 10 = 5$ T

c. $27 \times 6 = 162$ T **f.** $2 \times 18 = 20 + 9$ F

2. The mathematical sentence $n = 2 + 3$ is an open sentence. If we say that n represents 5, then the sentence is a true sentence. We say we have *solved the sentence* or we have *found the number represented by n*. For each of Ex. a–c, solve the mathematical sentence.

a. $n = 14 + 39$ 53 **b.** $67 - n = 60$ 7 **c.** $n = 13 \times 10$ 130

3. John was told to find the number represented by n in the mathematical sentence $n = 25 + 32$. John gave 53 as his answer. Was that correct? No If not, give the correct answer. 57

4. If n represents 8, then $n < 12$ is a true mathematical sentence. Is $n < 12$ a true mathematical sentence if n represents

a. 9? Yes **b.** 11? Yes **c.** 5? Yes **d.** 2? Yes **e.** 13? No

5. The members of $\{4, 7, 8, 10\}$ which make $n > 6$ a true sentence are 7, 8, and 10. Which members of $\{3, 5, 7, 9\}$ make $n > 6$ a true sentence? 7, 9

6. List within braces the whole numbers which make $n < 15$ a true sentence. {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14}

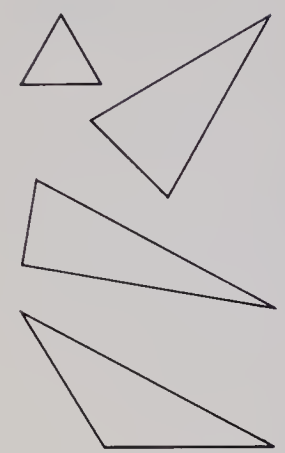
7. List within braces the counting numbers which make $n < 15$ a true sentence. {1,2,3,4,5,6,7,8,9,10,11,12,13,14}

8. Which even counting numbers make $n < 23$ a true sentence? 2,4,6,8,10,12,14,16,18,20,22 Which odd counting numbers make $n < 23$ a true sentence? 1,3,5,7,9,11,13,15,17,19,21

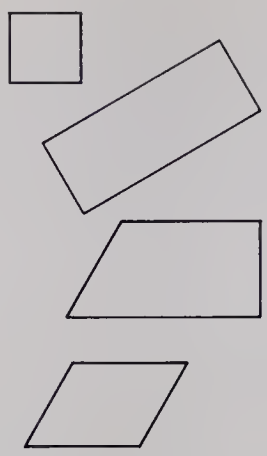
Polygons

Set 5

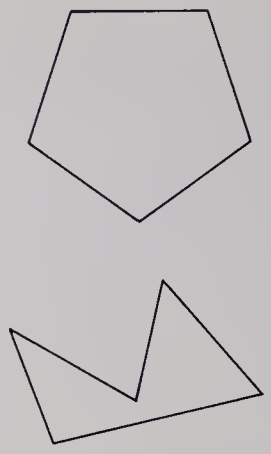
To be used after page 46



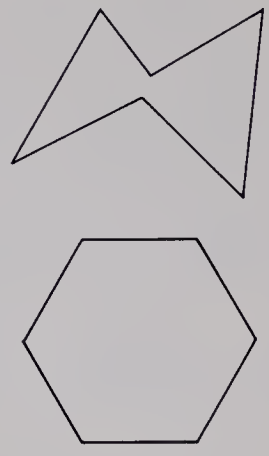
Triangles



Quadrilaterals



Pentagons



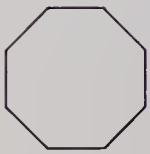
Hexagons

1. Each geometric figure shown above is a simple closed curve that is formed by line segments only. Such geometric figures are called *polygons*. The line segments are called the *sides* of the polygon. A triangle has 3 sides. Copy and complete the following:

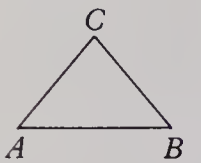
a. A polygon with 4 sides is a _? quadrilateral

b. A polygon with 5 sides is a _? pentagon

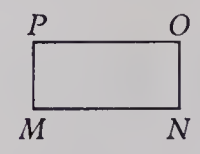
c. A polygon with _? 6 sides is a hexagon.

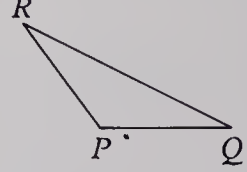


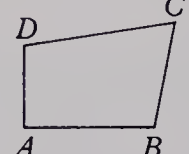
2. A polygon can have more than 6 sides. If the number of sides is 8, the polygon is called an octagon. Is the polygon shown at the left an octagon? Yes

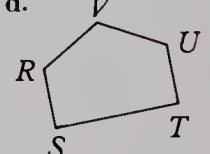


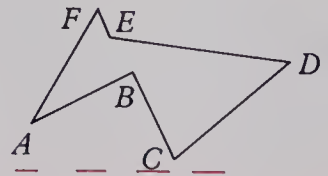
3. The sides of the polygon shown at the left are $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$. For Ex. a–f, tell whether the polygon is a triangle, a quadrilateral, a pentagon, or a hexagon and name the sides. See below.

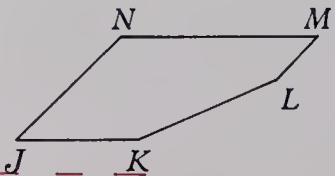
a. 

b. 

c. 

d. 

e. 

f. 

Ex. 3 a. quadrilateral; $\overline{PO}$, $\overline{ON}$, $\overline{MN}$, $\overline{MP}$ b. triangle; $\overline{QR}$, $\overline{PQ}$, $\overline{PR}$ c. quadrilateral; $\overline{CD}$, $\overline{AD}$, $\overline{AB}$, $\overline{BC}$ d. pentagon; $\overline{RS}$, $\overline{ST}$, $\overline{TU}$, $\overline{UV}$, $\overline{VR}$ e. hexagon; $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, $\overline{AF}$ f. pentagon; $\overline{JK}$, $\overline{KL}$, $\overline{LM}$, $\overline{MN}$, $\overline{JN}$

Prime Factors and Products of Primes

Set 6

To be used after page 68

1. Box A. A prime number which is a factor of a number is called a *prime factor* of that number. For example, 2 is a prime factor of 6. Name another prime factor of 6. 3

A
Factors of 6:
1, 2, 3, and 6
Prime Factors of 6:
2 and 3

2. The factors of 15 are 1, 3, 5, and 15. Which of these are prime factors? 3,5

3. List within braces the factors of 12. {1,2,3,4,6,12} Which of the factors of 12

a. are prime factors? 2, 3 **b.** are not prime factors? 1, 4, 6, 12

4. List within braces the factors of 42 which are prime factors. {2,3,7}

5. Box B shows a factor tree for expressing 30 as a product of primes. In making a factor tree for a number, the factors named in the final product expression should be prime numbers.

a. In (1), 30 is expressed as 2 × _? 15.

b. In (2), the factor 2 is shown again, and 15 is expressed as 3 × _? 5.

c. Does (2) show 30 expressed as a product of primes? Yes

6. Copy and complete the factor tree in box C.

7. Box D shows a factor tree for 12. Note that the prime factor 2 is named more than once in the final product expression for 12. Make a factor tree for 36 and then express 36 as a product of primes as in box D.

36 = 2 × 2 × 3 × 3

8. Express each of the following as a product of primes. Make factor trees if it will help you.

a. 28 (2 × 2 × 7) **b.** 42 (2 × 3 × 7) **c.** 150 (2 × 3 × 5 × 5) **d.** 300 (2 × 2 × 3 × 5 × 5) **e.** 480 (2 × 2 × 2 × 2 × 2 × 3 × 5) **f.** 520 2 × 2 × 2 × 5 × 13

9. Find the standard numeral for the number named by each of the following:

a. 2 × 3 × 3 × 3 × 7 (378) **b.** 5 × 5 × 13 (325) **c.** 3 × 7 × 7 × 7 1,029

Divisors Greater Than Ten

Set 7

To be used after page 72

A

$15\overline{)467} = 15\overline{)46 \text{ tens} + 7 \text{ ones}}$

45 tens

1 ten + 7 ones ------→ 17 ones

1 ten + 5 ones ------→ 15 ones

2 ones

B

$$15\overline{)467}$$

45

17

15

R 2

C

$467 = (15 \times 31) + 2$

1. Box A shows one way to find 467 ÷ 15.

a. Since we can not divide hundreds, we rename 467 as 46 tens + _?_ 7 ones.

b. *Divide tens:* The greatest multiple of 15 that can be subtracted from 46 is 45. When 46 tens are divided by 15, the quotient is 3 tens and the remainder is _?_ 1 ten.

c. 1 ten + 7 ones is renamed as _?_ 17 ones.

d. *Divide ones:* The greatest multiple of 15 that can be subtracted from 17 is _?_ 15.

e. For 467 ÷ 15, the quotient is 3 tens + 1 one, or 31, and the remainder is _?_ 2 ones, or 2.

2. Box B shows a better way to find 467 ÷ 15. We *think* the steps shown in box A but we do not write them. Study the work in the box which shows how we indicate the division of tens and ones. A 3 is written in ten's place in the numeral for the quotient to indicate 3 tens and a 1 is written in _?_ one's place to indicate 1 one.

3. Box C shows a checking sentence for the work in box B. Is the checking sentence a true sentence? Yes

Ex. 4–13. Copy and divide. Work as in box B and write and test a checking sentence for your work. Checking sentences shown below.

4. $27\overline{)361}$ 13, R10
5. $13\overline{)845}$ 65, R0
6. $71\overline{)809}$ 11, R28
7. $19\overline{)383}$ 20, R3
8. $55\overline{)937}$ 17, R2
9. $24\overline{)932}$ 38, R20
10. $36\overline{)11{,}245}$ 312, R13
11. $41\overline{)1{,}092}$ 26, R26
12. $65\overline{)5{,}632}$ 86, R42
13. $82\overline{)4{,}389}$ 53, R43

4. 361 = (27 × 13) + 10
5. 845 = (13 × 65) + 0
6. 809 = (71 × 11) + 28
7. 383 = (19 × 20) + 3
8. 937 = (55 × 17) + 2
9. 932 = (24 × 38) + 20
10. 11,245 = (36 × 312) + 13
11. 1,092 = (41 × 26) + 26
12. 5,632 = (65 × 86) + 42
13. 4,389 = (82 × 53) + 43

Intersection of Sets in Geometry

Set 8

To be used after page 85

1. Fig. 1. The segments shown do not have any points in common, so their intersection is { }, the empty set. On your paper, show three segments no two of which have any points in common. Sample answer:

Fig. 1

2. Fig. 2. Point E is the only point common to $\overline{AB}$ and $\overline{CD}$, so point E is the only member in the intersection of the two segments. We write, $\overline{AB} \cap \overline{CD} = \{E\}$. Refer to Fig. 3 and copy and complete Ex. a–c.

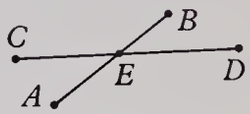


Fig. 2

a. $\overline{MN} \cap \overline{KJ} = \{\_?^{X}\}$ **b.** $\overline{PQ} \cap \overline{RS} = \{\_?^{W}\}$ **c.** $\overline{RS} \cap \overline{KJ} = \_?^{\{\ \}}$

3. Since segments are *sets of points*, the intersection of two segments is a *set* of points. Referring to Fig. 2, why is it incorrect to write, $\overline{AB} \cap \overline{CD} = E$? E does not name a set of points.

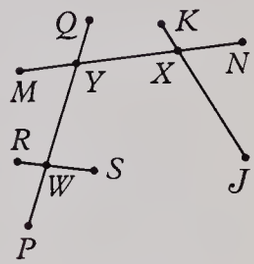


Fig. 3

4. The vertex of an angle is a point that belongs to each of the two sides of the angle. Is it the only such point? Yes

5. Copy and complete Ex. a–d by referring to Fig. 4. Remember to use braces.

a. $\overrightarrow{QP} \cap \overrightarrow{QR} = \_?^{\{Q\}}$ **b.** $\angle PQR \cap \overline{PR} = \_?^{\{P,R\}}$

c. $\overline{PR} \cap \{Q\} = \_?^{\{\ \}}$ **d.** $\angle PQR \cap \overline{QR} = \_?^{\overline{QR}}$

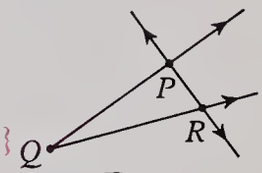


Fig. 4

6. Does the center for a circle belong to the circle? No Then the intersection of a circle and the set containing the center for the circle is the $\_?^{\text{empty}}$ set.

7. The end points of a chord for a circle are the only points common to the chord and the circle. The intersection of a diameter for a circle and the circle is the set containing the $\_?^{\text{end points}}$ $\_?^{\text{end points}}$ of the diameter.

8. Refer to Fig. 5 and copy and complete Ex. a–f. Remember to use braces.

a. $\{E\} \cap \overline{MN} = \_?^{\{\ \}}$ **d.** $\overline{MN} \cap \overline{MG} = \_?^{\{M\}}$

b. $\overline{ME} \cap \overline{EG} = \_?^{\{E\}}$ **e.** Circle $E \cap \overline{MN} = \_?^{\{M,N\}}$

c. $\overline{MN} \cap \overline{EN} = \_?^{\{N\}}$ **f.** Circle $E \cap \{E\} = \_?^{\{\ \}}$

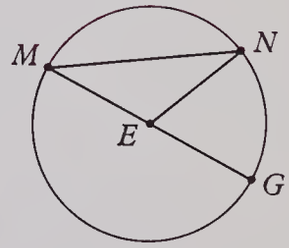


Fig. 5

Union of Sets in Geometry

Set 9

To be used after page 86

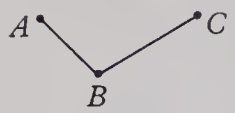


Fig. 1

1. The geometric figure (a set of points) shown in Fig. 1 is the union of segments AB and BC. Does this show that the union of two segments is not always a segment? Yes

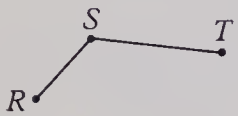


Fig. 2

2. Fig. 2. The geometric figure shown is the union of $\overline{RS}$ and $\overline{ST}$.

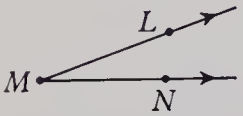


Fig. 3

3. On your paper, show two segments whose union

a. is a segment. Sample answers:

b. is not a segment.

4. Fig. 3. The union of rays ML and MN is angle LMN. We write, $\overrightarrow{ML} \cup \overrightarrow{MN} = \angle LMN$. Refer to Fig. 4 and copy and complete Ex. a–c.

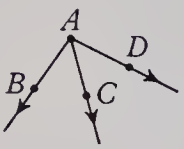


Fig. 4

a. $\overrightarrow{AB} \cup \overrightarrow{AC} =$? $\angle BAC$ b. $\overrightarrow{AB} \cup \overrightarrow{AD} =$? $\angle BAD$ c. ? $\overrightarrow{AC} \cup \overrightarrow{AD} = \angle CAD$

5. The triangle shown in Fig. 5 is the union of segments WX, XY, and ? $\overline{WY}$.

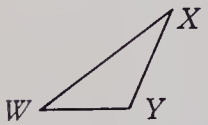


Fig. 5

6. Copy and complete the following using the word *segments* or the word *rays*.

a. A triangle is the union of three ? segments

b. An angle is the union of two ? rays

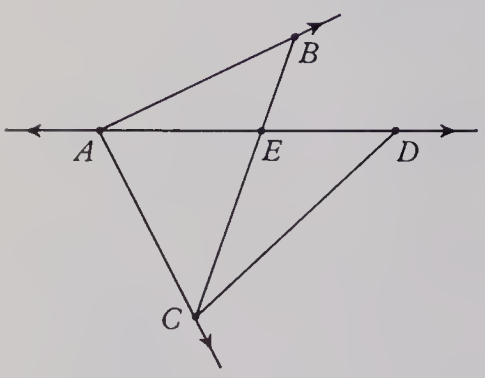
c. A pentagon is the union of five ? segments

d. A hexagon is the union of six ? segments

Fig. 6

7. If A, B, C, D, and E denote five different points, then the union of $\{A, C\}$ and $\{B, C, D, E\}$ has how many members? 5

8. Fig. 6. The union of triangle ABC and segment BC is ? triangle ABC.



Ex. 9–16. Refer to the figure shown at the left and copy and complete the statement.

9. $\overline{AE} \cup \overline{ED} =$? $\overline{AD}$ 13. $\overrightarrow{AD} \cup \overrightarrow{AC} =$? $\angle DAC$

10. $\overline{CE} \cup \overline{EB} =$? $\overline{CB}$ 14. $\overrightarrow{AD} \cup \overrightarrow{AB} =$? $\angle BAD$

11. $\overline{CB} \cup \overline{CE} =$? $\overline{CB}$ 15. $\triangle ABE \cup \overline{AB} =$? $\triangle ABE$

12. $\overrightarrow{AB} \cup \overrightarrow{AC} =$? $\angle BAC$ 16. $\triangle ACE \cup \overline{AC} =$? $\triangle ACE$

Subtracting with Mixed Forms

Set 10

To be used after page 114

A Subtract	B Subtract	C Subtract	D Subtract
$6\frac{3}{5}$ $2\frac{1}{5}$ $4\frac{2}{5}$	$8\frac{3}{4}$ $5\frac{1}{4}$ $3\frac{2}{4}$, or $3\frac{1}{2}$	$2\frac{1}{2} = 2\frac{3}{6}$ $1\frac{1}{3} = 1\frac{2}{6}$ $1\frac{1}{6}$	$15\frac{1}{4} = 15\frac{1}{4} = 14\frac{5}{4}$ $8\frac{1}{2} = 8\frac{2}{4} = 8\frac{2}{4}$ $6\frac{3}{4}$

1. Box A. Think of $6\frac{3}{5}$ as $6 + \frac{3}{5}$. Think of $2\frac{1}{5}$ as $2 + \frac{1}{5}$. $\frac{3}{5} - \frac{1}{5} = \frac{2}{5}$ and $6 - 2 = 4$, so $6\frac{3}{5} - 2\frac{1}{5} = 4 + \frac{2}{5} =$ _?_ $4\frac{2}{5}$

2. Solve each of Ex. a–c by subtracting in vertical form.
a. $n = 5\frac{6}{7} - 3\frac{2}{7}$ $2\frac{4}{7}$ **b.** $n = 14\frac{6}{8} - 9\frac{1}{8}$ $5\frac{5}{8}$ **c.** $n = 45\frac{9}{10} - 27\frac{6}{10}$ $18\frac{3}{10}$

3. Box B. Is the fraction $\frac{2}{4}$ in the mixed form $3\frac{2}{4}$ in *simplest form*? No $3\frac{2}{4} = 3 + \frac{2}{4} = 3 + \frac{1}{2} =$ _?_ $3\frac{1}{2}$

4. Find the number represented by n. Make sure the answer is in simplest form.
a. $9\frac{7}{10} - 5\frac{3}{10} = n$ $4\frac{2}{5}$ **b.** $n = 84\frac{5}{6} - 36\frac{1}{6}$ $48\frac{2}{3}$ **c.** $n = 72\frac{7}{12} - 55\frac{5}{12}$ $17\frac{1}{6}$

5. Box C. Before subtracting, both $2\frac{1}{2}$ and $1\frac{1}{3}$ were renamed so that the fractions in the mixed forms show the common denominator _?_ 6.

6. Find $16\frac{3}{5} - 7\frac{1}{3}$ by subtracting in vertical form as in box C. $9\frac{4}{15}$

7. Study the work in box D. Since $\frac{2}{4}$ can not be subtracted from $\frac{1}{4}$, $15\frac{1}{4}$ was renamed as $14\frac{5}{4}$.
$15\frac{1}{4} = (14+1)+\frac{1}{4} = (14+\frac{4}{4})+\frac{1}{4} = 14+(\frac{4}{4}+\frac{1}{4}) = 14+\frac{5}{4} =$ _?_ $14\frac{5}{4}$

8. Find $37\frac{2}{3} - 23\frac{5}{6}$ by subtracting in vertical form. Rename as in Box D if necessary. $13\frac{5}{6}$

Ex. 9–13. Copy and subtract. Remember to express your answer in simplest form.

9.	10.	11.	12.	13.
$15\frac{3}{7}$	$76\frac{3}{8}$	$329\frac{2}{3}$	$301\frac{8}{9}$	$66\frac{1}{5}$
$5\frac{1}{7}$	$14\frac{1}{4}$	$88\frac{5}{6}$	$289\frac{9}{10}$	$34\frac{1}{2}$
$10\frac{2}{7}$	$62\frac{1}{8}$	$240\frac{5}{6}$	$11\frac{89}{90}$	$31\frac{7}{10}$

Adding Three Rational Numbers

Set 11

To be used after page 115

A

$n = \frac{3}{7} + \frac{4}{7} + \frac{2}{7}$

$= \frac{3+4+2}{7}$

$= \frac{9}{7}$, or $1\frac{2}{7}$

B

$n = 3\frac{1}{2} + \frac{3}{4} + \frac{5}{8}$

$= \frac{7}{2} + \frac{3}{4} + \frac{5}{8}$ (1)

$= \frac{28}{8} + \frac{6}{8} + \frac{5}{8}$ (2)

$n = \frac{39}{8}$, or $4\frac{7}{8}$

C

$n = 1\frac{1}{3} + \frac{11}{15} + 2\frac{1}{5}$

$= \frac{?^{4}}{3} + \frac{11}{15} + \frac{?^{11}}{5}$

$= \frac{?^{20}}{15} + \frac{11}{15} + \frac{?^{33}}{15}$

$n = \frac{?^{64}}{15}$, or $-?- \; ^{4}\frac{4}{15}$

1. Box A. The addends are named with fractions which show a common denominator of -?-[7]. Then to find the numerator of the fraction for the answer, we add the -?- (numerators) of the fractions for the addends.

2. Study the work in box B for solving $n = 3\frac{1}{2} + \frac{3}{4} + \frac{5}{8}$. In (1), the addend $3\frac{1}{2}$ was renamed with a -?- (fraction). In (2), since the fractions do not show a common denominator, the addends were each named with a fraction showing denominator -?-[8]. Is 8 the L.C.D. of the fractions in (1)? Yes

3. Copy and complete the work in Box C.

4. Solve Ex. a–d as in boxes A–C.

a. $n = 1\frac{2}{3} + \frac{7}{12} + \frac{3}{4}$ 3

b. $n = 5\frac{1}{2} + 3\frac{1}{3} + \frac{5}{6}$ $9\frac{2}{3}$

c. $n = 4\frac{3}{8} + 2\frac{1}{3} + 1\frac{1}{12}$ $7\frac{19}{24}$

d. $x = 2\frac{3}{5} + 4\frac{2}{3} + 5\frac{1}{15}$ $12\frac{1}{3}$

5. Box D shows how to solve $n = 2\frac{1}{2} + 3\frac{3}{5} + 7\frac{3}{10}$ by adding in vertical form. Study the work shown.

$12\frac{14}{10} = 12 + \frac{14}{10} = 13 + \frac{?^{4}}{10} = 13 + \frac{2}{5} = 13\frac{2}{5}$

6. Solve the mathematical sentence in box D by renaming the addends with fractions as in box B. Compare your answer with the answer in box D. See below.

D

$n = 2\frac{1}{2} + 3\frac{3}{5} + 7\frac{3}{10}$

Add

$2\frac{1}{2} = 2\frac{5}{10}$

$3\frac{3}{5} = 3\frac{6}{10}$

$\underline{7\frac{3}{10} = 7\frac{3}{10}}$

$12\frac{14}{10}$, or $13\frac{2}{5}$

$n = 13\frac{2}{5}$

Ex. 7–11. Copy and add as in box D.

7.	8.	9.	10.	11.
$3\frac{1}{3}$	$12\frac{1}{8}$	$18\frac{2}{3}$	$104\frac{4}{5}$	$49\frac{3}{10}$
$6\frac{2}{5}$	$20\frac{3}{4}$	$7\frac{5}{7}$	$88\frac{2}{7}$	$30\frac{5}{12}$
$\underline{9\frac{4}{5}}$	$\underline{51\frac{1}{2}}$	$\underline{66\frac{2}{3}}$	$\underline{821\frac{3}{5}}$	$\underline{51\frac{4}{5}}$
$19\frac{8}{15}$	$84\frac{3}{8}$	$93\frac{1}{21}$	$1{,}014\frac{24}{35}$	$131\frac{31}{60}$

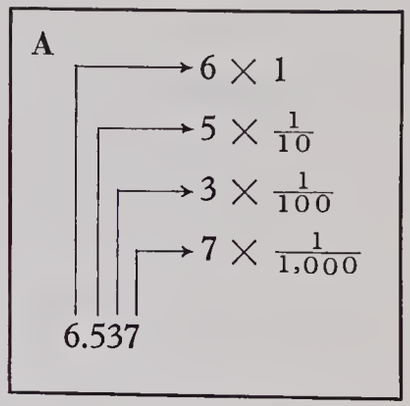
6. $n = 2\frac{1}{2} + 3\frac{3}{5} + 7\frac{3}{10}$ $n = \frac{5}{2} + \frac{18}{5} + \frac{73}{10}$ $n = \frac{25}{10} + \frac{36}{10} + \frac{73}{10}$ $n = \frac{134}{10}$, or $13\frac{2}{5}$

Decimals

Set 12

To be used after page 121

A



1. Look at the diagram in box A. In 6.537, the 6 is in one's place, the 5 is in tenth's place, the 3 is in hundredth's place, and the 7 is in thousandth's place.

2. In 6.537, the 7 indicates the number $7 \times \frac{1}{1,000}$, or $\frac{7}{1,000}$. Give similar statements for the other digits shown. See below.

3. The decimal 6.537 may be read, *six and five hundred thirty-seven thousandths.* For each of Ex. a–e, copy and complete the word description.

a. 0.05 _?_ *hundredths* (five)

b. 5.8 · _?_ *and* _?_ *tenths* (five, eight)

c. 0.025 _?_ *thousandths* (twenty-five)

d. 20.009 _?_ *and nine* _?_ (twenty, thousandths)

e. 8.572 *Eight and* _?_ *hundred* _?_ *thousandths* (five, seventy-two)

4. Write decimals for the numbers named in Ex. a–c.

a. Fifty-six and three tenths 56.3

b. One hundred and four hundredths 100.04

c. Seven thousand one and thirty-nine thousandths 7,001.039

B

$$\frac{3}{25} = \frac{4 \times 3}{4 \times 25}$$
$$= \frac{12}{100}$$
$$= 0.12$$

5. Study the work in box B for renaming $\frac{3}{25}$ with a decimal. The number $\frac{3}{25}$ was first renamed with a fraction showing a denominator which is a power of _?_. (10)

6. Rename each of Ex. a–e with a decimal.

a. $\frac{7}{10}$ 0.7 **b.** $\frac{37}{100}$ 0.37 **c.** $\frac{7}{25}$ 0.28 **d.** $\frac{3}{4}$ 0.75 **e.** $\frac{327}{500}$ 0.654

C

$$0.48 = \frac{4}{10} + \frac{8}{100}$$
$$= \frac{40}{100} + \frac{8}{100}$$
$$= \frac{48}{100}, \text{ or } \frac{12}{25}$$

7. Study the work in box C for renaming 0.48 with a fraction in simplest form. Both 48 and 100 were divided by _?_ (4) to obtain the fraction in simplest form.

8. Rename each of Ex. a–e with a fraction in simplest form. Work as in box C if necessary.

a. 0.7 $\frac{7}{10}$ **b.** 0.93 $\frac{93}{100}$ **c.** 0.88 $\frac{22}{25}$ **d.** 0.024 $\frac{3}{125}$ **e.** 0.3002 $\frac{1,501}{5,000}$

9. Copy and complete: $4.0371 = 4 + \frac{3}{?_{100}} + \frac{?_{7}}{1,000} + \frac{1}{?_{10,000}}$

2. The 3 indicates the number $3 \times \frac{1}{100}$, or $\frac{3}{100}$

The 5 indicates the number $5 \times \frac{1}{10}$, or $\frac{5}{10}$

The 6 indicates the number 6×1, or 6

Subtracting with Decimals

Set 13

To be used after page 164

A Subtract

$5.3 = 5 + \frac{3}{10} = 4 + \frac{13}{10}$

$\underline{3.4} = \underline{3 + \frac{4}{10}} = \underline{3 + \frac{4}{10}}$

$1 + \frac{9}{10}$, or 1.9

B Subtract

$5.3 = 5\frac{3}{10} = 4\frac{13}{10}$

$\underline{3.4} = \underline{3\frac{4}{10}} = \underline{3\frac{4}{10}}$

$1\frac{9}{10}$, or 1.9

4.

$6.8 = 6 + \frac{8}{10} = 5\frac{18}{10}$

$\underline{3.9} = \underline{3 + \frac{9}{10}} = \underline{3\frac{9}{10}}$

$2\frac{9}{10}$, or 2.9

1. Study the work in box A for finding 5.3 − 3.4. Since $\frac{4}{10}$ can not be subtracted from $\frac{3}{10}$, $5 + \frac{3}{10}$ was renamed $4 + \frac{?\ 13}{10}$.

2. Box B. Both 5.3 and 3.4 were renamed with mixed forms before subtracting. Do you see that the work in box B is much like the work in box A? Yes For each of Ex. a–e, copy and subtract as in box B.

4.

$6.8 - 3.9 = \frac{68}{10} - \frac{39}{10}$

$= \frac{29}{10}$, or 2.9

a.	b.	c.	d.	e.
2.7	6.3	14.35	75.93	138.4
1.5	4.8	9.14	17.28	93.9
1.2	1.5	5.21	58.65	44.5

C

$5.3 - 3.4 = \frac{53}{10} - \frac{34}{10}$

$= \frac{19}{10}$, or 1.9

3. Box C shows how to find 5.3 − 3.4 by first renaming with fractions. Copy and complete:

$0.35 - 0.17 = \frac{?\ 35}{100} - \frac{?\ 17}{100} = \frac{?\ 18}{100} = 0.18$

4. Find 6.8 − 3.9 first as in box A and then as in box C. See above.

D Subtract

$\begin{array}{r} 4\,(13) \\ \cancel{5.3} \\ 3.4 \\ \hline 1.9 \end{array}$

E Subtract

$\begin{array}{r} 3\ ?\,(17) \\ 3.\cancel{4}7 \\ 1.29 \\ \hline 2.\,?\,? \\ \quad 18 \end{array}$

5. To find 5.3 − 3.4, we may subtract as shown in box D. Study the work shown. We can not subtract 4 tenths from 3 tenths, so 5.3 is thought of as $4 + \underline{\ ?\ }^{13}\underline{\ }$ tenths. How is this shown? (13)

6. Copy and complete the work in box E.

7. Find 8.24 − 2.55 first as in box B and then as in box D.

Ex. 8–12. Copy and subtract. Work as in box D. See below.

8.	9.	10.	11.	12.
3.9	17.3	31.83	9.004	26.931
2.4	9.6	2.78	0.007	17.635
1.5	7.7	29.05	8.997	9.296

Ex. 13–15. Solve.

13. $n = 5.1 - 3.4$ 1.7 14. $n = 2.73 - 1.52$ 1.21 15. $n = 0.83 - 0.75$ 0.08

$\begin{array}{r} 7\ (11)(14) \\ 8.\cancel{2}\ \cancel{4} \\ 2.5\ 5 \\ \hline 5.6\ 9 \end{array}$

7. $8.24 = 8\frac{24}{100} = 7\frac{124}{100}$

$\underline{2.55} = \underline{2\frac{55}{100}} = \underline{2\frac{55}{100}}$

$5\frac{69}{100}$, or 5.69

Set 14

Answers will vary. Might be { }

1. List within braces the names of your brothers and sisters.

2. Is the set you described for Ex. 1 *finite* or *infinite*? finite

3. List within braces the multiples of 4 that are greater than 10 and less than 45. Is this set of multiples a finite set? Yes {12, 16, 20, 24, 28, 32, 36, 40, 44}

4. The set of even whole numbers is an infinite set. Is the set of odd whole numbers an infinite set? Yes

5. What whole number indicates the number of members in the empty set? 0 Is this number a counting number? No

Set 15

1. List within braces the members of the intersection of

a. {4, 5} and {5, 6}. {5}
b. {0, 5} and {4, 6}. { }
c. {2, 4, 7, 9} and {0, 5, 7, 8, 9}. {7, 9}
d. {6, 10} and {10, 12, 14}. {10}

2. List within braces the members of the union of

a. {3, 4} and {5}. {3, 4, 5}
b. {5, 7, 8} and {9, 10}. {5, 7, 8, 9, 10}
c. {0, 4} and {10}. {0, 4, 10}
d. {6, 8, 10} and {10, 12}. {6, 8, 10, 12}

3. List within braces the members of {0, 5, 7, 10} which are also members of the intersection of {5, 7, 10} and the set of multiples of 5. {5, 10}

4. Which members of the union of {3, 6} and {2, 4} are even numbers? 2, 4, 6

Set 16

$A = \{0, 4, 7\}$ $B = \{4, 7, 8, 9\}$ $C = \{7, 10\}$
$D = \{6, 12, 15\}$ $E = \{\}$ $F = \{0, 1, 2, 3, 4, 5, 6\}$

1. Copy and complete.

a. $A \cup B =$ _? $\{0, 4, 7, 8, 9\}$
b. $B \cap C =$ _? $\{7\}$
c. $E \cap B =$ _? $\{\}$
d. $B \cup C =$ _? $\{4, 7, 8, 9, 10\}$
e. $A \cap D =$ _? $\{\}$
f. $F \cup E =$ _? $\{0,1,2,3,4,5,6\}$
g. $A \cap C =$ _? $\{7\}$
h. $C \cup D =$ _? $\{6, 7, 10, 12, 15\}$
i. $A \cap F =$ _? $\{0, 4\}$

2. Which members of $A \cup B$ are odd numbers? 7, 9

3. Which members of $A \cap B$ are also members of C? 7

4. Copy and complete.

a. $\{1, 2, 3, 4, 5, 6, 7, 8\} \cap B =$ _? $\{4, 7, 8\}$
b. $\{2, 4, 6, 8, 10\} \cap A =$ _? $\{4\}$
c. $\{2, 4, 6, 8\} \cup C =$ _? $\{2, 4, 6, 7, 8, 10\}$
d. $\{0, 3, 6, 9, 12\} \cup D =$ _? $\{0, 3, 6, 9, 12, 15\}$

Extra Examples

Set 17

Row 1. Round to the nearest ten.
Row 2. Round to the nearest thousand.
Row 3. Round to the nearest dollar.

	a	b	c	d	e	f	g	h
1.	36 40	55 60	839 840	11 10	273 270	5,829 5,830	91 90	2,075 2,080
2.	2,312 2,000	3,529 4,000	32,692 33,000	978 1,000	1,002 1,000	28,800 29,000	44,500 45,000	1,499 1,000
3.	$5.95 $6.00	$42.50 $43.00	$88.25 $88.00	$2.05 $2.00	$1.98 $2.00	$99.99 $100.00	$250.50 $251.00	$0.51 $1.00

Set 18

I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII, XIII, XIV, XV, XVI, XVII, XVIII, XIX, XX, XXI, XXII, XXIII,

1. Write Roman numerals for the whole numbers from 1 through 25. XXIV, XXV
2. In the Roman numeral CXLV, the number indicated by XL is _?_. 40
3. What number is named by CXLV? 145
4. Find the number named by the Roman numeral.

a. XXXIII 33 b. XXIV 24 c. LXV 65 d. LVIII 58 e. CLXX 170 f. CCCLXIX 369

g. CCL 250 h. CXIV 114 i. DCXL 640 j. MCCXI 1,211 k. MMMCIV 3,104 l. CMXCIX 999

5. What number is named by MMCDXCIV? 2,494
6. Write a Roman numeral for the number.

a. 29 XXIX b. 74 LXXIV c. 43 XLIII d. 235 CCXXXV e. 447 CDXLVII f. 395 CCCXCV

g. 551 DLI h. 813 DCCCXIII i. 927 CMXXVII j. 2,645 MMDCXLV k. 1,949 MCMXLIX l. 2,045 MMXLV

Set 19

1. Write the standard numeral for the number.

a. 7^3 343 b. 5^4 625 c. 3^5 243 d. 8^3 512 e. 2^7 128 f. 15^3 3,375

g. 6^5 7,776 h. 3^8 6,561 i. 10^5 100,000 j. 20^4 160,000 k. 100^2 10,000 l. $1{,}000^2$ 1,000,000

2. Copy and complete.

a. $36 = {}^{6}?_{2}$ b. $64 = 4^{?}{}^{3} = 2^{?}{}^{6}$ c. $10^{?}{}^{4} = 100^2$ d. $625 = 25^{?}{}^{2} = 5^{?}{}^{4}$

Set 20

1. Write the standard numeral for the number.

a. $(5 \times 10^4) + (4 \times 10^2) + (8 \times 1)$ 50,408 b. $(3 \times 10^3) + (6 \times 10) + (7 \times 1)$ 3,067

c. $(3 \times 1{,}000) + (7 \times 100) + (5 \times 10)$ 3,750 d. $(6 \times 10^5) + (4 \times 10^3) + (8 \times 10)$ 604,008

2. Rename with an exponent form.

a. 100 10^2 b. 100,000 10^5 c. 10,000,000 10^7 d. 100,000,000 10^8 (or 100^4)

3. Write an expanded form using exponent forms to name the place values.

a. 4,620 b. 68,402 c. 125,000 d. 406,228

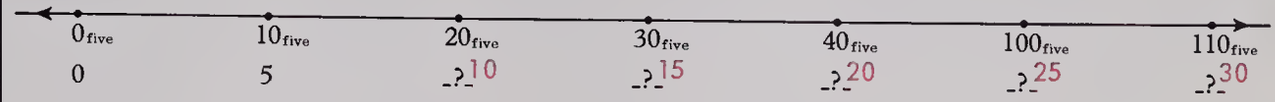
a. $(4 \times 10^3) + (6 \times 10^2) + (2 \times 10)$
b. $(6 \times 10^4) + (8 \times 10^3) + (4 \times 10^2) + (2 \times 1)$
c. $(1 \times 10^5) + (2 \times 10^4) + (5 \times 10^3)$
d. $(4 \times 10^5) + (6 \times 10^3) + (2 \times 10^2) + (2 \times 10) + (8 \times 1)$

Set 21

1. Find a base-ten numeral for the number.

a. 33_{five} 18 **b.** 101_{five} 26 **c.** 141_{five} 46 **d.** 43_{five} 23 **e.** 434_{five} 119 **f.** 322_{five} 87

2. Copy the number-line picture below and complete the labeling by writing base-ten numerals.



3. Copy and complete: $233_{five} = (2 \times \underset{25}{\_?\_}) + (\underset{3}{\_?\_} \times 5) + (3 \times 1)$

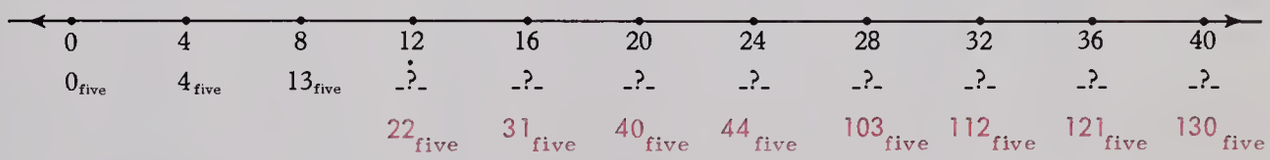
Set 22

1. Write a base-five numeral for the number.

a. 19 34_{five} **b.** 26 101_{five} **c.** 43 133_{five} **d.** 52 202_{five} **e.** 60 220_{five} **f.** 73 243_{five}

g. 79 304_{five} **h.** 86 321_{five} **i.** 95 340_{five} **j.** 107 412_{five} **k.** 112 422_{five} **l.** 124 444_{five}

2. Copy the number-line picture below and complete the labeling by writing base-five numerals.



Set 23

1. Write a base-ten numeral for the number.

a. 251_{seven} 134 **b.** 13_{four} 7 **c.** 212_{seven} 107 **d.** 444_{five} 124 **e.** 123_{five} 38

f. 11_{seven} 8 **g.** 30_{five} 15 **h.** 40_{eight} 32 **i.** 200_{four} 32 **j.** 77_{eight} 63

2. Copy and complete.

a. $9 = \underset{21}{\_?\_}_{four} = \underset{14}{\_?\_}_{five} = \underset{12}{\_?\_}_{seven} = \underset{11}{\_?\_}_{eight}$

b. $27 = \underset{36}{\_?\_}_{seven} = \underset{33}{\_?\_}_{eight}$

c. $55 = \underset{313}{\_?\_}_{four} = \underset{106}{\_?\_}_{seven}$

d. $100 = \underset{202}{\_?\_}_{seven} = \underset{144}{\_?\_}_{eight}$

3. The numeral 222 is a base-$\underset{five}{\_?\_}$ numeral for the number *sixty-two*.

4. Copy and complete.

$627_{eight} = (6 \times 64) + (\underset{2}{\_?\_} \times 8) + (7 \times \underset{1}{\_?\_})$

Extra Examples

Set 24

Write T or F.

1. $3 \times 4 = 7$ F
2. $5 \times 12 = 60$ T
3. $30 < 4 \times 7$ F
4. $15 \div 15 = 1$ T
5. $30 - 6 > 23$ T
6. $40 \div 10 \neq 4$ F
7. $5 \times 25 > 150$ F
8. $36 - 13 < 30$ T
9. $7 \times 8 > 5 \times 11$ T
10. $2 \times 30 \neq 4 \times 15$ F
11. $23 + 37 = 120 \div 2$ T
12. $56 < 18 + 31 + 5$ F

Set 25

Add.

	a	b	c	d	e	f	g	h	i
1.	67	38	92	47	98	13	77	24	69
	33	55	13	16	62	49	85	18	58
	100	93	105	63	160	62	162	42	127
2.	286	638	139	903	883	507	667	118	346
	350	87	289	170	666	109	392	465	157
	636	725	428	1,073	1,549	616	1,059	583	503

Set 26

Add.

	a	b	c	d	e	f	g	h
1.	1,356	5,892	8,903	$4.58	$3.75	$84.23	$46.75	$125.78
	687	4,880	8,108	9.95	8.88	7.98	0.89	15.65
	2,043	10,772	17,011	$14.53	$12.63	$92.21	$47.64	$141.43
2.	4,682	6,450	2,987	$9.19	$4.62	$46.98	$59.60	$465.75
	3,196	125	1,648	8.92	3.99	8.65	29.99	126.89
	7,878	6,575	4,635	$18.11	$8.61	$55.63	$89.59	$592.64

Set 27

Solve.

1. $n = 46 + 19$ 65
2. $n = 67 + 40$ 107
3. $n = 13 + 128$ 141
4. $n = 678 + 15$ 693
5. $25 + 637 = n$ 662
6. $39 + 299 = n$ 338
7. $n = 46 - 31$ 15
8. $n = 389 - 299$ 90
9. $2{,}389 + 5{,}802 = n$ 8,191
10. $n = 2{,}881 - 1{,}893$ 988
11. $n = 23{,}603 + 5{,}950$ 29,553
12. $n = 18{,}000 - 594$ 17,406

Set 28

Add.

a	b	c	d	e	f	g	h
79	93	409	138	4,829	7,372	4,007	15,720
24	15	68	99	556	6,927	6,009	1,903
33	55	117	9	1,880	1,115	223	47,081
136	163	594	246	7,265	15,414	10,239	64,704

Set 29

Add.

a	b	c	d	e	f	g	h
$6.79	$7.75	$0.88	$80.25	$31.69	$428.50	$98.29	$775.25
4.50	0.95	7.77	4.50	5.50	7.75	5.93	230.10
3.50	1.25	0.25	7.98	17.10	56.23	0.68	18.75
$14.79	$ 9.95	$ 8.90	$ 92.73	$54.29	$492.48	$104.90	$1,024.10

Set 30

Subtract.

	a	b	c	d	e	f	g	h
1.	293	321	5,720	4,373	7,320	929	23,637	56,942
	88	106	808	777	115	858	1,108	47,008
	205	215	4,912	3,596	7,205	71	22,529	9,934
2.	1,640	4,325	6,287	12,450	26,876	48,261	68,726	82,638
	767	497	1,460	6,785	7,467	8,472	49,809	81,759
	873	3,828	4,827	5,665	19,409	39,789	18,917	879

Set 31

Subtract.

	a	b	c	d	e	f	g	h
1.	$2.94	$394.80	$12.13	$13.28	$10.60	$12.28	$55.73	$920.50
	1.56	43.85	7.88	11.66	0.87	0.99	16.92	78.56
	$1.38	$350.95	$ 4.25	$ 1.62	$ 9.73	$11.29	$38.81	$841.94
2.	$6.75	$14.22	$22.98	$35.76	$77.65	$50.28	$167.82	$857.37
	0.98	8.69	16.29	0.88	76.87	49.79	159.78	96.79
	$ 5.77	$ 5.53	$ 6.69	$34.88	$ 0.78	$ 0.49	$ 8.04	$ 760.58

Set 32

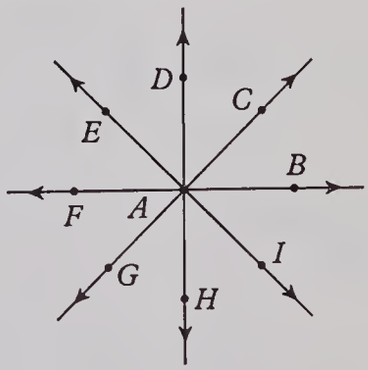
Find the number represented by the letter.

1. $n - 38 = 95$ 133
2. $x + 18 = 79$ 61
3. $x + 329 = 400$ 71
4. $650 + n = 724$ 74
5. $930 = x - 3{,}657$ 4,587
6. $n + 45{,}829 = 50{,}000$ ∨
7. $x - 17 = 1{,}107$ ∨ 1,124
8. $n + 408 = 1{,}265$ 857
9. $n = 285 + 9{,}389 + 555$ ∧ 10,229
10. $n = 4{,}729 + 5{,}550 + 13{,}005$ ∧ 23,284
11. $4{,}820 - x = 3{,}880$ 940
12. $16{,}405 + 2{,}650 + 9{,}789 = n$ ∧ 28,844

Set 33

Refer to the figure shown and indicate whether the angle named appears to be an acute angle, an obtuse angle, or a right angle.

1. $\angle BAD$ right
2. $\angle EAF$ acute
3. $\angle EAB$ obtuse
4. $\angle DAE$ acute
5. $\angle EAC$ right
6. $\angle GAH$ right
7. $\angle FAC$ obtuse
8. $\angle GAI$ right
9. $\angle BAI$ acute
10. $\angle FAI$ obtuse
11. $\angle HAB$ right
12. $\angle GAF$ acute
13. $\angle DAF$ right
14. $\angle GAB$ obtuse
15. $\angle CAI$ right

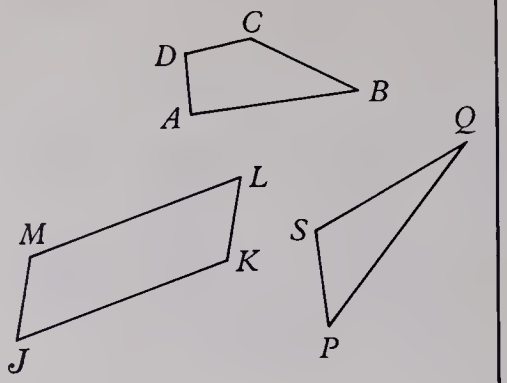


Extra Examples

Set 34

Write *T* or *F*. Refer to the figures shown.

1. Figure *ABCD* is a parallelogram. F
2. Figure *JKLM* is a rhombus. F
3. Figure *JKLM* is a parallelogram. T
4. Figure *PQS* is a scalene triangle. T
5. Figure *PQS* is an equilateral triangle. F



Set 35

Solve.

1. $n = 3 \times (7 + 2)$ 27
2. $n = 6 \times (10 + 5)$ 90
3. $x = 10 \times (13 + 7)$ 200
4. $x = 5 \times (30 + 6)$ 180
5. $x = 21 \times (5 + 15)$ 420
6. $n = 50 \times (8 + 12)$ 1,000
7. $n = 8 \times (13 + 12)$ 200
8. $9 \times (5 + 25) = n$ 270
9. $12 \times (4 + 8) = x$ 144

Copy and complete.

10. $17 \times (56 + 14) = (17 \times 56) + (\_?\_ \times 14)$ 17
11. $(6 \times 34) + (6 \times 15) = \_?\_ \times (34 + 15)$ 6

Set 36

Solve.

1. $n = 6 \times 400$ 2,400
2. $n = 70{,}000 \times 9$ 630,000
3. $n = 20 \times 500$ 10,000
4. $n = 26 \times 4{,}000$ 104,000
5. $300 \times 1{,}000 = n$ 300,000
6. $800 \times 400 = n$ 320,000

Copy and complete.

7. $50 \times 70 = (5 \times 10) \times (\_?\_ \times 10) = (5 \times 7) \times (10 \times \_?\_)$ 7, 10
8. $600 \times 30 = (6 \times 3) \times (\_?\_ \times 10) = 18 \times \_?\_$ 100, 1,000

Set 37

Multiply.

	a	b	c	d	e	f	g	h	i
	38	27	55	38	98	88	92	21	37
	9	4	23	11	14	21	83	73	44
	342	108	1,265	418	1,372	1,848	7,636	1,533	1,628

Set 38

Multiply.

	a	b	c	d	e	f	g	h	i
1.	38	94	17	83	77	25	83	15	46
	17	13	88	21	66	53	16	75	27
	646	1,222	1,496	1,743	5,082	1,325	1,328	1,125	1,242
2.	29	36	20	45	78	36	29	98	87
	64	57	70	60	59	73	79	56	68
	1,856	2,052	1,400	2,700	4,602	2,628	2,291	5,488	5,916

Set 39

Multiply.

	a	b	c	d	e	f	g	h
1.	304	358	839	109	2,341	8,482	985	$46.35
	× 2	× 7	× 17	× 39	× 8	× 26	× 55	× 23
	608	2,506	14,263	4,251	18,728	220,532	54,175	$1,066.05
2.	324	731	601	535	629	$5.95	$45.98	$47.33
	× 67	× 213	× 219	× 165	× 843	× 32	× 354	× 155
	21,708	155,703	131,619	88,275	530,247	$190.40	$16,276.92	$7,336.15

Set 40

Multiply.

a	b	c	d	e	f	g	h
5,748	6,118	4,645	1,119	7,004	5,503	995	14,849
× 86	× 223	× 372	× 995	× 809	× 666	× 586	× 25
494,328	1,364,314	1,727,940	1,113,405	5,666,236	3,664,998	583,070	371,225

Set 41

Solve.

1. $n \div 36 = 367$ 13,212
2. $n \div 108 = 392$ 42,336
3. $x \div 46 = 272$ 12,512
4. $x = 47 \times 938$ 44,086
5. $831 \times 329 = x$ 273,399
6. $n = 306 \times 278$ 85,068
7. $8{,}339 = x \div 345$ 2,876,955
8. $x \div 98 = 5{,}827$ 571,046
9. $n = 6{,}748 \times 453$ 3,056,844

Set 42

1. Express the number as a product of primes.

a. 26 2×13 b. 55 5×11 c. 40 $2 \times 2 \times 2 \times 5$ d. 27 $3 \times 3 \times 3$ e. 188 $2 \times 2 \times 47$ f. 240 $2 \times 2 \times 2 \times 2 \times 3 \times 5$ g. 625 $5 \times 5 \times 5 \times 5$

2. List within braces the prime factors of

a. 14. $\{2, 7\}$ b. 56. $\{2, 7\}$ c. 18. $\{2, 3\}$ d. 68. $\{2, 17\}$ e. 200. $\{2, 5\}$ f. 231. $\{3, 7, 11\}$ g. 256. $\{2\}$

Set 43

Copy and complete.

1. $200 = 2 \times 2 \times 2 \times 5 \times \_^{5} = 2^3 \times 5^{\_^{2}}$
2. $3 \times 3 \times 5 \times 11 \times 11 = 3^2 \times 5 \times \_^{11} = 9 \times 5 \times \_^{121}$

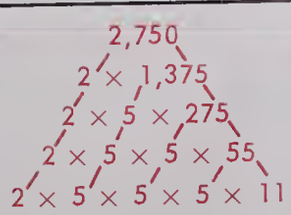
3. Make a factor tree for 2,750 and then express 2,750 as a product of primes using exponent forms.

4. Use exponent forms to express $3 \times 3 \times 3 \times 3 \times 7 \times 7 \times 13 \times 13$. $3^4 \times 7^2 \times 13^2$

5. Write the standard numeral for the number.

a. $2 \times 2 \times 2 \times 2 \times 2 \times 5$ 160
b. $7 \times 7 \times 7 \times 17 \times 17$ 99,127
c. $5^3 \times 7 \times 23$ 20,125
d. $2^4 \times 7^2 \times 11$ 8,624

3.



$2{,}750 = 2 \times 5^3 \times 11$

Extra Examples

Set 44

a	b	c	d	e	f
437, R0 7)3,059	239, R0 9)2,151	2,413, R0 6)14,478	3,114, R2 7)21,800	4,537, R2 8)36,296	6,492, R0 9)58,428

Set 45

	a	b	c	d	e	f
1.	424, R0 12)5,088	147, R0 25)3,675	523, R0 48)25,104	724, R18 32)23,186	729, R5 27)19,688	278, R4 64)17,796
2.	276, R0 18)4,968	178, R24 32)5,720	861, R5 45)38,750	625, R0 38)23,750	273, R12 56)15,300	148, R0 75)11,100

Set 46

	a	b	c	d	e	f
1.	54, R0 212)11,448	75, R21 134)10,071	87, R0 536)46,632	8, R4 987)7,900	82, R100 545)44,790	84, R0 376)31,584
2.	43, R0 166)7,138	56, R200 263)14,928	26, R0 584)15,184	46, R18 847)38,980	92, R26 265)24,406	65, R0 492)31,980

Set 47

a	b	c	d	e	f
231, R27 108)24,975	184, R0 223)41,032	197, R20 193)38,041	229, R123 347)79,586	238, R22 681)162,100	656, R0 274)179,744

Set 48

Find the arithmetic mean for the measures.

1. 35; 32; 33; 34; 31 33
2. 18; 19; 20; 19; 21; 17 19
3. 8; 10; 22; 17; 15; 24 16
4. 631; 626; 621 626
5. 1,232; 1,239; 1,245; 1,228 1,236
6. 2,642; 2,650; 2,640; 2,648; 2,645 2,645

Set 49

1. What is the mode for 24, 25, 23, 23, 24, 27, 21, 24, and 22? 24
2. Find the mean and the median for the measures.

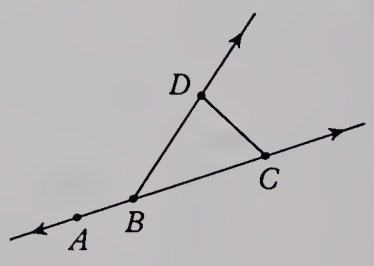
a. 65; 66; 63; 64; 63; 69; 61; 67; 66; 66; 65 mean: 65 ; median: 65

b. 103; 109; 113; 107; 110; 106 mean: 108 ; median 108

3. What is the mode for the measures in Ex. 2a? 66

Set 50

Refer to the figure shown.



1. Name three labeled segments which are subsets of
a. $\overleftrightarrow{AC}$. $\overline{AC}$, $\overline{AB}$, $\overline{BC}$ b. $\triangle BCD$. $\overline{BC}$, $\overline{CD}$, $\overline{BD}$
2. Name two labeled segments which are subsets of angle DBC. $\overline{BD}$, $\overline{BC}$
3. $\{B\_?\_\}$ is a 1-element subset of both $\overrightarrow{BC}$ and $\overrightarrow{BA}$.
4. Is ray BC a subset of ray AC? Yes

Set 51

Copy and complete.

	a	b	c	d	e	f
1.	$\frac{1}{2} = \frac{?}{8}$ 4	$\frac{1}{3} = \frac{?}{12}$ 4	$\frac{1}{5} = \frac{?}{20}$ 4	$\frac{1}{4} = \frac{?}{24}$ 6	$\frac{1}{7} = \frac{?}{21}$ 3	$\frac{1}{6} = \frac{3}{?}$ 18
2.	$\frac{2}{3} = \frac{?}{15}$ 10	$\frac{3}{4} = \frac{?}{12}$ 9	$\frac{2}{5} = \frac{?}{15}$ 6	$\frac{4}{7} = \frac{?}{28}$ 16	$\frac{5}{6} = \frac{?}{12}$ 10	$\frac{4}{5} = \frac{16}{?}$ 20
3.	$\frac{1}{7} = \frac{?}{35}$ 5	$\frac{1}{3} = \frac{?}{30}$ 10	$\frac{2}{3} = \frac{20}{?}$ 30	$4 = \frac{?}{1}$ 4	$7 = \frac{7}{?}$ 1	$15 = \frac{?}{1}$ 15

Set 52

Rename with a fraction in simplest form.

	a	b	c	d	e	f	g	h	i	j
1.	$\frac{8}{10}$ $\frac{4}{5}$	$\frac{5}{25}$ $\frac{1}{5}$	$\frac{8}{16}$ $\frac{1}{2}$	$\frac{4}{10}$ $\frac{2}{5}$	$\frac{3}{6}$ $\frac{1}{2}$	$\frac{4}{16}$ $\frac{1}{4}$	$\frac{5}{15}$ $\frac{1}{3}$	$\frac{10}{16}$ $\frac{5}{8}$	$\frac{7}{14}$ $\frac{1}{2}$	$\frac{20}{35}$ $\frac{4}{7}$
2.	$\frac{10}{30}$ $\frac{1}{3}$	$\frac{2}{10}$ $\frac{1}{5}$	$\frac{10}{15}$ $\frac{2}{3}$	$\frac{20}{25}$ $\frac{4}{5}$	$\frac{32}{100}$ $\frac{8}{25}$	$\frac{6}{30}$ $\frac{1}{5}$	$\frac{35}{50}$ $\frac{7}{10}$	$\frac{3}{15}$ $\frac{1}{5}$	$\frac{28}{88}$ $\frac{7}{22}$	$\frac{36}{48}$ $\frac{3}{4}$
3.	$\frac{2}{6}$ $\frac{1}{3}$	$\frac{14}{16}$ $\frac{7}{8}$	$\frac{25}{30}$ $\frac{5}{6}$	$\frac{6}{16}$ $\frac{3}{8}$	$\frac{10}{25}$ $\frac{2}{5}$	$\frac{6}{10}$ $\frac{3}{5}$	$\frac{2}{16}$ $\frac{1}{8}$	$\frac{22}{44}$ $\frac{1}{2}$	$\frac{44}{200}$ $\frac{11}{50}$	$\frac{150}{225}$ $\frac{2}{3}$

Set 53

Solve. Express answers in simplest form.

1. $n = \frac{2}{11} + \frac{5}{11}$ $\frac{7}{11}$
2. $\frac{21}{50} + \frac{19}{50} = x$ $\frac{4}{5}$
3. $\frac{11}{23} + \frac{12}{23} = n$ 1
4. $\frac{3}{5} + \frac{1}{5} = n$ $\frac{4}{5}$
5. $n = \frac{6}{15} + \frac{9}{15}$ 1
6. $n = \frac{42}{80} + \frac{13}{80}$ $\frac{11}{16}$
7. $x = \frac{15}{30} + \frac{5}{30}$ $\frac{2}{3}$
8. $n = \frac{6}{10} + \frac{4}{10}$ 1
9. $\frac{7}{100} + \frac{33}{100} = x$ $\frac{2}{5}$

Set 54

Copy and complete.

1. $\frac{1}{10} + \frac{?}{10} = \frac{9}{10}$ 8
2. $\frac{4?}{15} + \frac{9}{15} = \frac{13}{15}$ 4?
3. $\frac{7}{40} + \frac{?}{40} = \frac{31}{40}$ 24
4. $\frac{5}{12} + \frac{?}{12} = \frac{11}{12}$ 6
5. $\frac{9}{20} + \frac{?}{20} = \frac{11}{20}$ 2
6. $\frac{32?}{50} + \frac{17}{50} = \frac{49}{50}$ 32?

Set 55

Find the L.C.M. of

1. 4 and 8. 8
2. 5 and 15. 15
3. 9 and 12. 36
4. 10 and 15. 30
5. 3 and 7. 21

Solve. Express answers in simplest form.

6. $n = \frac{1}{5} + \frac{1}{3}$ $\frac{8}{15}$
7. $n = \frac{3}{4} + \frac{5}{6}$ $1\frac{7}{12}$
8. $x = \frac{7}{8} + \frac{2}{3}$ $1\frac{13}{24}$
9. $x = \frac{3}{10} + \frac{2}{5}$ $\frac{7}{10}$
10. $n = \frac{7}{30} + \frac{7}{10}$ $\frac{14}{15}$
11. $n = \frac{1}{9} + \frac{2}{5}$ $\frac{23}{45}$
12. $n = \frac{1}{7} + \frac{1}{5}$ $\frac{12}{35}$
13. $\frac{2}{3} + \frac{1}{5} = x$ $\frac{13}{15}$
14. $\frac{7}{20} + \frac{3}{10} = x$ $\frac{13}{20}$
15. $n = \frac{1}{10} + \frac{2}{7}$ $\frac{27}{70}$
16. $x = \frac{2}{3} + \frac{3}{4}$ $1\frac{5}{12}$
17. $n = \frac{3}{20} + \frac{1}{4}$ $\frac{2}{5}$
18. $n = \frac{4}{9} + \frac{1}{3}$ $\frac{7}{9}$
19. $\frac{1}{7} + \frac{1}{9} = n$ $\frac{16}{63}$
20. $x = \frac{2}{5} + \frac{3}{4}$ $1\frac{3}{20}$

Extra Examples

Set 56

Solve. Express answers in simplest form.

1. $n = \frac{7}{10} - \frac{4}{10}$ $\frac{3}{10}$
2. $x = \frac{5}{3} - \frac{2}{3}$ 1
3. $\frac{5}{8} - \frac{1}{5} = x$ $\frac{17}{40}$
4. $\frac{5}{7} - \frac{3}{14} = n$ $\frac{1}{2}$
5. $n = 1 - \frac{7}{8}$ $\frac{1}{8}$
6. $n = \frac{9}{10} - \frac{5}{10}$ $\frac{2}{5}$
7. $\frac{2}{3} - \frac{1}{7} = x$ $\frac{11}{21}$
8. $\frac{3}{4} - \frac{1}{3} = n$ $\frac{5}{12}$
9. $x = \frac{5}{8} - \frac{1}{8}$ $\frac{1}{2}$
10. $n = \frac{3}{4} - \frac{1}{5}$ $\frac{11}{20}$
11. $n = \frac{9}{5} - \frac{6}{7}$ $\frac{33}{35}$
12. $2 - \frac{1}{2} = x$ $1\frac{1}{2}$
13. $\frac{5}{12} - \frac{1}{3} = n$ $\frac{1}{12}$
14. $\frac{15}{16} - \frac{7}{8} = x$ $\frac{1}{16}$
15. $n = \frac{7}{9} - \frac{2}{3}$ $\frac{1}{9}$
16. $x = \frac{9}{10} - \frac{8}{15}$ $\frac{11}{30}$

Set 57

Solve. Express answers in simplest form.

1. $n - \frac{3}{12} = \frac{5}{12}$ $\frac{2}{3}$
2. $\frac{2}{3} = x - \frac{1}{7}$ $\frac{17}{21}$
3. $n + \frac{2}{7} = \frac{5}{6}$ $\frac{23}{42}$
4. $n + \frac{3}{10} = \frac{8}{10}$ $\frac{1}{2}$
5. $x + \frac{7}{12} = 1\frac{5}{12}$
6. $\frac{5}{4} = x + \frac{2}{3}$ $\frac{7}{12}$
7. $\frac{3}{4} - x = \frac{1}{2}$ $\frac{1}{4}$
8. $n - \frac{1}{3} = \frac{1}{5}$ $\frac{8}{15}$
9. $\frac{2}{11} + x = \frac{1}{2}$ $\frac{7}{22}$
10. $x - \frac{5}{6} = \frac{1}{6}$ 1
11. $\frac{3}{10} + x = \frac{1}{3}$ $\frac{1}{30}$
12. $\frac{1}{6} = \frac{7}{8} - n$ $\frac{17}{24}$

Set 58

Add. Express answers in simplest form.

	a	b	c	d	e	f	g	h	i
1.	$3\frac{3}{4}$	$5\frac{1}{7}$	$3\frac{2}{9}$	$7\frac{6}{13}$	5	$6\frac{7}{8}$	$4\frac{1}{8}$	$27\frac{9}{20}$	$56\frac{3}{25}$
	$2\frac{1}{4}$ 6	$17\frac{4}{7}$ $22\frac{5}{7}$	$5\frac{4}{9}$ $8\frac{2}{3}$	$3\frac{7}{13}$ 11	$3\frac{2}{3}$ $8\frac{2}{3}$	17 $23\frac{7}{8}$	$\frac{5}{8}$ $4\frac{3}{4}$	$35\frac{2}{20}$ $62\frac{11}{20}$	$16\frac{2}{25}$ $72\frac{1}{5}$
2.	$6\frac{1}{6}$	$9\frac{3}{8}$	$8\frac{5}{12}$	$9\frac{4}{15}$	16	$21\frac{7}{10}$	$19\frac{1}{5}$	$76\frac{3}{10}$	$125\frac{1}{20}$
	$7\frac{1}{6}$ $13\frac{1}{3}$	$15\frac{5}{8}$ 25	$4\frac{1}{12}$ $12\frac{1}{2}$	$4\frac{1}{15}$ $13\frac{1}{3}$	$5\frac{1}{7}$ $21\frac{1}{7}$	$54\frac{1}{10}$ $75\frac{4}{5}$	$20\frac{3}{5}$ $39\frac{4}{5}$	$25\frac{3}{10}$ $101\frac{3}{5}$	$250\frac{9}{20}$ $375\frac{1}{2}$

Set 59

Add. Express answers in simplest form.

	a	b	c	d	e	f	g	h	i
1.	$3\frac{1}{2}$	$5\frac{2}{3}$	$4\frac{3}{5}$	$5\frac{6}{7}$	$9\frac{2}{11}$	$\frac{3}{5}$	$5\frac{2}{3}$	$18\frac{9}{10}$	$27\frac{5}{12}$
	$4\frac{1}{3}$ $7\frac{5}{6}$	$6\frac{1}{5}$ $11\frac{13}{15}$	$9\frac{2}{7}$ $13\frac{31}{35}$	$\frac{3}{4}$ $6\frac{17}{28}$	$\frac{1}{2}$ $9\frac{15}{22}$	$4\frac{1}{2}$ $5\frac{1}{10}$	$7\frac{3}{4}$ $13\frac{5}{12}$	$14\frac{4}{5}$ $33\frac{7}{10}$	$29\frac{7}{8}$ $57\frac{7}{24}$
2.	$6\frac{5}{6}$	$2\frac{1}{3}$	$\frac{9}{11}$	$\frac{2}{5}$	$3\frac{6}{25}$	$17\frac{5}{8}$	$14\frac{6}{7}$	$25\frac{1}{2}$	$34\frac{1}{5}$
	$7\frac{3}{4}$ $14\frac{7}{12}$	$5\frac{9}{10}$ $8\frac{7}{30}$	$\frac{4}{5}$ $1\frac{34}{35}$	$\frac{7}{9}$ $1\frac{8}{45}$	$\frac{4}{5}$ $4\frac{1}{25}$	$39\frac{2}{3}$ $57\frac{7}{24}$	$\frac{13}{35}$ $15\frac{8}{35}$	$67\frac{2}{7}$ $92\frac{11}{14}$	$5\frac{8}{9}$ $40\frac{4}{45}$

Set 60

Solve. Express answers in simplest form.

1. $n - 2 = 5\frac{6}{7}$ $7\frac{6}{7}$
2. $x = 19\frac{1}{3} + 235\frac{1}{2}$ $254\frac{5}{6}$
3. $16\frac{1}{2} = n - 12\frac{3}{4}$ $29\frac{1}{4}$
4. $5\frac{7}{8} + 4\frac{5}{6} = x$ $10\frac{17}{24}$
5. $n - \frac{2}{3} = 6\frac{1}{2}$ $7\frac{1}{6}$
6. $7\frac{4}{9} = n - 5\frac{1}{3}$ $12\frac{7}{9}$
7. $x = 125\frac{1}{3} + 63\frac{1}{2}$ $188\frac{5}{6}$
8. $n - 25\frac{1}{2} = 8\frac{7}{10}$ $34\frac{1}{5}$
9. $7\frac{4}{5} + 16\frac{1}{2} = x$ $24\frac{3}{10}$
10. $29\frac{1}{4} = n - 125\frac{3}{8}$ $154\frac{5}{8}$
11. $n = 264\frac{3}{4} + 78\frac{1}{8}$ $342\frac{7}{8}$
12. $x - 19\frac{7}{10} = 6\frac{13}{15}$ $26\frac{17}{30}$

Set 61

Subtract. Express answers in simplest form.

a	b	c	d	e	f	g	h	i
$6\frac{2}{3}$	$7\frac{9}{10}$	$27\frac{5}{8}$	8	$4\frac{4}{5}$	27	$33\frac{5}{7}$	$24\frac{5}{6}$	37
$3\frac{1}{3}$ $3\frac{1}{3}$	$4\frac{5}{10}$ $3\frac{2}{5}$	$19\frac{3}{8}$ $8\frac{1}{4}$	$5\frac{2}{3}$ $2\frac{1}{3}$	$\frac{1}{5}$ $4\frac{3}{5}$	$8\frac{9}{10}$ $18\frac{1}{10}$	$6\frac{3}{7}$ $27\frac{2}{7}$	$18\frac{1}{6}$ $6\frac{2}{3}$	$29\frac{1}{12}$ $7\frac{11}{12}$

Set 62

Subtract. Express answers in simplest form.

	a	b	c	d	e	f	g	h	i
1.	$7\frac{1}{2}$	$9\frac{4}{5}$	$7\frac{2}{3}$	$21\frac{1}{2}$	$3\frac{1}{4}$	$67\frac{1}{2}$	$29\frac{4}{7}$	$435\frac{1}{2}$	$528\frac{7}{9}$
	$6\frac{1}{3}$ $1\frac{1}{6}$	$3\frac{1}{3}$ $6\frac{7}{15}$	$5\frac{1}{5}$ $2\frac{7}{15}$	$19\frac{2}{5}$ $2\frac{1}{10}$	$1\frac{7}{10}$ $1\frac{11}{20}$	$13\frac{3}{4}$ $53\frac{3}{4}$	$16\frac{2}{3}$ $12\frac{19}{21}$	$219\frac{1}{4}$ $216\frac{1}{4}$	$39\frac{1}{3}$ $489\frac{4}{9}$
2.	$8\frac{2}{5}$	$7\frac{6}{7}$	$8\frac{1}{4}$	$9\frac{2}{7}$	$43\frac{1}{5}$	$61\frac{1}{3}$	$30\frac{1}{2}$	$89\frac{7}{12}$	$230\frac{1}{5}$
	$5\frac{1}{3}$ $3\frac{1}{15}$	$3\frac{1}{3}$ $4\frac{11}{21}$	$\frac{2}{5}$ $7\frac{17}{20}$	$\frac{3}{4}$ $8\frac{15}{28}$	$19\frac{2}{3}$ $23\frac{8}{15}$	$7\frac{2}{5}$ $53\frac{14}{15}$	$16\frac{2}{3}$ $13\frac{5}{6}$	$33\frac{1}{4}$ $56\frac{1}{3}$	$98\frac{1}{9}$ $132\frac{4}{45}$

Set 63

Solve. Express answers in simplest form.

1. $n + 5\frac{6}{7} = 7\frac{1}{2}$ $1\frac{9}{14}$
2. $x + 3\frac{1}{3} = 5$ $1\frac{2}{3}$
3. $9\frac{1}{4} = n + 6\frac{3}{5}$ $2\frac{13}{20}$
4. $33\frac{1}{3} = n + 5\frac{1}{8}$ $28\frac{5}{24}$
5. $20 = x + 19\frac{2}{3}$ $\frac{1}{3}$
6. $x = 16\frac{5}{6} - 6\frac{7}{8}$ $9\frac{23}{24}$
7. $x + 45 = 62\frac{1}{2}$ $17\frac{1}{2}$
8. $15\frac{1}{5} - 3\frac{1}{7} = x$ $12\frac{2}{35}$
9. $35\frac{3}{4} = n + 21\frac{3}{8}$ $14\frac{3}{8}$

Set 64

Add. Express answers in simplest form.

a	b	c	d	e	f	g	h	i
$6\frac{1}{5}$	$13\frac{4}{9}$	35	$1\frac{5}{12}$	14	$15\frac{2}{11}$	$6\frac{3}{4}$	$40\frac{1}{5}$	$285\frac{1}{9}$
15	$7\frac{2}{9}$	$16\frac{1}{8}$	$\frac{7}{12}$	29	$13\frac{3}{11}$	$8\frac{3}{4}$	$21\frac{4}{5}$	$17\frac{8}{9}$
$7\frac{2}{5}$ $28\frac{3}{5}$	$6\frac{1}{9}$ $26\frac{7}{9}$	$17\frac{1}{8}$ $68\frac{1}{4}$	3 5	$62\frac{1}{2}$ $105\frac{1}{2}$	$29\frac{10}{11}$ $58\frac{4}{11}$	16 $31\frac{1}{2}$	$16\frac{3}{5}$ $78\frac{3}{5}$	$535\frac{7}{9}$ $838\frac{7}{9}$

Set 65

Add. Express answers in simplest form.

	a	b	c	d	e	f	g	h	i
1.	$6\frac{1}{2}$	$7\frac{1}{5}$	$5\frac{3}{4}$	$3\frac{1}{2}$	$7\frac{3}{7}$	17	$29\frac{3}{4}$	$4\frac{2}{9}$	$54\frac{1}{2}$
	$3\frac{1}{3}$	$2\frac{3}{4}$	$16\frac{2}{5}$	$6\frac{1}{12}$	$6\frac{2}{3}$	$9\frac{3}{11}$	$16\frac{1}{2}$	$18\frac{1}{3}$	$17\frac{3}{7}$
	$4\frac{1}{2}$ $14\frac{1}{3}$	9 $18\frac{19}{20}$	$3\frac{7}{10}$ $25\frac{17}{20}$	$4\frac{1}{4}$ $13\frac{5}{6}$	$5\frac{1}{3}$ $19\frac{3}{7}$	$4\frac{10}{11}$ $31\frac{2}{11}$	$7\frac{1}{3}$ $53\frac{7}{12}$	$29\frac{3}{5}$ $52\frac{7}{45}$	$5\frac{2}{5}$ $77\frac{23}{70}$

Solve. Express answers in simplest form.

	a	b	c
2.	$n = 4\frac{3}{5} + 6\frac{1}{2} + 12\frac{9}{10}$ 24	$5\frac{3}{8} + 14\frac{2}{3} + 10\frac{7}{12} = x$ $30\frac{5}{8}$	$34\frac{5}{6} + 12\frac{3}{4} + 10\frac{1}{2} = n$ $58\frac{1}{12}$
3.	$x = 3\frac{2}{3} + 18\frac{3}{4} + 15\frac{1}{2}$ $37\frac{11}{12}$	$15\frac{9}{10} + 16\frac{2}{3} + 5\frac{4}{5} = n$ $38\frac{11}{30}$	$x = 12\frac{5}{8} + 29\frac{7}{16} + 18\frac{3}{4}$ $60\frac{13}{16}$

Set 66

Rename with a fraction in simplest form.

	a	b	c	d	e	f	g	h	i	j	k
1.	0.3 $\frac{3}{10}$	0.4 $\frac{2}{5}$	0.8 $\frac{4}{5}$	0.12 $\frac{3}{25}$	0.48 $\frac{12}{25}$	0.06 $\frac{3}{50}$	0.004 $\frac{1}{250}$	0.0002 $\frac{1}{5,000}$	0.36 $\frac{9}{25}$	0.085 $\frac{17}{200}$	0.075 $\frac{3}{40}$
2.	0.2 $\frac{1}{5}$	0.6 $\frac{3}{5}$	0.5 $\frac{1}{2}$	0.05 $\frac{1}{20}$	0.96 $\frac{24}{25}$	0.25 $\frac{1}{4}$	0.025 $\frac{1}{40}$	0.0025 $\frac{1}{400}$	0.72 $\frac{18}{25}$	0.124 $\frac{31}{250}$	0.975 $\frac{39}{40}$

Rename with a decimal.

	a	b	c	d	e	f	g	h	i	j	k
3.	$\frac{5}{10}$ 0.5	$\frac{9}{10}$ 0.9	$\frac{17}{100}$ 0.17	$\frac{53}{100}$ 0.53	$\frac{31}{1,000}$ 0.031	$\frac{3}{1,000}$ 0.003	$\frac{7}{50}$ 0.14	$\frac{13}{500}$ 0.026	$\frac{3}{20}$ 0.15	$\frac{21}{25}$ 0.84	$\frac{13}{250}$ 0.052
4.	$\frac{2}{5}$ 0.4	$\frac{3}{4}$ 0.75	$\frac{17}{20}$ 0.85	$\frac{11}{20}$ 0.55	$\frac{9}{1,000}$ 0.009	$\frac{23}{1,000}$ 0.023	$\frac{7}{10}$ 0.7	$\frac{6}{250}$ 0.024	$\frac{9}{25}$ 0.36	$\frac{49}{50}$ 0.98	$\frac{3}{125}$ 0.024

Set 67

Rename with a mixed form.

	a	b	c	d	e	f	g	h	i	j	k
1.	2.5 $2\frac{1}{2}$	5.8 $5\frac{4}{5}$	$7\frac{37}{100}$ 7.37	$5\frac{3}{100}$ 5.03	$7\frac{1}{1,000}$ 7.001	$18\frac{2}{5}$ 18.4	$19\frac{1}{20}$ 19.05	$25\frac{4}{5}$ 25.8	$3\frac{3}{200}$ 3.015	$7\frac{19}{200}$ 7.095	$23\frac{1}{4}$ 23.25

Rename with a decimal.

	a	b	c	d	e	f	g	h	i	j	k
2.	$\frac{17}{10}$ 1.7	$\frac{27}{10}$ 2.7	$\frac{33}{10}$ 3.3	$\frac{107}{100}$ 1.07	$18\frac{13}{100}$ 18.13	$9\frac{7}{10}$ 9.7	$\frac{3}{2}$ 1.5	$\frac{5}{4}$ 1.25	$17\frac{2}{50}$ 17.04	$30\frac{1}{2}$ 30.5	$\frac{231}{100}$ 2.31
3.	$6\frac{3}{1,000}$ 6.003	$7\frac{3}{500}$ 7.006	$18\frac{2}{5}$ 18.4	$19\frac{3}{5}$ 19.6	$27\frac{9}{100}$ 27.09	$8\frac{91}{500}$ 8.182	$4\frac{4}{5}$ 4.8	$7\frac{37}{50}$ 7.74	$18\frac{9}{100}$ 18.09	$5\frac{3}{10,000}$ 5.0003	$8\frac{7}{5,000}$ 8.0014

Set 68

Round to the nearest whole number.

	a	b	c	d	e	f	g	h	i
1.	23.4 23	17.8 18	5.52 6	16.1 16	3.04 3	0.9 1	$16\frac{2}{3}$ 17	$7\frac{4}{5}$ 8	$19\frac{41}{80}$ 20

Round to the nearest tenth.

	a	b	c	d	e	f	g	h	i
2.	0.47 0.5	0.08 0.1	1.71 1.7	3.06 3.1	5.99 6.0 (or 6)	0.412 0.4	0.253 0.3	6.84 6.8	7.90 7.9
3.	0.08 0.1	13.09 13.1	14.66 14.7	7.37 7.4	8.95 9.0 (or 9)	8.05 8.1	0.71 0.7	15.085 15.1	6.109 6.1

Round to the nearest hundredth.

	a	b	c	d	e	f	g	h	i
4.	1.005 1.01	3.008 3.01	7.991 7.99	0.008 0.01	3.771 3.77	0.006 0.01	8.925 8.93	2.3055 2.31	6.0082 6.01

Round to the nearest thousandth.

	a	b	c	d	e	f	g	h	i
5.	0.0037 0.004	0.0009 0.001	5.6134 5.613	7.6815 7.682	0.0083 0.008	0.0055 0.006	7.6013 7.601	9.4162 9.416	8.9005 8.901

Set 69

Round to the nearest 10¢; to the nearest dollar.

	a	b	c	d	e	f	g	h	i
1.	$6.90 $7.00	$3.80 $4.00	$2.20 $2.00	$3.10 $3.00	$14.10 $14.00	$56.10 $56.00	$60.20 $60.00	$25.30 $25.00	$195.90 $196.00
	$6.85	$3.79	$2.17	$3.12	$14.09	$56.05	$60.19	$25.25	$195.91
2.	$5.55	$0.86	$8.04	$7.12	$25.33	$72.95	$22.69	$49.61	$200.75
	$5.60	$0.90	$8.00	$7.10	$25.30	$73.00	$22.70	$49.70	$200.80
	$6.00	$1.00	$8.00	$7.00	$25.00	$73.00	$23.00	$50.00	$201.00

Set 70

Solve. Express answers with mixed forms.

1. $n = 567 \div 60$ $9\frac{9}{20}$
2. $n = 15 \div 12$ $1\frac{1}{4}$
3. $x = 1{,}780 \div 360$ $4\frac{17}{18}$
4. $x = 79 \div 16$ $4\frac{15}{16}$
5. $n = 893 \div 16$ $55\frac{13}{16}$
6. $n = 4{,}560 \div 2{,}000$ $2\frac{7}{25}$

Set 71

Copy and complete.

1. 13 yd. = 39 ft.
2. 8 ft. = 96 in.
3. 33 ft. = 11 yd.
4. 5 bu. = 20 pk.
5. 48 qt. = 12 gal.
6. 91 da. = 13 wk.
7. 7 T. = 14,000 lb.
8. 3 gal. = 24 pt.
9. 240 in. = 20 ft.
10. 204 mo. = 17 yr.
11. 5 hr. = 18,000 sec.
12. 176 oz. = 11 lb.

Set 72

Copy and complete.

1. 51 in. = 4 ft. 3 in.
2. 75 qt. = 18 gal. 3 qt.
3. 50 oz. = 3 lb. 2 oz.
4. 3,926 lb. = 1 T. 1,926 lb.
5. 800 rd. = 2 mi. 160 rd.
6. 500 sec. = 8 min. 20 sec.

Set 73

Copy and complete by writing mixed forms.

1. 75 in. = $6\frac{1}{4}$ ft.
2. 37 ft. = $12\frac{1}{3}$ yd.
3. 25 qt. = $6\frac{1}{4}$ gal.
4. 371 oz. = $23\frac{3}{16}$ lb.
5. 300 da. = $42\frac{6}{7}$ wk.
6. 525 pk. = $131\frac{1}{4}$ bu.
7. 7,500 lb. = $3\frac{3}{4}$ T.
8. 4,380 da. = 12 yr.
9. 55 pt. = $6\frac{7}{8}$ gal.

Set 74

Copy and complete.

1. 7 m. = 7,000 mm.
2. 14 cm. = 140 mm.
3. 250 mm. = 25 cm.
4. 500 mm. = $\frac{1}{2}$ m.
5. 65 dm. = $6\frac{1}{2}$ m.
6. 25 dm. = 250 cm.
7. 1,500 m. = $1\frac{1}{2}$ km.
8. 2 km. = 200,000 cm.
9. 5 dm. = 500 mm.

Set 75

Copy and subtract.

1.

ft.	in.
15	8
6	5
9 ft.	3 in.

2.

T.	lb.
2	1,300
1	920
1 T.	380 lb.

3.

min.	sec.
18	35
14	16
4 min.	19 sec.

4.

gal.	qt.
425	2
307	1
118 gal.	1 qt.

5.

bu.	pk.
12	2
6	3
5 bu.	3 pk.

6.

hr.	min.
4	30
3	55
	35 min.

7.

km.	m.
6	500
3	525
2 km.	975 m.

8.

mi.	ft.
10	2,000
4	3,250
5 mi.	4,030 ft.

Set 76

Add.

	a	b	c	d	e	f	g	h
1.	0.7	0.9	3.6	15.2	92.9	138.5	1,632.5	2,649.7
	0.2	0.6	2.7	3.3	0.4	92.8	3,837.9	508.6
	0.9	1.5	6.3	18.5	93.3	231.3	5,470.4	3,158.3
2.	0.67	0.05	3.64	16.28	30.08	381.26	500.45	6,267.02
	0.22	0.89	0.88	6.07	0.77	9.95	7,367.77	956.98
	0.89	0.94	4.52	22.35	30.85	391.21	7,868.22	7,224.00

Set 77

Add.

a	b	c	d	e	f	g	h
0.6	3.3	4.1	6.29	1.01	65.51	230.07	863.55
0.7	6.2	27.5	3.88	0.77	16.93	584.33	5.63
0.3	5.9	33.5	0.21	13.38	345.88	114.75	69.28
1.6	15.4	65.1	10.38	15.16	428.32	929.15	938.46

Set 78

Solve.

1. $n = 0.75 + 2.38$ 3.13
2. $n = 3.82 + 14.57$ 18.39
3. $5.6 + 0.7 = x$ 6.3
4. $56.8 + 34.8 = x$ 91.6
5. $x = 0.65 + 3.47$ 4.12
6. $n = 1.68 + 23.73$ 25.41
7. $n = 0.35 + 4.73 + 0.37$ 5.45
8. $n = 0.7 + 3.4 + 56.9$ 61.0 (or 61)
9. $n = 6.78 + 0.05 + 2.36$ 9.19

Set 79

Subtract.

	a	b	c	d	e	f	g	h
1.	0.8	3.5	14.9	4.6	6.0	235.6	730.0	460.2
	0.3	0.2	9.8	0.9	4.4	94.8	18.5	138.7
	0.5	3.3	5.1	3.7	1.6	140.8	711.5	321.5
2.	1.1	2.1	15.9	1.8	2.45	16.47	15.09	9.25
	0.1	0.8	9.7	0.7	1.13	2.87	7.86	0.96
	1.0 (or 1)	1.3	6.2	1.1	1.32	13.60	7.23	8.29

Set 80

Subtract.

	a	b	c	d	e	f	g	h
1.	0.38	0.04	5.82	4.50	82.00	483.95	13.04	11.05
	0.36	0.01	3.77	2.25	8.12	15.04	10.05	5.95
	0.02	0.03	2.05	2.25	73.88	468.91	2.99	5.10
2.	4.03	12.21	93.10	21.96	714.16	40.00	200.08	92.32
	1.99	8.88	88.16	5.80	15.70	39.04	175.55	46.78
	2.04	3.33	4.94	16.16	698.46	0.96	24.53	45.54

Set 81

Add.

	a	b	c	d	e	f
1.	0.243	4.821	523.114	15.094	33.830	839.204
	0.035	0.005	57.830	6.007	14.996	220.767
	0.278	4.826	580.944	21.101	48.826	1,059.971
2.	0.801	8.365	358.076	53.002	98.2	870.45
	0.246	2.076	69.258	46.691	6.488	176.205
	1.047	10.441	427.334	99.693	104.688	1,046.655

Set 82

Add.

	a	b	c	d	e	f
	0.9231	4.0173	40.1903	5.5570	91.4424	820.0378
	0.0343	1.1140	0.0968	0.0095	4.8808	385.0099
	0.9574	5.1313	40.2871	5.5665	96.3232	1,205.0477

Set 83

1.	0.208	4.816	465.284	9.087	18.834	608.437

Copy the examples in Set 81 and subtract.

2.	0.555	6.289	288.818	6.311	91.712	694.245

Set 84

Copy the examples in Set 82 and subtract.

0.8888	2.9033	40.0935	5.5475	86.5616	435.0279

Set 85

Solve.

1. $n = 3.937 + 5.770$ 9.707
2. $0.9314 + 23.8813 = n$ 24.8127
3. $x = 3.731 + 867.993$ 871.724
4. $x - 5.678 = 13.331$ 19.009
5. $39.999 = 56 - x$ 16.001
6. $27 - x = 14.1482$ 12.8518
7. $3.89 + x = 7.77$ 3.88
8. $5.8812 + n = 356$ 350.1188
9. $n = 36 - 4.007$ 31.993
10. $n - 57.03 = 4.26$ 61.29
11. $14 - 1.06 = n$ 12.94
12. $0.0447 + n = 2$ 1.9553

Set 86

Find the area of each rectangular region. 5. 99 sq. ft. (or 11 sq. yd.) 6. 800 sq. dm. (or 8 sq. m.) 7. 560 sq. mm. (or 5.6 sq. cm.)

	1 (450 sq. ft.)	2 (153 sq. cm.)	3 (44,288 sq. mi.)	4 (1,139 sq. yd.)	5	6	7
Length	25 ft.	17 cm.	346 mi.	67 yd.	11 ft.	5 m.	8 cm.
Width	18 ft.	9 cm.	128 mi.	17 yd.	3 yd.	16 dm.	7 mm.

Set 87

Copy and complete.

1. 4 sq. ft. = 576 sq. in.
2. 150 sq. in. = $1\frac{1}{24}$ sq. ft.
3. 91 sq. yd. = 819 sq. ft.
4. 18 sq. ft. = 2 sq. yd.
5. 2 sq. m. = 20,000 sq. cm.
6. 4,000 sq. cm. = 0.4 sq. m.
7. 1 sq. mi. = 27,878,400 sq. ft.
8. 1 sq. mi. = 3,097,600 sq. yd.

Set 88 Rename with a mixed form.

	a	b	c	d	e	f	g	h	i	j	k	
1.	$\frac{13}{4}$ $3\frac{1}{4}$	$\frac{17}{12}$ $1\frac{5}{12}$	$\frac{19}{10}$ $1\frac{9}{10}$	$\frac{25}{2}$ $12\frac{1}{2}$	$\frac{17}{3}$ $5\frac{2}{3}$	$\frac{50}{8}$ $6\frac{1}{4}$	$\frac{29}{12}$ $2\frac{5}{12}$	$\frac{32}{5}$ $6\frac{2}{5}$	$\frac{200}{9}$ $22\frac{2}{9}$	$\frac{125}{30}$ $4\frac{1}{6}$	$\frac{62}{15}$	$4\frac{2}{15}$
2.	$\frac{10}{9}$ $1\frac{1}{9}$	$\frac{15}{4}$ $3\frac{3}{4}$	$\frac{20}{3}$ $6\frac{2}{3}$	$\frac{29}{10}$ $2\frac{9}{10}$	$\frac{20}{9}$ $2\frac{2}{9}$	$\frac{45}{6}$ $7\frac{1}{2}$	$\frac{31}{20}$ $1\frac{11}{20}$	$\frac{65}{50}$ $1\frac{3}{10}$	$\frac{36}{25}$ $1\frac{11}{25}$	$\frac{30}{12}$ $2\frac{1}{2}$	$\frac{33}{5}$	$6\frac{3}{5}$

Set 89 Copy and complete.

1. $3 \times \frac{1}{5} = \frac{3 \times ?^{1}}{5} = \frac{?^{3}}{5}$
2. $2 \times \frac{4}{9} = \frac{2 \times ?^{4}}{9} = \frac{?^{8}}{9}$
3. $17 \times \frac{1}{3} = \frac{17 \times ?^{1}}{3} = \frac{?^{17}}{3}$, or $5\frac{?^{2}}{3}$
4. $4 \times \frac{2}{7} = \frac{?^{8}}{7}$, or $1\frac{?^{1}}{7}$
5. $25 \times \frac{4}{5} = \frac{100}{?^{5}} = \_?^{20}$
6. $15 \times \frac{6}{9} = \frac{?^{90}}{9} = \_?^{10}$
7. $6 \times \frac{3}{4} = \frac{?^{18}}{4} = \frac{?^{9}}{2}$, or $4\frac{?^{1}}{2}$
8. $9 \times \frac{5}{6} = \frac{?^{45}}{6} = \frac{?^{15}}{2}$, or $7\frac{?^{1}}{2}$
9. $50 \times \frac{2}{3} = \frac{?^{100}}{3}$, or $\_?^{33\frac{1}{3}}$

Set 90 Solve.

1. $n = 4 \times \frac{1}{5}$ $\frac{4}{5}$
2. $7 \times \frac{1}{5} = x$ $1\frac{2}{5}$
3. $7 \times \frac{7}{8} = n$ $6\frac{1}{8}$
4. $n = 6 \times \frac{3}{5}$ $3\frac{3}{5}$
5. $29 \times \frac{2}{10} = x$ $5\frac{4}{5}$
6. $33 \times \frac{3}{11} = n$ 9
7. $n = 22 \times \frac{3}{7}$ $9\frac{3}{7}$
8. $x = 18 \times \frac{7}{8}$ $15\frac{3}{4}$
9. $15 \times \frac{9}{10} = x$ $13\frac{1}{2}$
10. $n = 9 \times \frac{7}{10}$ $6\frac{3}{10}$
11. $x = 135 \times \frac{2}{5}$ 54
12. $225 \times \frac{5}{6} = n$ $187\frac{1}{2}$

Set 91 Solve.

1. $n = \frac{2}{3} \times 5$ $3\frac{1}{3}$
2. $\frac{6}{11} \times 5 = x$ $2\frac{8}{11}$
3. $\frac{7}{9} \times 6 = x$ $4\frac{2}{3}$
4. $x = \frac{4}{5} \times 7$ $5\frac{3}{5}$
5. $x = \frac{2}{3} \times 312$ 208
6. $\frac{2}{7} \times 121 = n$ $34\frac{4}{7}$
7. $\frac{5}{8} \times 12 = x$ $7\frac{1}{2}$
8. $n = \frac{3}{4} \times 17$ $12\frac{3}{4}$
9. $x = \frac{5}{8} \times 24$ 15
10. $x = \frac{3}{7} \times 19$ $8\frac{1}{7}$
11. $\frac{5}{12} \times 4 = n$ $1\frac{2}{3}$
12. $\frac{3}{15} \times 12 = x$ $2\frac{2}{5}$

Set 92

	a	b	c	d	e	
1.	$3 \times 1\frac{1}{2}$ $4\frac{1}{2}$	$4\frac{1}{3} \times 7$ $30\frac{1}{3}$	$2 \times 6\frac{1}{3}$ $12\frac{2}{3}$	$20 \times 1\frac{2}{3}$ $33\frac{1}{3}$	$7\frac{1}{4} \times 5$	$36\frac{1}{4}$
2.	$8 \times 2\frac{1}{3}$ $18\frac{2}{3}$	$14 \times 5\frac{4}{5}$ $81\frac{1}{5}$	$2\frac{5}{12} \times 5$ $12\frac{1}{12}$	$8\frac{1}{7} \times 9$ $73\frac{2}{7}$	$6\frac{1}{10} \times 3$	$18\frac{3}{10}$
3.	$2\frac{3}{8} \times 5$ $11\frac{7}{8}$	$15 \times 5\frac{2}{5}$ 81	$7\frac{2}{3} \times 8$ $61\frac{1}{3}$	$9\frac{3}{4} \times 10$ $97\frac{1}{2}$	$6\frac{5}{6} \times 20$	$136\frac{2}{3}$

Set 93 Multiply.

	a	b	c	d	e	f	g	
1.	12	48	15	30	$3\frac{1}{7}$	$5\frac{2}{3}$	16	
	$2\frac{1}{2}$ 30	$3\frac{1}{3}$ 160	$6\frac{1}{5}$ 93	$7\frac{1}{10}$ 213	14 44	30 170	$8\frac{3}{4}$	140
2.	18	25	40	27	$6\frac{1}{2}$	150	156	
	$3\frac{2}{3}$ 66	$2\frac{2}{5}$ 60	$6\frac{3}{10}$ 252	$9\frac{2}{3}$ 261	32 208	$7\frac{3}{5}$ 1,140	$7\frac{1}{4}$ 1,131	

Set 94

Multiply.

	a	b	c	d	e	f	g
1.	$31\frac{2}{5}$	46	5	16	$7\frac{1}{5}$	20	14
	15 — 471	$23\frac{1}{2}$ — 1,081	$4\frac{1}{3}$ — $21\frac{2}{3}$	$4\frac{1}{3}$ — $69\frac{1}{3}$	4 — $28\frac{4}{5}$	$9\frac{1}{3}$ — $186\frac{2}{3}$	$19\frac{1}{3}$ — $270\frac{2}{3}$
2.	$17\frac{1}{10}$	$15\frac{1}{12}$	20	31	59	60	75
	12 — $205\frac{1}{5}$	25 — $377\frac{1}{12}$	$7\frac{1}{7}$ — $142\frac{6}{7}$	$3\frac{1}{10}$ — $96\frac{1}{10}$	$5\frac{1}{11}$ — 300 $\frac{4}{11}$	$13\frac{3}{4}$ — 825	$9\frac{4}{5}$ — 735

Set 95

Copy and complete.

1. $6 \times 5\frac{2}{3} = 6 \times \frac{?}{3}^{17} = \frac{6 \times ?}{3}^{17} = \frac{?}{3}^{102}$, or $\_?\underline{34}$
2. $9 \times 4\frac{1}{6} = 9 \times (4 + \frac{1?}{6?}) = (9 \times \underline{4}?\_) + (9 \times \frac{1}{6}) = 36 + \frac{?}{6}^{9} = 36\frac{?}{2}^{3}$, or $37\frac{?}{2}^{1}$

Solve.

3. $n = 140 \times 5\frac{1}{2}$ 770
4. $6\frac{2}{3} \times 30 = n$ 200
5. $x = 43 \times 4\frac{1}{2}$ $193\frac{1}{2}$
6. $x = 17 \times 6\frac{3}{5}$ $112\frac{1}{5}$
7. $9 \times 5\frac{1}{5} = n$ $46\frac{4}{5}$
8. $x = 220 \times 1\frac{1}{2}$ 330
9. $55 \times 5\frac{4}{5} = x$ 319
10. $n = 62 \times 2\frac{1}{3}$ $144\frac{2}{3}$

Set 96

	a	b	c	d	e
1.	$\frac{2}{3} \times \frac{1}{5}$ $\frac{2}{15}$	$\frac{1}{4} \times \frac{5}{6}$ $\frac{5}{24}$	$\frac{5}{12} \times \frac{3}{5}$ $\frac{1}{4}$	$\frac{2}{3} \times \frac{5}{12}$ $\frac{5}{18}$	$\frac{1}{4} \times \frac{5}{6}$ $\frac{1}{24}$
2.	$\frac{2}{5} \times \frac{8}{9}$ $\frac{16}{45}$	$\frac{1}{7} \times \frac{2}{3}$ $\frac{2}{21}$	$\frac{5}{6} \times \frac{3}{4}$ $\frac{5}{8}$	$\frac{1}{9} \times \frac{3}{4}$ $\frac{1}{12}$	$\frac{1}{7} \times \frac{7}{9}$ $\frac{1}{9}$
3.	$\frac{3}{4} \times \frac{9}{10}$ $\frac{27}{40}$	$\frac{4}{5} \times \frac{7}{8}$ $\frac{7}{10}$	$\frac{2}{3} \times \frac{9}{10}$ $\frac{3}{5}$	$\frac{11}{15} \times \frac{3}{4}$ $\frac{11}{20}$	$\frac{7}{12} \times \frac{9}{10}$ $\frac{21}{40}$

Set 97

Solve.

1. $n = \frac{2}{3} \times \frac{1}{8}$ $\frac{1}{12}$
2. $n = \frac{4}{11} \times \frac{3}{4}$ $\frac{3}{11}$
3. $\frac{5}{8} \times \frac{9}{10} = x$ $\frac{9}{16}$
4. $\frac{6}{7} \times \frac{2}{3} = n$ $\frac{4}{7}$
5. $n = \frac{3}{8} \times \frac{2}{3}$ $\frac{1}{4}$
6. $\frac{3}{10} \times \frac{5}{6} = x$ $\frac{1}{4}$
7. $\frac{4}{5} \times \frac{1}{10} = n$ $\frac{2}{25}$
8. $x = \frac{1}{7} \times \frac{5}{9}$ $\frac{5}{63}$
9. $n = 1\frac{1}{12} \times \frac{3}{5}$ $\frac{11}{20}$
10. $n = \frac{3}{10} \times \frac{3}{8}$ $\frac{9}{80}$
11. $\frac{2}{7} \times \frac{4}{5} = x$ $\frac{8}{35}$
12. $\frac{6}{7} \times \frac{8}{9} = x$ $\frac{16}{21}$

Set 98

Solve.

1. $n = 1\frac{1}{2} \times 3\frac{1}{3}$ 5
2. $14 \times 5\frac{1}{2} = n$ 77
3. $x = 17\frac{1}{2} \times 44$ 770
4. $n = 5\frac{1}{3} \times 6\frac{3}{4}$ 36
5. $x = 16\frac{2}{3} \times 4\frac{1}{7}$ $69\frac{1}{21}$
6. $3\frac{1}{2} \times 5\frac{2}{5} = x$ $18\frac{9}{10}$
7. $x = 6\frac{5}{7} \times \frac{3}{4}$ $5\frac{1}{28}$
8. $\frac{5}{6} \times 7\frac{3}{4} = x$ $6\frac{11}{24}$
9. $n = 19 \times 2\frac{1}{10}$ $39\frac{9}{10}$
10. $3\frac{3}{4} \times 1\frac{1}{2} = n$ $5\frac{5}{8}$
11. $n = 136 \times 5\frac{1}{8}$ 697
12. $7\frac{2}{3} \times 4\frac{3}{4} = n$ $36\frac{5}{12}$

Extra Examples

Set 99

	a	b	c	d	
1.	$\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$ $\frac{1}{24}$	$\frac{2}{5} \times \frac{3}{4} \times \frac{1}{7}$ $\frac{3}{70}$	$\frac{1}{9} \times \frac{3}{4} \times \frac{2}{3}$ $\frac{1}{18}$	$\frac{3}{4} \times \frac{2}{3} \times \frac{1}{5} \times \frac{5}{6}$ $\frac{1}{12}$	
2.	$\frac{3}{5} \times 15 \times \frac{2}{3}$ 6	$14 \times \frac{1}{2} \times \frac{3}{7}$ 3	$1\frac{1}{2} \times \frac{2}{3} \times 4\frac{2}{3}$ $4\frac{2}{3}$	$7\frac{1}{5} \times 50 \times 2\frac{2}{3} \times \frac{1}{2}$	480
3.	$\frac{5}{6} \times \frac{7}{9} \times 15$ $9\frac{13}{18}$	$\frac{3}{4} \times 16 \times \frac{2}{9}$ $2\frac{2}{3}$	$3\frac{2}{3} \times \frac{3}{8} \times 2\frac{4}{9}$ $3\frac{13}{36}$	$5\frac{2}{5} \times 25 \times 6\frac{1}{4} \times \frac{5}{8}$	$527\frac{11}{32}$

Set 100

Solve.

1. $n = \frac{2}{3} \times \frac{3}{5} \times 15$ 6	4. $x = 4\frac{1}{3} \times \frac{1}{2} \times 20$ $43\frac{1}{3}$	7. $x = 6 \times 4 \times 17 \times \frac{5}{12}$ 170	
2. $n = 48 \times 16 \times \frac{3}{4}$ 576	5. $\frac{1}{6} \times \frac{2}{9} \times \frac{3}{5} = x$ $\frac{1}{45}$	8. $n = \frac{5}{8} \times 3\frac{1}{2} \times \frac{3}{7} \times 4$ $3\frac{3}{4}$	
3. $n = \frac{4}{5} \times 5\frac{1}{5} \times 25$ 104	6. $\frac{7}{8} \times \frac{4}{5} \times 4 = x$ $2\frac{4}{5}$	9. $n = 29 \times 48 \times 3\frac{1}{12} \times \frac{1}{4}$	1,073

Set 101

Copy and complete.

1. $5 \div \frac{5}{6} = \frac{30}{?} \div \frac{5}{6} = \frac{30 \div ?}{6 \div ?} = 30 \div 5 =$ $\frac{?}{6}$ 30 5
2. $20 \div \frac{5}{8} = \frac{160}{?} \div \frac{5}{8} = 160 \div 5 = \frac{160}{?} =$ $\frac{?}{32}$ 160 5

Use the common-denominator method.

	a	b	c	d	e	
3.	$5 \div \frac{1}{2}$ 10	$16 \div \frac{4}{5}$ 20	$24 \div \frac{2}{3}$ 36	$9 \div \frac{1}{7}$ 63	$15 \div \frac{2}{3}$	$22\frac{1}{2}$

Set 102

Solve.

1. $n = 7 \div \frac{2}{3}$ $10\frac{1}{2}$	4. $50 \div \frac{1}{9} = n$ 450	7. $n = 14 \div \frac{2}{5}$ 35	10. $x = 24 \div \frac{4}{5}$ 30	
2. $x = 55 \div \frac{5}{6}$ 66	5. $x = 3 \div \frac{4}{9}$ $6\frac{3}{4}$	8. $19 \div \frac{2}{3} = n$ $28\frac{1}{2}$	11. $30 \div \frac{3}{7} = n$ 70	
3. $24 \div \frac{7}{8} = n$ $27\frac{3}{7}$	6. $n = 64 \div \frac{5}{6}$ $76\frac{4}{5}$	9. $75 \div \frac{3}{5} = x$ 125	12. $125 \div \frac{2}{3} = x$ $187\frac{1}{2}$	

Set 103

Solve.

1. $n = 8 \div 2\frac{1}{2}$ $3\frac{1}{5}$	4. $n = 3 \times 1\frac{2}{3}$ 5	7. $x \div 1\frac{1}{2} = 12$ 18	10. $n \div 6\frac{1}{3} = 300$	1,900
2. $n \times 3\frac{1}{5} = 500$ $156\frac{1}{4}$	5. $n = 7 \div \frac{3}{4}$ $9\frac{1}{3}$	8. $x \times 1\frac{3}{4} = 5$ $2\frac{6}{7}$	11. $n = 18 \times 5\frac{1}{5}$	$93\frac{3}{5}$
3. $x \div 4\frac{2}{3} = 16$ $74\frac{2}{3}$	6. $5\frac{1}{2} = 200 \div x$ $36\frac{4}{11}$	9. $6\frac{3}{4} \times n = 24$ $3\frac{5}{9}$	12. $125 \div x = 10\frac{1}{4}$	$12\frac{8}{41}$

Set 104

Solve.

1. $n = \frac{3}{4} \div 5$ $\frac{3}{20}$	4. $x = 4\frac{1}{2} \div 12$ $\frac{3}{8}$	7. $n \times 7 = 6\frac{2}{3}$ $\frac{20}{21}$	10. $x = 5\frac{1}{3} \div 2$ $2\frac{2}{3}$	
2. $n = \frac{3}{5} \div 7$ $\frac{3}{35}$	5. $x \div 3\frac{1}{3} = 36$ 120	8. $x \div 4 = 5\frac{3}{4}$ 23	11. $3\frac{3}{4} = x \times 8$ $\frac{15}{32}$	
3. $n = 5\frac{1}{3} \div 9$ $\frac{16}{27}$	6. $n \times 4 = 5\frac{1}{2}$ $1\frac{3}{8}$	9. $6 \times x = 1\frac{1}{5}$ $\frac{1}{5}$	12. $n \div 3\frac{1}{2} = 400$	1,400

Set 105 Use the reciprocal method.

	a	b	c	d	e
1.	$8 \div \frac{2}{7}$ 28	$15 \div \frac{3}{5}$ 25	$\frac{2}{9} \div \frac{5}{12}$ $\frac{8}{15}$	$\frac{3}{8} \div \frac{1}{4}$ $1\frac{1}{2}$	$\frac{8}{9} \div \frac{10}{11}$ $\frac{44}{45}$
2.	$10 \div \frac{3}{5}$ $16\frac{2}{3}$	$16 \div \frac{5}{6}$ $19\frac{1}{5}$	$\frac{7}{8} \div \frac{9}{10}$ $\frac{35}{36}$	$\frac{5}{12} \div \frac{2}{3}$ $\frac{5}{8}$	$\frac{7}{11} \div \frac{2}{7}$ $2\frac{5}{22}$

Set 106 Use the reciprocal method.

	a	b	c	d	e
1.	$\frac{8}{9} \div 4$ $\frac{2}{9}$	$1\frac{1}{2} \div 5$ $\frac{3}{10}$	$2\frac{2}{3} \div 1\frac{1}{2}$ $1\frac{7}{9}$	$4\frac{1}{5} \div 7$ $\frac{3}{5}$	$\frac{3}{4} \div 2\frac{1}{2}$ $\frac{3}{10}$
2.	$18 \div 1\frac{1}{5}$ 15	$2\frac{2}{7} \div 2\frac{2}{3}$ $\frac{6}{7}$	$5\frac{1}{9} \div 1\frac{2}{3}$ $3\frac{1}{15}$	$1\frac{1}{16} \div 2\frac{1}{4}$ $\frac{17}{36}$	$\frac{7}{15} \div 14$ $\frac{1}{30}$
3.	$6 \div 10\frac{1}{2}$ $\frac{4}{7}$	$5\frac{6}{7} \div 4\frac{1}{2}$ $1\frac{19}{63}$	$7\frac{1}{4} \div 2\frac{1}{3}$ $3\frac{3}{28}$	$4\frac{3}{8} \div 3\frac{3}{4}$ $1\frac{1}{6}$	$24 \div 3\frac{1}{3}$ $7\frac{1}{5}$

Set 107 Solve.

1. $n \times 5 = 3\frac{3}{4}$ $\frac{3}{4}$
2. $n = 28 \div \frac{2}{3}$ 42
3. $n = 100 \div 5\frac{1}{5}$ $19\frac{3}{13}$
4. $n \div 3\frac{1}{3} = 4\frac{1}{2}$ 15
5. $5 \div 6\frac{2}{5} = x$ $\frac{25}{32}$
6. $x = 33 \times 15\frac{2}{3}$ 517
7. $x = 3\frac{1}{2} \times 7\frac{2}{3}$ $26\frac{5}{6}$
8. $x = 5\frac{1}{2} \div \frac{6}{7}$ $6\frac{5}{12}$
9. $8\frac{1}{4} \div \frac{1}{2} = n$ $16\frac{1}{2}$
10. $n = 15 \times (\frac{1}{2} \times \frac{3}{4})$ $5\frac{5}{8}$
11. $n = \frac{2}{3} \div (1\frac{1}{2} - \frac{3}{8})$ $\frac{16}{27}$
12. $n = (\frac{4}{5} \div 2) \div \frac{3}{8}$ $1\frac{1}{15}$

Set 108 Copy and complete.

1. $\frac{2}{3} = \frac{?}{48}$ 32
2. $\frac{5}{6} = \frac{25}{?}$ 30
3. $\frac{3}{7} = \frac{?}{231}$ 99
4. $\frac{5}{12} = \frac{50}{?}$ 120
5. $\frac{2?}{9} = \frac{18}{81}$
6. $\frac{11?}{15} = \frac{66}{90}$
7. $\frac{14}{30} = \frac{7}{?}$ 15
8. $\frac{5}{3?} = \frac{185}{111}$
9. $\frac{55}{125} = \frac{11}{?}$ 25
10. $\frac{60}{88} = \frac{?}{22}$ 15
11. $\frac{68}{120} = \frac{17}{?}$ 30
12. $\frac{17}{51} = \frac{?}{3}$ 1

Set 109 Express the ratio with a fraction in simplest form.

1. 25 to 50 $\frac{1}{2}$
2. 34 to 66 $\frac{17}{33}$
3. 80 to 125 $\frac{16}{25}$
4. 55 to 135 $\frac{11}{17}$
5. 8 to 74 $\frac{4}{37}$
6. 3:15 $\frac{1}{5}$
7. 28:70 $\frac{2}{5}$
8. 14:70 $\frac{1}{5}$
9. 50:195 $\frac{10}{39}$
10. 7:112 $\frac{1}{16}$

Set 110 Multiply.

	a	b	c	d	e	f	g
1.	0.8	0.7	4.8	9.3	4.6	13.3	44.1
	6	9	2	8	8	7	8
	4.8	6.3	9.6	74.4	36.8	93.1	352.8
2.	23.3	45.5	60.4	75.1	70.7	347.7	713.8
	9	5	4	9	7	3	4
	209.7	227.5	241.6	675.9	494.9	1,043.1	2,855.2

Set 111

Multiply.

	a	b	c	d	e	f	g
1.	13.5	27.3	6.2	0.56	5.55	13.27	78.04
	9	8	12	7	3	9	7
	121.5	218.4	74.4	3.92	16.65	119.43	546.28
2.	28.3	40.1	5.16	7.29	25.75	821.06	928.6
	63	18	42	13	15	42	68
	1,782.9	721.8	216.72	94.77	386.25	34,484.52	63,144.8

Set 112

Multiply.

	a	b	c	d	e	f	g
1.	5.104	0.869	0.808	6.009	7.921	56.113	80.065
	4	7	2	5	6	7	3
	20.416	6.083	1.616	30.045	47.526	392.791	240.195
2.	0.253	0.073	1.117	8.016	0.989	34.028	17.884
	45	88	19	63	76	33	52
	11.385	6.424	21.223	505.008	75.164	1,122.924	929.968

Set 113

Solve.

1. $n = 16 \times 0.37$ 5.92
2. $n = 45 \times 0.9$ 40.5
3. $n = 2.17 \times 82$ 177.94
4. $5.001 \times 7 = x$ 35.007
5. $x = 6.668 \times 54$ 360.072
6. $n = 7.3 \times 185$ 1,350.5
7. $n = 23 \times 5.004$ 115.092
8. $x = 17.23 \times 521$ 8,976.83
9. $x = 5.5 \times 65$ 357.5
10. $x = 258 \times 0.73$ 188.34
11. $n \div 17 = 0.623$ 10.591
12. $x \div 481 = 0.743$ 357.383

Set 114

Solve.

1. $n = 10 \times 0.82$ 8.2
2. $x = 100 \times 5.3$ 530
3. $n = 100 \times 33.2$ 3,320
4. $x = 1{,}000 \times 7.93$ 7,930
5. $x = 2.3 \times 1{,}000$ 2,300
6. $n = 1{,}000 \times 2.6894$ 2,689.4
7. $x = 10{,}000 \times 0.84$ 8,400
8. $n = 0.57 \times 100$ 57
9. $x = 10{,}000 \times 6.025$ 60,250

Set 115

Multiply.

	a	b	c	d	e	f	g	h
1.	0.9	0.9	1.3	18.8	6.5	7.3	23.8	40.7
	0.7	0.8	0.5	9.1	0.8	1.5	0.6	2.6
	0.63	0.72	0.65	171.08	5.20 (or 5.2)	10.95	14.28	105.82
2.	7.5	2.5	3.6	22.7	53.2	23.3	60.5	16.3
	1.4	2.6	1.6	8.1	9.5	13.2	10.9	30.8
	10.50 (or 10.5)	6.50 (or 6.5)	5.76	183.87	505.40 (or 505.4)	307.56	659.45	502.04

Set 116 Multiply.

	a	b	c	d	e	f	g
1.	16.4	86.6	315.5	40.37	4.12	473.8	17.39
	9.7	3.4	15.7	5.2	3.5	0.32	6.1
	159.08	294.44	4,953.35	209.924	14.410 (or 14.41)	151.626	106.079
2.	13.4	0.13	5.802	13.524	41.37	6.33	4.132
	0.16	0.04	41	63	0.77	1.15	1.1
	2.144	0.0052	237.882	852.012	31.8549	7.2795	4.5452

Set 117 Multiply.

	a	b	c	d	e	f	g
1.	3.98	93.56	7.114	150.37	5.8294	4,780.3	0.00463
	2.25	4.83	0.52	4.213	0.8	5.113	50.07
	8.9550 (or 8.955)	451.8948	3.69928	633.50881	4.66352	24,441.6739	0.2318241
2.	8.58	64.72	1.275	32.79	4.0694	229.66	0.96787
	4.62	0.32	3.06	0.673	0.25	3.421	200.6
	39.6396	20.7104	3.90150 (or 3.9015)	22.06767	1.017350 (or 1.01735)	785.66686	194.154722

Set 118 Solve.

1. $n = 3.6 \times 1.23$ 4.428
2. $n = 5.682 \times 6.3$ 35.7966
3. $n \div 2.7 = 6.352$ 17.1504
4. $x \div 364 = 5.69$ 2,071.16
5. $5{,}438 \times 0.71 = x$ 3,860.98
6. $n = 4.23 \times 0.04$ 0.1692
7. $3.2 \times 7.934 = n$ 25.3888
8. $x = 5.6293 \times 15$ 84.4395
9. $x = 729.3 \times 0.09$ 65.637

Set 119

	a	b	c	d	e	f	g
1.	$4\overline{)4.8}$ 1.2	$5\overline{)6.5}$ 1.3	$2\overline{)0.8}$ 0.4	$9\overline{)65.7}$ 7.3	$4\overline{)9.2}$ 2.3	$5\overline{)72.5}$ 14.5	$6\overline{)17.4}$ 2.9
2.	$9\overline{)918.9}$ 102.1	$3\overline{)58.2}$ 19.4	$4\overline{)100.4}$ 25.1	$7\overline{)36.4}$ 5.2	$9\overline{)855.9}$ 95.1	$8\overline{)27.2}$ 3.4	$6\overline{)571.2}$ 95.2

Set 120

	a	b	c	d	e	f	g
1.	$5\overline{)8.85}$ 1.77	$6\overline{)17.82}$ 2.97	$5\overline{)2.4}$ 0.48	$4\overline{)8.9}$ 2.225	$3\overline{)0.09}$ 0.03	$4\overline{)0.26}$ 0.065	$16\overline{)0.032}$ 0.002
2.	$25\overline{)5.5}$ 0.22	$14\overline{)2.1}$ 0.15	$34\overline{)3.74}$ 0.11	$5\overline{)0.007}$ 0.0014	$41\overline{)214.43}$ 5.23	$83\overline{)107.9}$ 1.3	$25\overline{)3.575}$ 0.143

Set 121 Solve.

1. $n = 1.08 \div 9$ 0.12
2. $n = 0.042 \div 7$ 0.006
3. $n \times 8 = 1.04$ 0.13
4. $13.86 \div 7 = x$ 1.98
5. $x \times 66 = 72.6$ 1.1
6. $5 = 1.975 \div x$ 0.395
7. $x = 9.117 \div 9$ 1.013
8. $n \times 24 = 0.0048$ 0.0002
9. $25 \times n = 4.2$ 0.168
10. $11 \times n = 29.854$ 2.714
11. $n = 241.62 \div 5$ 48.324
12. $24 = 0.768 \div n$ 0.032

Extra Examples

Set 122

Find the unknown factor to the nearest tenth.

	a	b	c	d	e
1.	$2.7 \div 4$ 0.7	$33.6 \div 5$ 6.7	$23.4 \div 19$ 1.2	$2.319 \div 38$ 0.1	$29 \div 12$ 2.4
2.	$43.8 \div 8$ 5.5	$75.2 \div 6$ 12.5	$54.3 \div 17$ 3.2	$16 \div 6$ 2.7	$8.75 \div 12$ 0.7

Find the unknown factor to the nearest hundredth.

3.	$5.37 \div 9$ 0.60	$0.384 \div 37$ 0.01	$5 \div 7$ 0.71	$82.6045 \div 27$ 3.06	$8.004 \div 31$ 0.26
4.	$0.964 \div 4$ 0.24	$7 \div 11$ 0.64	$36.8 \div 22$ 1.67	$91.006 \div 35$ 2.60	$200.639 \div 13$ 15.43

Find the unknown factor to the nearest thousandth.

5.	$9 \div 17$ 0.529	$2.6341 \div 2$ 1.317	$0.6923 \div 55$ 0.013	$88.5 \div 20$ 4.425	$0.0536 \div 4$ 0.013

Set 123

Solve.

1. $n = 6.8 \div 10$ 0.68
2. $n = 5.3 \div 100$ 0.053
3. $n = 0.62 \div 100$ 0.0062
4. $n = 7.812 \div 100$ 0.07812
5. $n = 0.05 \div 1{,}000$ 0.00005
6. $n = 68.451 \div 10^4$ 0.0068451
7. $n = 0.09 \div 10^5$ 0.0000009
8. $n = 2.0013 \div 10^3$ 0.0020013

Set 124

Copy and complete.

1. $0.5\overline{)1.8} = 5\overline{)\_?\_}$ 18
2. $1.8\overline{)0.93} = 18\overline{)\_?\_}$ 9.3
3. $0.38\overline{)5} = 38\overline{)\_?\_}$ 500
4. $3.68\overline{)0.014} = 368\overline{)\_?\_}$ 1.4
5. $0.05\overline{)281.6} = 5\overline{)\_?\_}$ 28,160
6. $0.42\overline{)0.00056} = 42\overline{)\_?\_}$ 0.056

Set 125

	a	b	c	d	e
1.	12.1 $0.4\overline{)4.84}$	31.3 $0.9\overline{)28.17}$	181.1 $0.3\overline{)54.33}$	0.011 $1.1\overline{)0.0121}$	0.004 $15.2\overline{)0.0608}$
2.	0.72 $3.4\overline{)2.448}$	120.8 $2.5\overline{)302}$	22.3 $4.7\overline{)104.81}$	201 $20.1\overline{)4{,}040.1}$	31.2 $25.8\overline{)804.96}$

Set 126

	a	b	c	d	e
1.	2.1 $2.2\overline{)4.62}$	1,280 $0.7\overline{)896}$	230.5 $0.003\overline{)0.6915}$	8,000 $0.008\overline{)64}$	168 $0.025\overline{)4.2}$
2.	2.4 $0.87\overline{)2.088}$	0.03 $5.21\overline{)0.1563}$	44 $6.01\overline{)264.44}$	2 $3.005\overline{)6.01}$	23.51 $0.33\overline{)7.7583}$

Set 127

Find the unknown factor to the nearest tenth.

	a	b	c	d	e
1.	$4 \div 5.3$ 0.8	$8.2 \div 1.7$ 4.8	$0.09 \div 0.7$ 0.1	$1 \div 4.72$ 0.2	$8.92 \div 3.9$ 2.3

Find the unknown factor to the nearest hundredth.

2.	$3 \div 8$ 0.38	$4.4 \div 5.6$ 0.79	$31 \div 44.2$ 0.70	$42.7 \div 0.07$ 610.00	$361 \div 0.009$ 40,111.11
3.	$5.5 \div 3.6$ 1.53	$27 \div 0.044$ 613.64	$0.089 \div 3.1$ 0.03	$7.093 \div 6$ 1.18	$0.72 \div 0.025$ 28.80

Set 128

Use $C \approx 3.14 \times d$ for Row 1. Use $C \approx \frac{22}{7} \times d$ for Row 2. Express each answer in the form "The circumference is about . . . ."

	a	b	c	d	e	f
Diameter	1. 5 cm. ∧ 15.70 cm.	23 in. ∧ 72.22 in.	7.1 in. ∨ 22.294 in.	0.7 yd. ∧ 2.198 yd.	8.23 ft. ∨ 25.8422 ft.	356.2 in. ∨ 1,118.468 in.
Diameter	2. 14 in. ∨ 44 in.	$\frac{7}{11}$ ft. ∨ 2 ft.	$\frac{14}{15}$ yd. $\frac{14}{15}$ yd.	$3\frac{1}{2}$ cm. ∨ 11 cm	$8\frac{2}{3}$ m. $27\frac{5}{21}$ m.	$15\frac{5}{7}$ mm. $49\frac{19}{40}$ mm.

Set 129

Use $d \approx C \div 3.14$ for Row 1. Use $d \approx C \div \frac{22}{7}$ for Row 2. Express each answer in the form "The diameter is about . . . ."

	a	b	c	d
Circumference	1. 31.4 in. 10 in.	21.98 ft. 7 ft.	3.454 cm. 1.1 cm.	0.7222 yd. 0.23 yd.
Circumference	2. 440 cm. 140 cm.	$3\frac{1}{2}$ in. $1\frac{5}{44}$ in.	$3\frac{2}{3}$ ft. $1\frac{1}{6}$ ft.	$\frac{25}{28}$ mm. $\frac{25}{88}$ mm.

Set 130

Express the ratio with a fraction showing denominator 100.

1. 23 to 50 $\frac{46}{100}$
2. 15 to 300 $\frac{5}{100}$
3. 16 to 400 $\frac{4}{100}$
4. 9 to 25 $\frac{36}{100}$
5. 4 to 5 $\frac{80}{100}$
6. 16 to 50 $\frac{32}{100}$
7. 3:4 $\frac{75}{100}$
8. 300:400 $\frac{75}{100}$
9. 7:10 $\frac{70}{100}$
10. 156:200 $\frac{78}{100}$
11. 17:20 $\frac{85}{100}$
12. 160:1,000 $\frac{16}{100}$

Set 131

Copy and complete.

1. $\frac{3}{5} = \frac{?^{60}}{100} = \frac{60}{}?\_\%$
2. $\frac{52}{200} = \frac{?^{26}}{100} = \frac{26}{}?\_\%$
3. $\frac{7}{25} = \frac{?^{28}}{100} = \frac{28}{}?\_\%$
4. $\frac{7}{50} = \frac{?^{14}}{100} = \frac{14}{}?\_\%$
5. $\frac{620}{1{,}000} = \frac{?^{62}}{100} = \frac{62}{}?\_\%$
6. $\frac{25}{500} = \frac{?^{5}}{100} = \frac{5}{}?\_\%$

Set 132

Express the ratio with a per cent form.

1. 3 to 100 3%
2. 57 to 100 57%
3. 55 to 500 11%
4. 79:100 79%
5. 14:25 56%
6. 12 to 200 6%
7. 125:500 25%
8. 19 to 20 95%
9. 44:200 22%
10. 5 to 20 25%

Set 133

Solve. Express answers with per cent forms.

1. $n = 17\% + 53\%$ 70%
2. $n = 2\% + 45\%$ 47%
3. $n = 100\% - 47\%$ 53%
4. $x = 1 - 29\%$ 71%
5. $1 - 50\% = x$ 50%
6. $n = 89\% - 17\%$ 72%
7. $n = 53\% - (47\% + 2\%)$ 4%
8. $n = 6\% + 28\% + 37\%$ 71%
9. $n = 1 - (88\% + 9\%)$ 3%

Set 134

Find the area of each parallelogram region.

	1	2	3	4	5	6
Length of base	1,311 sq. in. 57 in.^	1.37 sq. cm. 2.74 cm.^	$2\frac{1}{32}$ sq. ft. $3\frac{1}{4}$ ft.^	9,987.3 sq. m. 729 m.^	3,630 sq. yd. 275 yd.^	2.3 in.v 64.216 sq. in.
Height	23 in.	0.5 cm.	$\frac{5}{8}$ ft.	13.7 m.	$13\frac{1}{5}$ yd.	27.92 in.

Set 135

Find the area of each triangular region.

	1	2	3	4	5	6
Length of base	1.61 sq. in. 2.3 in.^	8,352 sq. cm. 87 cm.^	$27\frac{3}{12}$ sq. ft. $15\frac{2}{3}$ ft.^	0.1684 sq. mm. 8.42 mm.^	0. 295 yd.v 1,947 sq. yd.	119335 sq. cm. 0.823 cm.^
Height	1.4 in.	192 cm.	$3\frac{1}{2}$ ft.	0.04 mm.	13.2 yd.	0.29 cm.

Set 136

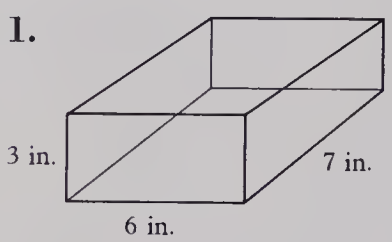
Find the lateral area of each right prism.

	1	2	3	4	5	6
Perimeter of base	414 sq. ft. 23 ft.^	64.8 sq. yd. 7.2 yd.^	5.37 cm.v 91.29 sq. cm.	7.56 sq. m. 8.4 m.^	1,263 in.v 690,861 sq. in.	936.2 sq. cm. 6.04 cm.^
Length	18 ft.	9 yd.	17 cm.	0.9 m.	547 in.	155 cm.

Set 137

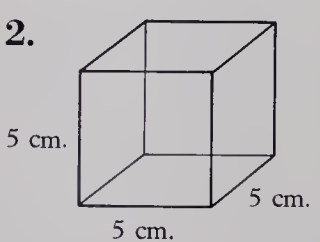
Find the total area of each rectangular prism.

1.



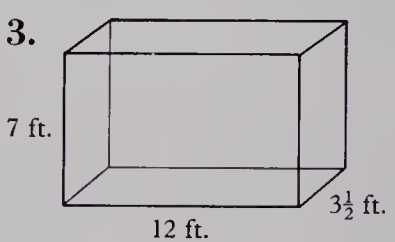
1. 162 sq. in.

2.



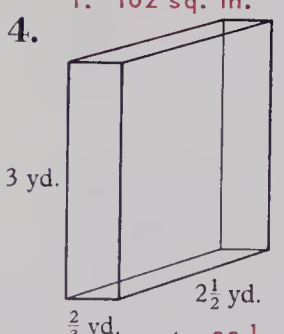
2. 150 sq. cm.

3.



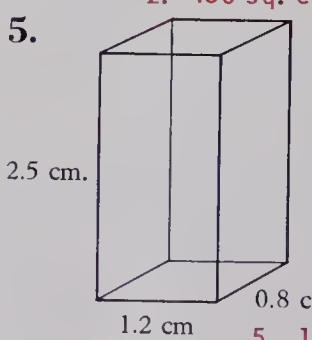
3. 301 sq. ft.

4.



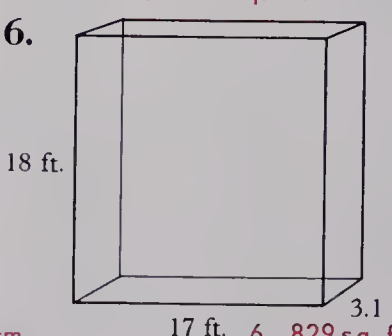
4. $22\frac{1}{3}$ sq. yd.

5.



5. 11.92 sq. cm.

6.



6. 829 sq. ft.

These suggestions will help you in solving problems.

(1) Read the problem carefully and select the needed information.

(2) Write a mathematical sentence that shows the relationship expressed in the problem.

(3) Estimate the answer.

(4) In finding exact answers, do as much as you can mentally.

(5) In checking your answer, compare it with your estimate.

(6) Decide whether or not your answer is reasonable.

(7) Write a sentence that answers the question in the problem.

Set 138

For use after page 33

1. Ted wanted to buy a new camera which cost $8.95. Ted only had $7.36, so his father paid the difference. How much did his father pay? $1.59

2. On a camping trip, Ted took 67 pictures and his father took 48. Ted took how many more pictures than his father? 19

3. Before starting on the trip, Ted weighed 87 pounds. When he returned, he weighed 83 pounds. How many pounds did Ted lose? 4

4. On a sales trip, Mr. James traveled 435 miles the first day, 386 miles the second day, and 175 miles the third day. How many miles in all did he travel on this trip? 996

5. At the book shop, Mrs. James bought 3 books which cost $3.45, $5.98, and $2.88. What was the total cost of the 3 books? $12.31

6. Mrs. James read the first 67 pages of a 276-page book. How many pages were left for her to read? 209

Set 139

For use after page 35

1. At a florist shop, Jane bought a plant which cost $1.75 and some moss which cost $0.89. How much change did Jane receive from $5.00? $2.36

2. Jane's family started on a 650-mile trip. They traveled 256 miles the 1st day, 272 miles the 2d, and then finished the trip the 3d day. How many miles did they travel the 3d day? 122

3. While on the trip, Jane spent $2.98 for a new swimming suit and $3.69 for a new beach robe. Her brother, Eric, bought a new book for $1.25. Jane spent how much more than her brother? $5.42

4. At the beach, Jane found 14 snail shells and Eric found 23. They decided to keep 28 of the shells. How many shells did they throw away? 9

Set 140

For use after page 78

1. Miss Miller used 41 sheets of paper from a package of 500 sheets. Then she distributed the remaining sheets equally among 27 pupils. How many sheets did each pupil receive? 17

2. For washing and waxing Mr. Steven's car, Bob, Ted, and Jim earned $4.20 which they shared equally. After spending $0.39 of his share for a model, how much did Bob have left? $1.01

3. For every 25 tickets that Mr. Jeb sells, he receives 3 free tickets. If he sells 350 tickets, how many free tickets will he receive? 42

4. Mr. Jeb and three of his friends spent a total of $59.76 on gasoline for their trip. Mr. Jeb paid $7.50 and the rest of the expense was shared equally by his three friends. How much did each of his three friends pay? $17.42

5. Jerry bought 5 pairs of socks. He received $1.15 change from a $5 bill. How much did each pair of socks cost? $0.77

6. At a bake sale, Joan's class sold 32 large cakes and 9 small cakes. They received $15.05, of which $12.80 was for the large cakes. What was the selling price of each small cake? $0.25

Set 141

For use after page 80

1. During a 5-day period, the Blake Bakery sold 1,170 loaves of bread. What was the mean number of loaves sold per day? 234

2. The bakery sold 17 cakes on Monday, 16 on Tuesday, 23 on Wednesday, 18 on Thursday, and 21 on Friday. What was the mean number of cakes sold per day? 19

3. On a trip to a national park, the members of the Oakdale Camera Club took 175 pictures the first day, 123 pictures the second day, and 206 pictures the third day. For that 3-day period, what was the mean number of pictures taken per day? 168

4. During a 4-day holiday period, there were 384 car accidents. What was the mean number of accidents occurring per day? 96

5. From the school library, pupils borrowed 56 books on Monday, 58 on Tuesday, 31 on Wednesday, and 39 on Thursday. What was the mean number of books borrowed per day? 48

6. Find the sum of 37 measures if the mean is 156. 5,772

7. Find the mean of 26 measures if their sum is 858. 33

8. Find the mean number of dollars: $32.50, $36.00, $31.75, $33.40, $35.80 $38.80, $39.90 $35.45

Set 142

For use after page 109

1. Cindy lives $\frac{3}{4}$ mi. from school. To get to school, she rides $\frac{5}{8}$ mi. and then walks the rest of the way. How far does Cindy walk in going to school? $\frac{1}{8}$ mi.

2. Nancy's mother baked a batch of cookies for the party. At the party, the boys ate $\frac{3}{5}$ of the cookies, and the girls ate $\frac{1}{3}$. Together, they ate what part of all the cookies? $\frac{14}{15}$ of the cookies

3. After shopping for $\frac{3}{10}$ hr., Nancy visited with friends for $\frac{2}{5}$ hr. What was the total time Nancy spent shopping and visiting that day? $\frac{7}{10}$ hr. (or 42 min.)

4. Jack caught 2 fish which weighed $\frac{3}{16}$ lb. and $\frac{5}{8}$ lb. Tom caught one fish which weighed $\frac{15}{16}$ lb. The weight of Tom's fish was how much greater than the total weight of Jack's 2 fish? $\frac{1}{8}$ lb.

5. At the market, Jim's mother bought $\frac{1}{2}$ lb. of bacon and $\frac{3}{8}$ lb. of sliced ham. How many pounds of meat did she buy in all? $\frac{7}{8}$ lb.

6. One evening John completed his homework in 1 hour. If he spent $\frac{1}{3}$ hr. on his spelling and $\frac{1}{2}$ hr. on his mathematics, how long did he spend on his history? $\frac{1}{6}$ hr.

Set 143

For use after page 116

1. Mrs. Stevens bought a $1\frac{1}{2}$-oz. jar of cinnamon, a $1\frac{1}{8}$-oz. jar of cloves, a $1\frac{1}{4}$-oz. jar of nutmeg, and a 4-oz. container of pepper. How many ounces of spices did she buy in all? $7\frac{7}{8}$

2. Mr. Stevens bought two sleeping bags, one which weighed $7\frac{3}{4}$ lb. and one which weighed $6\frac{13}{16}$ lb. What was the total weight of the two sleeping bags? $14\frac{9}{16}$ lb.

3. The heavier sleeping bag (see Ex. 2) weighed how much more than the lighter sleeping bag? $\frac{15}{16}$ lb.

4. Before the Beaver Club's weekend hike, Gary weighed $92\frac{1}{4}$ lb. After the hike, he weighed $90\frac{3}{8}$ lb. How many pounds did Gary lose on the hike? $1\frac{7}{8}$ lb.

5. Mr. Stevens mixed $5\frac{1}{2}$ lb. of bluegrass seed with $1\frac{3}{8}$ lb. of clover seed. If he used $4\frac{3}{4}$ lb. of this mixture to seed his lawn, how much did he have left? $2\frac{1}{8}$ lb.

6. Mr. Stevens can mow his lawn in $\frac{3}{4}$ hr. less time than it takes Mr. Bly, his neighbor, to mow his lawn. If it takes Mr. Bly $2\frac{1}{5}$ hr., how long does it take Mr. Stevens? $1\frac{9}{20}$ hr.

7. Find the sum of 45, $13\frac{3}{7}$, $37\frac{2}{3}$, 56, and $20\frac{2}{7}$. $172\frac{8}{21}$

8. From $81\frac{2}{3}$, subtract the sum of $57\frac{3}{4}$ and $11\frac{1}{3}$. $12\frac{7}{12}$

9. Which is greater, $34\frac{1}{2} + 41\frac{2}{3}$ or $80 - 4\frac{1}{8}$? $34\frac{1}{2} + 41\frac{2}{3}$ $(= 76\frac{1}{6})$

Set 144

For use after page 190

1. Mrs. Fine wants to make curtains for her kitchen windows. For each pair of curtains, she needs $2\frac{2}{3}$ yd. of material. How many yards of material does she need for 3 pairs of kitchen curtains? 8

2. How many yards of material are needed for 5 pairs of curtains for the recreation room if $2\frac{1}{4}$ yd. are needed for each pair? $11\frac{1}{4}$

3. At $12\frac{1}{2}¢$ per pound, how much would 7 pounds of bananas cost? $87\frac{1}{2}¢$ (88¢)

4. What is the weight of 24 boxes of salt if each box of salt weighs $1\frac{5}{8}$ lb.? 39 lb.

5. Mary's mother uses $1\frac{3}{4}$ cups of milk in making 1 cake. How many cups of milk are needed for 5 cakes? $8\frac{3}{4}$

6. Find the cost of a $14\frac{1}{2}$-lb. turkey if turkey costs 39¢ per pound. $565\frac{1}{2}¢$ (566¢ or $5.66)

7. Mary rides her bicycle $1\frac{1}{5}$ mi. in making the round trip to and from school. In making 4 such trips, how far does Mary ride her bicycle? $4\frac{4}{5}$ mi.

8. Find the perimeter of a square if the length of each side is $14\frac{1}{3}$ in. $57\frac{1}{3}$ in.

9. Find the perimeter of a regular octagon if the length of each side is $34\frac{1}{7}$ ft. $273\frac{1}{7}$ ft.

Set 145

For use after page 199

1. The Elite Candy Shop made $22\frac{1}{2}$ lb. of fudge for a special sale. During the first hour of the sale, $\frac{2}{3}$ of the fudge was sold. How many pounds of fudge were sold during the first hour? 15

2. If 1 package of assorted candies contains $1\frac{1}{4}$ lb. of chocolate drops, how many pounds of chocolate drops do $3\frac{1}{2}$ packages contain? $4\frac{3}{8}$

3. What is the cost of 240 cardboard boxes if 1 cardboard box costs $1\frac{3}{4}¢$? $4.20

4. The cost of 48 boxes at $1\frac{3}{4}¢$ per box is how much more than the cost of 60 boxes at $1\frac{1}{5}¢$ per box? 12¢

5. If each pound of assorted nuts contains $\frac{1}{4}$ lb. of peanuts and $\frac{3}{8}$ lb. of almonds, then 24 lb. of assorted nuts contains how many pounds of peanuts 6 and how many pounds of almonds? 9

6. The product of 72 and $16\frac{2}{3}$ is how much less than 1,500? 300

7. From $4\frac{1}{2}$, subtract the product of $1\frac{1}{2}$ and $2\frac{2}{3}$. $\frac{1}{2}$

8. Find the area of a rectangular region $3\frac{1}{4}$ in. by $15\frac{1}{2}$ in. $50\frac{3}{8}$ sq. in.

9. A rectangular region which measures 46 ft. by $\frac{2}{3}$ ft. is how much larger than a rectangular region which measures $4\frac{1}{5}$ ft. by $5\frac{2}{3}$ ft.? $6\frac{13}{15}$ sq. ft.

Set 146

For use after page 217

1. In making some blueberry pies, Mrs. Gardner used 18 cups of blueberries. If $4\frac{1}{2}$ cups were needed for one pie, how many pies did she make? 4

2. If it takes Bill $1\frac{1}{3}$ hr. to build one rabbit cage, how many hours would it take him to build 5 rabbit cages? $6\frac{2}{3}$

3. Bill bought $\frac{3}{4}$ lb. of small nails and $1\frac{1}{3}$ lb. of large nails. How many pounds of nails did he buy in all? $2\frac{1}{12}$

4. In walking to and from school 5 times, Jay walks a total of $2\frac{1}{3}$ mi. How far is it to and from school? $\frac{7}{15}$ mi.

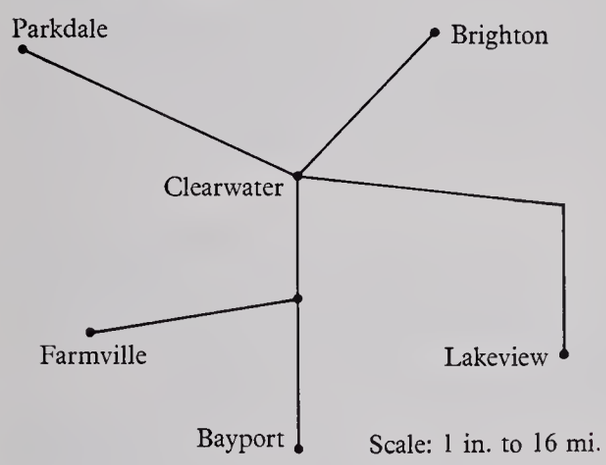
5. Find the mean weight for the following: $4\frac{1}{2}$ lb., $5\frac{2}{3}$ lb., $4\frac{1}{3}$ lb., $5\frac{1}{4}$ lb. $4\frac{15}{16}$ lb.

6. If a $1\frac{1}{3}$-lb. package of assorted nails costs 36¢, what is the cost per pound? 27¢

7. Find the total weight of 144 hammers if each hammer weighs $1\frac{1}{4}$ lb. 180 lb.

Set 147

For use after page 220



Bus route from Clearwater	Measurement on map	One-way distance
to Bayport	$1\frac{1}{8}$ in.	18 mi.
to Farmville	$1\frac{3}{8}$ in.	22 mi.
to Parkdale	$1\frac{1}{4}$ in.	20 mi.
to Brighton	$\frac{7}{8}$ in.	14 mi.
to Lakeview	$1\frac{3}{4}$ in.	28 mi.

1. Copy and complete the table shown above at the right. See table. Use your ruler and the given scale to obtain the information you need.

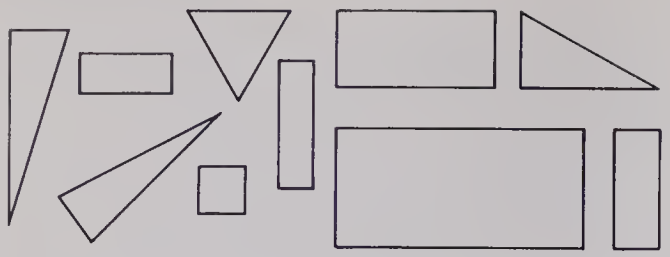
2. The one-way bus trip from Clearwater to Brighton takes 20 min. The trip from Clearwater to Parkdale takes $1\frac{1}{2}$ times as long. How many minutes does it take to go from Clearwater to Parkdale? 30

3. In a scale drawing with a scale of *1 in. to 28 ft.*, the floor plan of a room is $\frac{7}{8}$ in. long and $\frac{5}{8}$ in. wide. Find the real dimensions of the room. Length: $24\frac{1}{2}$ ft.; Width: $17\frac{1}{2}$ ft.

4. The real dimensions of a living room are 30 ft. by 20 ft. A floor plan of the living room with a scale of *1 in. to 8 ft.* should be how long and how wide? $3\frac{3}{4}$ in. long and $2\frac{1}{2}$ in. wide

Set 148

For use after page 223



1. Express with a fraction in simplest form the ratio of the number of triangles to the number of rectangles shown above. $\frac{3}{4}$

2. Find the ratio of the number of prime factors of 60 to the number of factors of 60. 4 to 12, or 1 to 3

3. Find the ratio of the number of vowels in the word *ratio* to the number of letters in the word *ratio*. 3 to 5

4. Find the ratio of the number of multiples of 5 less than 31 to the number of primes less than 31. 7 to 10

5. In a certain classroom, the ratio of the number of girls to the number of boys is 2 to 3. There are 14 girls in that classroom. How many boys are there? (Hint: $\frac{2}{3} = \frac{14}{?}$) 21

6. The ratio of the number of robins that Frank saw the first day of Spring to the number that Jim saw was 4 to 5. If Jim saw 15 robins, how many robins did Frank see? 12 What was the ratio of the number of robins Jim saw to the total number they both saw? 5 to 9

Set 149

For use after page 229

1. To get to and from work, Mr. Wyman drives 8.3 miles per day. How far does Mr. Wyman drive in five days? 41.5 miles

2. The mean for the weights of 5 hams was 3.7 lb. Find the total weight of the 5 hams. 18.5 lb.

3. What is the total weight of 9 small bearings if each bearing weighs 1.6 oz.? 14.4 oz.

4. The perimeter of a square with each side 3.1 in. long is how much greater than the perimeter of a rectangle 2 in. by 1.8 in.? 4.8 in.

5. What is the sum when 3.7 is added to the product of 6.6 and 3? 23.5

6. Mrs. Wyman bought seven 5.5-lb. bags of onions. How many pounds of onions did she buy in all? 38.5

7. Find the perimeter of a regular octagon if the length of each side is 8.8 cm. 70.4 cm.

8. Find the area of a rectangular region 0.7 ft. by 7 ft. 4.9 sq. ft.

9. Find the perimeter of a triangle with each side 3.9 in. long. 11.7 in.

10. From the product of 4.8 and 3, subtract the sum of 6.6 and 0.9. 6.9

11. Which is greater, 4×3.8 or $5.6 + 2.7 + 6.2$? 4×3.8 (= 15.2)

Set150

For use after page 245

1. Find the cost of 1 basketball if 7 basketballs cost $45.08. $6.44

2. If the total cost of 12 folding camp chairs was $9.48, what was the cost of each chair? $0.79 (or 79¢)

3. The cost of food for a hike, $18.60, was shared equally by 12 boys. How much was each boy's share? $1.55

4. The rainfall for a 5-day period was recorded as 3.7 in. To the nearest $\frac{1}{10}$ inch, what was the mean number of inches per day? 0.7 in.

5. The perimeter of a square is 9.278 in. To the nearest $\frac{1}{100}$ inch, find the length of one side. 2.32 in.

6. With his new motor scooter, John traveled 161.8 miles on 5 gallons of gasoline. To the nearest $\frac{1}{10}$ mile, what was the mean number of miles traveled per gallon? 32.4

7. To the nearest $\frac{1}{100}$ centimeter, find the length of a rectangular region whose width is 24 cm. and whose area is 806.5992 sq. cm. 33.61 cm.

8. If the perimeter of a regular octagon is 1.3 ft., find the length of one side to the nearest $\frac{1}{1,000}$ foot. 0.163 ft.

9. To the nearest $\frac{1}{10}$ gram, find the mean for the following: 5.28 gm., 5.42 gm., 4.98 gm., 5.14 gm., 5.25 gm. 5.2 gm.

Set 151

For use after page 250

1. Jack, a 4-H Club member, raised 385.4 bu. of corn on 4.7 acres. What was the mean average of the numbers of bushels raised per acre? 82

2. In one month, Duke's Diner used 32.5 ounces of pepper. If the pepper was contained in 2.5-oz. boxes, how many boxes of pepper were used? 13

3. At a rate of 15.8 mi. per gallon, how far can Mr. Buck's car travel on 22 gallons of gasoline? 347.6 mi.

4. Find the area of a rectangular region whose length is 3.4 cm. and whose width is 2.46 cm. 8.364 sq. cm.

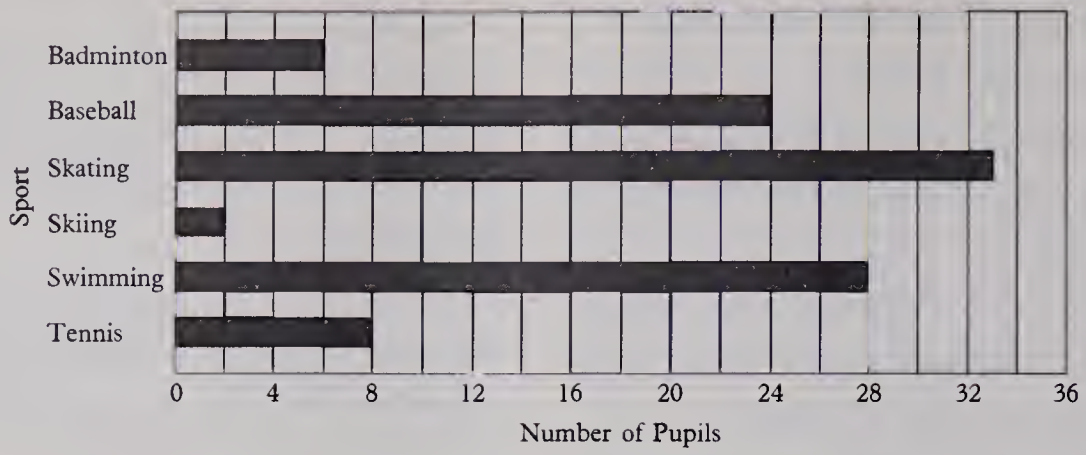
5. Last year, the Bell family spent 0.22 of Mr. Bell's yearly salary for food. If Mr. Bell's yearly salary was $5,650 last year, how much did the Bell family spend for food? $1,243

6. The population of the town of Parkville is 0.55 of the population of Grantsburg. The population of Parkville is 704. What is the population of Grantsburg? 1,280

7. To the nearest $\frac{1}{100}$ inch, find the width of a rectangular region whose length is 13.25 in. and whose area is 109.021 sq. in. 8.23 in.

Set 152

For use after page 279



Sports Liked by the 36 Pupils in the Edison School 6th Grade

Study the bar-graph shown above.

1. Each unit on the horizontal scale represents how many pupils? 2

2. Tell which sport is liked best and indicate the number of pupils who like that sport. Skating; 33

3. Tell which sport is liked least and indicate the number of pupils who like that sport. Skiing; 2

4. Write a fraction in simplest form that indicates what part of the 36 pupils like baseball. $\frac{2}{3}$

5. If only 10 pupils had liked skating, the bar for "Skating" would have been how many units long? 5

6. The number of pupils who like badminton is what per cent of the number of pupils who like baseball? 21%

Set 153

For use after page 295

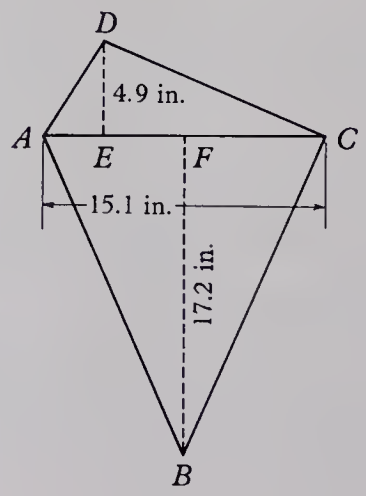
Refer to the figure at the right.

1. $\overline{DE}$ is perpendicular to $\overline{AC}$. Find the area of $\triangle ACD$. 36.995 sq. in.

2. $\overline{BF}$ is perpendicular to $\overline{AC}$. Find the area of $\triangle ABC$. 129.86 sq. in.

3. The area of $\triangle ABC$ is how much greater than the area of $\triangle ACD$? 92.865 sq. in.

4. Using your answers for Ex. 1–2, find the area of the polygonal region $ABCD$. 166.855 sq. in.



NOTES

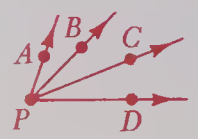
ANSWERS TO EXTRA ACTIVITIES SETS 154–159

Set 154

Sample answers:

1. • A • C • B

$\overrightarrow{AB}, \overrightarrow{BA}, \overrightarrow{BC}, \overrightarrow{CB}, \overrightarrow{AC}, \overrightarrow{CA}$

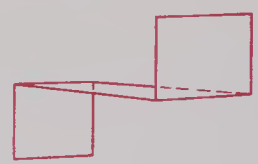
2. 

$\angle APB, \angle BPC, \angle CPD, \angle APC, \angle APD, \angle BPD$

Set 155

Sample answers:

1. He used a plane to make the edge of the board smooth.

2. 

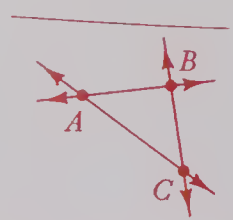
Set 156

Sample answers:

1.

2. acute

obtuse

3. 

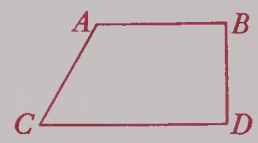
$\angle BAC, \angle ACB, \angle CBA$

4. Answers will vary according to the diagram.

Set 157

Sample answers:

1.

2. 

Parallel sides $\overline{AB}$, $\overline{CD}$
nonparallel sides $\overline{AC}$, $\overline{BD}$

Set 158

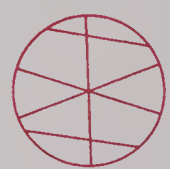
Sample answers:

1.

No

2.

Yes

3. 

Set 159

1. Sample answer: 8, 9, 9, 10, 11
2. 26
3. Sample answer: 13, 15, 19
4. Sample answer: 2, 6, 8, 10, 12

Set 154

1. On your paper, show three points not in the same line and label them A, B, and C. Then name as many rays as possible using the labeled points.

2. On your paper, show four rays that have point P as a common end point. How many angles have you pictured? Then show and label a point on each ray and name the angles you have pictured.

Set 155

1. Look up the word *plane* in a dictionary. Then write a sentence in which *plane* does not refer to a set of points.

2. Fold a card to suggest three planes such that two of the planes do not seem to have any points in common.

Set 156

1. Using the corner of a sheet of paper, picture two right angles that have a common end point.

2. On your paper, show 4 acute angles and 4 obtuse angles.

3. On your paper, show three points not in the same line and label them A, B, and C. Show three lines which contain any two of these points. Then name three angles using the labeled points.

4. Which of the angles that you named for Ex. 3 appear to be acute angles?

Set 157

1. On your paper, show a rhombus that is not a square.

2. On your paper, show a trapezoid and label the vertices A, B, C, and D. Then name the sides which

   a. are parallel.

   b. are not parallel.

Set 158

1. On your paper, show three points in the same line. Then try to show a circle which contains the three points. Can you show such a circle?

2. On your paper, show three points which do not lie in the same line. Then show a circle which contains the three points. Can you show such a circle for any three points not in the same line?

3. On your paper, show a circle. Then show three diameters and two chords which are not diameters.

Set 159

1. List five measures such that the mode for the measures is 9.

2. If three of four measures are 24, 26, and 27, and the mode is 26, name the fourth measure.

3. Write three different measures such that the median for the measures is 15.

4. Write four different measures such that the median for the measures is 8.

Set 160

1. A 1-element set has 2 subsets. A 2-element set has 2^2, or 4, subsets. A 3-element set has 2^3, or 8, subsets. Using this pattern, see if you can determine the number of subsets a 4-element set has.

2. List and count the subsets of {a, b, c, d}. Does the number of subsets check with your answer for Ex. 1?

Set 161

1. On your paper, show a circle and a chord for the circle. Label the end points of the chord A and B. Explain why chord AB is not a subset of the circle.

2. Name two rays which are subsets of an angle RST.

Set 162

1. On your paper, show a simple closed curve. Then show another simple closed curve which lies in the interior for the first curve.

2. On your paper, show a line segment and then show a simple closed curve which intersects the segment in 5 different points.

3. On your paper, show an angle and then show a simple closed curve such that each side of the angle intersects the curve in 2 points.

Set 163

1. Explain how a rectangular region differs from a rectangle.

2. On your paper, draw and lightly shade a model of a quadrangular region which is not a rectangular region.

3. On your paper, show a rectangular region and then show a segment which intersects the rectangular region in only 1 point.

Set 164

1. On your paper, show an acute angle and then, using a protractor, find the measure of the angle to the nearest 10 degrees.

2. On your paper, show an obtuse angle and then follow the instructions given above for Ex. 1.

Set 165

1. Explain why it is not possible for each of the three angles of a triangle to have a measure of 70 in degrees.

2. Explain why 2 of the 3 angles of a triangle can not be obtuse angles.

3. Determine the greatest *whole number* that may be the sum of the measures in degrees of 2 angles of a triangle.

4. Which 3 members of {15, 20, 25, 145, 170} could be the 3 measures in degrees of the angles of a triangle?

ANSWERS TO EXTRA ACTIVITIES SETS 160–165

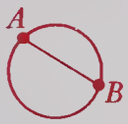
Set 160

1. 2^4 or 16 subsets

2. $\{\ \}, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}$; Yes

Set 161

1. Only points A and B belong to the circle.



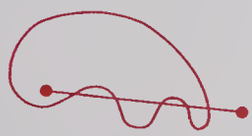
2. $\overrightarrow{SR}, \overrightarrow{ST}$

Set 162

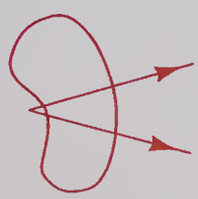
Sample answers:

1.

2.



3.



Set 163

1. A rectangular region is the union of a rectangle and its interior.

2. Sample answer:

3. Sample answer:

Set 164

1. Answers will vary.

2. Answers will vary.

Set 165

1. Because the sum of the measures would be 210 in degrees and it must be 180 in degrees.

2. Because the sum of the measures would be greater than 180 in degrees.

3. 179

4. 15, 20, 145

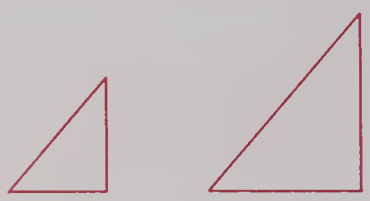
ANSWERS TO EXTRA ACTIVITIES SETS 166–173

Set 166

1. Sample answers:

2. They have a circular shape and have the same size. No

3.



Set 167

1. Sample answer:

2. The sum of the measure of the angles would be 270 in degrees and it must be 180.

3. 4

Set 168

1. 24

2. Sample answer:
   16 ft. by 2 ft.
   32 ft. by 1 ft.

3. 33 sq. in.

Set 169

1. Answers will vary.

2. $1\frac{1}{2}$ hr.

Set 170

1. Answers will vary.

2. (The drawing should be 2 in. by 3 in.)

Set 171

Answers will vary.

Set 172

Because water was chosen as the standard for specific gravity, and its specific gravity is 1.0.

Set 173

1. 5 cm.

2. 3.141592
   3.141593

Set 166

1. On your paper, show three line segments which appear to be congruent.

2. Explain why two pennies could be thought of as models of congruent circular regions. Could a penny, a nickel, and a dime be thought of as models of congruent circular regions?

3. On your paper, show two right triangles which are not congruent.

Set 167

1. On your paper, show two acute angles which are not congruent.

2. Explain why the three angles of a right triangle can not be congruent.

3. A rectangle has how many congruent angles?

Set 168

1. Draw a model of a rectangular region that is 24 in. by 4 in. Determine how many square regions, each 2 in. by 2 in., the rectangular region can be partitioned into.

2. A rectangular region is 8 ft. long and 4 ft. wide. Give the dimensions of two other rectangular regions which have the same area as this one.

3. Find the mean for the areas of three rectangular regions whose dimensions are 16 in. by 2 in., 6 in. by 5 in., and 37 in. by 1 in.

Set 169

1. Write the names and watt-ratings of some electrical appliances that you have in your house.

2. A 200-watt light bulb operating for $7\frac{1}{2}$ hours uses the same amount of electrical energy as a 1,000-watt toaster operating for how may hours?

Set 170

1. Find some scale drawings of animals. Write the names of the animals and indicate the scale that was used for each drawing.

2. Using a scale of *1 in. to 2 ft.*, make a scale drawing of a 24 in.-by-36 in. rectangular window screen.

Set 171

Make a map of some region that you have visited. Show at least 2 cities and indicate the distance between them.

Set 172

Explain why the specific gravity of water is 1.0.

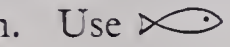
Set 173

1. If the circumference of a circle is 5π cm., what must be the diameter for the circle?

2. Using a dictionary, find the decimal for π to six decimal places. Which is greater, π or 3.141593.

Set 174

1. Suppose that each in the graph on page 275 represented 25 willow leaves. Then the graph would show that _?_ willow leaves were found on Monday.

2. Bob has 4 goldfish, Tim has 3, and Terry has 5. Make a pictograph to show this information. Use  to represent 1 goldfish.

Set 175

1. Make a pictograph to show how many boys there are in each row of your classroom using to represent 1 boy. Then make another pictograph using to represent 2 boys.

2. Find how many Representatives there are from each of the five most populated states. Then make a pictograph to show your information using to represent 2 Representatives.

3. How many Senators are there from each state? Would this be a good subject for a pictograph?

Set 176

To the nearest inch, find the height of each of five pupils in your class. Make a table to show the information and then make a vertical-bar graph to go with the table. Label the vertical scale so that each unit represents 10 (inches).

Set 177

Make a line graph to show your performance on 4 consecutive tests in one of your subjects for this year.

Set 178

The population of the United States for each of four census years was:

1810	7,239,881	1830	12,866,020
1820	9,638,453	1840	17,069,453

Make a line graph to show this information. Round each number to the nearest million and let each unit on the vertical scale represent 2 (millions of people).

Set 179

Of the money that Fred earned cleaning the garage, 20% was spent for a ball, 10% for candy, 10% for ice cream, and 60% for a book. Make a divided-bar graph to show this information.

Set 180

The union of 2 sets is $\{2, 3, 4, 6, 7, 8\}$ and their intersection is $\{4, 6\}$. One of the two sets is $\{2, 3, 4, 6\}$. Draw a Venn diagram to show this and then list within braces the members of the other set.

Set 181

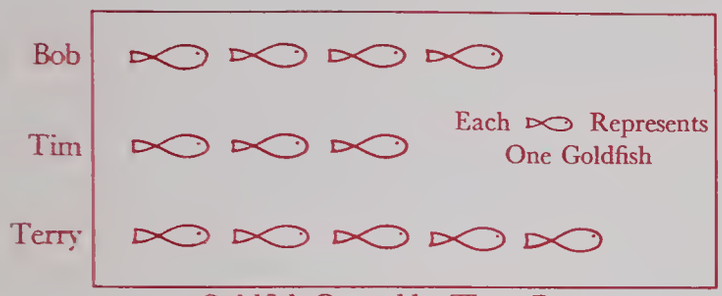
Using a straightedge, pencil, and compass, and starting with a model for a circle, construct a square; a regular hexagon; a regular octagon.

ANSWERS TO EXTRA ACTIVITIES SETS 174–181

Set 174

1. 75

2.



Goldfish Owned by Three Boys

Set 175

1. Answers will vary.

2.

Representatives from the Five Most Populated States

3. 2
No

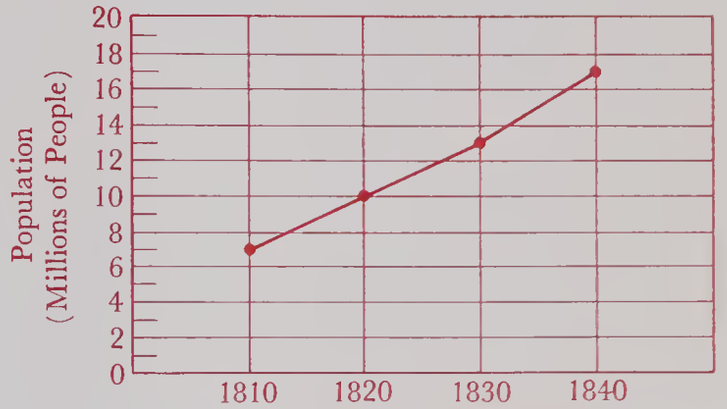
Set 176

Answers will vary.

Set 177

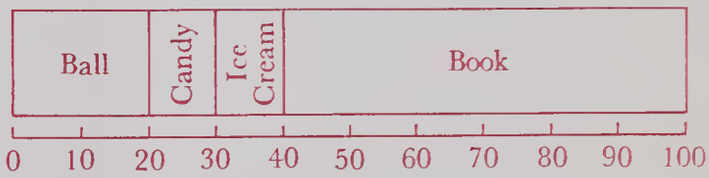
Answers will vary.

Set 178



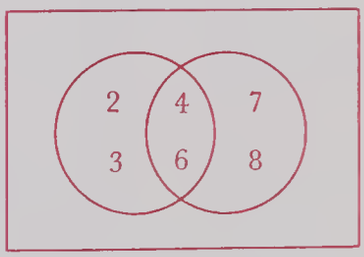
Census Year Population
of United States for Four Census Years

Set 179



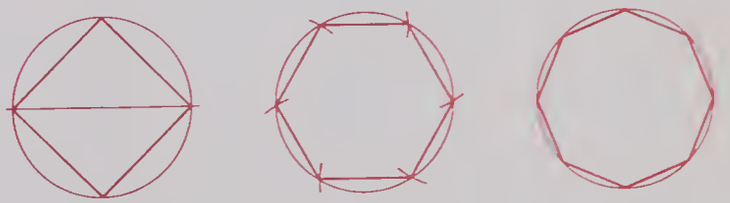
How Fred Spent His Earnings

Set 180



{4, 6, 7, 8}

Set 181



NOTES

Tables of Measurement

Length

12 inches (in.) = 1 foot (ft.)

3 ft.
36 in. } = 1 yard (yd.)

$5\frac{1}{2}$ yd.
$16\frac{1}{2}$ ft. } = 1 rod (rd.)

5,280 ft.
1,760 yd.
320 rd. } = 1 mile (mi.)

Area

144 square inches (sq. in.) = 1 square foot (sq. ft.)

9 sq. ft.
1,296 sq. in. } = 1 square yard (sq. yd.)

$30\frac{1}{4}$ sq. yd. = 1 square rod (sq. rd.)

160 sq. rd. = 1 acre (A.)

640 A. = 1 square mile (sq. mi.)

Counting

12 things = 1 dozen (doz.)

12 doz.
144 things } = 1 gross

Time

60 seconds (sec.) = 1 minute (min.)

60 min. = 1 hour (hr.)

24 hr. = 1 day (da.)

7 da. = 1 week (wk.)

30 da.
4 wk. } = 1 month (mo.)

365 da.
52 wk.
12 mo. } = 1 year (yr.)

366 da. = 1 leap year

10 yr. = 1 decade

100 yr. = 1 century

Weight

16 ounces (oz.) = 1 pound (lb.)

2,000 lb. = 1 ton (T.)

Dry Measurement

2 pints (pt.) = 1 quart (qt.)

8 qt. = 1 peck (pk.)

4 pk. = 1 bushel (bu.)

Liquid Measurement

2 cups = 1 pint (pt.)

2 pt. = 1 quart (qt.)

4 qt. = 1 gallon (gal.)

METRIC SYSTEM

Length

10 mm. = 1 cm.

10 cm. = 1 dm.

1,000 mm.
100 cm.
10 dm. } = 1 m.

1,000 m. = 1 km.

Abbreviations

gm.—gram

mm.—millimeter

cm.—centimeter

dm.—decimeter

m.—meter

km.—kilometer

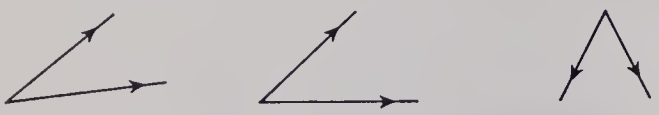
Symbols

$=$	is equal to	
$\neq$	is not equal to	
$\approx$	is approximately equal to	page 254*
$<$	is less than	page 27
$>$	is greater than	page 27
$\leq$	is less than or is equal to	page 31
$\geq$	is greater than or is equal to	page 31
$\cap$	intersection	page 9
$\cup$	union	page 9
$\{\}$	the empty set	page 3
$-$	segment	page 36
$\leftrightarrow$	line	page 38
$\rightarrow$	ray	page 39
$\angle$	angle	page 39
$\triangle$	triangle	page 85

Terms with Illustrations

The example or model which appears with each term illustrates the idea.

Acute Angle (page 43)*

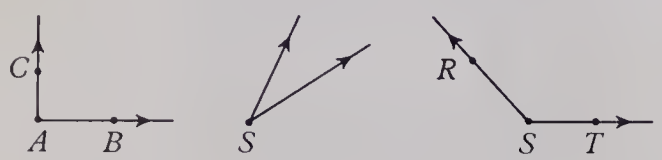


Addends-Sum Relationship (page 30)

$9 = 7 + 2$ $\quad$ $9 - 2 = 7$

$9 - 7 = 2$ $\quad$ $9 = 2 + 7$

Angle (page 39)



Angle CAB $\quad$ Angle S $\quad$ Angle TSR

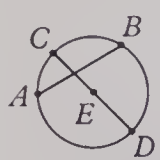
Associative Property of Addition (page 28)

$(3 + 5) + 2 = 3 + (5 + 2)$

Associative Property of Multiplication (page 54)

$(4 \times 6) \times 5 = 4 \times (6 \times 5)$

Chord for a Circle (page 48)



$\overline{AB}$ and $\overline{CD}$ are chords for circle E.

Circumference of a Circle (page 254)

The distance around a circle is called the circumference of the circle. $C = \pi d$

Closed Curve (page 45)

Commutative Property of Addition (page 28)

$8 + 21 = 21 + 8$

Commutative Property of Multiplication (page 54)

$3 \times 14 = 14 \times 3$

Composite Number (page 67)

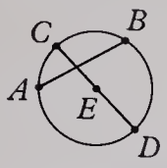
4, 6, 8, 9, 10, 12, 14, 15, 16, and 18 are the composite numbers that are less than 20.

Congruent Figures (page 135)

Figures which have the same size and shape are said to be congruent.

*Sampling of pages where idea used.

Diameter for a Circle (page 48)



$\overline{CD}$ is a diameter for circle E.

Distributive Property (page 55)

$4 \times (2 + 5) = (4 \times 2) + (4 \times 5)$

Division Terms (page 71)

divisor

5 ← quotient

$8\overline{)43}$ ← dividend

40

R 3 ← remainder

Empty Set (page 3)

$\{\ \}$ The empty set has no members.

Equivalent Sets (page 4)

$\{4, 6\}$ and $\{3, 7\}$ are equivalent.

$\{7\}$ and $\{5, 8\}$ are not equivalent.

Expanded Form (page 20)

$542 = (5 \times 100) + (4 \times 10) + (2 \times 1)$

$542 = (5 \times 10^2) + (4 \times 10) + (2 \times 1)$

Exponent Form (page 19)

The exponent form 5^2 names the number 25.

Factor of a Number (page 66)

The factors of 8 are 1, 2, 4, and 8.

Factors-Product Relationship (page 58)

$15 = 3 \times 5$ $\qquad$ $15 \div 5 = 3$

$15 \div 3 = 5$ $\qquad$ $15 = 5 \times 3$

Fraction (page 94)

7 ← numeral for the numerator

9 ← numeral for the denominator

Greatest Common Factor (page 98)

The greatest common factor of 16 and 40 is 8.

Greatest Possible Error (page 156)

The greatest possible error for a measurement to the nearest inch is $\frac{1}{2}$ in.; to the nearest $\frac{1}{2}$ inch is $\frac{1}{4}$ in.

Identity Element for Addition (page 28)

0; $\quad 3 + 0 = 3, \quad 0 + 3 = 3$

Identity Element for Multiplication (page 56)

1; $\quad 3 \times 1 = 3, \quad 1 \times 3 = 3$

Intersection of Two Sets (page 8)

$\{3, 4\} \cap \{2, 4, 7\} = \{4\}$

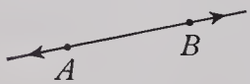
Least Common Denominator (L.C.D.) (page 104)

The least common denominator of $\frac{1}{4}$ and $\frac{5}{6}$ is 12.

Least Common Multiple (L.C.M.) (page 104)

The least common multiple of 4 and 6 is 12.

Line (page 38)



Line AB; $\overleftrightarrow{AB}$

Mathematical Sentence (page 27)

True: $5 + 13 = 18$, $13 > 4$

False: $40 - 10 = 25$, $5 < 3$

Open: $n = 5 \times 3$, $n > 40$ (page 29)

Mean for a Set of Data (page 81)

2 in., 5 in., 3 in., 2 in.

$$4\overline{)12}$$

Mean — 3

12

Median for a Set of Data (page 81)

$$\frac{13 \text{ ft.}, 14 \text{ ft.}}{2}, 15 \text{ ft.}, \frac{16 \text{ ft.}, 19 \text{ ft.}}{2},$$

Median — 15

Member of a Set (page 3)

The members of {3, 7} are 3 and 7.

Mode for a Set of Data (page 81)

2 in., 2 in., 2 in., 3 in., 3 in., 4 in.,
Mode — 2

Multiple (page 6)

0, 3, 6, and 9 are the *multiples* of 3 that are less than 10. 3, 6, and 9 are the *counting-number multiples* of 3 that are less than 10.

Number (page 5)

Whole numbers: {0, 1, 2, 3, · · ·} (page 6)
Counting numbers: {1, 2, 3, · · ·} (page 6)
Rational numbers: $\{\frac{0}{1}, \frac{1}{1}, \frac{1}{2}, \frac{2}{1}, \frac{3}{1}, \frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \cdots\}$ (page 94)
Negative numbers: $\{^{-}1, ^{-}2, ^{-}3, \cdots\}$ (page 312)
Integers: $\{\cdots, ^{-}3, ^{-}2, ^{-}1, 0, ^{+}1, ^{+}2, ^{+}3, \cdots\}$ (page 313)

Numeral (page 15)

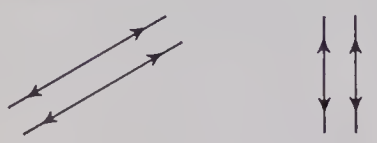
Standard numeral: 0, 5, 37 (page 15)
Fraction: $\frac{1}{2}, \frac{3}{4}, \frac{9}{5}$ (page 94)
Mixed form: $1\frac{1}{2}, 5\frac{2}{3}$ (page 110)
Exponent form: $2^3, 5^2, 3^6$ (page 19)
Decimal: 0.2, 5.72, 13.337 (page 120)
Per Cent form: 2%, 15%, 37% (page 258)

Obtuse Angle (page 43)

One-to-One Correspondence (page 4)

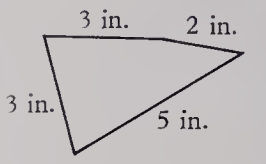
{5, 8, 9}
{7, 10, 20}

Parallel Lines (page 44)



Parallelogram (page 47)

Perimeter of a Polygon (page 150)



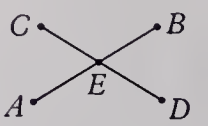
Perimeter is 13 in.

Perpendicular Lines (page 43)

Plane (page 41)

A flat surface is a model of part of a plane.

Point of Intersection (page 42)



Point E is a point of intersection of $\overline{AB}$ and $\overline{CD}$.

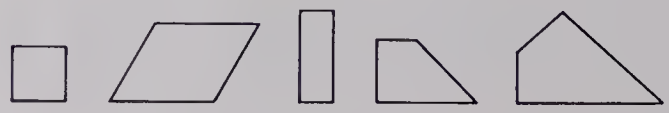
Polygon (page 46)

Prime Number (page 67)

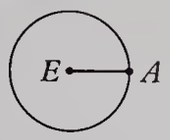
2, 3, 5, 7, 11, 13, 17, and 19 are the prime numbers that are less than 20.

Prism (page 298)

Quadrilateral (page 47)



Radius for a Circle (page 48)

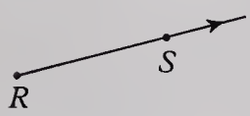


$\overline{EA}$ is a radius for circle E.

Ratio (page 222)

The ratio of the number of letters in the word *prime* to the number of members in {4, 6, 9, 10, 11, 12} is 5 to 6.

Ray (page 39)

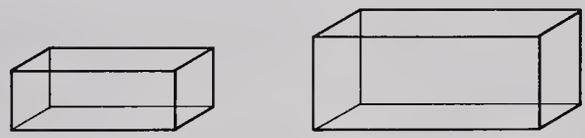


Ray RS; $\overrightarrow{RS}$

Reciprocal of a Number (page 212)

The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.

Rectangular Prism (page 299)

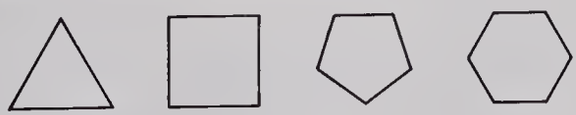


Region (page 90)

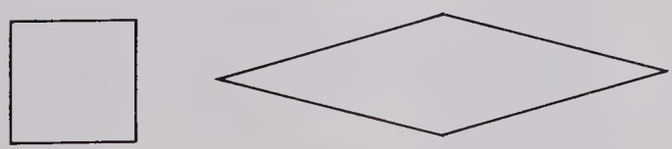
A region is the union of a simple closed curve and its interior.

region $ABCD$

Regular Polygon (page 134)



Rhombus (page 47)



Right Angle (page 43)

Segment (page 36)

Segment AB; $\overline{AB}$

Set (page 2)

Finite: {3, 5} (page 7)

Infinite: {2, 4, 6, · · ·} (page 7)

Simple Closed Curve (page 45)

Simplest Form (page 97)

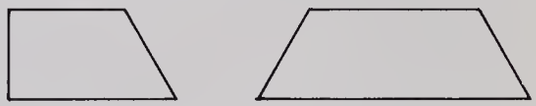
$\frac{2}{3}$ is a fraction in simplest form.

$\frac{6}{9}$ is a fraction not in simplest form.

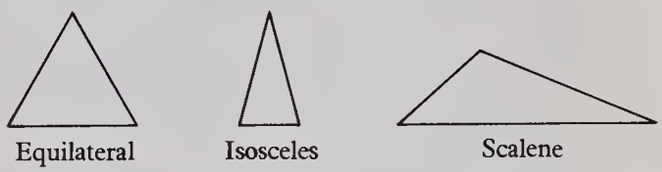
Subset (page 83)

{2, 4} is a subset of {1, 2, 3, 4}.

Trapezoid (page 47)



Triangle (page 47)



Union of Two Sets (page 8)

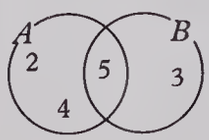
$\{3, 4\} \cup \{2, 4, 7\} = \{2, 3, 4, 7\}$

Unit of Precision (page 154)

For $3\frac{0}{4}$ in., the unit of precision is $\frac{1}{4}$ inch.

For 7.3 yd., the unit of precision is $\frac{1}{10}$ yard.

Venn Diagram (page 290)



$A = \{2, 4, 5\}$

$B = \{3, 5\}$

$A \cup B = \{2, 3, 4, 5\}$

$A \cap B = \{5\}$

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